Select/Special Topic in Classical Mechanics
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Module No. # 10
Lecture No. # 34
Classical Electrodynamics (iv)

Greetings, we will in this class conclude the discussion on classical electrodynamics which is a brief overview of Maxwell's equations and how it connects to the special theory of relativity. What we will be discussing here is, examine the trajectories of charged particles in electromagnetic fields, but as seen by 2 different observers. One who is in this frame which is a black frame and the other who is in this blue frame (Refer Slide Time: 00:38) which is moving at a constant velocity with respect to the black frame.

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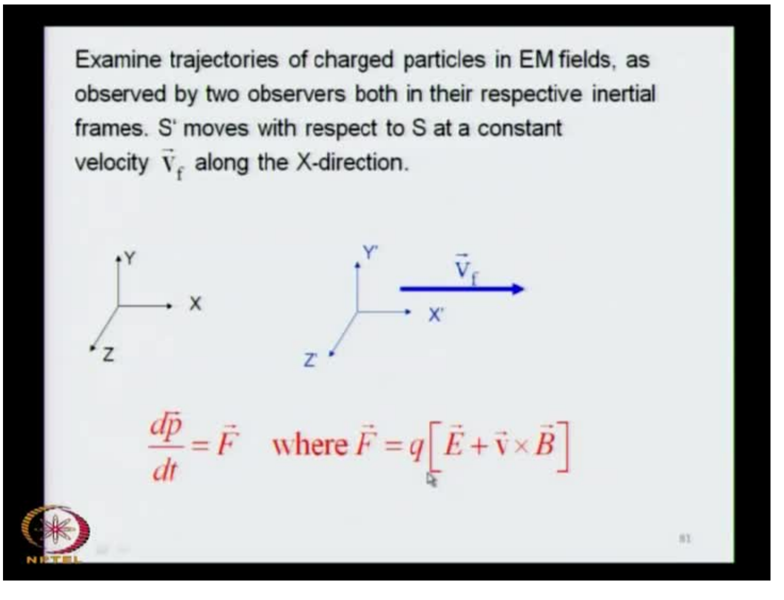

I want to emphasize that this velocity is constant. So, both are inertial frames of references; there are no pseudo forces which are involved. But, what is electric field and magnetic field for one observer is not the same electromagnetic field for the other observer and that is the new thing that we are going to learn.

The equation of motion of course, is given over here (Refer Slide Time: 01:18) that the rate of change of momentum is force; so it is a completely classical Newtonian idea between cause and effect. The force that we are talking about is the electromagnetic force namely, the Lorentz force and this is something that you need over and above Maxwell's equations, other than the boundary conditions that are required to solve problems of charged particle dynamics in electromagnetic fields.

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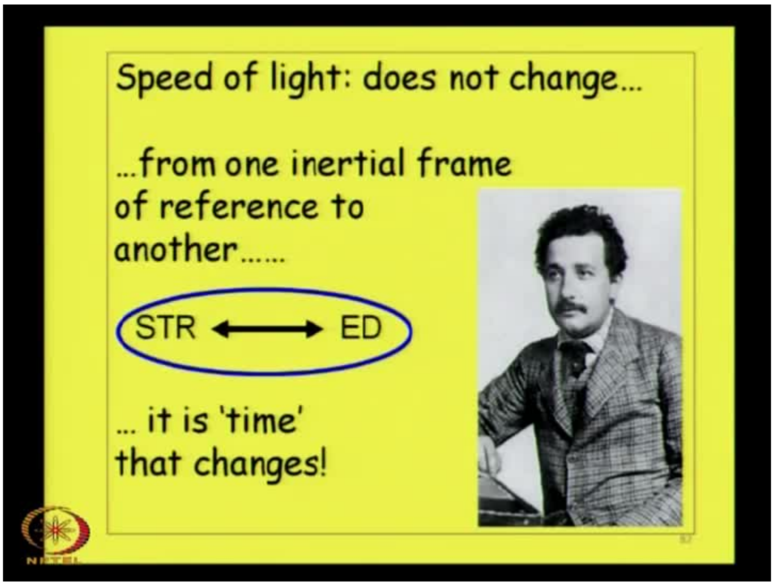

The governing idea is that the speed of light does not change from one inertial frame to another. What changes is time and in fact also, space intervals; so these are the governing ideas. Einstein was a very young person when he formulated this in 1905; he must have been about 26 years old I would think so, something like that and this connects the special theory of relativity with the with laws of electrodynamics.

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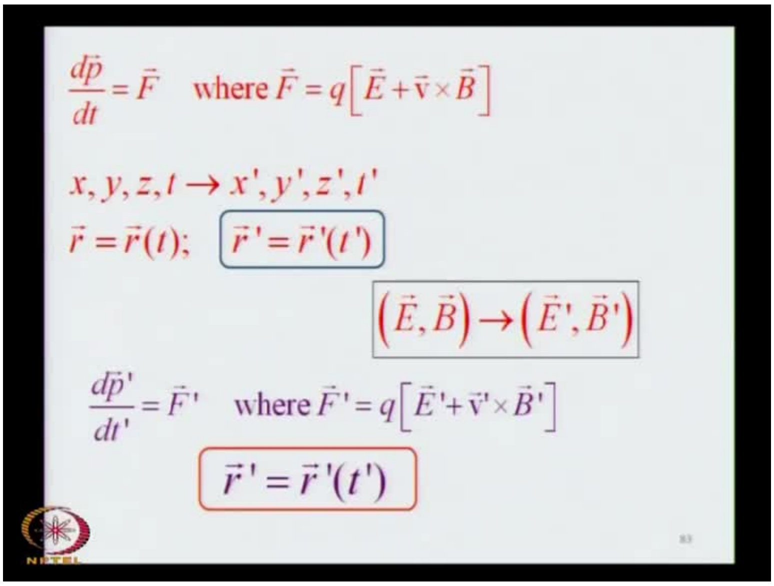

So, like I mentioned in the previous class, as a result of Lorentz transformations x y z and t which is time for the observer in the unprimed frame of reference, must transform to x prime, y prime, z prime, t prime. These transformations are essentially the Lorentz transformations that we are talking about, not Galilean and therefore, t prime is different from t. The trajectory of the particle is given by how the function the position vector changes as the function of time. Whereas, for the observer in the primed frame the trajectory will be given by how the position vector in the primed frame changes as the function of time in the primed frame which is t prime which being different from t.

The connection comes from the fact that what is E and B for the first observer is a different mix of E and B to the second observer (Refer Slide Time: 03:35), which is E prime and B prime which is not the same E prime; is not equal to E and B prime; is not equal to B; that is the essential idea and therefore, the observer in the second frame of reference can get his trajectory by setting up an equation of motion in the primed frame which is d p prime by d t prime. Notice that you have the primed symbols appearing over here because the times are different and then, automatically every other parameter also has got a different meaning.

Not only that because, E and B transform to E prime and B prime, the force is given by F prime rather than F because, E is no longer E, it is E prime; v is no longer v, it is v prime;

B is no longer B, it is B prime; but then, in the primed frame you are still talking about the same electromagnetic interaction namely the Lorentz force; there is no pseudo forces.

The solution to this will be given by the position vector in the primed frame as a function of t prime and now If you solve this equation of motion in the unprimed frame and carry out transformations to the primed frame by Lorentz transformations and then plot r prime as a function of t prime, you will get the trajectory in the primed frame. The other way of getting the trajectory in the primed frame is to set up the equation of motion in the primed frame and express the solution r prime as a function of t prime in the primed frame. Now, these are again completely 2 different methods.

The first method is a Lorentz transformations of the coordinates. The second method involves the relativistic transformations of the electromagnetic field from E B in 1 frame to E prime B prime in the other and then setting up a different equation of motion. If we have done this correctly, then the solution obtained using this method must agree with the solution obtained using the other method. The two should generate the same curves in their respective spaces.

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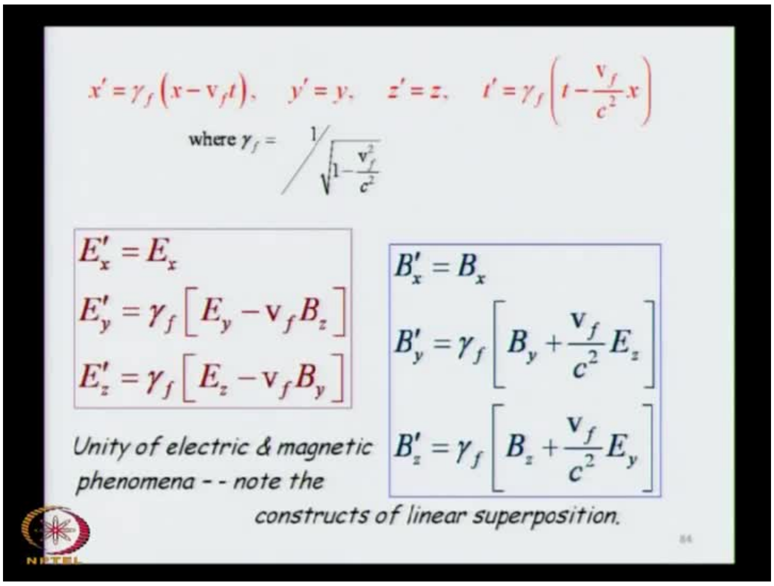

So, this is the test that r prime is a function of t prime obtained using Lorentz transformations of the coordinates x y z t or the relativistic transformations of E B to E prime B prime and solving a new equation of motion generate, you know, Congruent space curves giving the trajectories. So, here we have the Lorentz transformations which we have discussed already in unit 6 and these are the transformations of the Electromagnetic field.

So, what is E x is E x prime to the other; but, what is E y is not E y prime to the other. E y prime is made up of a mix of E y and B z and then there is a scaling factor which comes from this gamma (Refer Slide Time: 07:16). Likewise, E z prime is not the same as E z, but it is a mix of E z and B y. The mixing coefficient is this velocity and then on top of it, there is a scaling factor which is this gamma. So, E prime is not the same as E; rather it involves superposition of components of E with components of B. Let us look at how the magnetic field transforms.

Now the magnetic field: what is B x B y B z for the observer in the unprimed frame of reference, is a different field given by B x prime B y prime B z prime. These 3 components give you the magnetic field in the primed frame and B x prime is equal to B x all right, but B y prime is not equal to B y; it is a mix of B y and E z and B z prime is a mix of B z and E y.

These transformation equations which tell you how to go to E prime B prime from E and B again are statements of the unity of the electromagnetic phenomenon you are actually constructing the superposition adding pieces. One piece is E y the other piece is B z and you mix E y with B z and get E y prime.

Now when do you mix quantities? Only when they are the same; otherwise, you cannot. Of course, the coefficients velocity and everything are such that they take care of the units and dimensions of what you are adding, so that everything will have consistent dimensions. You always add a term to another only when it has got the corresponding dimensions not otherwise.

So all these things are properly balanced and that some of these are, some details that I think are good exercises for you to work out for yourself, just to make sure that everything is agreeing appropriately. Over here mind you, that there is a square of the speed of light in the denominator and that will also take care of the units, dimensions, everything (Refer Slide Time: 10:00). you So, you have to work this out in full details to convince yourself that the relationships are dimensionally correct.

I like to see the unity of electromagnetic phenomenon expressed by these equations very much. In fact, I feel that this is a stronger statement of the unity of electricity or magnetism than Maxwell's equations because, Maxwell's equation express the curl of E, a function of E in terms of a function of B and vice-versa. Over here, you actually mix them and you do know right from your high school days that you mix things only when they belong to the same kind. So, this is in a certain sense a stronger statement of the unity of the electric and the magnetic phenomena which was achieved by Maxwell and then this becomes a precursor to the search for unification of the physical interactions.

So, as the nuclear weak interaction is already unified with the electromagnetic interactions, that is what is called as a electro weak interaction and it belongs to the general formalism in physics in which frontier research is being carried out to understand the unification of all the fundamental interactions.

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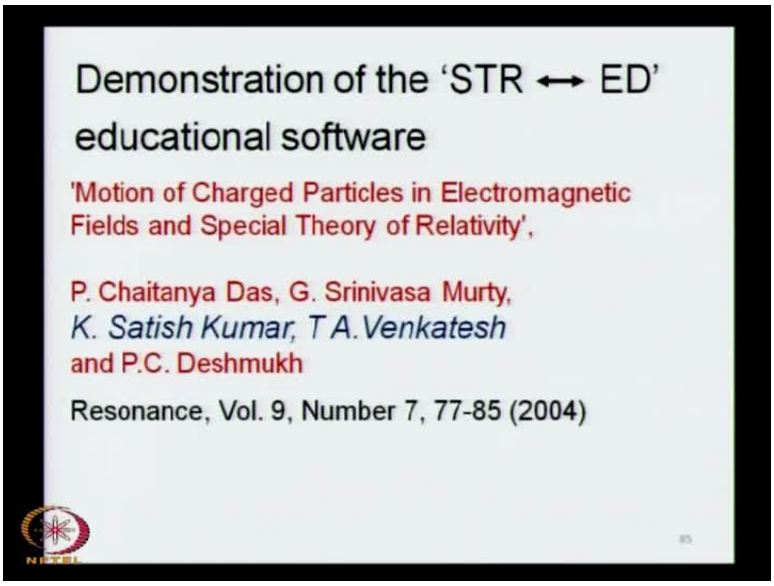

We will consider some examples now. These are constructs of linear super positions (Refer Slide Time: 11:55). So, that is the very big feature over here and many years ago I was discussing this in the physics 101 course here in IIT Madras when 2 students, Satish and Venkatesh came up and we had some discussions which led to development of an educational software which was subsequently published in this issue of Resonance, Volume 9, Number 7. It was published much after Satish and Venkatesh actually left IIT Madras because, for some reason the software was developed but we never got the time to write it up and then the technology became obsolete. The graphic software which they had developed was no longer being used so Chaithanya Das and Srinivas Murty rewrote it later and then we published it in 2004.

So, what I will do is, I will demonstrate this particular software. What this does is, it solves the problem of charged particle dynamics in electromagnetic fields using 2 different methods: one in which this solution is obtained in 1 frame of reference then you carry out the Lorentz transformations of x y z t to x prime, y prime, z prime, t prime; you get a new position vector r prime and you plot it as a function of t prime.

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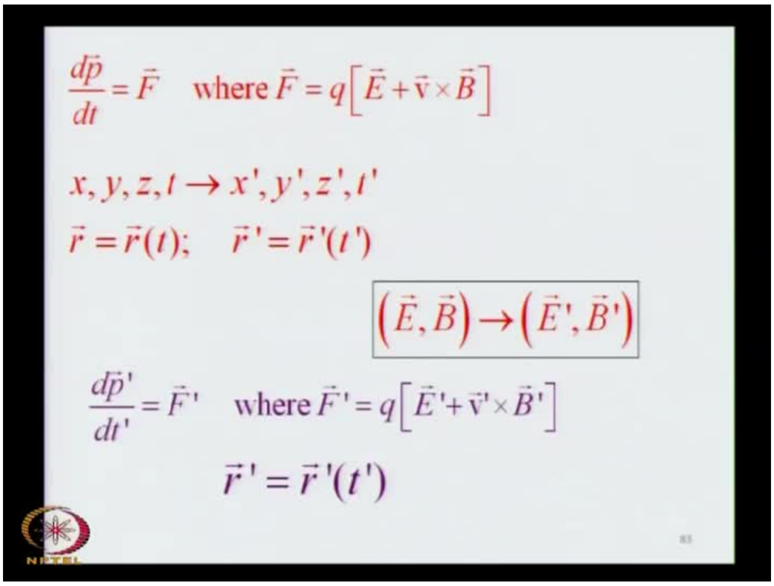

The second method which is completely independent of the first method is, it employs the transformation E B to E prime B prime and I already wrote those transformation relations in the previous slide. Then you set up a new equation of motion with the new E prime and B prime. It is this one; this is the new equation of motion which is set up with a new E prime and B prime and then it is this equation which is solved to get the trajectory r prime as a function of t prime. So, these are completely 2 different methods. Then there is a graphics package which projects r prime as a function of t prime and in fact, these projections can be made with the screen identified as the x y plane or with the y z plane or the z x plane.

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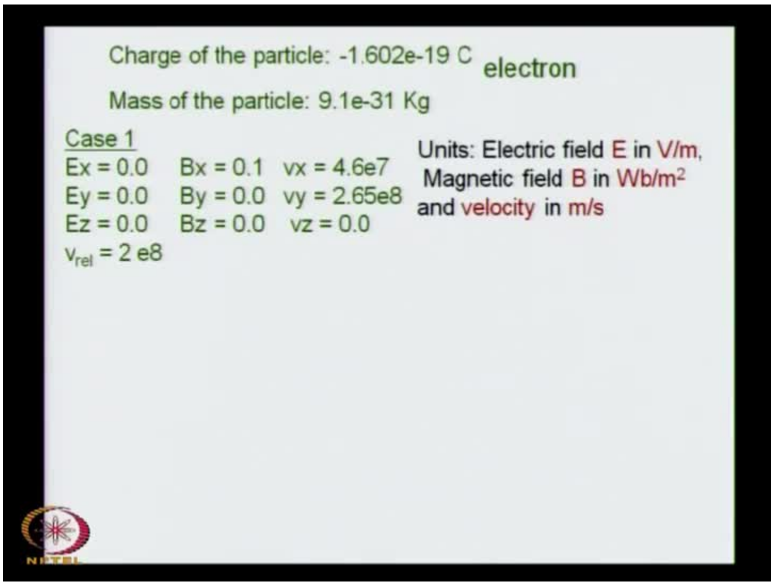

So, you can rotate the coordinate system, so that you,… because, on a screen you can see the trajectory only on a 2 dimensional flat surface. But, you can chose that flat surface to be either the x y plane or the y z or the z x so all of these features are incorporated in this very nice educational software which was first prepared by Satish and Venkatesh and subsequently by Chaithanya das and Srinivas Murthy. This software can in fact be downloaded from my website at this link (Refer Slide Time: 15:15), so that you can run this these programs yourself. But, we will show you a few test cases and what we will do is consider an electron; we have the its charge and mass and we will place it in different electromagnetic fields and I am going to invite Gagan and Jobin to demonstrate this for you, because they know how to run this software much better than what I do.

Gagan and Jobin will you please come over and demonstrate this software?

So this is the first case we are going to demonstrate, in which all the components of electric field is actually 0 and we have a B x which is 0 point 1 weber per meter square;

0 point 1 that is measured in terms of webers per meter square

Per meter square

And the initial velocities are given v x is 4 point 6 into ten raise to ten raise to 7

And v y is 2 point 65 into 10 raised to 8.

This is the initial velocity of the electron

Of the electron

Of the electron

Yes and this v relative that is 2 into 10 raise to 8

This is

So this is the velocity of an observer of a second observer

Yes

Who is moving with respect to the first observer

Yes

At 2 into 10 to the 8

Meter per second

Is that meter per second

Meter per second

So then that is fairly high speed because 3 into 10 the 8 is about the speed of light so this is at 2 third speed of light, right?

2 thirds yes

Yes

So, this is the fairly high relativistic speed and which is two thirds the speed of light; so let us put this electron in this field.

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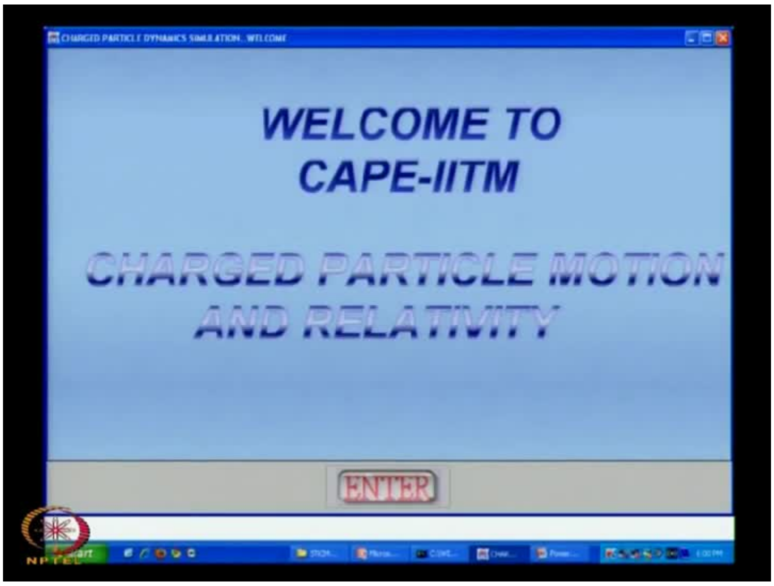

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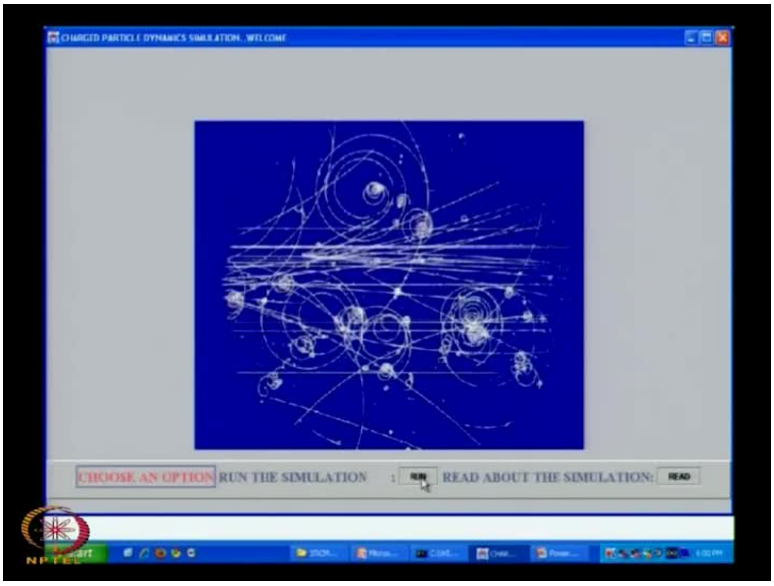

So this is the software which was developed by Satish Venkatesh Chaithanya das and Srinivas Murthy. What is this CAPE IIT M? That is computer aided physics education I believe; computer aided physics education at IIT Madras; so that is the CAPE IIT M comprise alright. What you doing now Jobin?

This we are just putting the electric field along x axis

Y axis

And these are the parameters that you chose?

Yes

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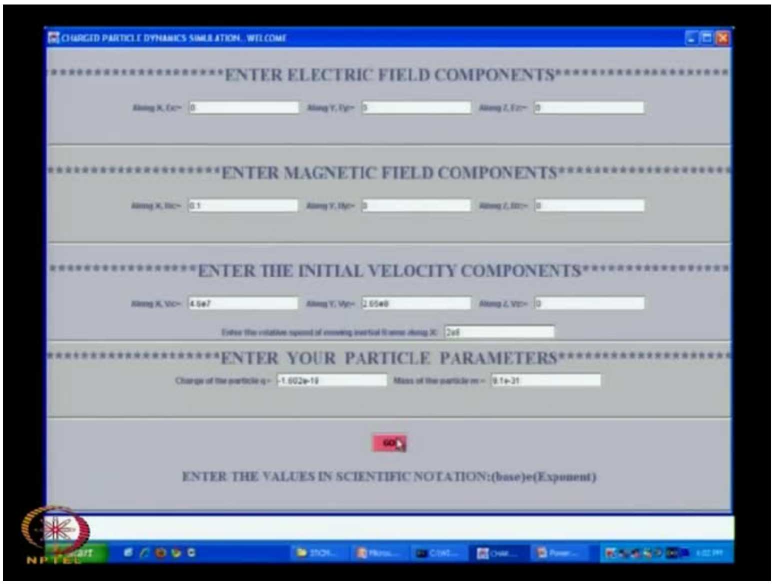

So mention your specifying the value of the electric field and

Shift tab

Magnetic field 0 point 1 1 0

All right

And then

B x is 4 point 6

B y and B z are both 0

4 point 6 into 10 raise to 7 that is the x component of velocity

This is the initial velocity of the electron?

Electron yes

And v y is 2 point 65 into 10 raise to 8 2 pint 65 into 10 raise to 8

65

So all right,... so the initial velocity of the electron has got an x component and a y component and 2 point 65 into 10 to the 8 and what is v z?

It is 0

V z is 0

Yes

And that will be displayed it is 2 into 10 raise to 8 meter per second.

And enter into you particle parameter so this Gagan's electron.

Yes this is the charge of the particle.

Yes

Minus 1 point 6 into 10 to the power minus 19

So coulombs

Yes

And next is the charge mass of the electron

Mass of the electron

That is 9 point 1 2 10 the power 31 kilo gram

Yes now go alright

Continue

So what is the program doing? It has already finished the calculation?

Yes

Yes

And now you get

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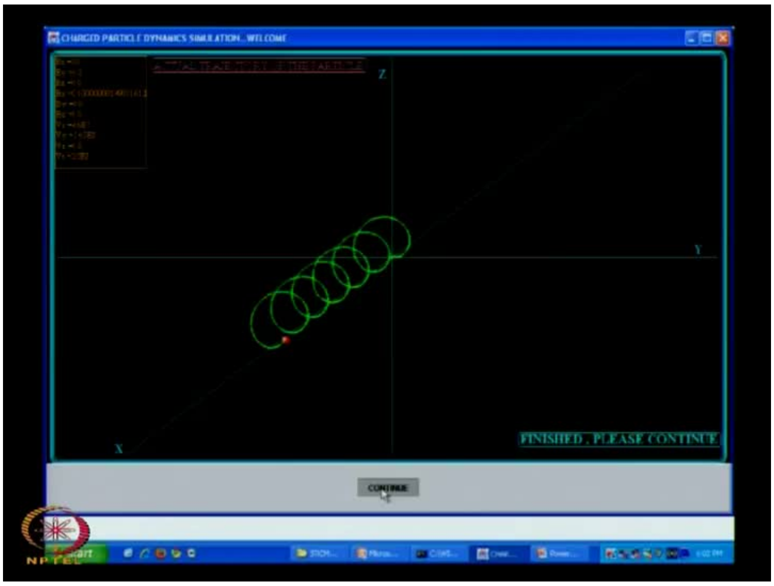

In the particle is initially the particle is here at the origin.

X equal to 0 y equal to 0 and z equal to 0.

Now let us run this

This is the trajectory that you see (Refer Slide Time: 19:11).

As observed from the s frame or the rest frame you can say.

This is the projection on the x y plane.

Yes

And this is on the y z plane

Yes yes

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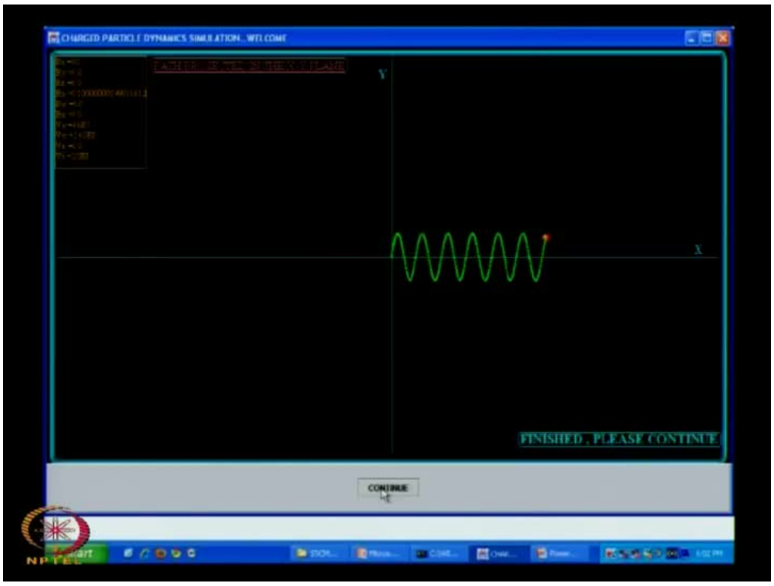

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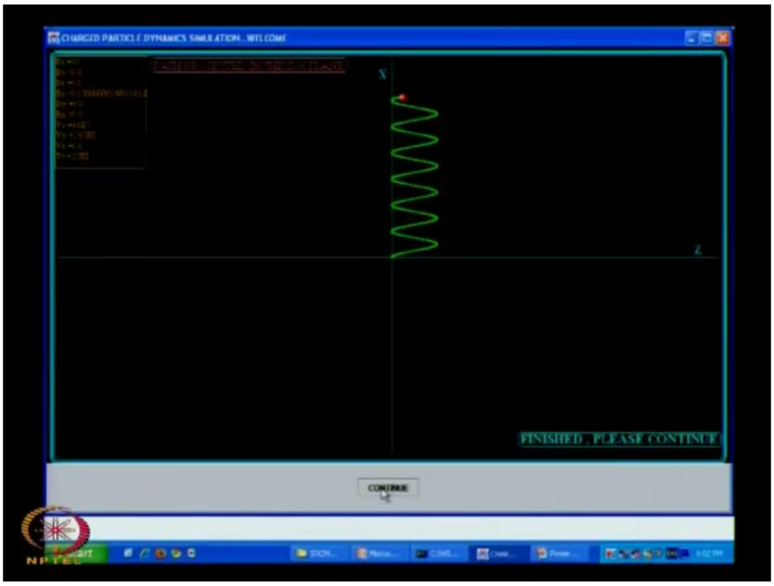

This is the projection of that trajectory on now this on the z x plane.

Z x plane yes

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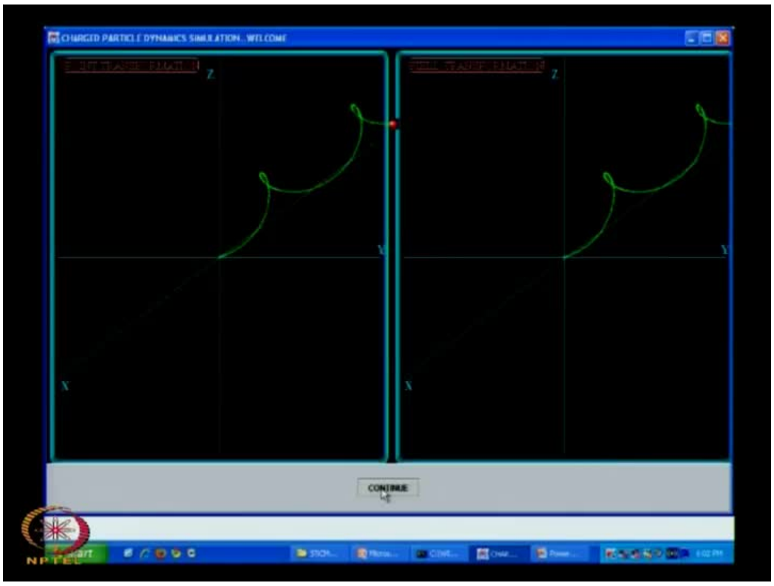

And what is this?

This is the trajectory of the particle what the observer and the prime to reference frame c.

Now the same trajectory as would be seen by another observer who is moving with respect to the first observer at the velocity that you specified earlier (Refer Slide Time: 19:45).

Yes

And that trajectory will be different and it is shown to be what it is but there are 2 panels over here what is that?

One is using this field transformation method.

And another using point transformation method.

So on the left panel you get the solution not by solving any equation of motion you have already solved that equation for the first observer.

Yes

And you just carry out the Lorentz transformations of x y z t to x prime, y prime, z prime, t prime and plot r prime as a function of t prime that is given in the panel on the left.

Left

Whereas what you have on the panel on the right is you setup a new equation of motion not with E and B but with E prime and B prime. Integrate F prime equal to d p prime by d t prime and then plot r prime as a function of t prime and at least on the y z plane the trajectories are Congruent. So, thank you Gagan and Jobin and then let me carry on this discussion.

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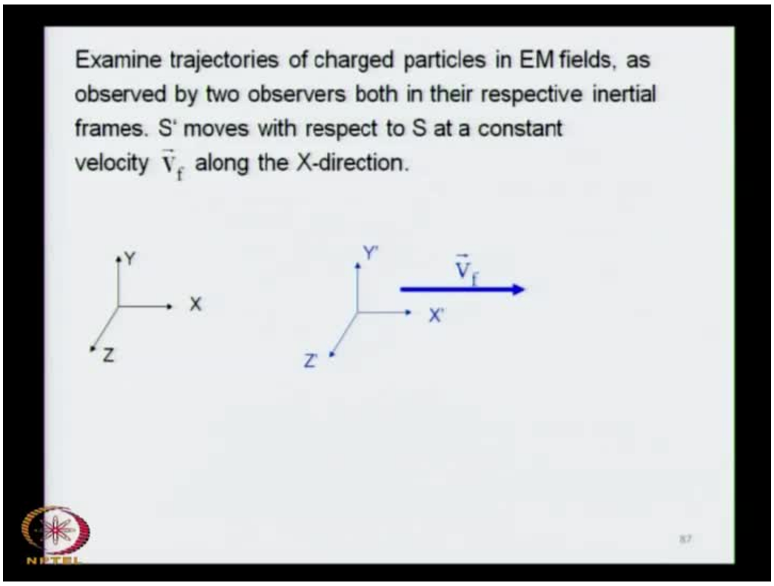

So, essentially what we did was to compare the observations of 2 observers; 1 in the black frame and the other in the blue frame; the blue frame moving at a constant velocity with respect to the black frame and we obtained the solutions using completely different techniques. One using Lorentz transformations of the x y z t to x prime, y prime, z prime, t prime the other in which we set up a new equation of motion with the Lorentz force being F prime which is charge times E prime plus v prime cross B prime and then integrating that.

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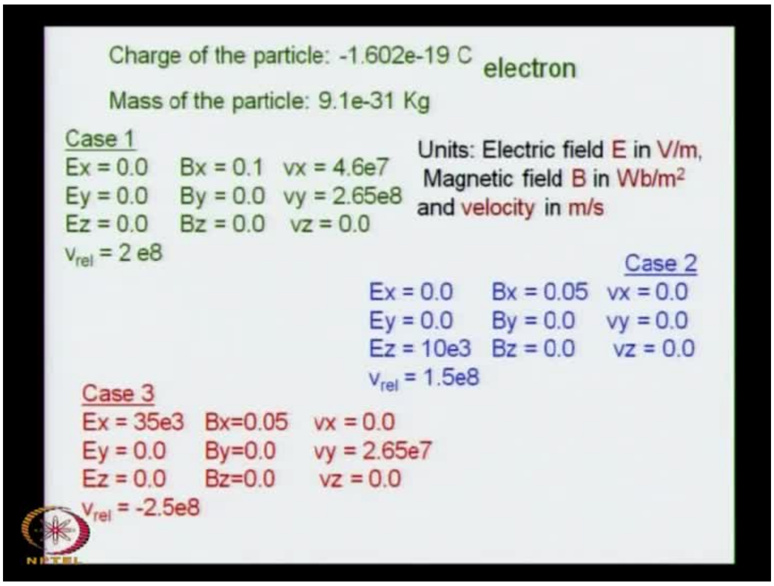

We found that the 2 solutions are quite Congruent to each other these are the some of the cases that we considered to illustrate this and thanks to Satish, Venkatesh, Chaithanya das and Srinivas Murthy and also to Gagan and Jobin for showing this nice software to us.

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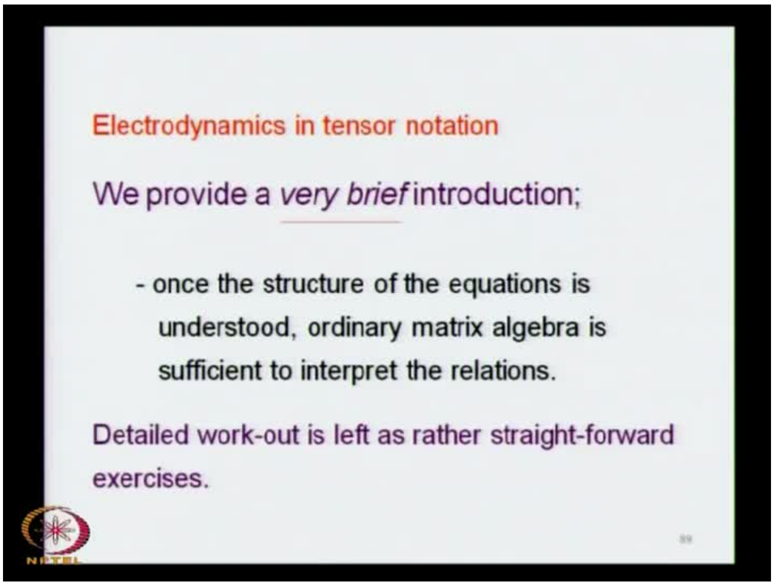

I will conclude by giving a very brief summary of writing the equations of electro dynamics and tensor notation. It is not something on which I will spend any bit of time but, it is something that you should be familiar with because, once you have seen this you will then be able to read literature on this subject from any source.

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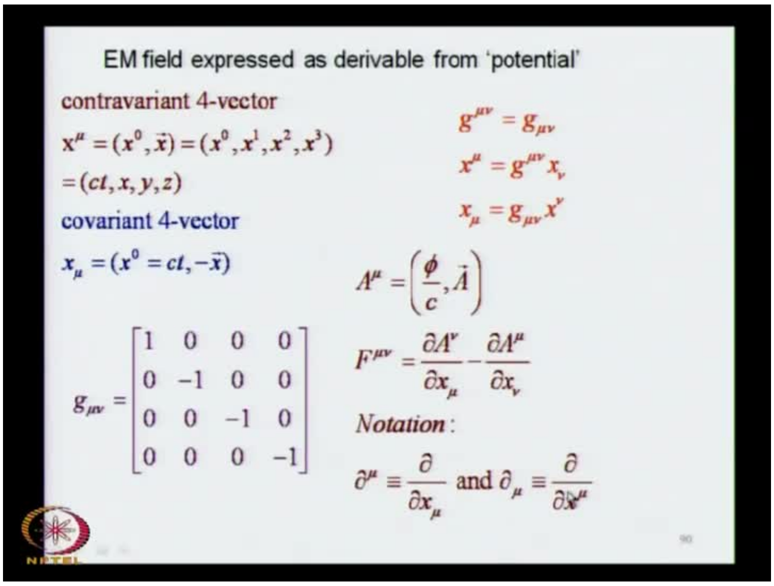

So, you need a little bit of familiarity with the notation; there is no new physics in it there is only a new notation. Let me just introduce you very briefly to the notation. We do know that the electromagnetic field is expressible in terms of potentials; you can derive it from potentials. In tensor notation what we do is, write Contravariant vectors in this form (Refer Slide Time: 23:13). So, x mu takes these 4 values x 0, x 1, x 2, and x 3 so mu takes 4 values 0 1 2 3 and the covariant notation is appears as a subscript over here the Contravariant notation appears as a super script over here. So this is the relationship between the Contravariant and the covariant notation. The signature that I make use of is this 1 minus 1 minus 1 minus 1. This is what distinguishes the space time continuum from an ordinary extension of the Euclidian space to 4 dimension. So, it is not an ordinary extension of the Euclidian 3 dimension space to 4 dimension. This is the space time continuum coming out of the special theory of relativity and this is the signature of this metric g.

You can go from the covariant notation to the Contravariant notation in a simple fashion. By using this matrix notation, you can just use this as matrix - like matrix algebra and these are the relations which you use to get the electromagnetic field from the potentials.

As you can see here you have the derivatives of A mu and these derivatives the A mu itself is a 4 vector (Refer Slide Time: 24:45).

The 3 components of what we call as a magnetic vector potential and this is the electro electric scalar potential together they give you 4 components which are A mu which are A 0 A 1 A 2 and A 3 and these are derivable you know when you take the derivatives of this electromagnetic potential, you get the corresponding electromagnetic field.

So the electromagnetic field is now written over here the right hand side gives you the derivatives of the potentials and the left hand side gives you the components of the electromagnetic field (Refer Slide Time: 25:25). The notation is sometimes made a little more [gibberish] you can write this del by del x mu as del mu and this one as del mu notice that there are covariant and Contravariant indices to keep track of.

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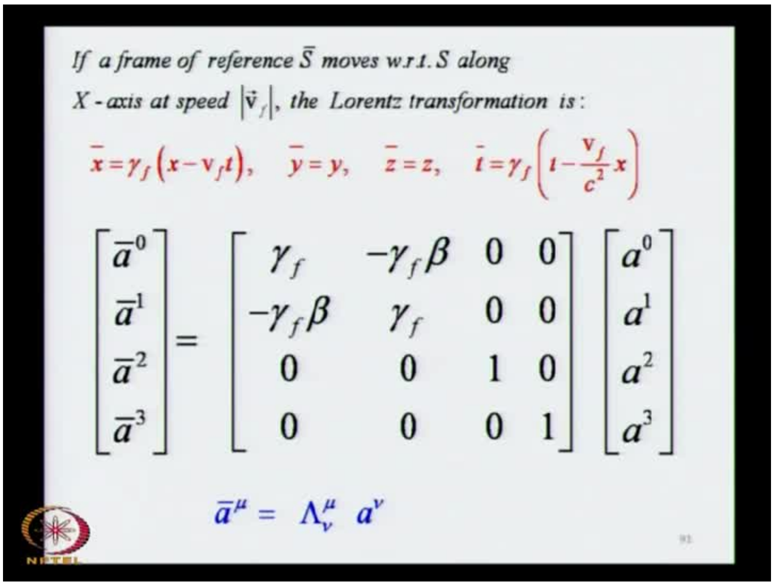

So now these are the Lorentz transformations that we had I have used a barred frame rather than a primed frame but, it is the same thing. I just needed the different notation to express the fact that you have 1 frame of reference which is moving with respect to another. This is the transformation which gives the Lorentz transformation and you can go from the parameters in one frame to the other by carrying out these transformations and you can write them in a very compact manner by this transformation matrix.

Lambda - this is the upper case lambda and this is the transformation matrix and you can go from 1 frame the normal frame without the bar to the corresponding parameters in the other frame which has got a bar or the primed frame if you like.

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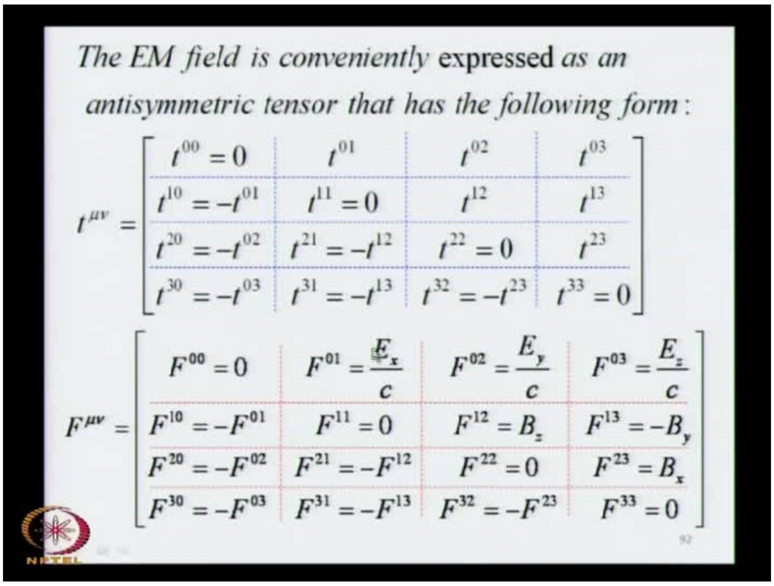

So, on the left hand side I could have used a primed a mu prime instead of a bar mu; it is the same thing. The electromagnetic field in fact, is very conveniently expressed as an anti-symmetric tensor which has got a very simple form. Notice that there is some sort of a symmetry about the diagonal but, this is more like anti-symmetry because the terms over here have got opposite signs. So if you go equidistant away from the diagonal then the corresponding terms have got opposite sign. So, this is t 2 3 and this will be minus t 2 3. So, t 3 2 is equal to minus of t 2 3; so when you inter change the indices over here you pick up a minus sign that is what makes this tensor to be anti-symmetric.

So in this notation you can write the electromagnetic field the electromagnetic field in our usual vector notation is made up of E x E y and E z so they are appearing over here; this is E x this is E y and this is E z (Refer Slide Time: 28:08) and in terms of this anti symmetric tensor the electromagnetic field is expressed by this 4 by 4 matrix which is an anti-symmetric tensor. Again, you have 0s along the diagonal just like this typical anti symmetric tensor.

Again you have an anti-symmetry over here so if f 0 1 is E x over c, then when you interchange 0 and 1 so instead of the element in the second column and first row, you go to the element in the second row and the first column, you get minus of F 0 1 so this will be minus E x over c. Likewise, you will have other components to be given by the corresponding anti symmetry.

So this is just a matter of notation. Essentially, what you have written is the electromagnetic field; there is nothing new in it here. This is the electromagnetic field over here; so you have the x component over here, you have got the y component over here, you have got the z component over here, then you have got the magnetic field over here; so the B z is here the B y is here and the B x here.

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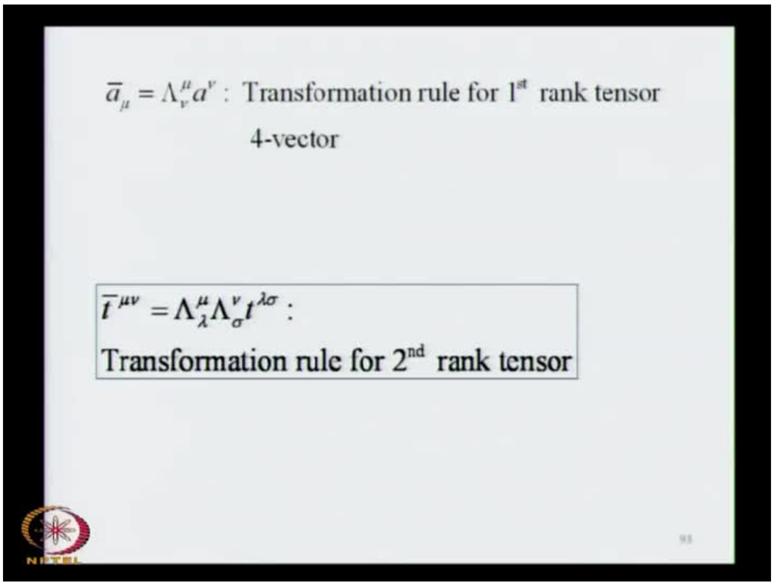

So this is just the electromagnetic field there is nothing new in it and the compact transformation relation can be written for a first rank tensor which is a 4 vector now instead of a 3 vector. Now, this is the transformation rule for the first rank tensor the electromagnetic field is a second rank…

Yes

(( ))

Yes

(( ))

(( ))

The corresponding transformation rule for the second rank tensor is given by this because there are 2 indices to play with in this case. So, this is the transformation rule for the second rank tensor; this lambda we have already defined.

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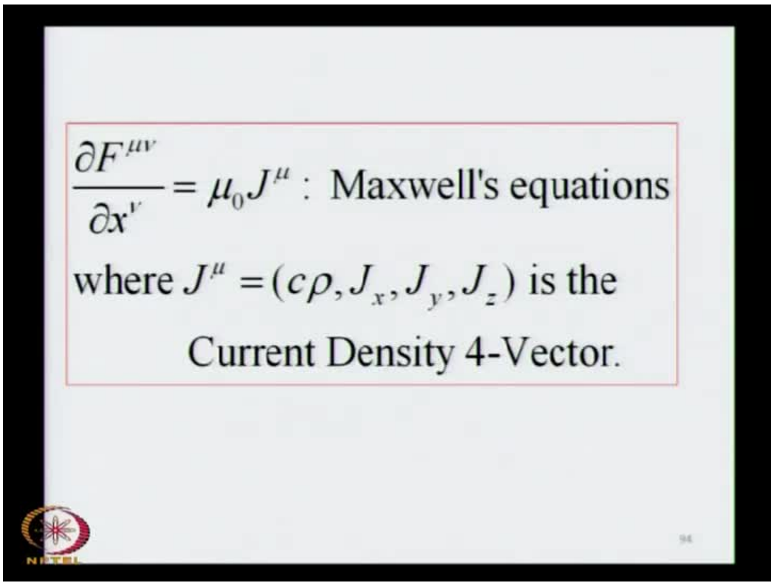

And Maxwell's equation, the same Maxwell's equations that you have seen earlier, are written in a very compact manner and in this particular form; so this is a set of equations (Refer Slide Time: 30:38) these are the Maxwell's equations; J mu is the density, this is the current density 4 vector so it has got the charge density as well as the current density which we have met already in the vector notation in which Maxwell's equations we developed earlier.

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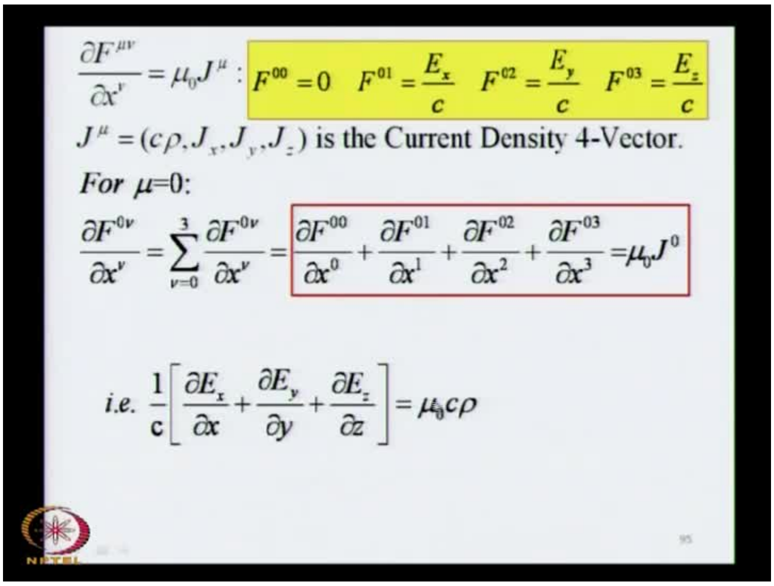

Let us just satisfy ourselves that this notation gives us consistent relationships which we have seen earlier in the ordinary vector notation. So, you have the Maxwell’s equations and you have got the current density 4 vector and if you just take the particular case, put mu equal to 0 then, all I have done over here is to put mu equal to 0. So, this is del by del x nu of F 0 nu and this index nu which appears twice is the one over which you carry out the summation. This is sometimes called as the Einstein summation convention. You must sum over this index nu going from 0 to 3. So, let us do that; so the term corresponding to nu equal to 0 is here; this is nu equal to 1 term, this is nu equal to 2 term, and this is nu equal to 3 term and Maxwell’s equations take this form.

Now, let us see if it is the same as we have seen earlier because we do know that these are the electromagnetic field terms. So, F 0 0 we have already seen is 0 this is coming from the anti-symmetric tensor for the electromagnetic field F 0 1 is E x over c, F 0 2 you have to take it is derivative with respect to x 2 and then F 0 3 is e z over c and you have to take it is derivative with respect to x 3 and you already know what these terms are. So, just take those derivatives term by term and what you get means the first term will gives you 0 because F 0 0 is 0. So, from the remaining 3 terms you get del E x by del x, del E y by del y, del E z by del z you have got a one over c here you get mu 0 c times rho on the right hand side but this c you could move to the right side so you will get mu 0 c square and that is 1 over epsilon 0.

So, what you get essentially is that the divergence of E is rho by epsilon 0. In other words, this equation gives you the complete family of Maxwell's equations and you can put in the other values of mu and do this as an exercise for yourself and satisfy yourself that all of Maxwell's equations can be written in a very compact form in this tensor notation which you will find being used in a lot of literature on classical electrodynamics; so it is good to be familiar with that.

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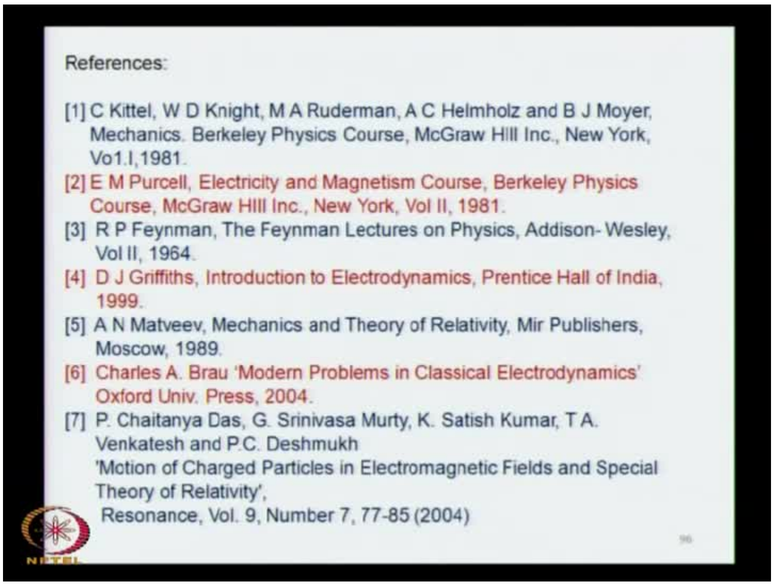

So with that I will like to give you a set of references which you could use to read these topics further. Berkeley physics course is one of my favorite sources. The volume 1 which is by Kittel Knight and Ruderman and Purcell volume 2 which belongs to the same series of Berkeley physics course. These 2 are excellent sources; Feynman lectures are of course, great Griffiths book also gives you all the essential tools and then there is a nice book by Matveev mechanics and the theory of relativity And this is a nice source to read about current problems of current interest; modern problems and classical electrodynamics.

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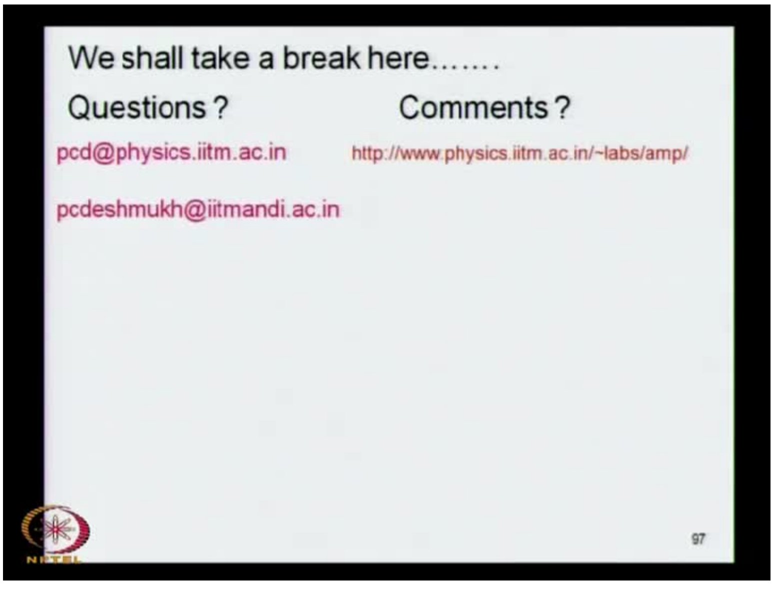

Then, the reference to the software which I mentioned the CAPE IIT M, the computer aided physics education program that we have. it is it is not a huge program but there are few attempts made by some students who have the enthusiasm to develop some educational software, they contribute to this and with that we pretty much conclude this unit.

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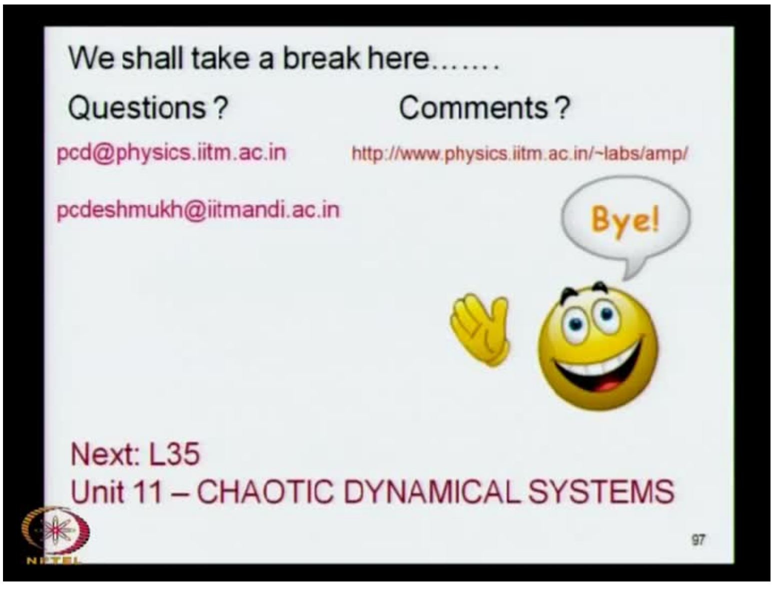

If there are any questions, comments, I will be happy to take. Otherwise, we will go over to the unit 11 in the next class which will be on chaotic dynamical systems.