Select / Special Topics in Classical Mechanics

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Module No. # 10

Lecture No. # 31

Classical Electrodynamics (i)

Greetings, we will begin discussion on unit 10, which is on classical electrodynamics. Our classical electrodynamics is a huge topic and in any undergraduate physics program, there is a full course on classical electrodynamics, which one would teach over like 40 or 50 or even 60 lectures and this would be followed by a second course in electro dynamics, which would be yet again another 40 or 50 or 60 lectures. So, this is coming as a very small part of our course on select topics in classical mechanics or special topics in classical mechanics; however you call it.

And this is less than 10 percent of this course, so we will have just about 3 or so classes of this. And what I will do is, summarize some of the essence which is contained in the very foundations of classical electrodynamics, which are very nicely enunciated in the famous Maxwell's equations.

So, we try to introduce the Maxwell's equations in a very compact simple manner, which will only contain the essence. So, I will not take you through the historical development nor will I take you through the very complete rational development of the subject, but give you just a very quick bird's eye view of Maxwell's equations.

Now, the reason it is included in a course on classical mechanics, because, at this point, we will not take into account the quantum effects in any manner around. Just the way we ignored quantum mechanics in dealing with the mechanics of either point particles or fluid motion; until now, we will continue to ignore quantum mechanics in this course and of course, toward the end of the course, I will make a few comments on what would perhaps provide a bridge between classical mechanics and quantum mechanics; so, that is something in the passing that I will do toward the end of this course.

But essentially, our subject remains essentially confined to classical domain, in which we do not include quantum mechanics, we do include the special theory of relativity, which we did at some length in unit 6. So, we at least got some introduction to the Lawrence transformations, we discussed the twin's paradox. So, all these things we did already in unit 6.

So, the special theory of relativity does belong to the domain of classical mechanics, the quantization of the radiation field does not and the special theory of relativity is very intimately connected to Maxwell's equations and to the laws of electrometric theory and to electrodynamics; so, they will go hand in hand and I will comment on some of that in this unit on classical electrodynamics.

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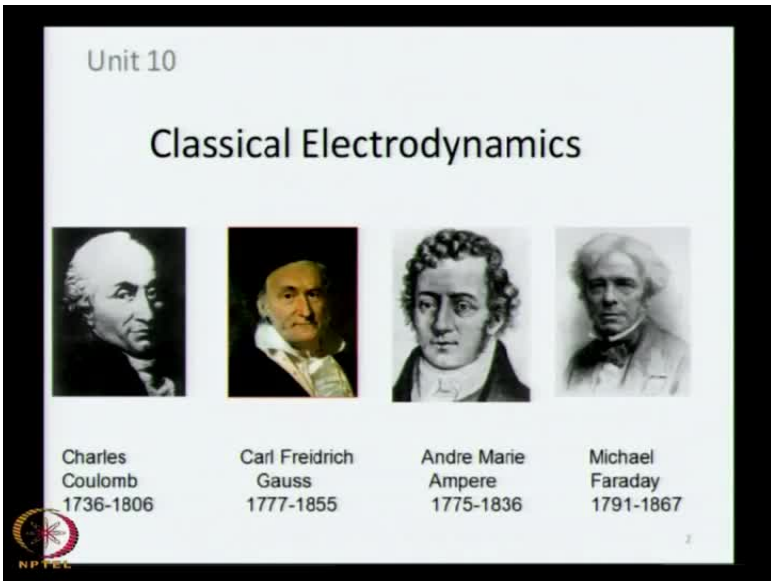

So, these are the main contributors to the development of empirical knowledge, which is based on the experiments that these joints of physics carried out in their times - Charles Coulomb, in the mostly in the 18 century, he lived until 1806; and then by carl gauss in the 19 century; and then also in the 19 century, Andre Ampere; and then also Michael faraday.

So, these were brilliant experimentalist and they carried out some very fascinating experiments and from these experiments, they compiled a lot of information, which lead them to formulate what we would call as a law; in the sense, that it always worked no violation to this is found and therefore you call it as a law; so, there is a Coulombs law; there is a Gauss's law; there is an Amperes law and there is a Faraday law or the Faraday Lenz law as we will call it.

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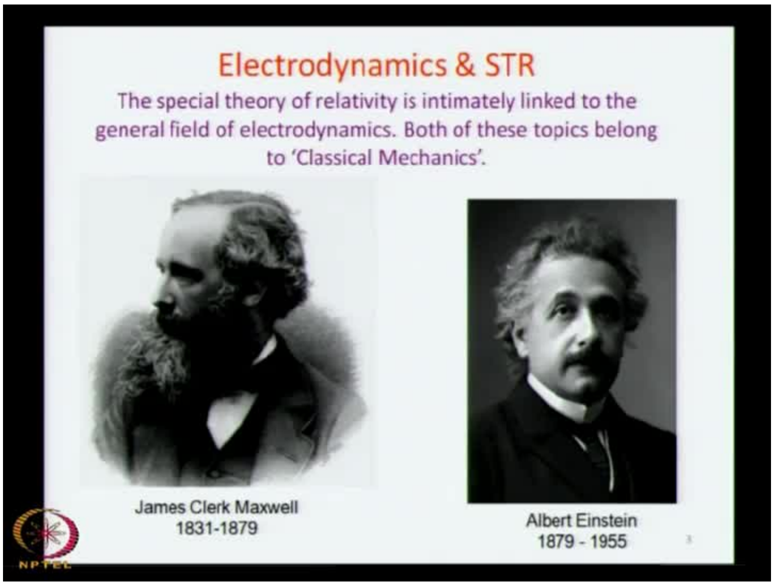

And then what Maxwell did was to integrate all of this in another formulation, which is really the theory of electrodynamics, which is the classical theory of electrodynamics and this what Maxwell did was to put it all together, in what a famously known as Maxwell's equations and they connect very nicely to the special theory of relativity - they go in fact, hand in hand - and the complete recognition of this relationship comes, of course, from Einstein's work, wherein he formulated the special theory of relativity in one of his major contributions to physics, in the year 1905, which is known as a magic here, because the same year he explained the Brownian motion, he also offered an explanation for the photo electric effect and in the same year, he also formulated the special theory of relativity and what inspired him to formulate the special theory of relativity was, in fact the laws of electrodynamics more than anything else.

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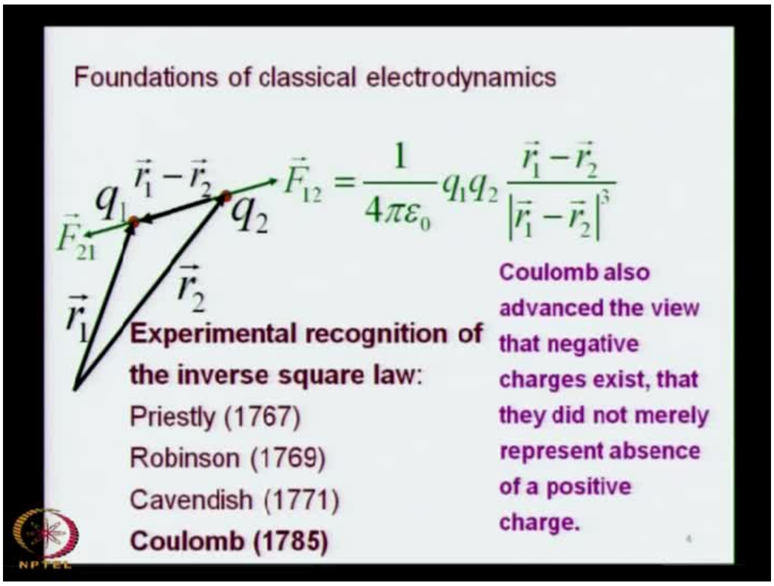

So, let us start looking at the foundations of electrodynamics and what we have over here are two charges: one is a charge q 1 and the other is charge q 2; they have position vectors respectively r 1 and r 2 with respect to some origin, which is the origin of a frame of reference and the excite forces on each other. So, the charge 2 exerts a force F 21, which is a force by the charge 2 on the charge 1; and likewise, there is a force F 12, which is a force on the charge 2 by the charge 1 and these two forces, so for our discussion, I consider these two charges to be of the same sign; so, that they would repel each other and this force is given by the inverse square law, q 1 q 2 by distance square; so, the distance square is written as a distance cube, because there is a distance in the numerator as well other than the direction, which is r 1 minus r 2.

So, this is once again a one over distance square law; this is known as the Coulombs law; and this is an empirical observation, this came out of observations which were carried out by priestly, Robinson, Cavendish, and most importantly by Coulomb, who really provided some sort of a very detailed catalog of how these charges…, you know what kind of force they exit on each other, how to estimate this and that if the two charges are light charges, they repel each other; if they are run like, they attract each other; but the force is nevertheless given by the one over distance square law; so, this is the Coulombs law, it was essentially a result of observations.

It did not come out of any theory, there was no theoretical model, which provided the basis for it; but it was a result of various experiments done predominantly by priestly, Robinson, Cavendish, and Coulomb; and it lead to the recognition that, two charges exert a force, which causes the inverse square of the distance between the two charges and their proportionalities involve this permittivity of free space and so on or whatever be the medium between them.

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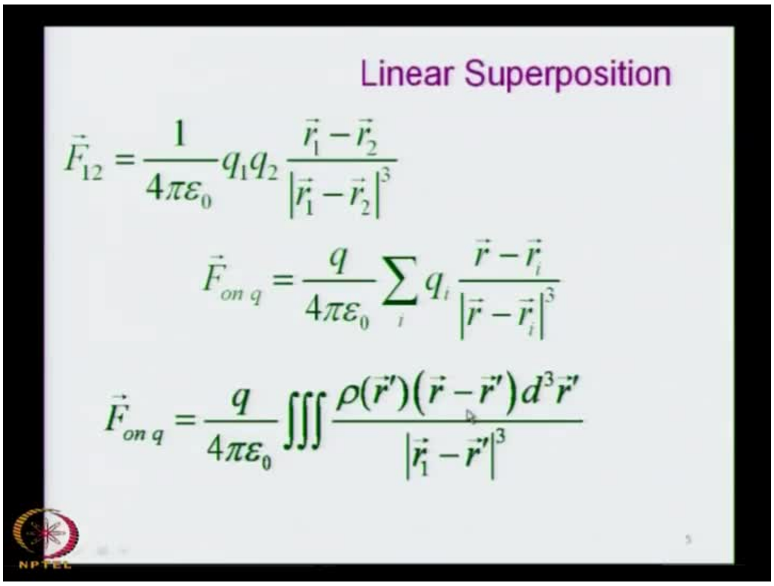

Now, what is important about the Coulombs law and all these goes into the foundations of classical electrodynamics is that, if there is a charge q and it is influence it is under the influence of not just another charge q prime or whatever but under the influence of a number of charges; so, there may be other several charges which could influence the dynamics of the charge q; if this were to happen, then what holds is the principle of superposition?

Now, this is an extremely important property of the Coulombs law that a principle of superposition holds. So, this charge q would be under the influence of a number of charges, so you will sum over all those charges, I going from one through end whatever be the total number of charges and then there will be a 1 over inverse square corresponding to each distance between the charge q and the q I, which is the ith charge.

So, the first important thing to recognize about the Coulombs law is a principle of superposition; not only that, if these charges are not point charges, but they are spread out, they are smeared out in space; then of course, the other charges must be recognized as volume integrals of the charge densities.

So, the volume integral of the charge density comes if rho r prime is a charge density. So, this is charge per unit volume, this multiplied by the volume element d 3 r prime gives you the net charge in that volume element. And this multiplied by this 1 over r square, which is contained in this numerator r minus r prime vector divided by the cube of the distance, which is what gives you the 1 over r square; so, this together once again is essentially the inverse square law, but it has been adopted not just for point charges, but also for continuous charge distributions.

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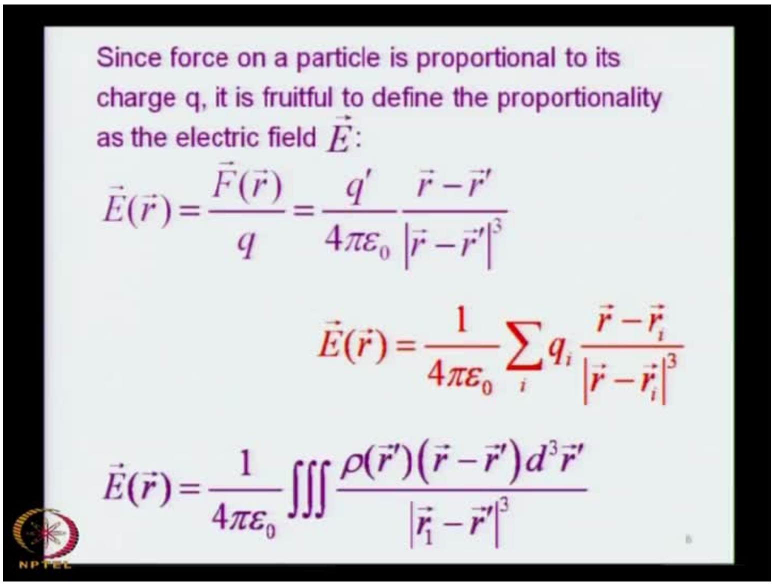

Now, we do make use of the idea of the charge density and the forces that are generated by this; but it is often fruitful to define a quantity, which is independent of the test charge q; so, what we do is, define a quantity which is that force per unit charge and this is what we call as the intensity of the field and this intensity of the field is given by this relationship.

So, this is again, it goes as the product of the two charges, but the test charge is now normalized to unity in our system of units and then you have once again the statement of the inverse square law, but normalize to the test charge being of unit magnitude in our system of units.

This is now extended to the to the situation, where you may have several charges. So, you have the principle of superposition, which will of course hold good for this and then of course also when the charges are smeared out in space. So, you deal with charge densities in this particular case.

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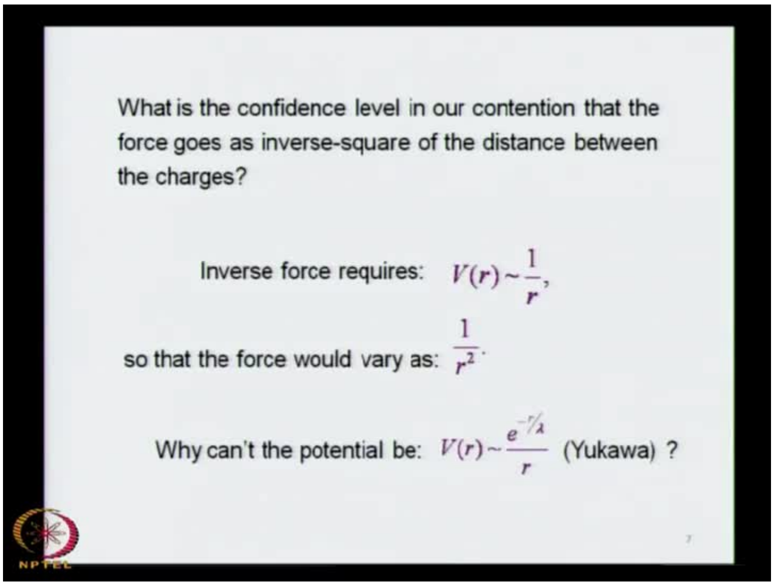

Now, let us ask this question, how do we know that the force between two charges really goes as 1 over distance square you write 1 over r to the power 2. Yes, it comes from experiments, but could it be 1 over r to the power 2 plus epsilon, where epsilon is some tiny quantity what is our confidence level in saying that it is exactly 1 over r square is there any possibility that the 1 over distance square needs to be corrected by no matter how small a correction.

So, it could be in principle be 1 over r to the power 2 plus epsilon, where epsilon is some small quantity and do we have any estimate of this epsilon, is it exactly 0, or is it just a small quantity, is it something that we are ignoring; what we are asking is, how exact is the so called inverse square r.

Now, another way of asking this question is that, if the force is to be inverse square, which goes as 1 over r square, then the corresponding potential will go as 1 over r; so, the same question we can ask in terms of the condition on the potential, what is our confidence level in saying that the potential goes exactly as 1 over r, if it is slightly different from 1 over r; then again the corresponding force that would be generated by taking the negative gradient of this potential would not be exactly 1 over r square, but it could depart from it and what we are asking is exactly what is our confidence level.

For example, why could the potential not be a Yukawa potential, because this is also - it has a form, it has got 1 over r - it has got a form which is not very different from the 1 over r potential of the Coulomb case. And could we ask why the potential could not be a Yukawa potential. Now, this potential is called as Yukawa, because Yukawa used it in some other context, but I will not go in to that,

But the form of the potential which is e to the minus r over lambda by r is also a candidate to represent this; and we are asking why the potential could not be a Yukawa potential; so, this is the question we are addressing.

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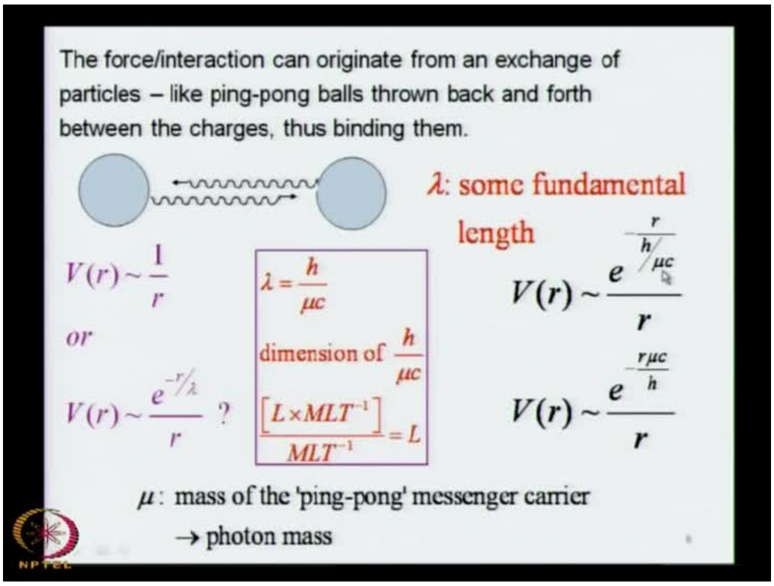

And let this analyzes this question, because this goes into the heart of the Coulombs law. We must recognize that, if there is an interaction between two charges, then this interaction is mediated by some carriers, because otherwise how will this interaction take place. So, it could be something like ping pong balls, which are going back and forth between these two charges and these are the messenger carriers and we know now that these are the photons which convey this message.

And they make the two charges - talk to each other - interact with each other through this so called 1 over r square law. So, let us look at these messenger particles, which are the photons and the picture we have in mind is something of this kind, that these messenger carriers are being exchanged between these two particles.

And you have if you were to select some kind of a Yukawa type potential. Then, let us look at the form of this lambda, this lambda - of course - will have to have the dimensions of links right, because it is coming as a denominator to this e to the minus r.

So, it will have to have some dimension of length, because it comes from the interaction of two charges from a very fundamental interaction; one would expect that, this lambda must be some very fundamental quantity coming, which has got the nature of length it must have the dimensions of length.

So, let us construct some quantity, from fundamental constants; the fundamental constants that we choose are, the Planck's constant, the photon mass m, and the speed of light; because if you take this ratio of angular momentum divided by the mass of the photon multiplied by the speed of light in the denominator, then the dimension of this quantity turns out to be length and it is if fundamental length, because it is coming from fundamental constants.

(()) What exactly the photon?

That is exactly what we are above to discuss right; that is precisely question I am coming to, the fundamental quantity that one can generate to construct length is coming from h over m c or mu c, where mu is the mass of the photon, which is being exchanged between these two charges and this is the carrier particle.

So, this would be the photon mass. And let us ask what it would be like. So, now, if we put this fundamental constant h over mu c for lambda in the Yukawa potential, so we have this minus r over lambda lambda being h over mu c.

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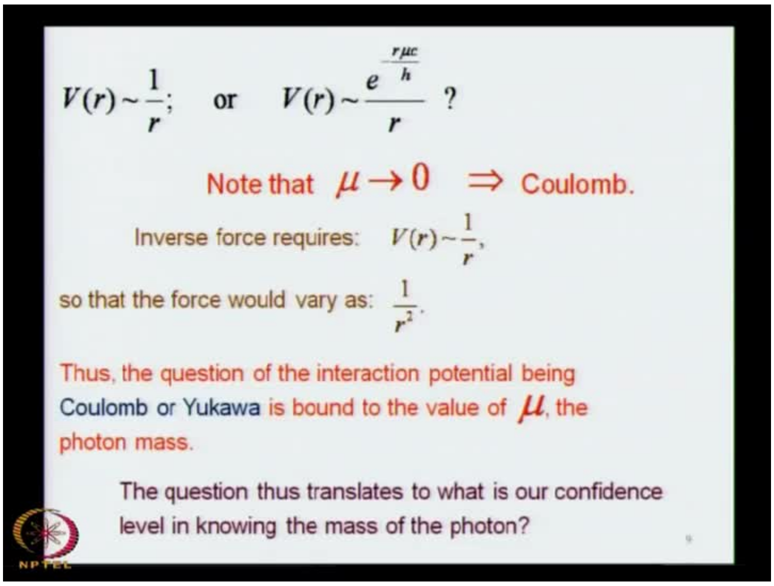

So, now, if this expression, the Yukawa potential becomes e to the minus r mu c over h divided by r; so, let us look at this term further, the question that we had asked, is the potential 1 over r has Coulomb would claim or is it given by the Yukawa type potential, as a candidate potential that we want to consider for comparison. And this question boils down to the fact that, if mu was 0, this is the mass of the photon; if mu was 0, then e to the 0 would be 1 and you would get 1 over r; which means, that the question of the potential being a strict 1 over r potential, is intimately connected with how good an approximation, we can make to the mass of the photon; if the potential is 1 over r, then the force would be 1 over r square. So, the validity of the 1 over r square force, it is connected with the validity of the 1 over r potential, which is connected with the claim with the mass of the photon is 0. These are all connected to each other and you cannot answer one without the other.

So, this is the summary, that the inverse force requires the potential to be 1 over r and the mass of the photon now becomes a quantity of fundamental interest; and how accurately we know this, is now connected with the potential being exactly the Coulomb potential that we call as the 1 over r potential.

So, our question translates to what is our confidence level in claiming, that the mass of the photon is 0; the same question can be reformulated not in terms of the Coulomb potential, but in terms of the mass of the photon, and these two questions are completely equivalent in this context.

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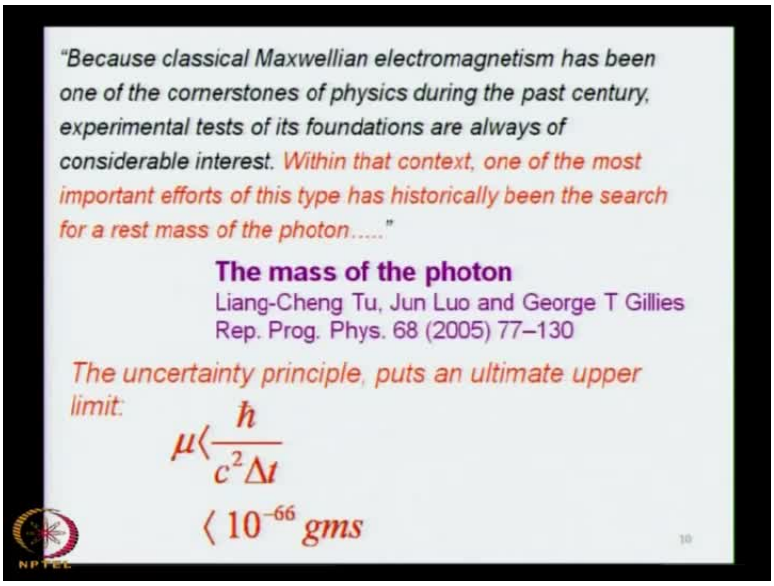

And I will like to coat from this very nice people in reports in progress in physics published in 2005, by Liang Cheng Tu, and Jun Luo, and George Gillies on the mass of the photon. And I will invite you to read this article, it is a fairly advanced article or technical article, but some of you may be interested in reading this.

And what they point out in this article is that, "Because classical Maxwellian electromagnetism has been one of the cornerstones of physics during the past century, experimental tests of its foundations are always of considerable interest. Within that context, one of the most important efforts of this type has historically been the search for a rest mass of the photon". Now, you understand the importance of this particular issue and those of you would like to read further will certainly like to read this article, in report some progress in physics, by Liang Cheng Tu and his coauthors.

So, now, let us get an estimate on the mass of the photon and if you consider simple uncertainty relationships, because the energy of the photon, which is which you could write as mc square m being the mass of the photon; and this energy and time would provide you a pair of canonically conjugate variables; so, this in quantum mechanics are connected by the uncertainty principle and again this is something that, I will not get into any detail, because this does require a background in quantum mechanics, which is really beyond the scope of this course.

This course being an introductory course in classical mechanics nevertheless, I will like to mention over here that, just the way there is an uncertainty between position and momentum in quantum theory, which comes or which is famously recognized has the Heisenberg principle of uncertainty.

Why is that, you are considering only the Yukawa potential.

Well, it is just to consider a possible candidate to suggest a departure from the 1 over r potential, which is somewhat similar to the 1 over r potential, but also somewhat different. So, the question we are raising is, is it a straight 1 over r potential or can it be anything else; and you can consider any departure from 1 over r; so, if you just distort the one over r potential by any function of r, you will have a departure from the Coulomb potential from the 1 over r potential or you can take n different forms; and of course, we are not going to discuss n different forms.

What I wanted to suggest is that, if you take any one of these departures, just to illustrate the idea as to what it really translates to; and the Yukawa potential is a very good candidate, because it has some of the features of the Coulomb potential.

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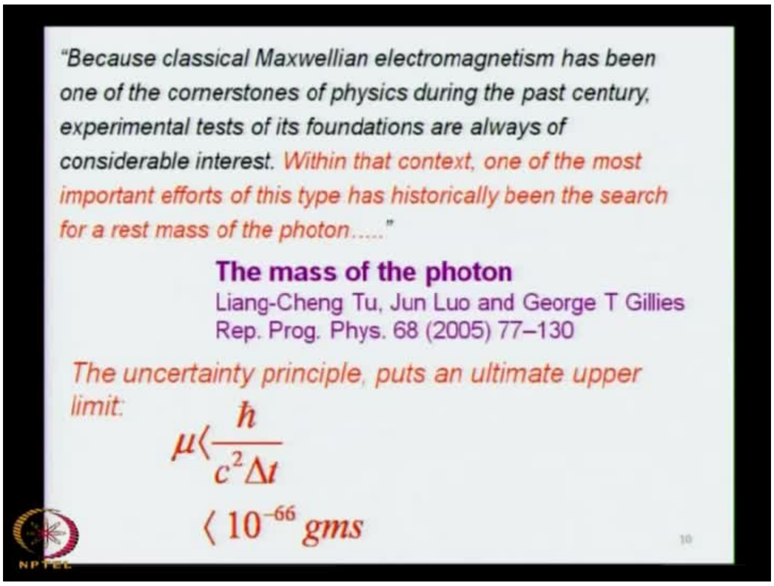

So, we already do know that, the 1 over r gives us very good results, which is why priestly, Coulomb, Cavendish, everybody systematically catalog the data and they recognizes and in fact, called it is a law, which is why we today call it as a Coulombs law.

So, a slight departure from this, just to suggest a slight departure from this, I took the case of the Yukawa potential and what we find is that, if you consider this kind of a departure, then the question of the accuracy of the 1 over distance square law boils down to the accuracy with which the mass of the photon is known. Yes, you have a question.

Why should we take lambda equal to h by mu c equal to you could take some other parameters.

You could, but we do know that, it is coming from a fundamental interaction. So, if it is a quantity of fundamental importance, it better come from fundamental constants of nature.

So, you can play with some fundamental constants of nature, which are known fundamental constants nature and the Planck's constant is known, the speed of light is known right; and how would you manipulate them to get the dimensions of length. So, this is an obvious way of doing it. So, if you consider… Yes.

We are suggesting that, we can also try it in some other manner.

You could, if there is something of some interest, we have not derived it from any first principle. So, you are quite free to try out something else, if you can come up with a viable candidate; it will not be very easy. Because there are only a few fundamental constants, are they must be such that, they give you the correct dimension, because you want at the dimension of length to come out of it. So, if you come up with anything else it is quite likely that, it will turn out to be completely equivalent to what we have already done.

So, let us come back to this uncertainty principle and what I would like to point out is that the uncertainty between energy and time is expressed by a relationship, which looks very similar to the Heisenberg principle of uncertainty between position and momentum, but it is not exactly that, because there is no operate of a time in quantum mechanics; but this is a matter of detail and I will certainly not have the scope to go into this, but if anybody is interested, you can perhaps send me an email or ask a question after the class.

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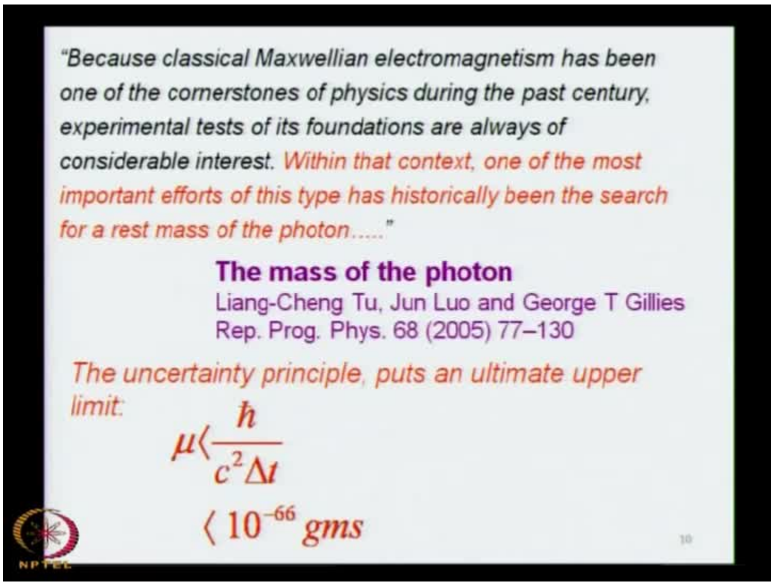

So, if you consider the uncertainty between energy and time, then you will find that, from the time uncertainty which comes from the level widths of the excited states from which - you know - if there is a transition to a lower state and there is a release of a photon; so, if you take that as a measure of the time uncertainty, then the mass of the photon would be less than 10 to the minus 66 grams that does not guaranty, that it is 0.

It does guaranty, that it is very small. So, it does guaranty, that the photon rest mass is extremely tiny, but any experimental verification of this will be very intimately connected to the validity of the 1 over r square law. So, these are some of the fundamental features of the Coulombs law, which I wanted to highlight, because we certainly do not have the time to go into a very detailed formulation of the theory of the electrodynamics; in three or four classes, we want to summarize the essence of Maxwell's equations as an integral part of this full course on classical mechanics or it is not a full course on everything in classical mechanics, but a full course on the first course in classical mechanics, where we only meet some of the introductions.

So, even the Hamilton Jacobi theory and so on, we have not dealt with. So, in this first course, so this is the first course after the high school that students will be taking, but what is important at this point to recognize for us, is at the inverse square law, the Coulombs law, which is a cornerstone of the electromagnetic theory, is intimately connected to the confidence level with which you can claim, that the photon, mass must be 0.

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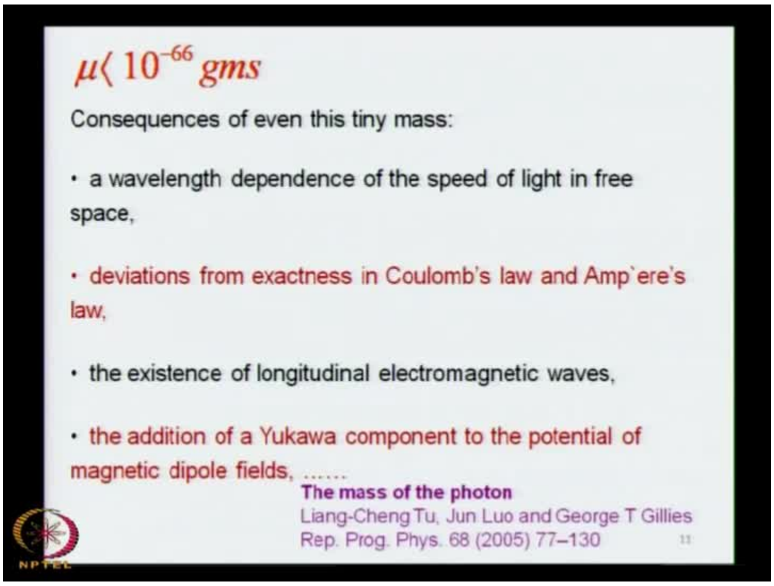

Now, there are other ways of arguing that, the photon mass must be 0, many of them come from the special theory of relativity and so on and I will not be discussing those complex issues.

But, if they were to be a tiny mass, no matter how small, now 10 to the minus 66 plenty of 0s right; but even if it were to be a very tiny mass, it could have major consequences on our understanding of the laws of electrodynamics on not only on just these two parameters of the inverse square law or the photon mass, but the wavelength, for example, would depend on the speed of light, even in free space. So, this will have amazing consequences, and then there is amperes law, which I will be talking about that will need that will need to be modified to a certain extent.

We always talk about electromagnetic waves being transfers in nature. One of the consequences that we will have to deal with is, we will have to consider the possibility of existences of longitudinal electromagnetic waves; so, this will have very major consequences on the entire formulation of electromagnetic theory.

And then, of course, there would be terms like the Yukawa potential, which the such components will have to be added to the potential of a magnetic dipole field and so on. So, some of these details are discussed at great length in this article, which are have already referred to and you might want to read that out.

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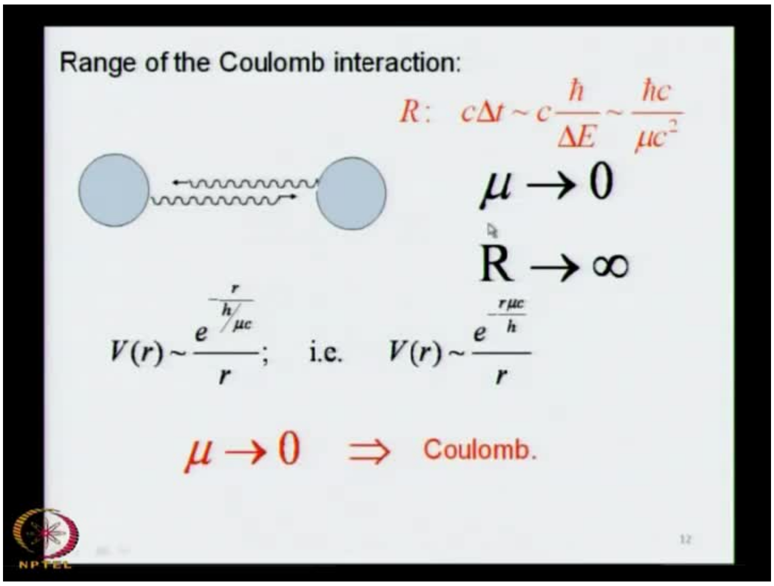

Now, let us consider some other consequences. Now, what is the range of the Coulomb interaction; now, that as you can see the range would be given by the speed times, the time right speed into time is the distance; so that is a range and if you go on these scales, then if you exploit - the relationship - the uncertainty relationship, which are refer to the previous slide, then you find that this range goes as h cross c over mu c square; in other words, as mu tends to 0 if the photon mass goes to 0 then the range becomes infinite.

So, the claim that the Coulomb potential has got an infinite range again is connected with the mass of the photon; so, it plays an extremely central role in the learning of electro dynamic theory. So, as mu tends to 0, R would tend to infinity, and also as mu tends to 0, as a photon mass tends to 0, you immediately see that the potential tends to the Coulomb potential; so, that will be connected with the confidence level of the 1 over r square.

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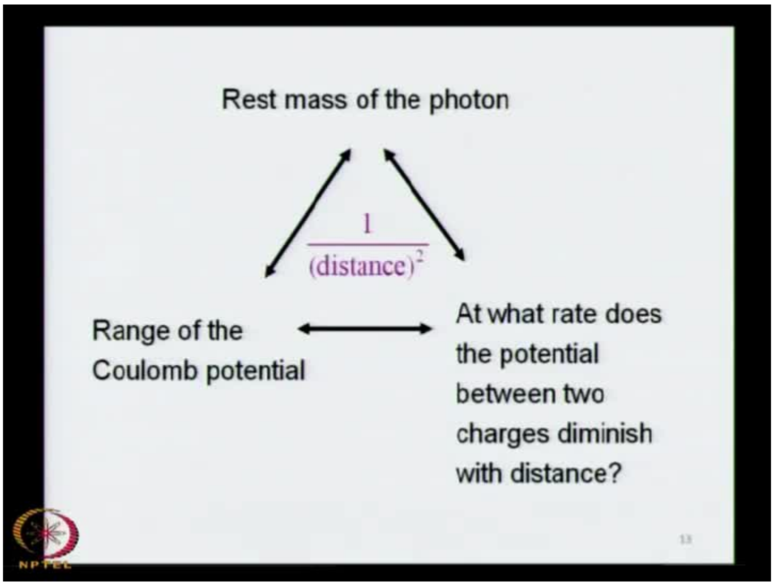

So, these are the three things - the rest mass of photon, this is connected with the range of the Coulomb potential, as we have already seen. And then, at what rate does the potential between the two charges diminish with distance? Does it go exactly as 1 over distance? Or is there another function of r which must multiply this? So, all these questions are central to our claim that, there is in fact an inverse square law. So, all of these are connected with each other.

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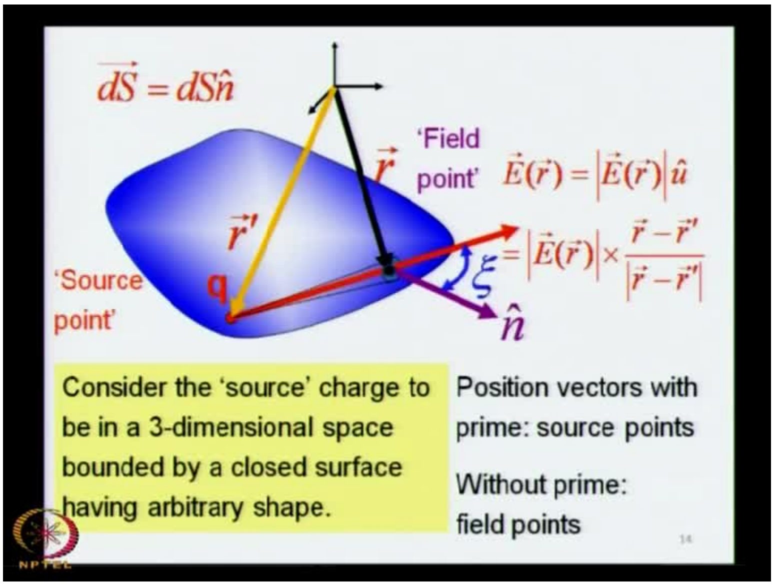

Now, let us proceed with this understanding of the Coulombs law. And we now have a source point, this is a source of electric field, there is a charge q and it generates a certain influence at a point in space, where the electric field generated by this charge exist; so, you called this as a field point.

The position vector of the field point is denoted by the position vector r, in this coordinate frame of reference and in the same coordinate frame of reference, the position vector of the source is denoted by r prime. So, the primed quantity denotes the source and the unprimed quantity denotes the field point - in our notation.

And the intensity of the electric field, which we have already consider, is a magnitude of the electric field times the unit vector in which this field is directed and we are considering field generated by positive charges, just for the sake of you know our convention; so, we also consider the influence it would exert on a unit test charge; so, it will go in the direction r minus r prime; so, this is the direction in which the intensity would act. So, this is the magnitude of the unit vector. And what we do now is, we consider this source charge to be in a three-dimensional space bound by a close surface of an arbitrary shape; now, this is a completely mathematical idea.

This charge q we consider to decide in a 3 dimensional space; and in this space, we consider a surface around, this charge, this surface we consider may have any shape; it must be a close surface, so it could if the charge is somewhere over here, then around it, I would take a surface, it could have any irregularities, but it must be a close surface; so that in this close surface the charge q resides.

So, this is a mathematical idea of a certain surface, which encloses the charge q. So, let us consider such a surface and this is the three-dimensional object, which surrounds which has trapped the charge q inside it, as our terminology develops we will call this as a Gaussian surface, because it was Gauss would invented these techniques for the mathematical analysis of the field influence.

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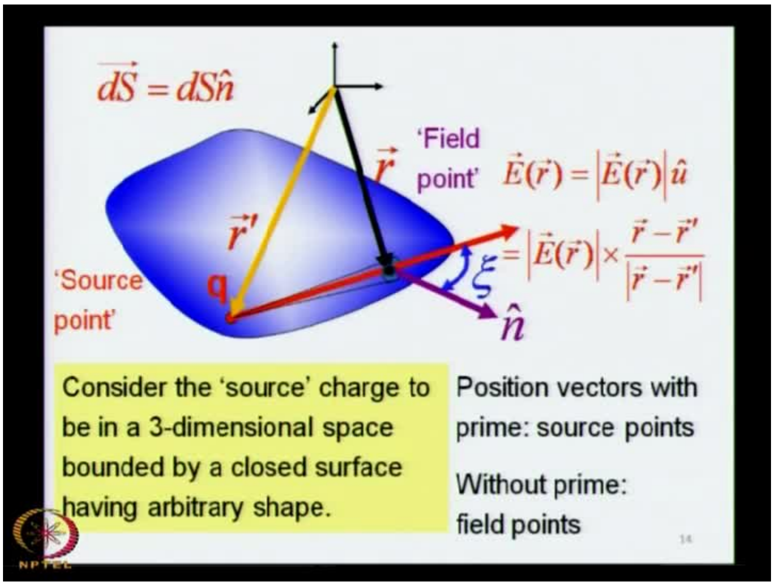

So, we consider such a surface, which is a close surface and the charge q is sitting inside and this surface has got some arbitrary shape; there is no particular geometrical shape, it may have some wiggles ups and downs and so on. So, in the direction of this point at which we are considering the influence, so let me go back one stroke of the mouse, so there is a certain field point and if you were to see the influence along this direction, one would tend to think of a cone with its vortex at the source point and the flat face of this cone being orthogonal to this direction.

But this surface is not oriented, such that, this surface would go over this flat surface, because it is a surface of some completely arbitrary shape; it may have some ups and downs, so it is not going to have a piece, which is exactly flat to close that cone. This is not the kind of cone that you are going to see, the surface element on this Gaussian surface to subtend at the source point q; it could be some other form, which will have some other irregularity, which would intersect this surface. It would so, this is what you see this intersection with this Gaussian surface, is not going to be the rim of that cone, but it will have some irregular shape, so keep that in your mind. And this surface, it will not only have an irregular edge rather than the circular rim of the cone; if you take a normal to that surface element, it will not be in the direction of this unit vector u, which is along this red arrow.

The unit normal to that surface could be something else and it could make a certain angle with the unit vector u and this angle let us say xi, is the geometrical here everybody. So, the actual surface that we have considered has an arbitrary shape. The normal to the surface is the unit vector, which is orthogonal to the surface at that point and that will not in general be in the direction of the unit vector u, which is the direction of the intensity of the field.

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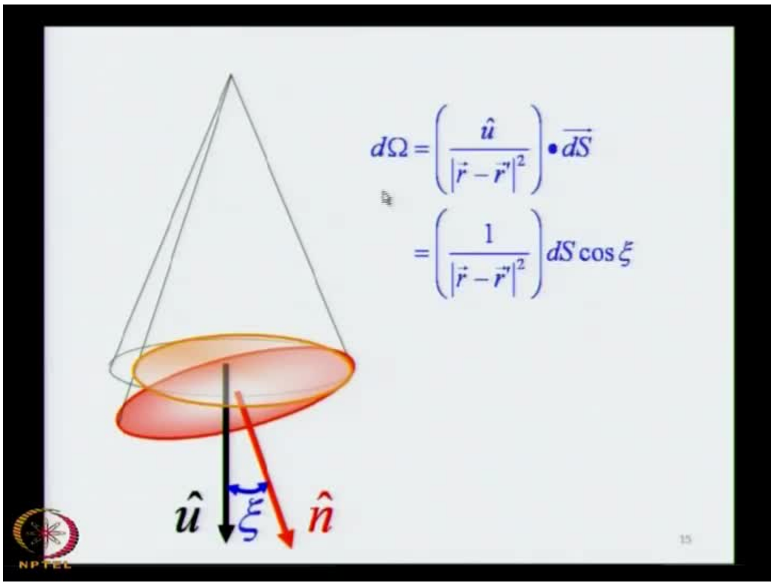

So, now, to look at this particular geometry closely, we look at this cone, and if we have a shape which is not going exactly along the rim of the cone, but it makes a certain angles xi, so these are the two angles, these are the two unit vectors that we consider in the previous slide. So, let me just go back to this to remind you that, the unit vector u is along this red line and the unit vector n which is a normal to this Gaussian surface, is along this purple line - purple or indigo or whatever be the colour, whatever be the name of the color you want to give - it purple more or less indigo, magenta, pink, I do not know I always get confused.

So, this is the unit vector u, this is the unit vector n which makes a certain angle and if you notice the angle subtend by this tilted form at this vortex, this solid angle is now slightly less than the solid angle which this this cone with a flat face with u along the direction of the intensity would make; and the extinctive which it would be less, will be given by the cosine of the angle xi.

Because this cosine xi, it has got a value equal to 1 when xi is 0; so, if n was exactly along u, you would get the same angle as the cone, as the first cone that we talked about; but when xi is not 0 when the surface element has got a unit normal, which is not in the direction of the electric intensity, then there will be a scaling down which will be given by the cosine xi and that is coming from the projection of this infinitesimal surface element with the unit vector u, so you get a dS cosine xi, so this is the solid angle which is skilled down by the cosine factor.

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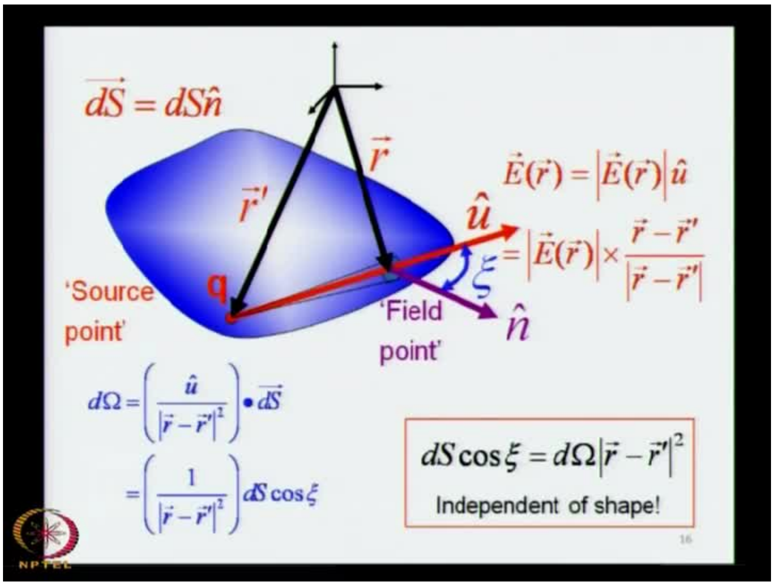

So, this is elementary geometry and we will use that over here; so, this this geometry tells us that, this solid angle subtended by this surface element, is dS over distance square times this cosine xi and what is important is that, it is completely independent of the shape; because we never claimed that, this particular shape which represents an elemental area on the surface of this imaginary surface that we constructed has any regular shape; so that details of the shape are quite irrelevant to this. So, if you multiply by this distance square on both side of this equation, you get dS cosine xi equal to d omega distance square and this relation is independent of the shape.

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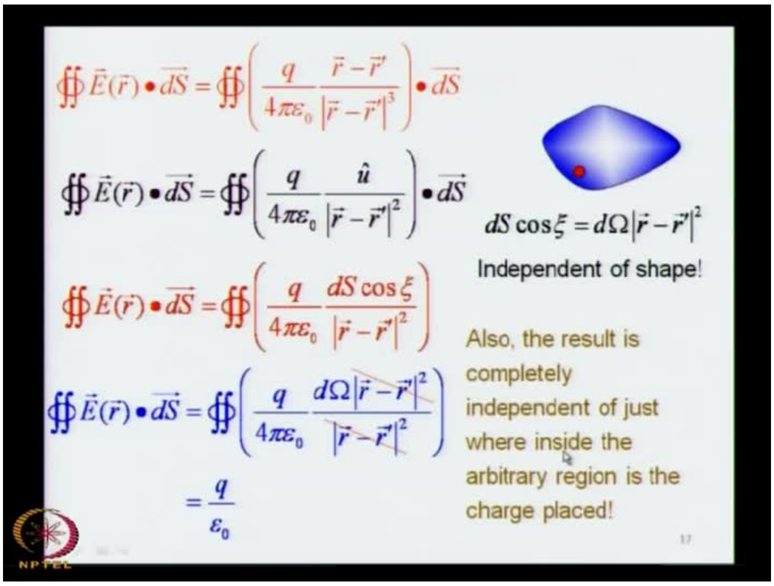

So, now, let us construct this surface integral and since we have done this in details in unit 8 and then also in unit 9, we used it in unit in the previous units - the surface integrals and the volume integrals. So, we construct this surface integral over a close surface, regardless of the shape of the surface. Then this goes as the Coulombs 1 over r square law dotted with this surface element, but what we do know is that, this surface element dS dot u, which is coming from here is given by this dS cosine xi; this relationship we have just arrived at, we will exploit it, because cosine xi, we have already determined, we have already analyzed it in the previous slide; so, this relation becomes q over 4 pi epsilon 0 and dS cos xi o is nothing but this, you have you have got this q over 4 pi epsilon 0 and this is your d omega r minus r prime modulus square; this is equal to dS cosine xi independent of the shape.

So, this dS cos xi is represented by this d omega distance square and now you can cancel the distance square, which comes in the numerator as well as the denominator, and then you can get the q over 4 pi epsilon 0 as a constant outside the surface integral and all you have to do is to integrate this solid angle over the entire close surface, which is exactly 4 pi; and that 4 pi cancels this 4 pi in the denominator and what comes out of this is the Gauss's law, this in fact is the first law of Maxwell.

So, we have gotten this Gauss's law by doing very simple geometrical analysis and which we have relayed on the Coulombs law. The Coulombs law we have relayed on the accuracy with which we can claim the photon mass to be 0, which we have learned is connected with the range of the Coulomb potential and within the frame work of the robustness of these approximations or accuracies depending on what is correct.

By playing with this geometry, we get this surface integral of E dot dS over a close surface to be exactly equal to q over epsilon 0 and this is a result which is completely independent of where inside the surface you have located the charge, because this charge could have been anywhere inside that close surface; that surface in the first place to begin with never did have any regular shape.

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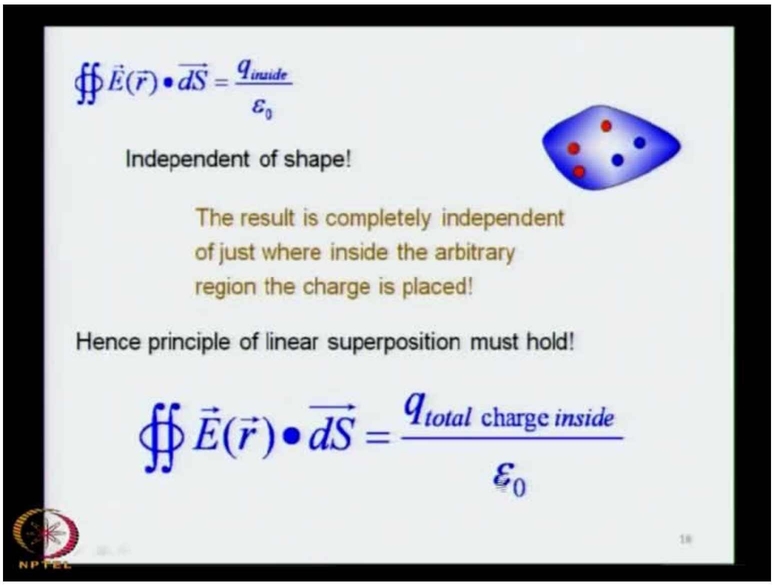

So, there would be no question - about talking - about claiming that, this charge should be at the center; and if it is not at the center, because the shape may not have any center of symmetry at all, then it is anywhere in the center, any anywhere inside that surface; so, if you consider a close surface like this very irregular shaped object, that charge could be here it, could be somewhere else, it could be anywhere, it could be here, it could be here, it could be here, it could be here as a matter of fact, since it could be here, you can have one charge over here, another over here, and a third over here, and a fourth over here, and a fifth over here.

So, in that case, you will need to carry out the sum total of all of these charges, because we know that the principle of superposition holds for the Coulombs law, so it was hold good in this case as well,

So, all you are getting is that the surface integral of E dot dS is equal to 1 over epsilon 0 - this is the permittivity of free space - the electric permittivity of the free space in the denominator and in the numerator you must have the total charge, which is inside that surface.

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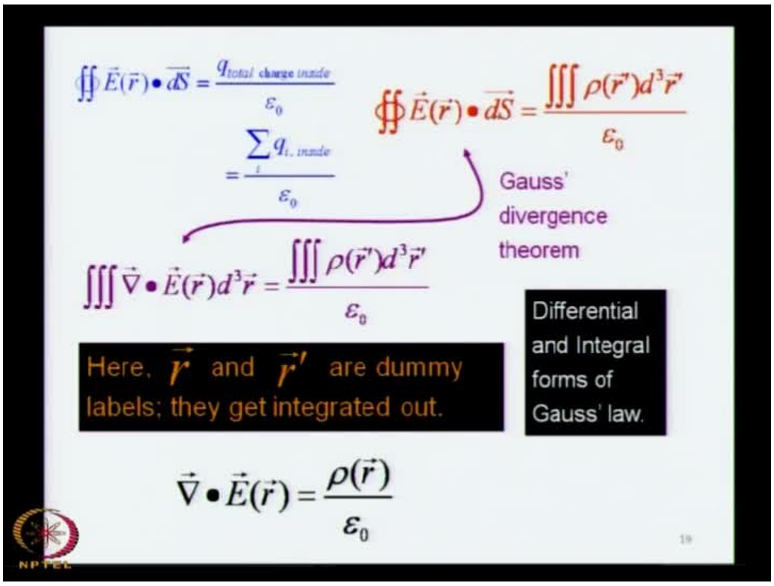

So, the total charge inside the surface, you can right as a sum of all the charges inside or the charges which are inside could be because of certain charges which are smeared out in space, so they could actually be charged densities and then you should then integrate the charge density by multiplying the charge density, charge per unit volume by the volume element and integrate over the entire volume, where the charges may reside inside that surface and you have this relationship that the surface integral E dot dS is equal to a volume integral of the charge density divided by epsilon 0.

So, all we have done is to exploit the principle of superposition, there is nothing else. The only thing that we have done is, at principle of superposition, we as applied to - discrete charges - discrete point charges or charges which are smeared out in space; if they are smeared out in space, they will then have to be integrated out; but this surface integral E dot dS is by Gauss's divergence theorem, which we studied, I believe in unit 9 or was it in unit 8, 1 or the 2 by Gauss's divergence theorem which we applied to fluid mechanics; this is equal to the volume integral of the divergence of this vector.

This comes from vector calculus, it holds good for any vector field and if it holds good for any vector field, why would it not for the electric intensity field; so, we apply the Gauss's divergence theorem and the left hand side of this surface integral is now replaced by the divergence of this electric intensity. And now, you have got two volume integrals, these are integrals over a certain volume, which volume are we talking about, essentially the same volume which is inside that close surface.

So, the volume integral that we are talking about on the left hand side is no different from the volume integral on the right hand side so far as the region of space is concerned; they are over essentially over the same regions of space, that region of space is indicated by the variable r on the left hand side and by the variable r prime on the right hand side; but does it matter, these are dummy labels which get a integrated out and they get integrated out they must go over and pick every point in exactly the same regional space, which means that the two volume integrals are necessarily equal; which also means that the corresponding integrant must be exactly equal.

So, the corresponding integrants are the divergence of E at the point and this must correspond to 1 over epsilon 0 times the charge density also at the same point. So, here we have got a law, which is in fact called as a Gauss's law; but this is a global form, because you have to integrate over the whole surface; here you consider only a particular point, so this is sometimes called as the point form of the Gauss's law, also this is sometimes called as the integral form of the Gauss's law, this is sometimes called as the differential form, because essentially it is a gradient operator, the differential operator, which plays the big role in this.

So, these are the differential and integral forms of the Gauss's law which appear in the Maxwell's equation, in fact, as the first equation of James Clerk Maxwell.

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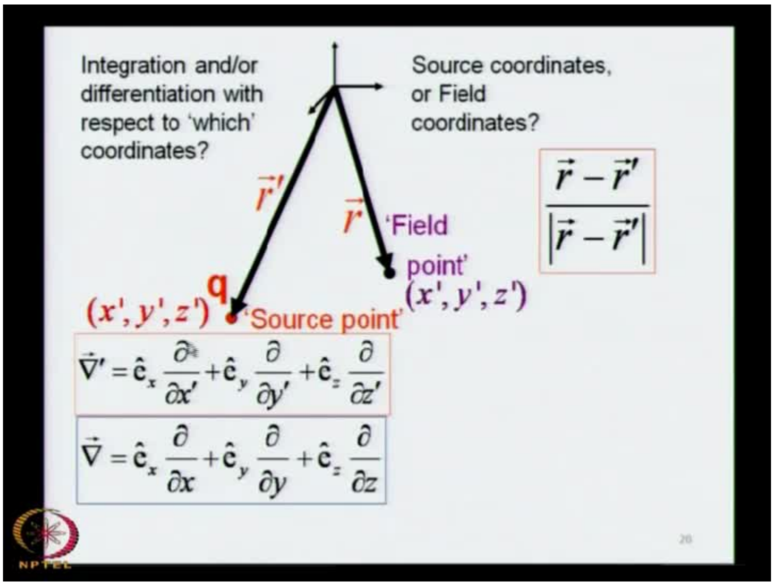

You must keep track of the fact that, when you carry out integration or differentiation with respect to either r or r prime, you are carrying out if it is over r, it is a field point, if it is over r prime, it is a source point right, so you have to remember that. So, a gradient with respect to the source point will be denoted by this prime, because if you consider electromagnetic fields or electrical fields and we consider the changes in that electric field, because you are moving around the source, you move the source from here to some other neighboring point; then you will be talking about the gradients with respect to the source point.

But you could also consider the effect at different field points, not at here but over at some other point; so, then it would be sorry the field point, is denoted as r; so, there should be no prime over here, I am sorry about that.

So, there is this x prime, y prime, z prime; this is the position of vector r prime over here, this is the position of vector r; so, there should be no prime over here, this is just x y z as a field point.

So, the gradient with respect to the field point will be given by this; the Cartesian unit vectors are constant vectors, but the derivative operators here the differentiation with respect to the variables y, here the differentiation with respect to x, and likewise also with y prime and y and z prime and z; so, when taking the gradients, you have to be very careful about what is it, that you are taking the derivative with respect to

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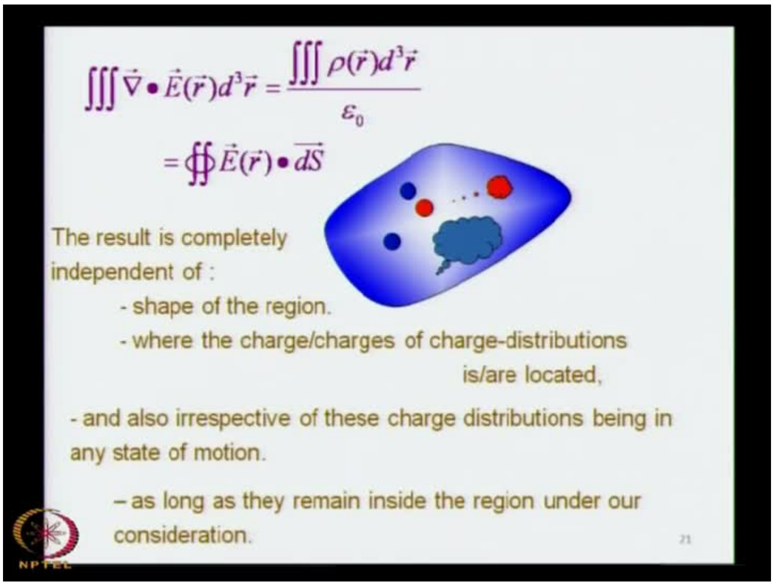

Not only that, the charges could also be smeared as I pointed out; so, you can have discrete charges, smeared out charges and the result that we have got is completely independent of the shape of the region and it does not depend on where these charges inside the surface are located; they could be located over here or here or here or anywhere inside. And if they are located anywhere inside would it matter; they are moving around inside, it would not a matter.

So, the Gauss's law would still be valid, even if the charges were to be moving inside as long as they remain completely inside that surface. So, it is independent of any state of motion of the charges, but they must remain completely inside the regional space, that is under consideration.

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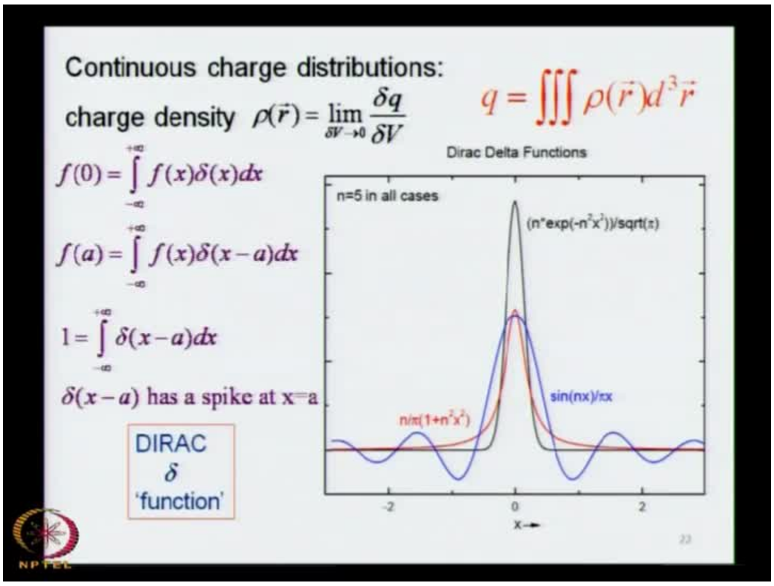

So, the fact that the charges under consideration are inside the region is of crucial importance. Now, one thing I would like to point out is that, when we talk about charge densities, you are talking about charge per unit volume in the limit volume going to 0.

And if you look at this quantity, if the volume becomes 0, no charge would reside inside it and you would have that numerator going to 0 and the denominator going to 0, are you looking at an indeterminate quantity, but the ratio could still be can be determined, and it will have an existence which will be well defined quantity and it is to that extend somewhat similar to the Dirac delta function.

And let me just define it over here, that the Dirac delta function is defined by these relations that, if you have a function of x, then the Dirac delta delta x is such that, if you construct this integral, this is one-dimensional function, that I have taken the idea can be easily extended to three-dimensions or even more and two variables other than just x; you can have a Dirac delta function in the momentum space, in the energy space, etcetera, etcetera and that is the matter of detail no matter, what the variable is?

So, here I consider one-dimensional derive delta function and if you multiply this function f of x with delta x and integrate the product from minus infinity to plus infinity, you get the value of the function at x equal to 0.

And this definition is completely equivalent to stating it; in the second equation, that if you have integrate f of x with delta not of x but of x minus a; so, as you can easily see, if a were 0, you will get the first relation; but when a is not 0, you have a more general expression for the Dirac delta, which is written in the second equation, the two are completely equivalent with the difference that, the second is a little more general, the first is a special case of the second with a equal to 0; so, this is the Dirac delta function.

And if f of x were equal to unity, then you get this relation, that if you integrate just the Dirac delta function itself then the integral of this function is equal to 1; if you integrate from minus infinity to plus infinity, so obviously you can see that this is the function, which has got a spike at x equal to a.

So, sometimes it is like a function, which becomes narrower and narrower in one-dimensional space, but it height increases as this becomes narrow and if you think of this is a rectangle, then you can think of the area of the rectangle giving you the area under the curve to be given by the product of the height of this rectangle with the width of the rectangle.

And as the width becomes narrower and narrower, shrinks to the point of 0, as the width becomes infinitesimally small and if the height becomes infinitely large, then the product can still be finite; so, it is an idea of this kind; sometimes, I like to refer to it as an expert function, because an expert is often defined as somebody who knows more and more about less and less and just like the Dirac delta function in the limit, he can know everything about nothing.

So, I do not know if Jobin remembers this, but he has drawn these pictures for me of the Dirac delta function for different… There are different representations, it is not just the rectangle, that is one of the representation there are many other representations like this n pi 1 over n square plus n square x square, which is given by this red curve, there is a blue curve which is sin n x over pi x and there is an another one which is given by this expression black.

And they all have the common feature, that they become narrower and narrower, as they be get narrower and narrower, their height increases and this is the kind of function that the Dirac delta function is and the charge density would pretty much remind you of this kind of a situation.

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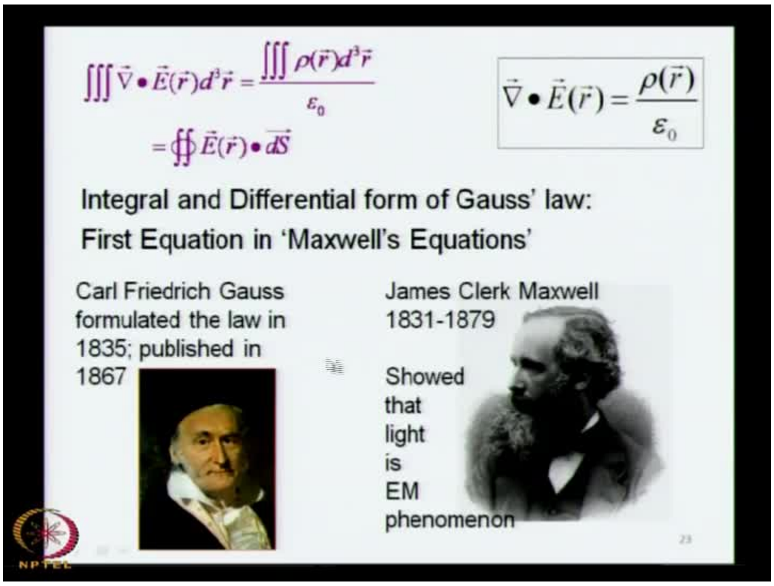

So, what we have is the charge density appearing in the Gauss's law and this is the first equation in Maxwell's equations. You can write it as these volume integrals or the surface integral, the reduction of the volume integral to the surface integral is a matter of elementary vector, calculus which we have discussed in considerable detail in an earlier unit of this course and this is the famous Gauss's law which was formulated in the 19 century, in 1835, but published more than 30 years after that.

So, I do not know why he did not formulated for those many years, why he did not publish it for those many years. And then it was included in Maxwell's theory, in which Maxwell went on to show that light is an electromagnetic phenomenon; so, that is the rest of the story for this unit 10 and at this point we will take a break.

If there are any questions or comments, I will be happy to take; otherwise, we will take a short break and in then next class, we will discuss the Oersted ampere law, which then gets incorporated in Maxwell's formulation as a second law second or you know it gets into the Maxwell's scheme of electrodynamics.

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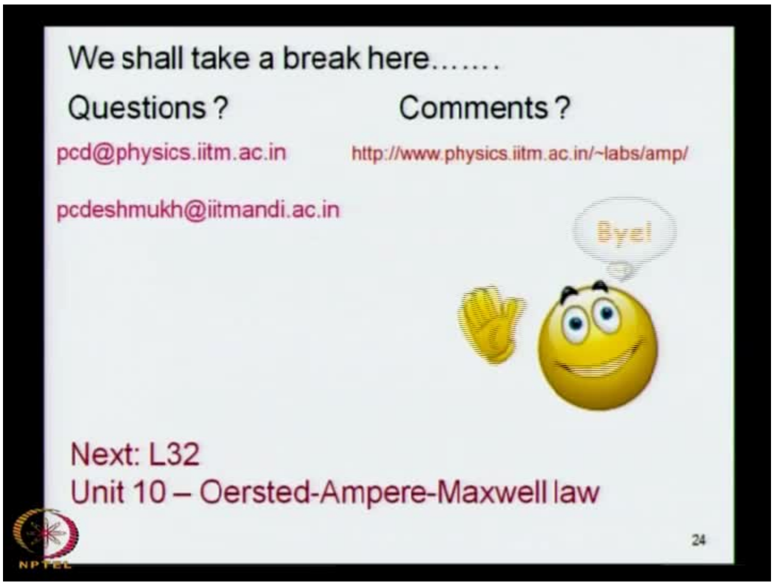

So, these are the four major equations of Maxwell and we will give you a summary of all of them; so, the Gauss's law is, what we did in today's class; and in the next class, we will do the Oersted ampere Maxwell law. Any question, comments or we just take a break, everybody wants a break. So, goodbye for now.