Select/Special Topics in Classical Mechanics

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Module No. # 10

Lecture No. # 32

Classical Electrodynamics (ii)

Greetings. So, we will continue our discussion on classical electrodynamics. In the previous class, we did the Gauss's law, and now, we will do the Oersted-Ampere-Maxwell law and we will formulate the Maxwell's equations. And you will find, there we will exploit the machinery in vector calculus that we have developed in the previous units. In fact, part of a reason we spent considerable time developing this machinery, part of the motivation came from this anticipation of the application of those tools in developing the laws of electrodynamics.

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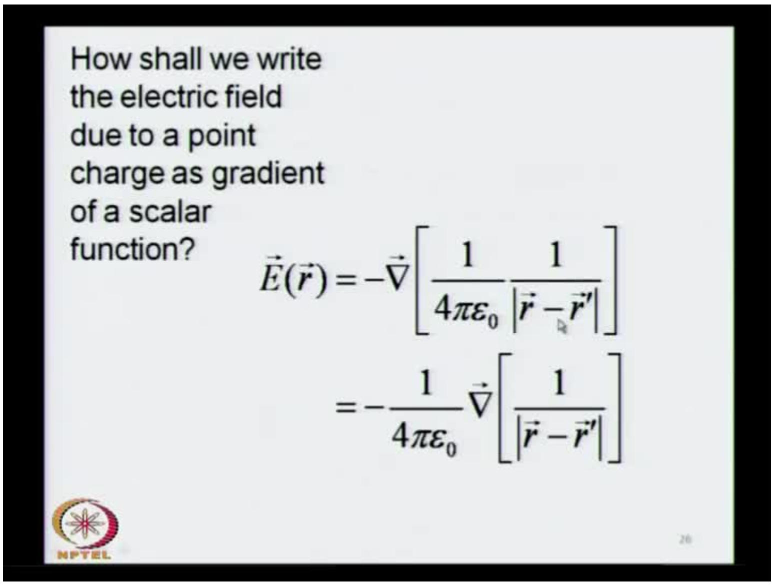

So, we will see that all of it will come together; the first thing we will do is to express the electric field due to a point charge as gradient of a scalar function. So, this is an idea that we have used earlier as well that field is often expressed as the gradient of a potential in the case of conservative fields. So, this is the field that we want to express, electric intensity and this is expressed as gradient of this 1 over distance potential, which is the coulomb potential.

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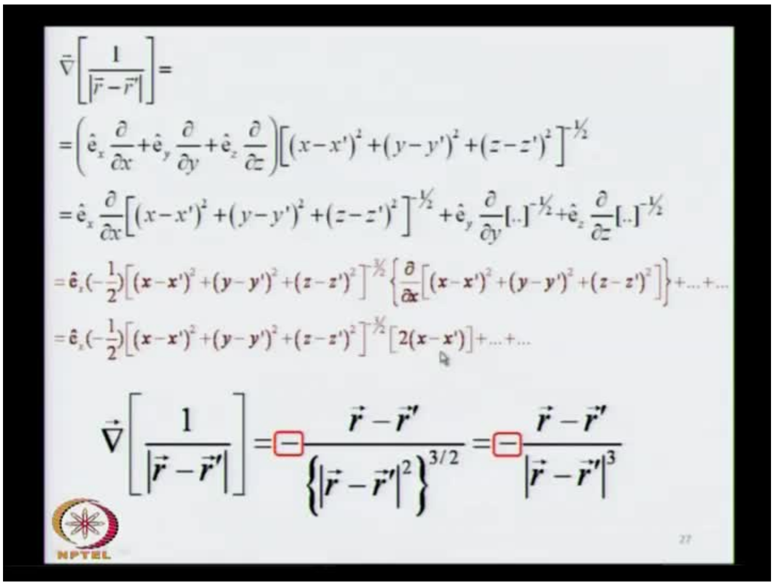

So, let us get this gradient explicitly; this is the 1 over distance. So, I write the expressions of the gradient in the Cartesian geometry, in the Cartesian coordinate system and this is the 1 over distance, this is the operand on which the gradient operator will operate and this 1 over distance, this distance is a square root of the distance square and the distance square is r, the inner product of r minus r prime with itself or the scalar product of r minus r prime with itself.

So, it will be x minus x prime square plus y minus y prime square plus z minus z prime square; so this gives you the square and then the square root will give you the distance itself; the minus sign will put it in the denominator where we wanted. So, let us now, take this derivative; there are three operators here, the e x del by del x and then you have got the e y del by del y; so there are three partial derivative operators and they come as a sum of these three operators. So, we will carry out this process of determining these derivatives term by term; so the first term is e x del by del x, and then you have got the 1 over distance plus e y del over del y operating on the 1 over distance, and the e z del by del z operating on 1 over distance. So, these are the three terms and now, it is the simple process of getting this derivative.

So, this is something that you can do very easily and you just do it term by term; so del over del x of this whole term will give you minus half coming from here and then this is to the power minus half minus 1, which is minus 3 by 2 and then you must take the derivative of this term over here, which is 2 times x minus x prime and there are similar contributions from the derivative with respect to y and the derivative with respect to z.

So, here is the minus sign; this is coming from here; here is the r minus r prime, because this x minus x prime times, this unit vector e x will give you the x component of r minus r prime and there are similar contributions from the y and the z terms. So, you get the r minus r prime in the numerator and you get the 1 over distance square to the power 3 by 2 coming from here; this is now, written in the denominator, because of this minus sign and here you have the result, which is minus of this ratio and this is distance square to the power 3 by 2; so this will be distance cube. So, essentially you have got 1 over distance square law, which is a Coulomb's law.

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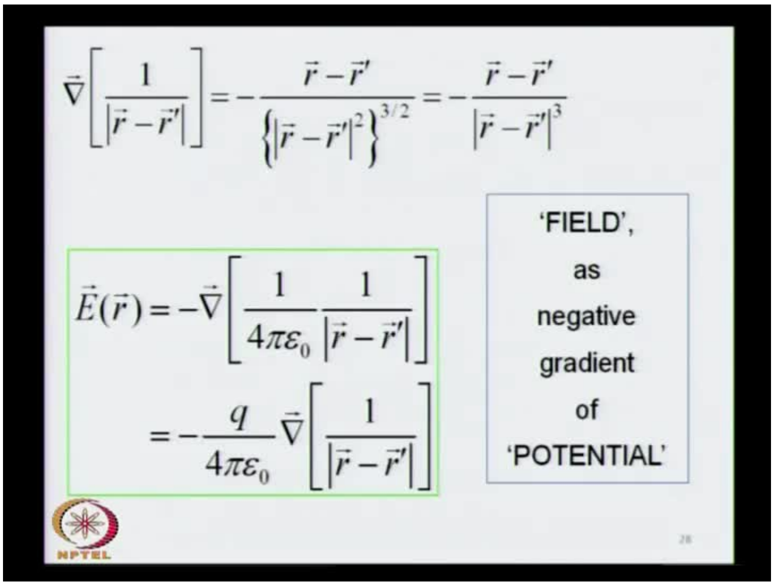

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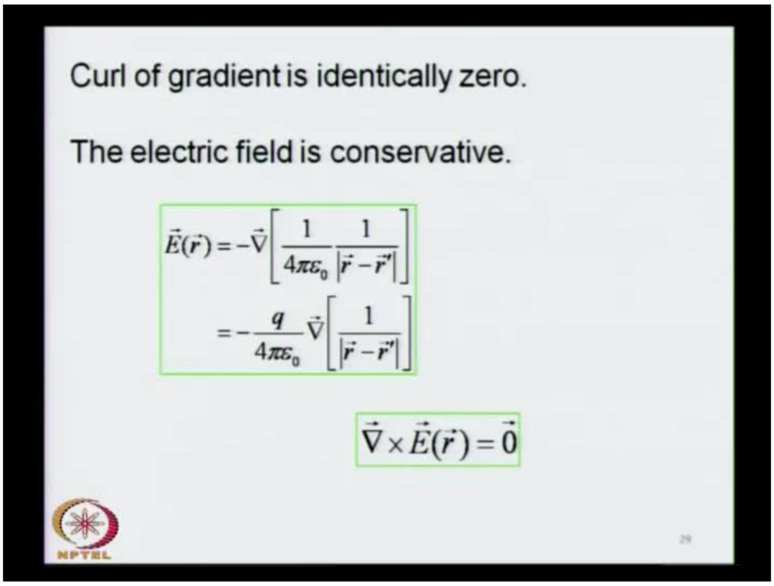

So, we have essentially succeeded in writing the field as a negative gradient of potential and expressed it in Cartesian geometry, Cartesian coordinate frame of reference one could do so in any other coordinate system which just as match is much ease. Now, the curl of a gradient is identically 0; the electric field is conservative, because we have expressed the electric field as gradient of a potential and it is irrotational and therefore, it corresponds to a conservative field; we have discuss these ideas earlier as well.

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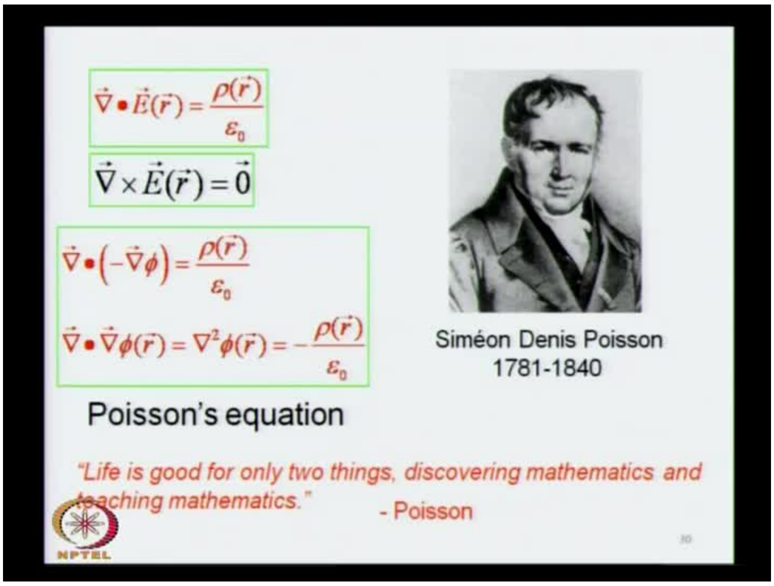

So, this is expression for the irrotational electric field. Let me quickly recapitulate a consequence of the Gauss's law, which we discussed in the previous class. You have got the divergence of E, as the charge density divided by the permittivity of free space; you also have the curl of E to be the null vector. So, essentially you have expressed both the divergence of E as well as the curl of E and you need these two quantities to express and E vector field. In addition to this, you need certain boundary condition and this is really the statement of the Helmholtz theorem, that to express any vector field, you must specify the divergence as well as the curl, and that is the reason in Maxwell's equations what you do find or the divergence and the curl of the electric field and also the divergence and the curl of the magnetic field, because both need to be specified.

So, here, further electric field, we already have the results for divergence and curl of the electric field and then if you express this first equation, which is the divergence of the electric field, but the electric field itself is the negative gradient of the potential. So, you can write this as minus of del square phi equal to rho over epsilon 0 or del square phi equal to minus of rho over epsilon 0 and this relation is known as Poisson's equation. This is one of the very important equations to be solved, if you are interested in determining the potential at a given point due to a certain charge density, which is smeared out in space.

What you must add to this information is of course the boundary conditions, because without the boundary conditions you cannot solve a second order differential equation or any differential equation; you must specify the boundary conditions. So, you need the differential equation and here, you have it and you can get the potentials from different charge distributions and this is the matter of detail, but once you have this relationship in front of you, you could in principle get the electric potential due to any charge distribution.

Let me quote points over here – "the life is good for only two things, discovering mathematics and teaching mathematics". So, yesterday, we had one of the Bernoulli say that it would be much better for physics, if they were to be no mathematicians. And now, we find Poisson saying, the life is good only for two things one has to discover mathematics and the other is teach mathematics, other than that life is useless.

So, anyhow, mathematics of course is very intimately connected with our studies of physics and it plays an extremely important role, is a very language of physics in a certain sense and it is important to get the concepts in physics, but it is equally important to formulate them; extremely rigorously and if there any approximations involve, then one must know exactly what these approximations are.

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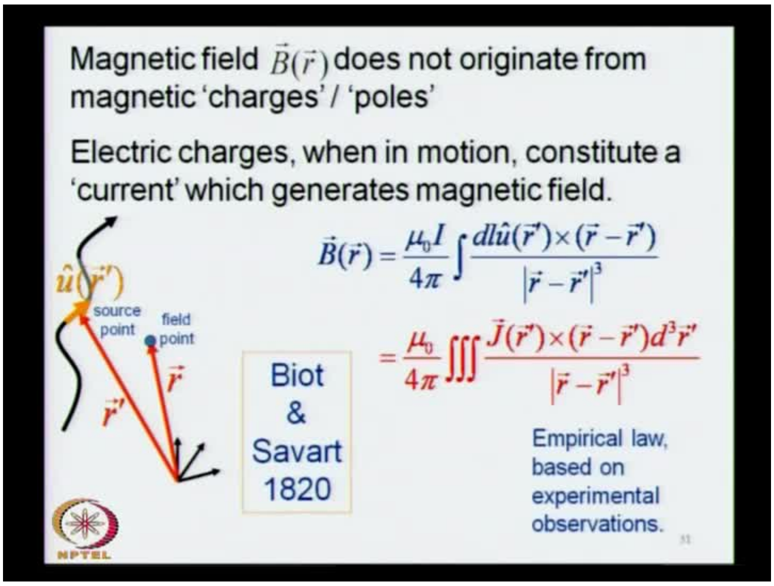

Because approximations are an important integral part of the development of almost every physical theory, but then one must know what these approximations. So, the regard in mathematics is very important and I do like a few things other than mathematics, rather than, I will not discovered any mathematics, but teaching mathematics is an important and fun part of teaching physics, but I like a few other things as well.

Special even it comes to things that I should not be eating. So, anyhow, let us also consider now, the magnetic fields and at these points we are talking about magnetic field as if it is different from an electric field, but let us do it for a while. Subsequently, we will learn the unity of electricity and magnetism, which is what Maxwell's theory really teaches us. So, this point we will consider that magnetic field, which does not quite originate from anything, which has any analogy with electrical charges, the analog is sometimes called as a magnetic pole, but then these magnetic poles really do not exist, because there are no isolated magnetic poles.

But magnetism results from the electric charges, when these are in motion. Now, this is classical electrodynamics mind you. In quantum mechanics, magnetism could originate from the spin angle of momentum for example; so these are different situations. So, we will keep ourselves within the domain of classical electrodynamics; wherein we would have magnetic fields generated by electric charges in motion, which is what an electric current is. An electric current is essentially a charge in motion and its generates a magnetic field and we will now discuss in this context, an empirical law very similar to the Coulomb's law very similar, but formulated for influences generated by flowing charges, by currents.

So, suppose, you have a current and there is a certain wire and no matter what the shape of the wire is and there is a current, which is flowing in this wire from bottom to the top as suggested by this arrow. So, you can consider the length of this wire as made up of tiny pieces and you have a tiny piece over here, from the bottom of this arrow to the top of this arrow and this little tiny arrow is in the direction of the current; it has got a certain direction, which will change from point to point on this wire. Because over here, this direction is over here, in the direction in which this pointer is suggesting; there is whereas over here, it is pointing toward the right with this screen; whereas over here it is pointing toward the left of this screen. So, this direction will change from point to point depending on how the wire is bent or wiggled or circled. And you think of this wire to be made up of infinitesimally tiny pieces, one after the other place in a series along which the charge moves, thereby generating the current through the wire.

Now, an empirical law is one which is based on experimental observations; it has no theoretical foundation that such a current generates a certain influence. So, the cause is here, the effect is around it that is the influence that I am referring to. The influence it generates is at the field point; the cause of this influence is the current flowing in the wire, which is where the source of this influences. So, each element, when it size becomes infinitesimally small is located about a certain point, which is called as a source point. So, the source point we denote by the primed vectors and the field point by the unprimed vectors. And the influence we are talking about, which is an empirical law is known as the Biot-Savart law, which was formulated in 1820.

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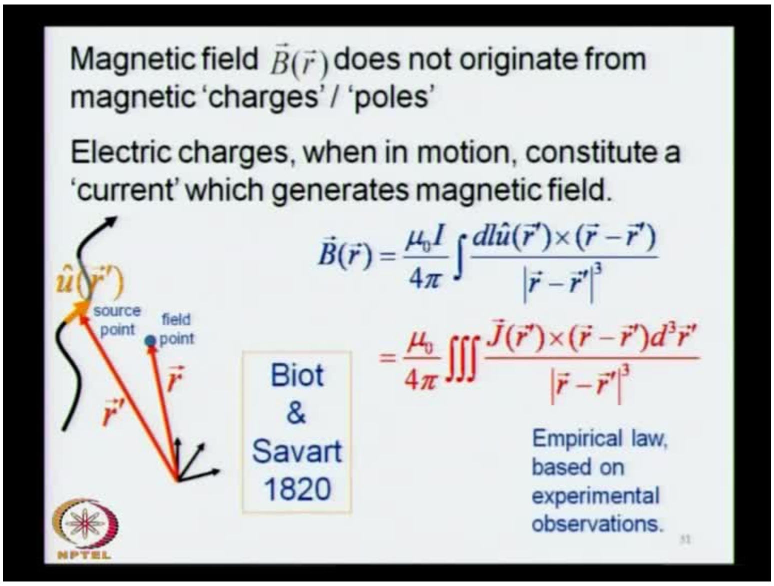

What is this law? That the existence of a current I, and it is this I, the existence of a current in this piece of element, generates a certain magnetic field which is the influence it generates and this influence is the magnetic field, at the field point B of r and this is equal to the properties of the sources. What is the property of the source? It depends on the current; it also depends on the properties of the medium. So, the medium in question over here is free space; so, it is a magnetic permeability of the free space, which appears in this expression, then there are constants likes 1 over 4 pi.

And how is it related to the geometry? It is generated by these elements over here, then to the next element, then to the next element and to the next element. So, it is given by a super position of the influences generated by all of these elements spread over the entire length of the wire; so you must integrate over the length of the wire.

So, here is an integral over the length of the wire and this element has got a length dl, which comes over here and it is given by a certain integrant, which is made up of the cross product of the direction u at r prime. This is the unit vector, which is pointed from left to right; it is also pointing right toward and toward the top, bottom to the top. So, those are the two orientations; you can always specify it in a given coordinate system by expressing it in any convenient coordinate system, Cartesian, cylindrical polar, spherical polar no matter what does a matter of detail.

But it depends on the cross product of this unit vector and then, here you have a term which has got an appearance just like the inverse square. Because you have got r minus r prime, which is r, is this vector, which is the field point and this is the source point; so r minus r prime is this direction from the source to the field.

So, this is the cube of that distance of the denominator and the vector r minus r prime in the numerator say, if you consider the direction alone, it would be like 1 over distance square. So, to that extent it is very similar to the Coulomb's law and you construct this integral and what you get out of it, is the magnetic field at the field point generated by this entire conductor. You must integrate over the entire loop; I have shown only a segment of this wire, it should of course at some point in somewhere and perhaps, close down and comeback to itself, it could have a source of you know, there should be a source current like, battery for example or power supply whatever.

Now, this electric current could also be a volume current not just going along a linear dimension, but it may have a certain cross section. There could be a beam of particles, which are in motion; you normally talk about the electric current as something in which a conventional positive charge would move, physically what moves inside? Metals are really the electrons, of course, in semiconductors there are also these absence of electrons which hope, which is the moment of the whole; so that is a matter of detail. But you have a certain conventional direction of an electric current and that is the one I am referring to, physically it could come from the motion of electrons, it could come from the motion of the absence of electrons, the way these holds up, and so on.

And these could take place over a volume, rather than, just a line and which case there would be a volume charge density, which is denoted by J at a source point r prime. So, the same expression, which is given as a single integral over the entire length; if it is coming from a cross section of charges; so if you have got charges spread out over here, and this whole thing is propagating; so you have got a certain surface of distribution of charges, which is travelling.

Then you have got a triple integral of this kind over a volume and it is essentially the same law, which is written over here with the difference that, this is in high dimensions in which you have got a cross section of charges, which is propagating. And this is how you calculate, how you determine the influence or magnetic influence at the field point due to a current source. So, this is the magnetic field given by the Biot-Savart law.

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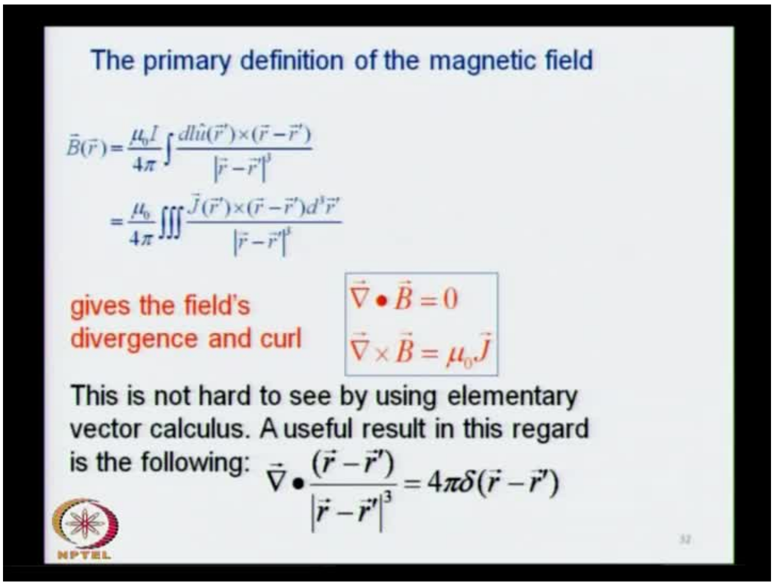

So, this is the primary definition of a magnetic field and if you just, use this definition and use the vector algebra, the vector calculus that we have already developed and you already know how to get the divergence of a vector and how to get the curl of a vector. You must keep track of what the gradient operator is? The differentiation in the gradient operator must take derivatives with respect to the field points not the source points; one can very easily make a mistake there.

So, if you avoid those mistakes find the gradient find the gradient operator, write it down explicitly, no matter what coordinate system you are using, and all you do is use the primary definition as a magnetic field, which is given by a law, which has got some resemblance to the Coulomb's law. The resemblance is essentially in terms of the fact that both are based on the principle of super position, but since this is due to a current which is extended in space, you must carry out the integration over the entire current loop, but it is essentially a principle of super position.

So, you go ahead and get the divergence of this quantity and it will turn out to be 0. I leave it as an exercise for you to work it out. So, write down this expression, which comes from Biot-Savart law, get this divergence you will find that is 0, and gets its curve and you will find it that it’s exactly mu 0 J. The process of getting the divergence and curl of magnetic field from the Biot-Savart law does not involve any new law of electrodynamics; it is just using vector calculus. You know how to get the divergence, you know how to get a vector, how to get the curl of a vector and we have already discuss that in previous units; in fact, this was part of the reason, part of the motivation that we did it. So, all you do is to get the divergence of the magnetic field, get the curl and you already have the Maxwell’s equations. And just as once, you have Newton’s equations, you can solve any problem in classical mechanics; once you have Maxwell’s equations, you can solve any problems in electromagnetic theory, you do have to add the Lorentz force law and a few other things and the boundary conditions for example that is the matter of small detail.

So, this is coming straight out of vector calculus and some of the properties which you will find, rather useful in this context is this expression, that if you take the divergence of this vector, it will generate at delta function. I have already introduced the delta function in the previous class; so some of these things are left as exercises. But this is something that one would do; one would actually prove this, if this were to be a full course on electromagnetic theory, which it is not.

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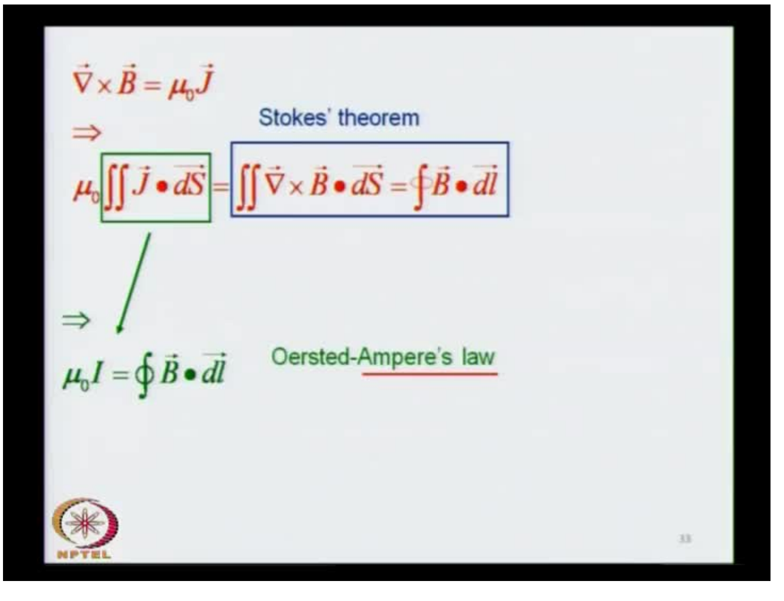

Now, you have got the curl of B, which is given by mu 0 J. Now, let us take the surface integral of J dot Ds, but the mu 0 J is nothing but the curl of B, this comes from the Biot-

Savart's law; no new physics added to it, no new law added to that. So, you get the surface integral of the curl of B given by this expression, but from the Stokes theorem, we know that the surface integral of the curl of the vector is given by the circulation of that vector about the close loop. But there is a relationship, which the direction of orientation of the surface must maintain with the direction in which the circulation is evaluated; mind you, it is a right hand screw rule convention that we had discussed. Because if you carry out that circulation, which is the line integral of B dot dl, that line integral can be carried over a close path by traversing a that close path along one way or in the opposite way.

So, then, you will obviously get an opposite sign; but if you do not make that mistake, if you carry out that circulation, if you determine that circulation using the right hand screw rule, so that if you turn a right hand screw in the direction along which the circulation is determined, then the forward motion of that right hand screw will be in the direction of the infinitesimal vector area, which goes into the left hand side of the Stokes theorem.

So, these two directions, direction of dS and direction of dl are connected by the right hand screw convention. So, we have discussed this earlier; this is just a gentle reminder that you do not forget that. And now, having exploited the Stokes theorem, we can now write this, J dot dS, this will give you the integration. This integration will generate the net current, because this is the current density which is integrated out; you get the net current. So, you get mu 0 times the net current equal to the circulation of B dot dl and this is known as the Oersted-Ampere's law.

So, this came from, this was recognized by Oersted, who did various experiments. It was recognized by Ampere also and this was obtained, this was this rule was formulated on the basis of empirical observations carried out by Oersted and Ampere; so this is known as the Oersted Ampere's law.

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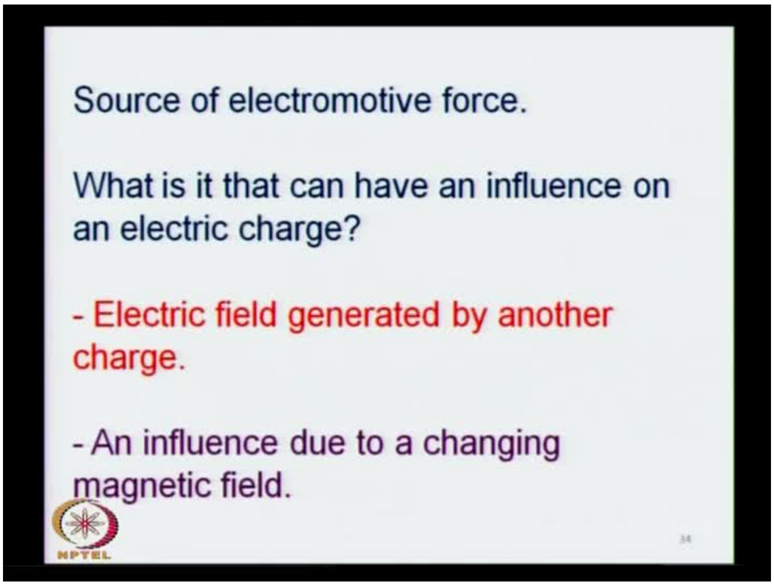

Now, one of the things what are the things that can have an influence on an electric charge? One charge The existences of a charge, which will generate some sort of an influence. So, this will be expressed in terms of an electric field, which is generated by another charge. But you can also have an influence generated by a changing magnetic field; if you change the magnetic field in a certain region, you can generate an influence on an electric charge. So, let us discuss this idea little bit further.

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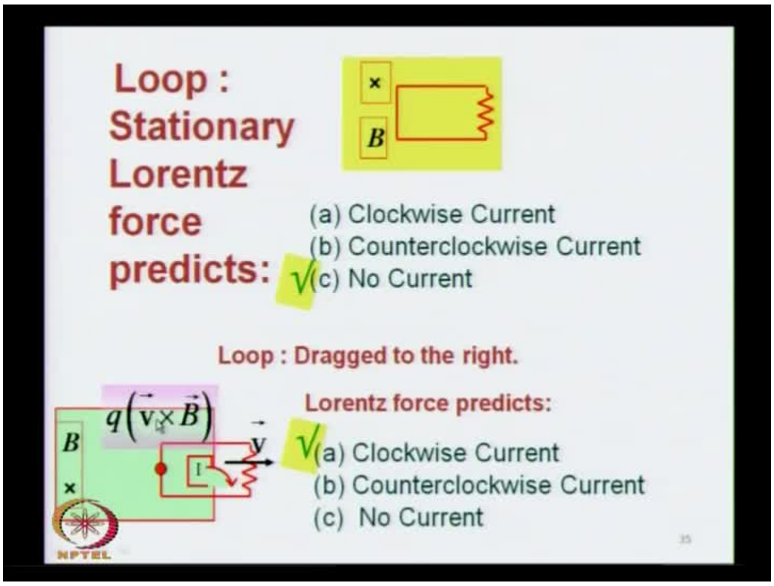

So, we take this certain loop, this is an electric conducting material, this is a loop, which is placed in a region in which there exist some magnetic field, which is a vector field and this magnetic field, in this figure that I am using is pointed into the plane of the screen; it is orthogonal to the plane of the screen and into the plane of the screen; so it is shown by this cross which is like the tail of an arrow, which you would see if you were to have short an arrow into this screen, do not do that.

Now, let us ask this question, that if you have a certain region of space and which you have got a magnetic field, which is orthogonal to the plane of the figure into the plane of the figure that is the direction, and in this field you have an electric loop a conducting loop, which is l static, it is not moving, then will there be any current in the loop? If so, will the current to be clockwise or will it be anticlockwise, and the directions clock wise and anti-clockwise are with reference to, how this loop appears to you; so, will there be any current at all? No, so that is the correct answer.

Now, let us do the same thing, but this time, we drag the loop to the right, we physically pull it from left to right. And now, let us ask the same question what will be the direction of the current in the loop? If there will be any current in the electric, if there will be any electric current at all in the loop?

In the first example that we consider, we already agreed that there will be no current, but this time the difference is that we are dragging the loop from the left to the right; so will there be a current now? Yes, and what will be the direction of the current? It will be clockwise and why will it be clockwise? Clockwise is correct. If you were to think of a positive charge over here, then this charge would gain a velocity from left to the right. The reason is, this charge is bound to the wire and you are physically dragging the wire from left to the right and other than Maxwell's equations, I have to I had mentioned earlier, that you must add the Lorentz force.

So, the Lorentz force will generate this q into v cross B force on discharge; so now, the velocity of discharge is to the right, the magnetic field is into the plane of this screen. So, v cross B would be a force, which is pointing up ward in this figure and therefore, discharge will be accelerated from the bottom to the top setting up a current in the clockwise direction in this loop.

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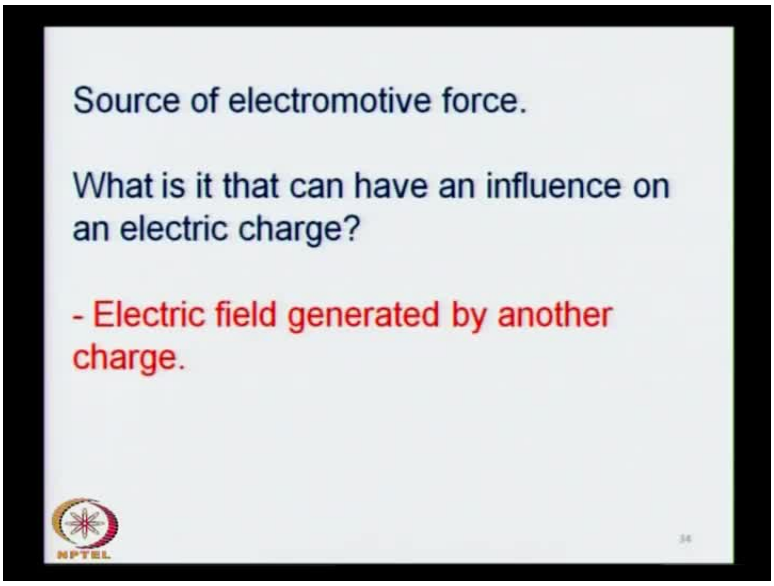

So, the source of current in this case is the Lorentz force, which is v cross B and is a result of this v cross B force, this is the question I raised earlier. Let me go back to that slide one more time, because this really is extremely important consideration. What is it that can have an influence on an electric charge? It is not the electric field generated by another charge in this case, of course, if you have another electric charge, you can generate that influence. But here, you are making a charge move, not by bringing in another charge from somewhere and making it influence discharge generating a force on it as a result of which discharge would be accelerated, but by dragging this loop to the right; so this is the origin of the influence you have generated.

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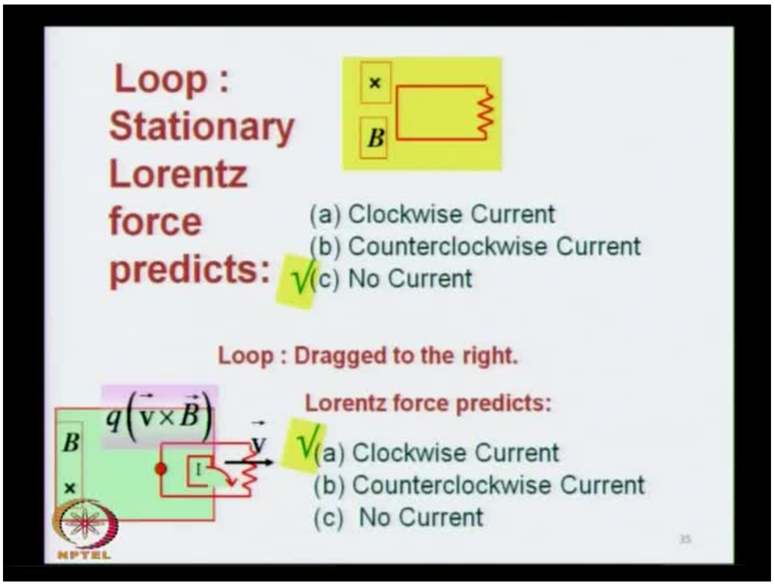

So, this is I putting it in some sort of and influence, which you can say is due to some electromotive force which is making it move.

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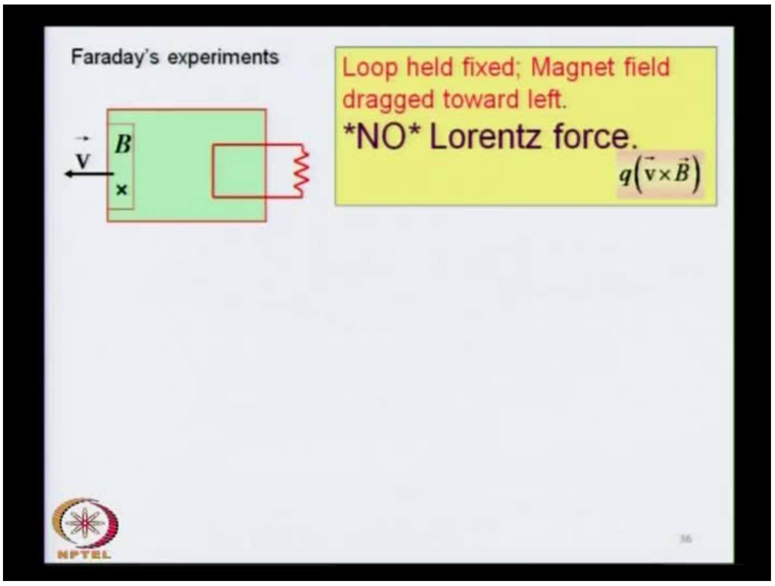

So, now, you will have a clockwise current, if you physically drag this current loop to the right. Now, let us do this experiment again, this time you hold the loop constant, but what you do is drag the magnetic field itself to the left, how do you do that? If you are generating that magnetic field by keeping this thing in a north pole and a south pole or between two poles of a horseshoe magnet and you take that horseshoe magnet and move it to the left.

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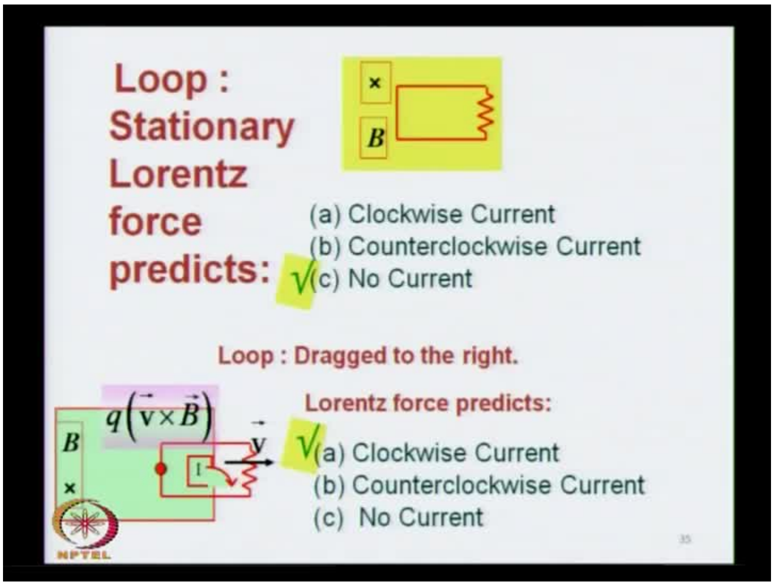

So, what you are moving now? Not the loop, but the magnetic field and if you are not dragging the current loop, this charge which had a quiet of velocity v, why did it acquire that velocity v? Because this loop was drag to the right, if this loop were not drag to the right it would not have any velocity. So, if that loop is static, there cannot be any v cross beta, because the velocity of the charge is 0.

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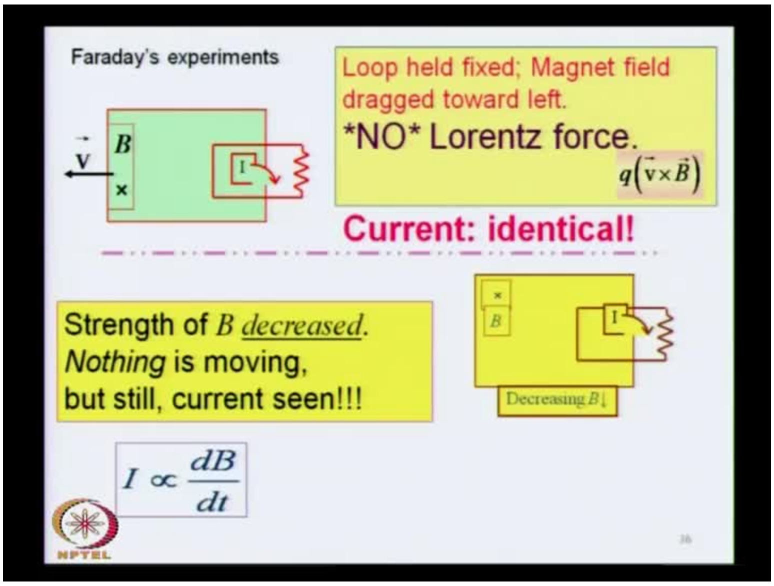

Now, there cannot be any Lorentz force; watch out this is very critical. You will nevertheless have a similar current, you have exactly the same current in the clockwise direction, but it is not because the charge is moved physically to the right, the charge velocity 0, you have not moved, it is stationary.

Now, you move neither you do not move the current loop nor you do not move the magnetic field, but what you do is change the strength of the magnetic field; you can do that. For example, you can generate the magnetic field by some, you know changing current and that would change the magnetic field. So, if it changes the magnetic field, then you have got a current, which is proportional to dB by dt and as a result of this change in the magnetic field once again, you could have a current set up in the loop.

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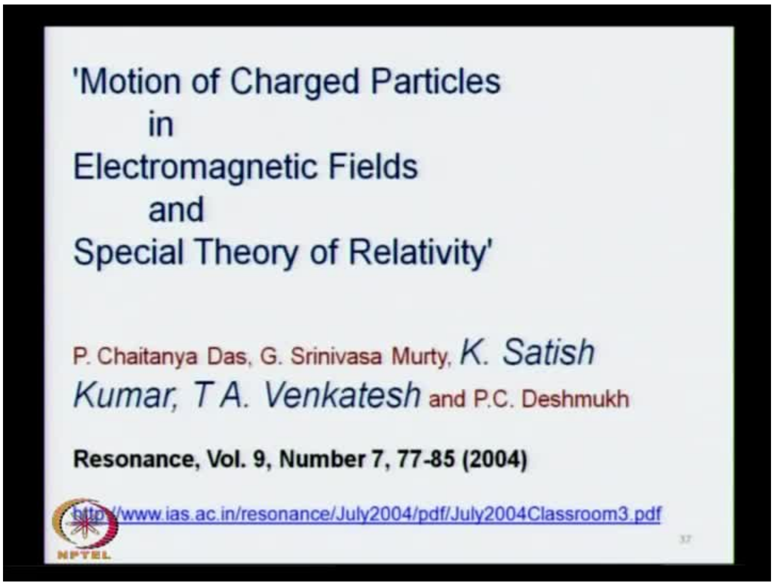

And I will like to invite you to read this article, which has been written by two, rather four of my former students in particular by Satish and Venkatesh, who wrote very nice software to demonstrate some very fascinating experiments in which you have charge particles in electromagnetic fields and you will find this is very closely connected to the special theory of relativity which is what I am getting down.

And this articles has been published resonance in several years ago in 2004 and the program they wrote was rather old in fact, they did it many years ago, but never published it and then Chaitanya das and Srinivasa Murthy read it that work and developed a new software and they have also contributed to this and we are going to demonstrate this software in the next class.

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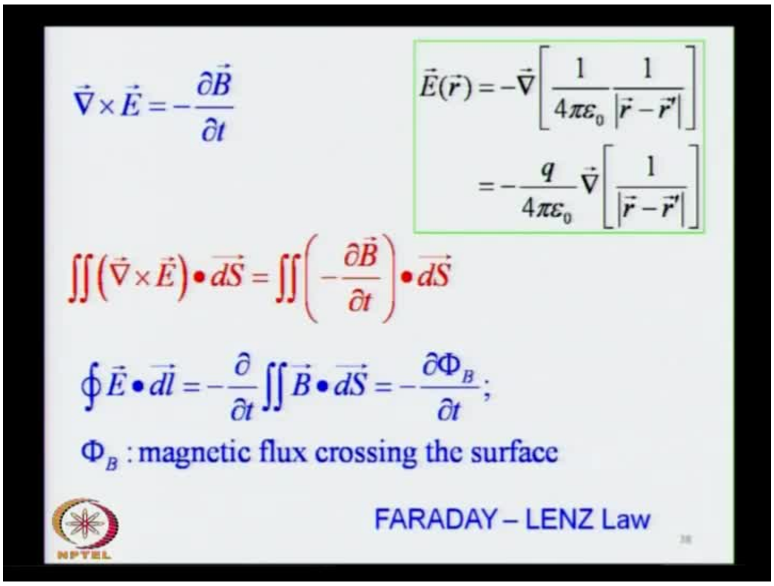

But that is something, so I will come into little bit more about this in the next class. But for the time being, let me once again invoke this idea, that you have got the electric field which is expressed as a gradient of the potential; this idea we are quite familiar with already. But now, as a result of the changing magnetic field, we can get a curl of the E. So, this is a new kind of a source of generating and influence on electrical charges.

So, if you change this relationship and this is this also appears as an important element in Maxwell’s formulation of electromagnetic theory, if you now take this relation and construct the surface integrals of both sides of this relationship, the curl of E is minus del B by del t it is coming because of the rate of change of the magnetic field, which can have a similar influence certain charges and these two surface integrals will therefore, become equal, but then through the Stokes theorem the left hand side which is the curl of the vector, you take the surface integral of the curl of the vector which we know from Stokes theorem is equal to the circulation of that vector about a close path once again, the direction of the circulation is connected with the direction of the surface element through the right hand screw rule and what you essentially have is the surface integral of this time derivative.

But this integration is over space, this is differentiation over time and you consider these two be independent of each other. So, you can take this as the time derivative of the surface integral B dot Ds, which you identify as the magnetic flux crossing that area.

So, you got this circulation E dot dl to be given by minus del by del t of phi B, which is the magnetic flux. This is known as the Faraday-Lenz law and this was arrived at by Faraday and Lenz from the observations. They did a variety of experiments of the kind that I suggested, that you have currents and you have got magnetic fields, you move one you change the other and use some experiments of this kind; they did a number of experiments of this kind and they actually arrived at this law that the circulation of E dot dl is equal to the negative time derivative of the magnetic flux.

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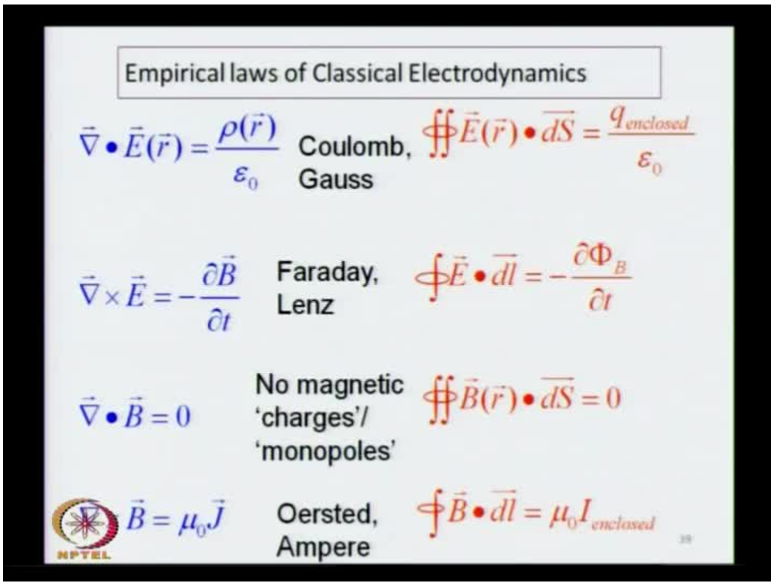

So, let us compile what we have learnt so far and these are all empirical laws. So, you have got the Coulomb's law corresponding to which using simple geometry, we got the Gauss's law. So, this is the coulomb Gauss law, then we have the Faraday-Lenz law, then we had the fact from the Biot-Savart law, we found that the divergence of the and the curl of the magnetic field is given by these two relationships and whenever the divergence is 0, the field is called solenoidal and the magnetic field in this is solenoidal, which is the statement of the fact that there are no monopoles, because there is no point in space from where you can have a divergence either positive or negative.

So, there is no analog of positive and negative charges in magnetism and its integral expression is B dot Ds. So, the expressions on the right hand side written in this red are completely equivalent to the expressions on the left written in blue. These are the points forms, these are the globally forms, these are differential forms in which you take the key mathematical operator is the differential operator, which is sitting in this gradient and over here this formulation is called as the integral form. Because these are global forms in which you have to carry out the integrations, but mathematically they are completely equivalent.

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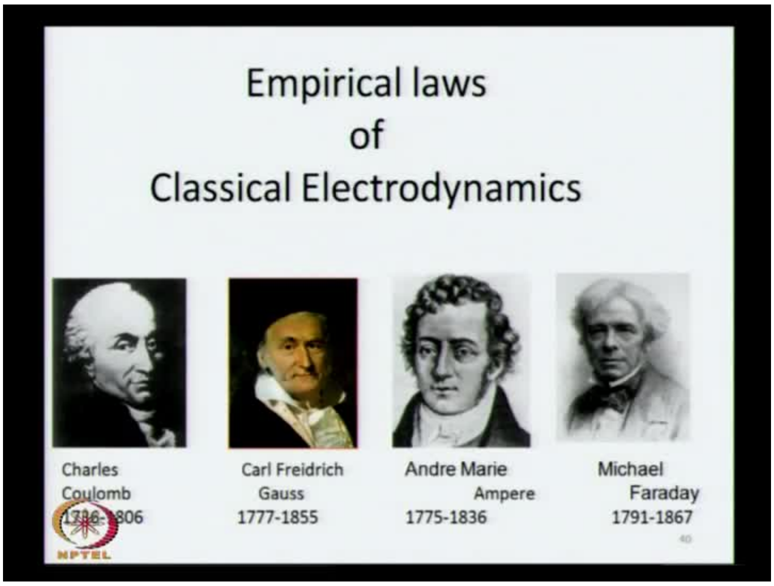

So, these are all empirical laws of classical electrodynamics or they are due to Charles Coulomb, Gauss, Andre Ampere and Michel Faraday, brilliant experimentalist; the basis for these laws is completely experimental, there is no theory.

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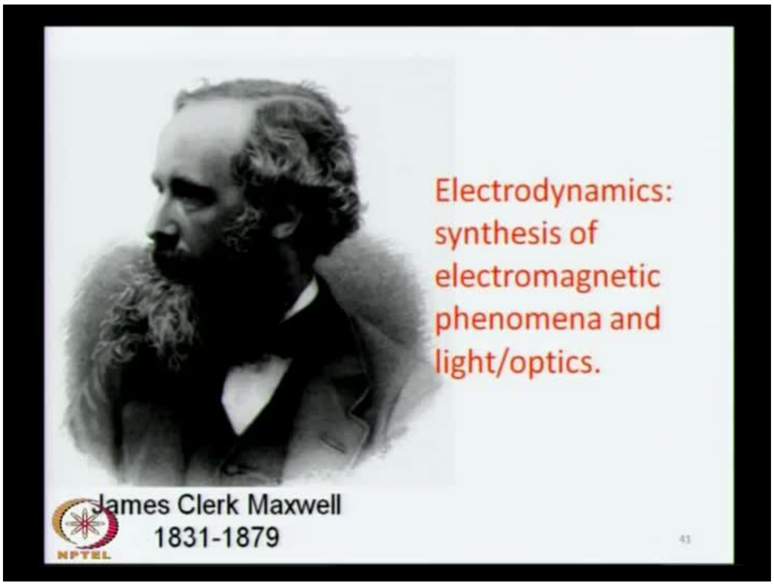

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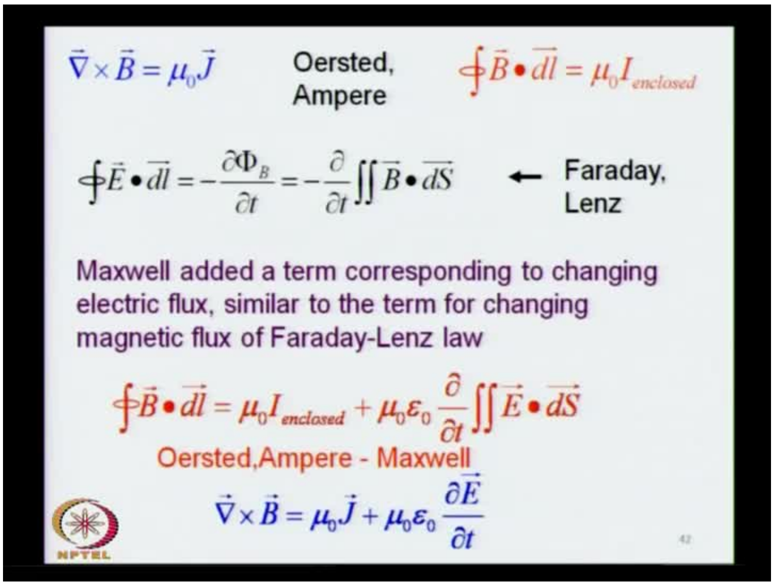

Now, what James Clerk Maxwell did, was to synthesis the electric and the magnetic phenomena with each other and also with light; so we will see how that happens. So, let us consider the Oersted Ampere law and the Faraday Lenze law; what Maxwell did was to added term corresponding to changing electric flux, that over here you have got changing magnetic flux. Nobody had ever considered a term involving changing electric flux, Maxwell did that. What Maxwell did was to take the Faraday-Lenz law and added a term, similar to the term that you find in the Faraday-Lenz law and he stuck it in this Oersted Ampere’s law.

So, he draw his inspiration from here; so this circulation B dot dl, which is equal to mu 0 I, he added a term corresponding to the time derivative of the electric flux. How did he get it? Intuition? Possibly, yes. Symmetry, he started recognizing this symmetry in the electric phenomena and the magnetic phenomena; he started recognizing in fact, the unity of the two; he came pretty close, but not quite to discover this special theory of relativity, that had to wait Einstein and that is an extremely important and an integral part of our story and classical electrodynamics.

So, this is the term that Maxwell added and now, we will call this as the Oersted, Ampere - Maxwell equation. This was not based directly on any experimental observations, but on what Maxwell proposed as a theory of electromagnetic phenomena, which integrated electric phenomena with magnetic phenomena and with light and it went in a few steps, we will see those steps very quickly.

So, this is the integral form and the corresponding differential form which you can get by manipulating these terms using the vector calculus you already know, you do not use anything other than the divergence theorem or the Stokes theorems, the Gauss’s divergence theorem or the Stokes theorem and what you essentially get is that the curl of B is given by mu 0 J plus mu 0 epsilon 0 del E by del t. So, this term, which is similar a Faraday-Lenz term, this was added by Maxwell.

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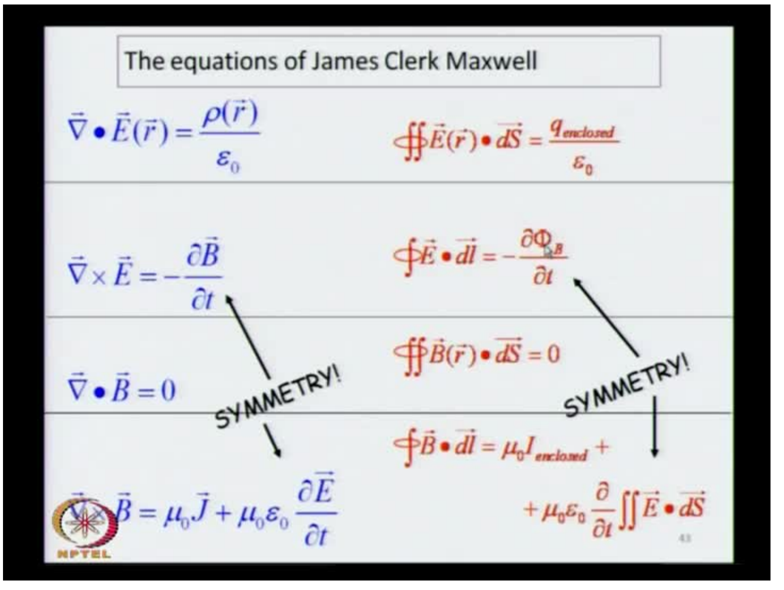

So, now, we have Maxwell's equations already, you have got the divergence of E equal to rho 1 epsilon 0, you have got the curl of E which goes is minus del B by del t, you get the divergence of B is equal to 0 and now, this Ampere, Oerstead- Ampere law is modified to include this term and when you do that, you already have Maxwell's equations.

Look at the symmetry of these two terms, they have got exactly the same form; one involves the time derivative of the magnetic field, the other has got the time derivative of the electric field; this symmetry is crucial, it has got deep consequences.

The corresponding integral forms, which you can get once again by using the theorems in vector calculus, but we now, I have considerable acquaints with, give you Maxwell's equations in the integral form they are essentially the same and once again you find that there is a symmetry. But now, here, you talk about the magnetic flux and here you have the electric flux, but once again you have essentially symmetry between electricity and magnetism.

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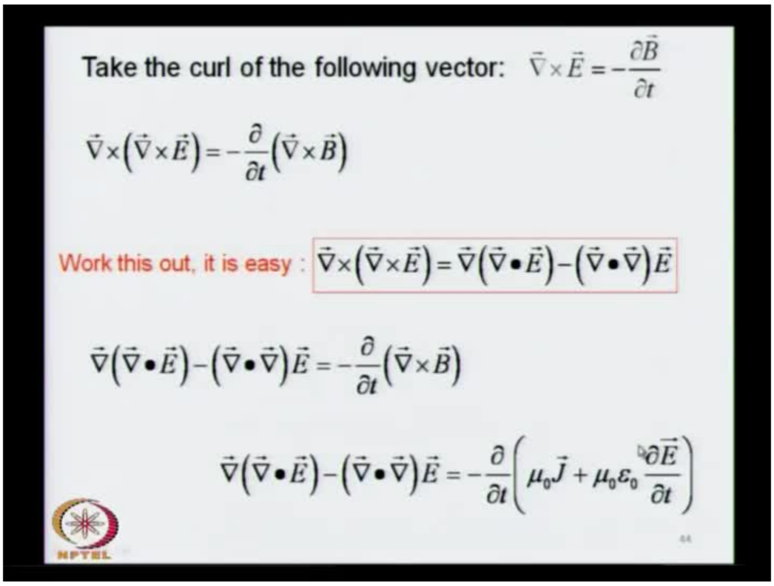

Now, let us play with this a little bit more. You have this expression curl of E equal minus del B by del t; now, let us take the curl of this vector, the curl of a vector is a vector and you can take its curl; so let us take the curl of this quantity; so you take the curl of curl of E.

So, again you can swap the positions of the gradient operator with the time derivative operator. So, you get the negative sign, you preserve and you take the time derivative of the curl of B and if you work this out term by term again, which is a matter of doing elementary vector calculus.

And now, we have used the ideas of divergence and curl in discussing fluid mechanics, the Bernoulli's principle in setting up the equation of motion for a fluid, the equation of continuity. So, we have done all these; so we have considerable experience with using vector calculus; we taking the curl of a vector, we taking the divergence of a vector.

So, I will not go through the intermediate steps, I leave that as an exercise and you find that if you take that curl of the curl of E, the result is the gradient of the divergence of E minus this Laplacian the del dot del operating on E. So, here is your result, the curl of E is nothing but minus del B by del t. So, you have this result we are all together and now, from the earlier expression, we have the curl of B to be given by mu 0 J plus this term which Maxwell added.

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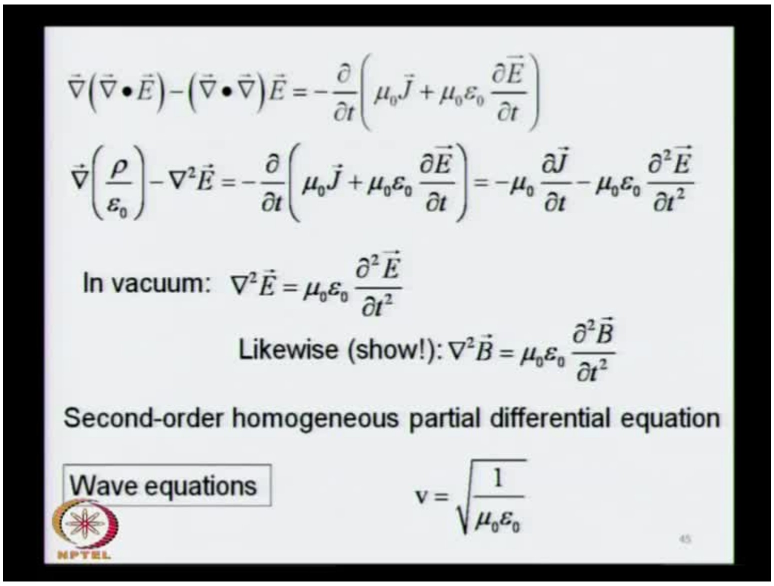

So, this is the term, which was added Maxwell out of symmetry. So, we now, have the complete expression for this. So, let us take this equation to the top of the next slide which is over here, so that we do not lose any term. And now, we know that the divergence of E is rho over epsilon 0 this is the Gauss’s law as we know it. This is the Laplacian of E on the left hand side, on the right hand side you have got mu 0 times del J by del t and then, the second term is mu 0 epsilon 0, but you have got the first time derivative of E operated upon by another time derivative operator del by del t. So, you have got the second derivative of E with respect to time.

So, what you have? If you specialize this equation to vacuum in which there is no charge density and no current; so these two terms drop out, you get del square E coming from here to be equal to mu 0 epsilon 0 del two E by del t 2, because this minus sign cancels this minus sign and rho and J are both 0. So, essentially, what you have is the wave equation.

Now, I have arrived at this for the electric field you can play with the Maxwell’s equation, do exactly the same kind of vector calculus which is very elementary and you will get as very similar expression for del square B, which is equal to mu 0 epsilon 0 times del 2 B by del t 2

Now, this is the second order homogeneous partial differential equation, which connects the space derivatives of the electric and magnetic fields with the time derivates of the electric and magnetic fields. Essentially it is a wave equation and the wave propagates at a certain speed, which depends on properties of vacuum, the speed of propagation and this is nothing to do with electromagnetic theory; it has to do with wave theory, no matter what wave you are talking about.

So, these two equations they represent waves and these waves propagate at a speed which is determined completely by properties of vacuum and by nothing else, and this is amazing. Because whenever you talk about any speed, you always ask speed with respect to what, with respect to which observer. And here, no matter which observer you are talking about, the speed is the same because it is determined by properties of vacuum mu 0 and epsilon 0 is square root of 1 over mu 0 epsilon 0. So, if you have got one observer who is here and another observer, who is moving with respect to this observer at a constant velocity, then the speed of an object like, if this were a car or some other object and if this were in motion, then the speed of this car relative to this observer and with reference to this observer would be different. Do we all agree that? The speed of a car measured to with respect to one observer who is stationary, who is standing on the road and another observer, who is moving at a constant velocity, there is no acceleration of the second observer and in the absence of any acceleration of the second observer, then no Pseudo forces to talk about both are in inertial frames of references.

Because the frame of reference of the second observer is also in inertial frame of reference, because it is moving at a constant velocity with respect to another inertial frame and it does not matter, what the relative speeds are. Because the property of this wave is that this wave propagates at a velocity which is determined completely by properties of vacuum and nothing else, it is square root of 1 over mu 0 epsilon 0. Now, this is an amazing result.

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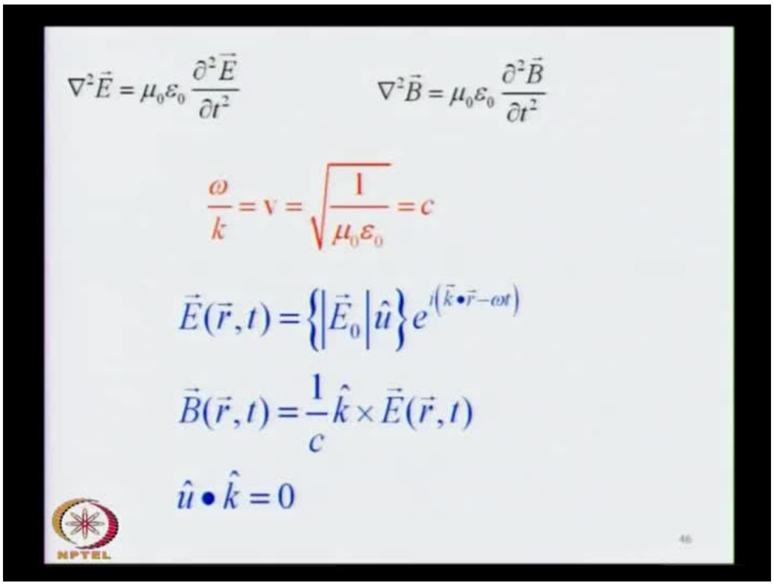

So, here, let us look at this velocity, further this square root of 1 over mu 0 epsilon 0 which is what gives you the speed of this is given by properties of vacuum. This is what we call as the speed of light and if you solve these wave equations and find you the solution for E and B these solutions are given by this and you immediately see that the propagation direction k is orthogonal to E and B.

So, if you just study these three relations, which comes straight out of elementary vector calculus, you find that the electric field and the magnetic field must be orthogonal to each other and the propagation of the wave is orthogonal to the electromagnetic field which is why you refer to it as a transverse weigh.

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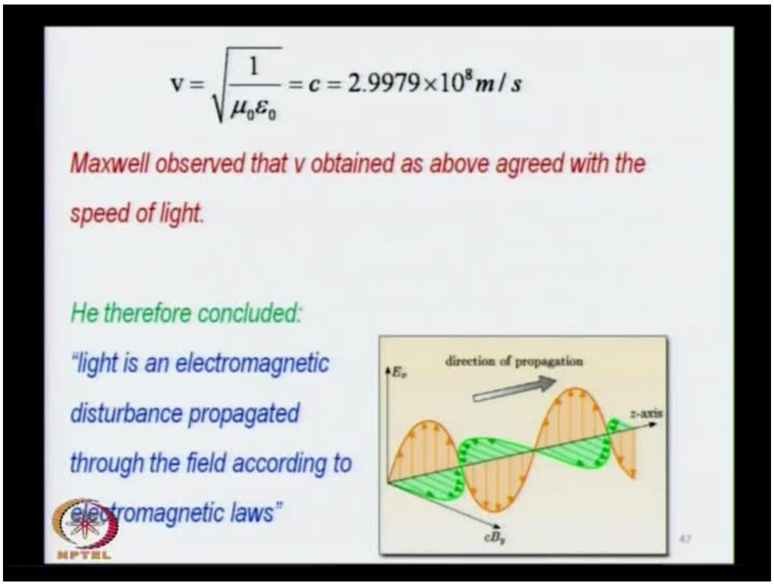

What is the value of the weight? This is very nearly equal to 3 into 10 to the power 8 meters per second and what Maxwell observed is that, this speed agrees, rather well with the speed of light, as was known at the time. Today's measurement give you very precise value, but the values which were available at Maxwell time agreed with this and that led Maxwell to conclude that light is an electromagnetic disturbance, which propagates through the field according to electromagnetic laws, this is how ended up synthesizing electricity magnetism and light.

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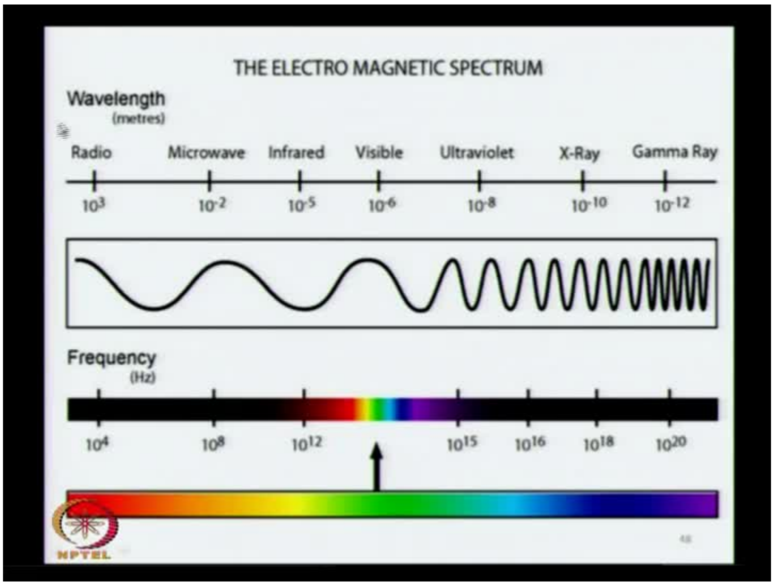

And he converges on the view point that the electromagnetic field is a transverse, has got transverse properties and it propagates at the speed of light. So, light is no different from the electromagnetic phenomenon and this is the electromagnetic spectrum that we now know and you are similar with this terminology, then you have got the long wavelengths the radio waves, then you have the microwaves infrared visible and they become shorter and shorter and they get into the gamma rays and the corresponding frequencies of course one increasing, because the product of the frequency and the wavelength is what gives you the speed of the wave, which is speed of light.

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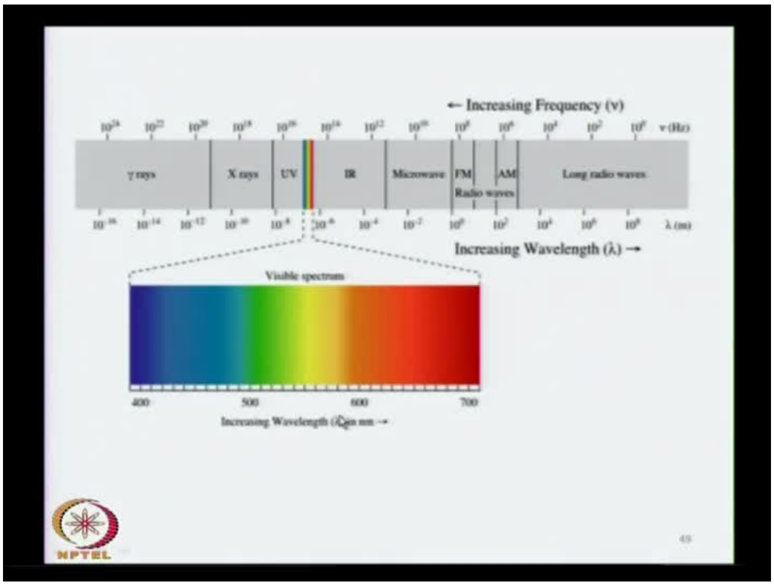

This is the visible range, which is the small part of the electromagnetic spectrum and we are able to see colors from violet to red from the shortest wavelength to the longest one’s. This is all that we can see, one of you may be able to see one or two extra wavelengths on either side of the spectrum or may be more, but there is not particularly important.

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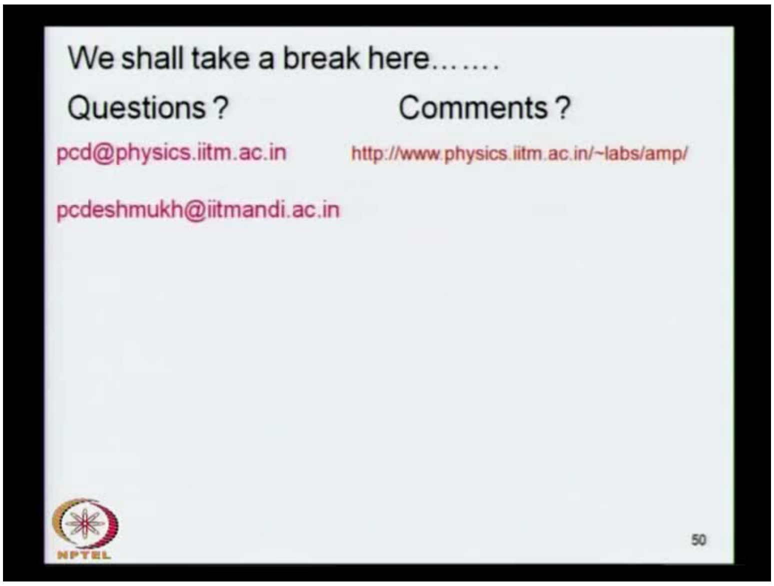

So, at this point we will take a break and in the next class, we will recognize these connections. I believe, you already have a hint that the special theory of relativity will play a huge role, because this electromagnetic wave is now travelling at a speed which is determined completely by the properties of vacuum.

And in fact, the provocation for Einstein to arrive at the special theory of relativity, provocation is perhaps, not a very nice work, the inspiration if you like. It came from the symmetry in the electromagnetic phenomena, it came from this amazing symmetry of the experiment that we consider, that you have a magnetic field, you have got a conductor; you move the conductor, you have got a current certain; you do not move the conduct this conducting element, but move the magnetic field even then you have the same effect.

But the sources of these effects seems to be so different and they come from an idea of symmetry, which Maxwell recognized. In fact, it was this symmetry which inspired Maxwell to plug in that extra term del E by del t similar to del B by del t he just plugged it in the formalism.

So, in the next class, we will discuss this further and we will see how the loss of electrodynamics is intimately connected to the special theory of relativity with STR. So, you have already have a hint, you already saw that the electromagnetic phenomenon, the electromagnetic wave is propagating at the speed of light irrespective of which observer in which inertial frame is measuring it.

So, if you have got one observer, who is stationary and another observer, who is moving at a constant velocity with respect to the first observer. Just imagine yourself doing this, if you are measuring the speed of a car, you are on a road, you are in traffic and there is a car in front you and you measure the speed of the car in front of you with respect to yourself and you measure the speed and your friend is also measuring the speed of the same car, but he standing on the road and not moving in a any car.

So, the sum and difference of the relative velocities of you and your friend will come into play in the estimation of the velocity of the target car those will be different. But here light is propagating at a speed, which is determined not by which observer you are talking about in which frame of reference, but only by properties of vacuum.

So, this goes into the special theory of relativity; we discuss this at some length in unit 6 and you will now be able to connect all the pieces together and see how it connects to the laws of electrodynamics and that will be in our next class. So, until then good bye; if there are any questions I will be happy to take otherwise, send me an email.