Select/Special Topics in Classical Mechanics
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Module No. # 11
Lecture No. # 35
Chaotic Dynamical Systems (i)

Greetings; we will begin unit 11 and this is really the last major unit of this course on select topics in classical mechanics. This course has grown out of the p h 101 which I have been teaching at IIT Madras and also gave this course at IIT Hyderabad and now at Mandi.

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This topic has usually not been included in the IIT 101 curriculum but, it is it is a nice thing to learn at this level. It has got extremely important applications - this topic of Chaotic Dynamical Systems. It has very major applications in the field of science and also engineering; so certainly very important to learn.

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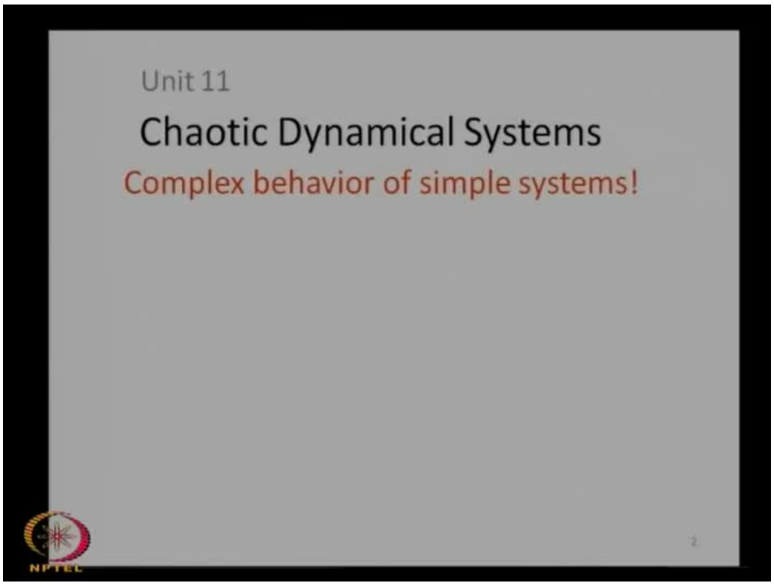

So, let us begin our discussion on this and let me begin by telling you what we are going to deal with are very simple systems essentially. But, simple systems having very complex behavior and this is a very strange thing that you expect simple systems to have very simple behavior. If you look at a simple pendulum for example, you think that okay, you know everything about it. but it turns out that certain consequences of some details like the initial conditions and so on lead to very complex behavior, complex dynamics and the importance of this subject can perhaps be gauged by examining a remark made by this gentleman whose picture you see on the screen.

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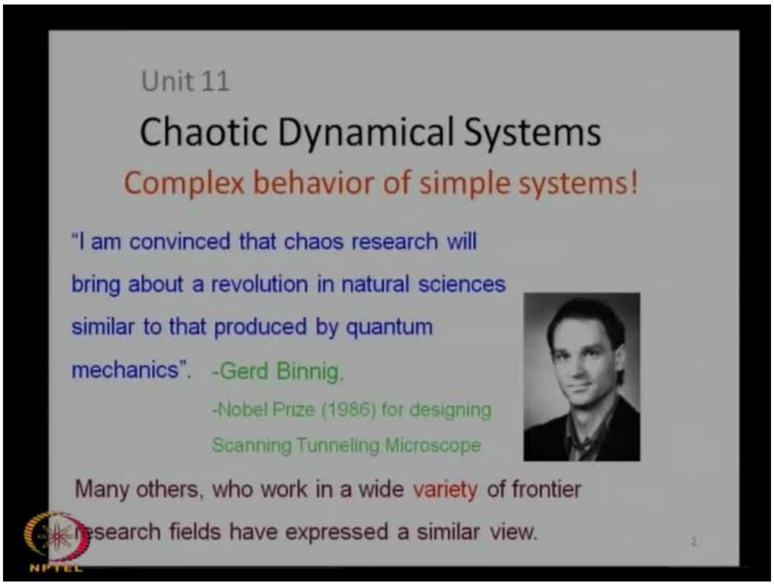

I do not know if anybody recognizes him; he is Gerd Binnig he got the Nobel~~le~~ prize in 1986 for designing the scanning tunneling microscope and let me begin by quoting him. What he says is, that "I am convinced that chaos research will bring about a revolution in natural sciences similar to that produced by quantum mechanics". We all know that quantum theory has had a huge impact on science, subsequently on engineering and subsequently on technology and everything that we are using around us including these communication channels and so on. They are completely determined by quantum laws, by quantum physics and other theories also like the theory of relativity for example and this has really absolutely revolutionized science and engineering and technology everything. What Gerd Binnig tells us is that the research in the field of chaos is quite likely to have a similar impact on developments in science and technology.

That is part of the reason, part of my motivation to introduce this subject early on as a part of this course which is essentially an undergraduate course which is something which is it is designed as a first course after high school. And at this stage, it is a good idea to get some exposure to the discipline of Chaotic Dynamical Systems. Our treatment will be relatively elementary. It is not our intention to get into too many details but basically to introduce students early onto this field.

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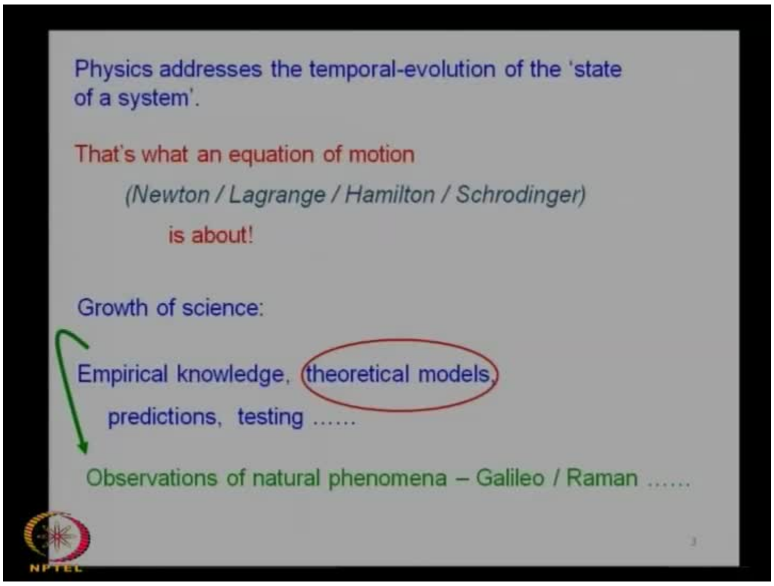

Now, this is what physics is about; that we consider physical systems and then examine how they evolve with time. This is the central problem in mechanics. How do you characterize a system and how does it evolve with time? This is the fundamental problem in mechanics. It is the same in classical mechanics as well as in quantum theory. In classical mechanics, you describe the state of a system. The first problem is, how do you characterize it categorically, clearly? Then ask how does it evolve with time and we know the answer; we have been discussing this throughout this course that a system in classical mechanics is described by a point in the phase space. You need 2 parameters to describe it - the position and the momentum and how the position and the momentum evolve with time is given by the rate at which the position changes which is d q by d t and the rate at which the momentum changes which is d p by d t and our the notation that we have been using is we write the q dot as d q by d t p dot as d p by d t.

So once we setup the equation for q dot and p dot, we have complete knowledge about how the system evolves with time. Of course, we have to provide the initial conditions and so on. So, these are the Hamilton's equation for q dot and p dot as well. Essentially, the same information is contained in the Newton's equations and also in the Lagrangian equations; except that those are second order differential equations. We also know and this is this really belongs to the realm of quantum theory but, we do know that position and momentum cannot really be determined completely, accurately, simultaneously because, measurement of one causes disturbance in such a way that if you measure the momentum then these 2 measurements are not compatible. You can certainly measure position; you can go ahead and measure the momentum. Also, it is not that you cannot do it. You will get an answer, but if you come back and repeat these measurements in some arbitrary order - first you measure the position, then you measure the momentum, then you come back and measure the position again. If you perform these measurements in arbitrary order, then you do not recover the values that you had got earlier and then you really do not know which value you can admit.

That is the reason the description of a state by q and p does not hold anymore. Then you have to represent the system by what is called as the state vector and how this state vector evolves with time is given by the Schrodinger equation. But, notice that the basic problem remains the same. How do you characterize the system? If it is not by q and p then it is by state vector and how does the system evolve a time? So, characterization of the system and it is temporal evaluation is the central problem in mechanics, both classical as well as quantum and the equation of motion is then either Newton, Lagrange, Hamilton or Schrodinger.

Now, let us ask ourselves, how do we learn about scientific laws? What is a scientific law at all? What we do is we compile a lot of empirical knowledge there is experimental data we systematically analyze it. If you look at most events in nature, there is so little that you can predict. When you wake up early in the morning for example, your hair is all tossed up in some random manner, right? It is very irregular. So, nature by itself will lead to unpredictable behavior of physical processes but certain things, in nature are predictable.

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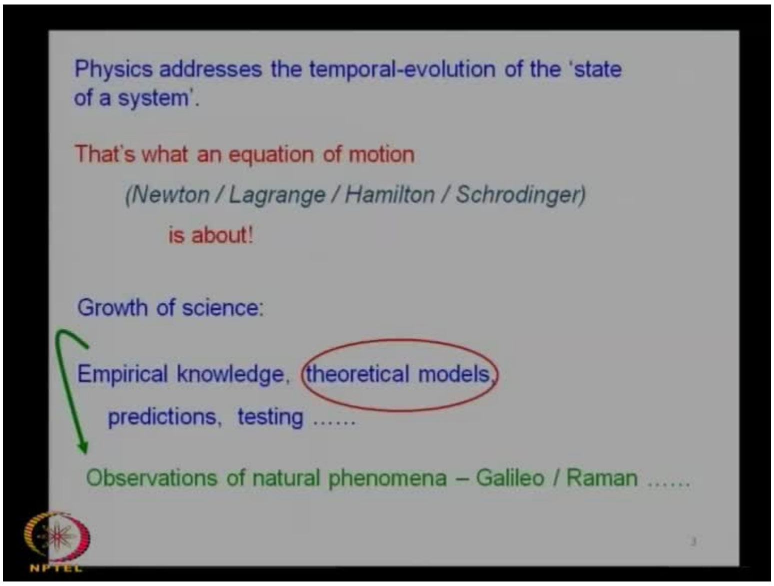

If I take this bottle and let go, it is accelerated downward at a certain rate which is measured in meters per second per second and whether I drop this object, or this object, or this laptop, or this table, or whatever everything is going to get accelerated downward at exactly the same rate which is 9 point 8 meters per second per second it is also independent of who does this experiment. Whether it is done by me or you, it is independent of what is dropped; whether it is a piece of stone, or a pen, or a water bottle or whatever and this is what a law of nature is. So, we are looking amidst everything that is unpredictable; what are the commonalities which can be predicted? And if, we are able to identify that then we say that we have discovered a law of nature.

How do we get these laws? We get them by empirical knowledge. For example, we carry out a large number of experiments. Galileo did these experiments; he looked at how these objects fall from the top of the mast of a ship. We discussed Galileo's experiments and by observing the regularities in that, he discovered the law of inertia. That is what we call as the first law of mechanics. Then, you having discovered these laws, you look for other laws and then you develop theoretical models. So, the Newton's laws for example, which is based on the principle of causality and deterministic mechanics.

You are able to make predictions, you can test them and all of this leads to formulation of laws of physics. either they come from observations by compiling empirical information and Galileo, people like C V Raman they were grand masters at this or else, you develop. in addition to that you systematically organize it, develop theoretical models the way Newton did Hamilton did, Lagrange did, Schrodinger did. Based on that you make predictions; then, you carry out experiments and this is the technique of science; this is how you develop science and use it to generate more knowledge. So, these theoretical models are to be developed and we develop this by taking advantage of empirical data or by taking recourse to intuition and so on. Now, this is the way science grows.

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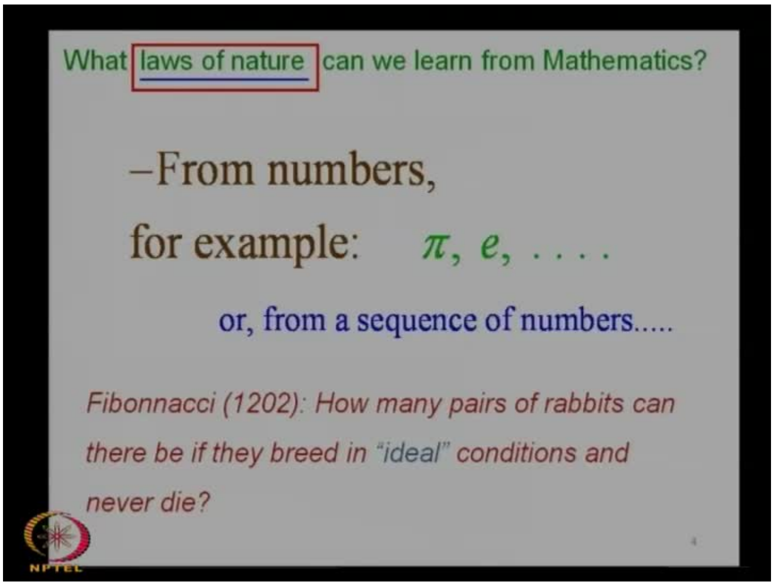

Now, we want to raise a different question, that, what is it that we can learn from mathematics? Can we learn laws of nature from mathematics? To make this idea little more concrete, let me give you a specific example. What is it that you can learn from numbers, such as pi, e, all of you are familiar with these numbers and you know that they connect immediately to various physical objects that you look around; their properties, they relate to geometry. For example, they relate to certain physical laws. The exponential function appears in a large number of physical laws, which I am sure all of you are familiar with and these are numbers and these numbers have in them information about the laws of nature; they connect to the physical laws in a certain sense.

What about sequence of numbers? There is some information sitting in these numbers. There is some other kind of information which is sitting in a sequence of numbers and to give you an example, I will discuss the Fibonacci sequence. Now, this is known after

Fibonacci and this was published in a book in the 13 th century at the beginning of the 13th century in the year 1202. The question he addressed was, how many pairs of rabbits can there be if they breed in ideal conditions and never die. Now, we need to define what is meant by ideal conditions, so let us do that.

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So, the ideal condition is that the Rabbits first of all never die, that is number 1. Second, the female always produces 1 new pair of Rabbits every month; it takes 1 month and at the end of 1 month it will deliver 2 Rabbits and these 2 will always be a male and a female. Now, this is not how normally things happen because there could be 2 males or 2 females but we are talking about in a certain scenario for the formulation of this problem and for the formulation of this problem we consider that the female will produce 1 new pair every month and the new pair will always be a male and a female. The original Rabbits keep getting older but nobody, no rabbit will ever die. This is our operational definition of what we are going to call as an ideal situation.

The question is how many pairs will there be in 1 year? I have taken some pictures form this website which I have referenced over here (Refer Slide Time: 15:15) but, there is a lot of information available in a large number of books and internet and so on and you can be careful about choosing your source.

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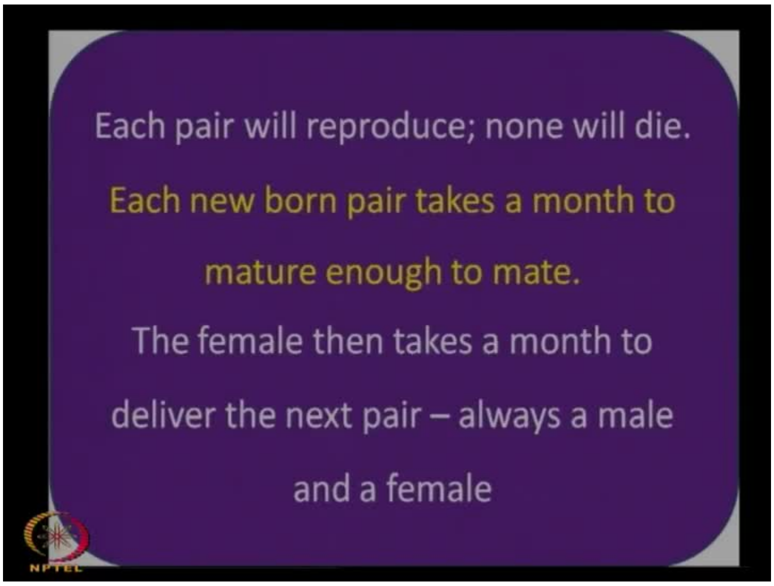

But this is a nice source where there I found some nice pictures so I will show you those pictures. Our first consideration is that each pair will reproduce; it is not that there will ever be a pair which will not reproduce, that is also a part of our consideration of what is called as an ideal situation. Each newborn pair will take a month to be mature enough to mate and after that it will take a month for the female to deliver a pair - 1 of which will be a male and the other a female. Now, that is the scenario we have. The question is, how many Rabbits will there be in this scenario after 1 year?

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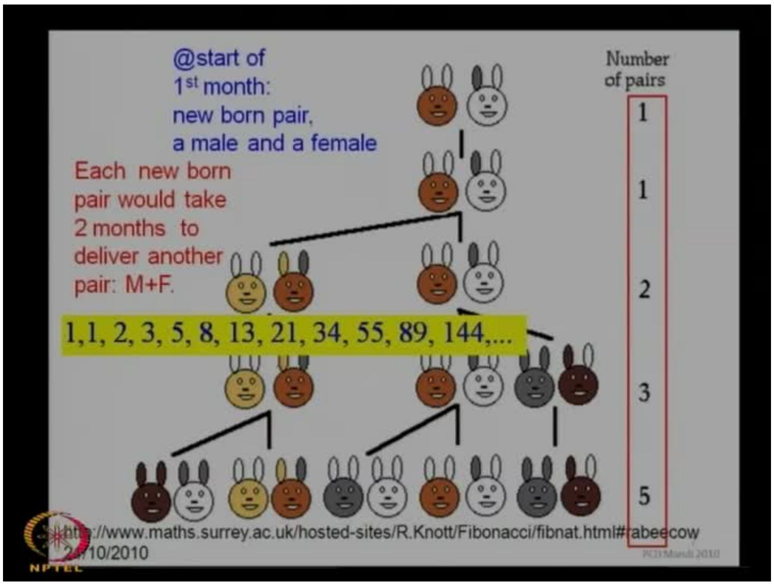

So, let us begin with this. At the beginning of the month, there is a newborn pair - a male and a female and they are here at the end of the first month. They are mature enough to mate at the end of another month, after that they continue to live here (Refer Slide Time: 16:48) and they deliver a pair a male and a female that is the picture. So, you begin with 1 pair of Rabbits; after the first month, you still have 1 pair of Rabbits and at the end of the second month, you have 2 pairs; now this is the process. Now, what about another month from now? What is going to happen after another month? After another month, this pair would be mature enough to mate, but would not have produced any pair and this pair would have produced another pair so, the total number of Rabbits is now 1 2 and 3 pairs at the end of the third month (Refer Slide Time: 17:35). What about the fourth month? Then this would continue to live; it will deliver a pair. This would continue to live and this would be continue to live and be mature enough to mate but this pair which already is matured would produce another pair which is over here.

So, the total number of pairs will be 1, 2, 3, 4 and 5. Now, there is something very peculiar about the sequence. What is peculiar about it? If you noticed, is this sequence you have 1 here, then 1 here, then the sum of these 2 gives you 2; then if you take the 2 preceding numbers, 1 plus 2 you get 3. If you take this number and add it to the preceding number which is 3 plus 2 you get 5 (Refer Slide Time: 18:35). This is how the sequence develops and this is known as the Fibonacci sequence. So, you have 1 1 plus 1 will give you 2 2 plus 1 gives 3 3 plus 2 gives 5 5 plus 3 gives 8 8 plus 5 gives 13 and so on. So, you now have a sequence of numbers; the questions we raised is there something about the laws of nature that we can get from the from a sequence of number we have already seen that there is some physical information sitting in some numbers; some individual numbers like pi and e and now we are examining if there is some information sitting in the sequence of numbers.

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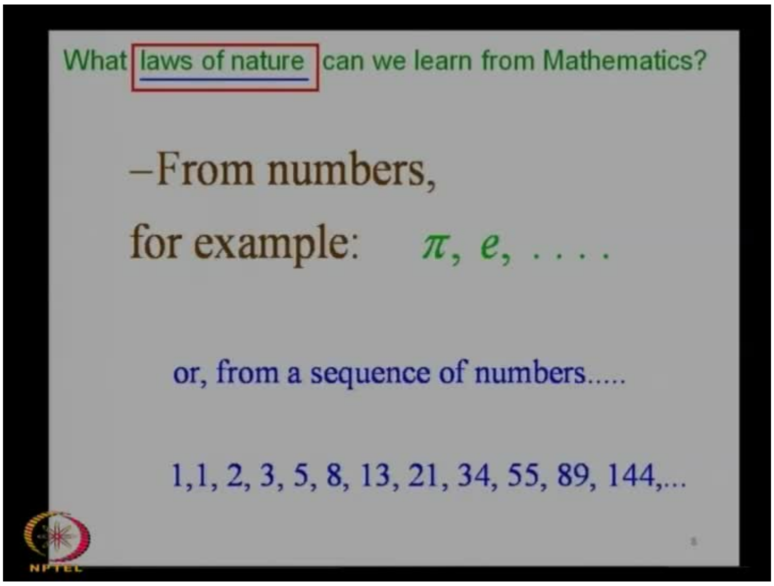

So, this is the sequence which is called as the Fibonacci sequence and it goes on and on so if you take 55 and 89 then the sum of these numbers will give you the next number which is a 144 so the whole sequence is generated through this.

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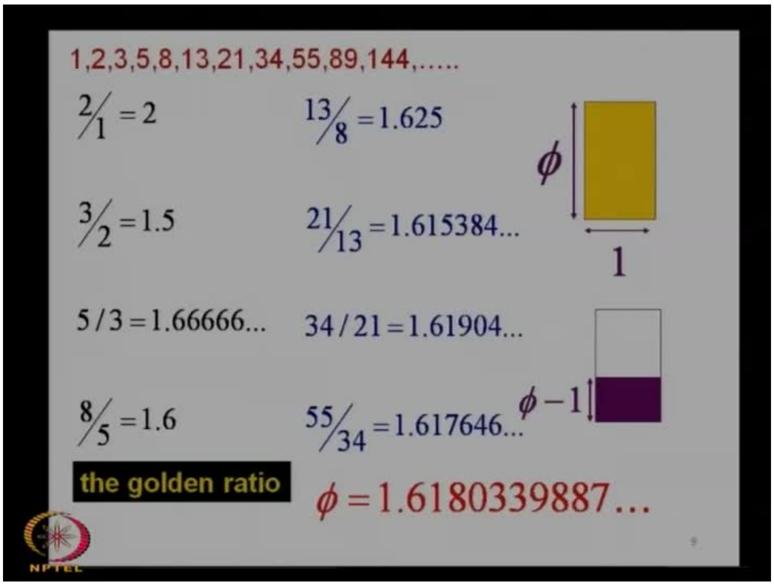

There is another peculiar feature of this. Now, if you take the ratio of 2 successive Fibonacci numbers, you take the first pair 2 divide by 1 you get 2; you take the next pair which is 3 divide by 2 you get 1 point 5; you get the next pair which is 5 divided by 3 you get 1 point 6 you take the next which is 8 by 5 and you get 1 point 6. Now, see notice what is happening; the numbers are slowly converging. The ratios that you get from this are slowly converging and if you keep going to the next pairs of successive ratios of successive numbers 13 by 8 21 by 13 and so on, notice that they are slowly converging. They are not blowing up, they remain in the vicinity of 1 point 6 1 7 and so on. If you go high enough as n tends to high enough, then it converges to a ratio which is a constant which is called as the golden ratio and this ratio is 1 point 6180339887. And, or leave it as an exercise for you to get these to as many places of decimal as you might enjoy getting it and then it just settles down over there and this is called as the golden ratio.

Now this is a very famous number; you can construct a rectangle with 1 side equal to phi, this ratio the symbol used for this is phi, the other side is 1. So, if you take the ratio of the sides it is equal to the golden ratio and the rectangle which is which has got sides the length and the breadth in this ratio, in this proportion, is called as the golden rectangle and this has got some very peculiar properties.

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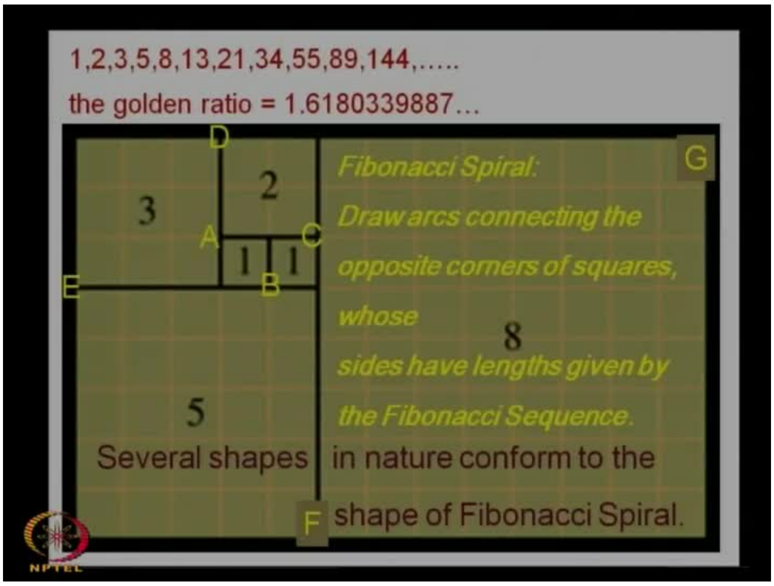

This is the golden ratio. What we will do is, we will construct squares in which we begin with this small square over here this is 1; it is a square of side 1. We place on this edge another square of the same side; then on the sum of these we place this square which has got the side whose length is equal to the sum of these 2 (Refer Slide Time: 22:22). Now, on this side we place this square whose side is equal to this and this. So, essentially what we are doing is following the squares according to the Fibonacci sequence. Do you see that over here, you have a square of side 3 you have a square of side 2 over here so 3 plus 2 is 5. So, a square of side 5 will sit on this edge and a square of side 5 plus 3 will sit on this edge. This is how you can cascade this process and you can go on and now you have understood how I have built this assembly of squares.

Now, let us label some corners; so we begin with the first corner, I label it as A then I go to the diagonal, label it as B then I go to the next diagonal label it as C then I label the next diagonal as D then the next diagonal point as E then F and then G. You see how these corners are labeled? So, just notice how these corners are labeled. So, A to B to C to D to E to F to G and it can go on and once you exhaust the 26 alphabets you can start labeling them as A 1 A 2 A 3 or anything; so it will go on and on.

Now, what I suggest to do is you connect these letters in that sequence. Join A to B construct a curve a mental curve in your mind from A to B to C to D to E to F to G and you will notice that what you are drawing in your mind is a spiral. This is called as a Fibonacci spiral. What is very strange and this comes as the big surprise is that there are many things in nature which have got these spirals which have exactly an identical shape as the Fibonacci spiral.

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Let me show you some examples. There are good number of such shapes and if you just Google Fibonacci spiral, you will get a large number of excellent pictures and I suggest that you do that and I will show you some of those pictures. Look at the shell for example, this is a sunflower (Refer Slide Time: 25:10), these are some cones and you can see the patterns over here and they all follow the Fibonacci spiral exactly.

Now so why this should happen is a very interesting question but that it does happen and if it does then, it is telling us something about nature. There is some information about the physical laws of nature which is sitting in this sequence of numbers and therefore, to understand science, to develop science it is important not only to carry out experiments, compile empirical data and deduce laws of physics laws of nature out of it, but we can also learn something about nature, about natural laws by studying numbers and also by sequence of numbers; that is the idea that I want to put across.

Fibonacci is also known; this was not his real name. He was known as Leonardo of Pisa and he published his book called liber abaci in 1202. But it is interesting to note that the Fibonacci sequence was in fact known before Fibonacci, it is known after Fibonacci.

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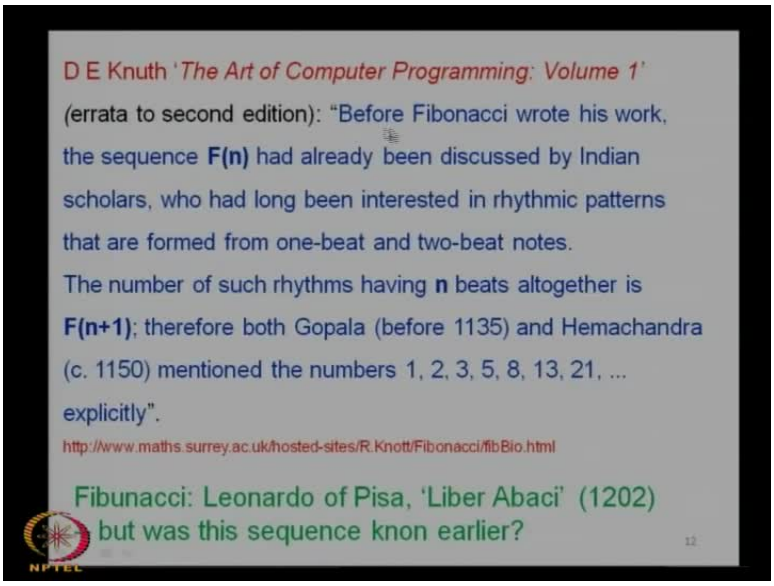

But the sequence of numbers has been known earlier. I will quote from Knuth and this is from his website at surrey and he has written in the art of computer programming volume. One in the errata to the second edition, he writes that before Fibonacci wrote his work the sequence F n which is the Fibonacci sequence had already been discussed by Indian scholars who had long been interested in rhythmic patterns that are formed from 1 beat and 2 beat notes. So, once you have got the 1 beat and then you have got the 2 beat and then you take the combination 1 plus 2 you get the 3 beats and then you get the combination 2 plus 3 and you get the 5 beats and you generate the Fibonacci sequence or what we call as the Fibonacci sequence. But, this was known to Indian scholars before Fibonacci and this has been discussed in certain rhythms in the work of Gopal before 1135 and by Hemachandra and there are a number of other references. I have not researched this in any great detail; but those of you who are interested might find it quite interesting to follow this lead and this certainly was known before Leonardo of Pisa found this out.

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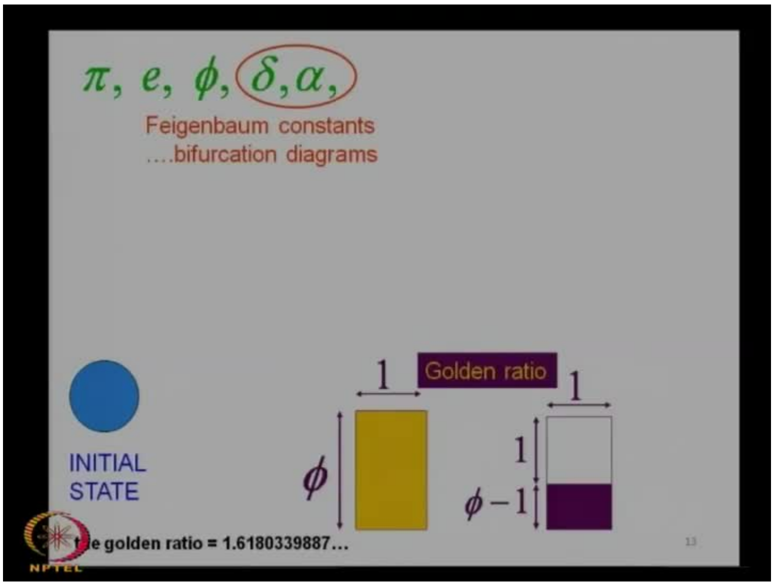

So, now what we have learnt is that we have learned something from these numbers pi or e and now this is the golden ratio of phi and now there are some other numbers over here from which we can learn something and these are delta alpha and these are known as Feigenbaum constants and they come from bifurcation diagrams these are central these diagrams are of central interest in chaos theory.

Now, this is what the problem is about that if you have got a certain system and you describe it is initial state and the system evolves over a passage of time and this is our central problem in mechanics. It evolves to a certain state and why does it get there?

Because, there is some abstract attractor to that so I have introduced a term called as an attractor. I will define it more precisely as the discussion progress as the discussion progresses. Let me start building a heuristic idea of what an attractor is, so system would evolve it head on and after a certain passage of time it will get to some other state and it will evolve along a certain path in the phase space. It may be because of how you describe the system evolution; perhaps in Newtonian mechanics you would use the principle of causality and determinism and then say that forces have acted on it and therefore, the system has accelerated. This being the initial condition, this being the force, it has evolved in this direction and this is the final state to which it has evolved. Or, you could explain it using the Hamilton's principle, saying that this evolution along this path is what minimizes the action integral. The integral l d t got minimized along this path and then it reaches a certain destination which is what I am referring to as an attractor.

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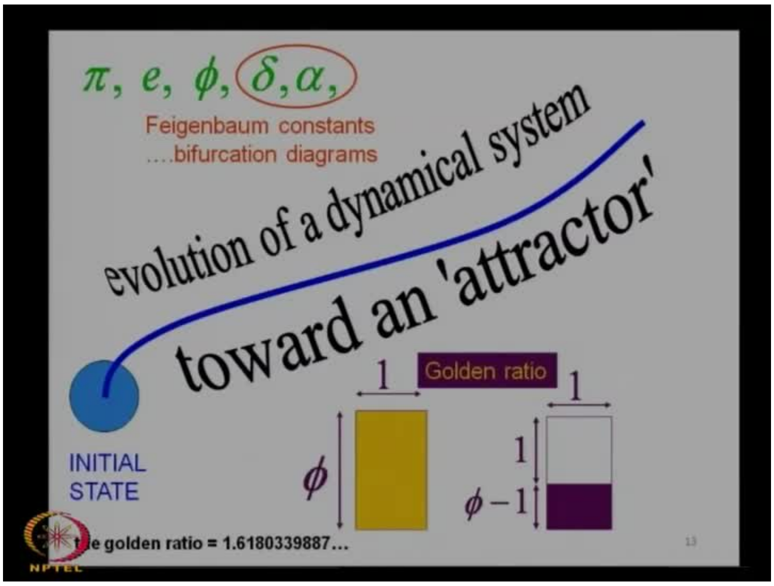

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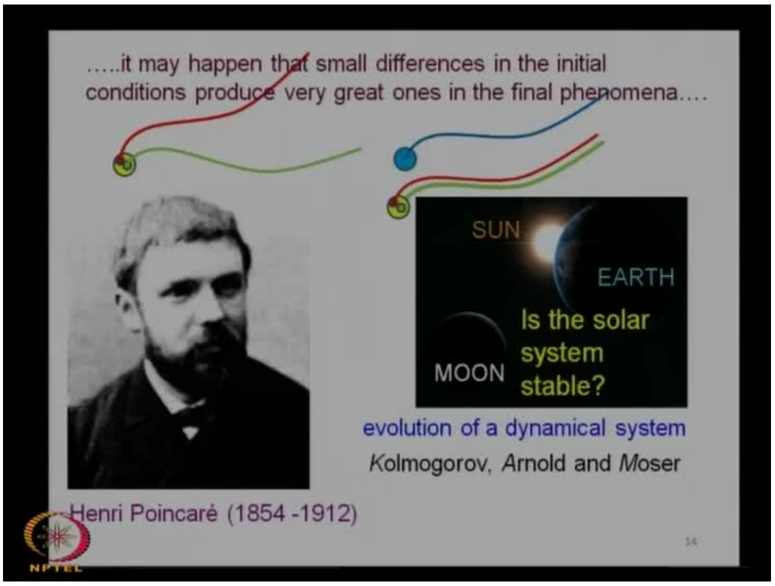

So, the system evolves and a change in the system is what makes it dynamical. So, this is the meaning of a dynamical system. A dynamical system is one that changes over a passage of time. It evolves over a passage of time toward an attractor; so is the meaning taking some form in your minds that the system will evolve toward an attractor. This is our central problem in mechanics.

A question that was posed by the king of Sweden early, toward the beginning of the last century was, if the solar system is stable and the solar system is made up of the sun and the earth and there are several other planets and then some of these planets have their own moons, earth is not the only planet which has got it is own moon, there are other planets also which have their moons. So, the solar system has got many such features; the question that was asked is the solar system stable? The reason this question is asked because, that seasons change periodically you have the same flowers coming up in the same season, the same fruits coming up in the same season and the next year the phenomenon repeats.

Is the same with the monsoon season? For example, you get the rain and you also based on this you are able to make predictions and all this is connected to the fact that the earth makes one round around the sun in 1 year. Now, if this is exact, if it goes on that Elliptic orbit exactly and if this is repetitive, if it is guaranteed, if you can ensure that yes this is going to happen for ever and ever and ever again then, you can predict that the seasons are going to remain more or less the same and so on. There are other parameters and I am not going to get into global warming and so on; but there are so far as at least the sun rise and the sun set is concerned, you can predict that it will be periodic with a certain predictable periodicity.

Now, this question was posed it was addressed by Henri Poincare and it turned out that he analyzes and he said that it is not so and this was in fact the beginning of the research on chaos. So, this was the earliest problem that if you have a system which evolves over a passage of time and this is the same picture that I am showing over here (Refer Slide Time: 33:00). This is the initial state of the system and this system evolves along a certain path and this is I am using this idea in a certain abstract sense. So, you do not have to really think about it as a diagram in the phase space or a diagram in the configuration space. It is the certain mental picture of the temporal evaluation a time sequence over which a system evolves that passage of time.

Now, consider a situation that there are 2 initial conditions. 1 denoted by this 1 circle over here and the other denoted by this other circle over here and both are very close to each other (Refer Slide Time: 33:43). These are the initial conditions and these initial conditions are extremely close to each other and if the initial condition was this upper 1, then the system would evolve along this red path. If the initial condition was this lower 1 the system would evolve along the green path. But, notice that the 2 paths are practically on top of each other. They are close to each other and if the initial conditions are close enough, if they are infinitesimally close, the paths will… are infinitesimally close and they remain close after 5 seconds, after 10 seconds, or after 10 minutes, or after 10 hours or 10 billion years, the system continues to evolve along a path which is not very far from where it would be. If it were to evolve along a slightly different path if the initial condition was slightly different, now this is our common expectation.

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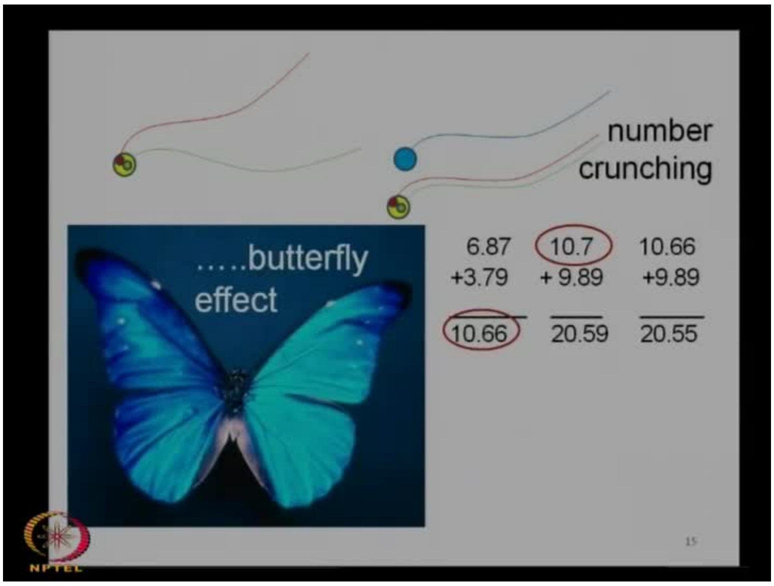

Now, what happens is that sometimes, if the initial conditions are only marginally different it does not guarantee that the evaluation will remain marginally different and let me illustrate this idea by this picture (Refer Slide Time: 33:13). Here you have initial conditions which are very close to each other; the same closeness as we had in this figure (Refer Slide Time: 33:20). But, the evolution - sort of divergence which means, that over a passage of time you will not be able to predict given what the initial conditions were. This is an essential element in chaos theory. This was developed by Henri Poincare, then Kolmogorov Arnold Moser and several others; this is sometimes called as the Butterfly effect and this is the classic, it is a wonderful term and it has been used by various scholars on chaos theory.

The idea is that if a butterfly flutters somewhere in this room and then there are some weather fluctuations in this room, the molecules of the air over here will kick off a cascade of operations. One molecule kicking the other and we do not know how these effects will really cascade over a passage of time and over a distance of time; over a spatial distance as well. What it can lead to is generate a storm somewhere in South Africa, who knows? This is called as the butterfly effect - that the flutter of a butterfly in some part of the world can actually cause a storm in some other part of the world; now, this is called as the butterfly effect.

And it We have to be careful about it because as the scientist we all have to use computers and computers do this arithmetic. Whatever be the science you are doing you may be integrating some function, then it is it boils down to some number crunching there is some arithmetic; some addition, subtractions, but then at some level of accuracy you start rounding off. You may consider the numbers accurate to the second decimal place or may be to the eighth decimal place but, may be to the sixteenth; but at some level, you would start rounding off and then you start getting different number. So, if you round off this addition of 6 point 87 to 3 point 79 and you round off 10 point 66 to 10 point 7 and then you add to this another number over here (Refer Slide Time: 37:52), you get 20 point 59 but if you take the original number you get 20 point 55. You do not know how these errors, the small differences really will add up to in large calculations. So, you might as well get a butterfly effect which will end up giving you a completely nonsense answer.

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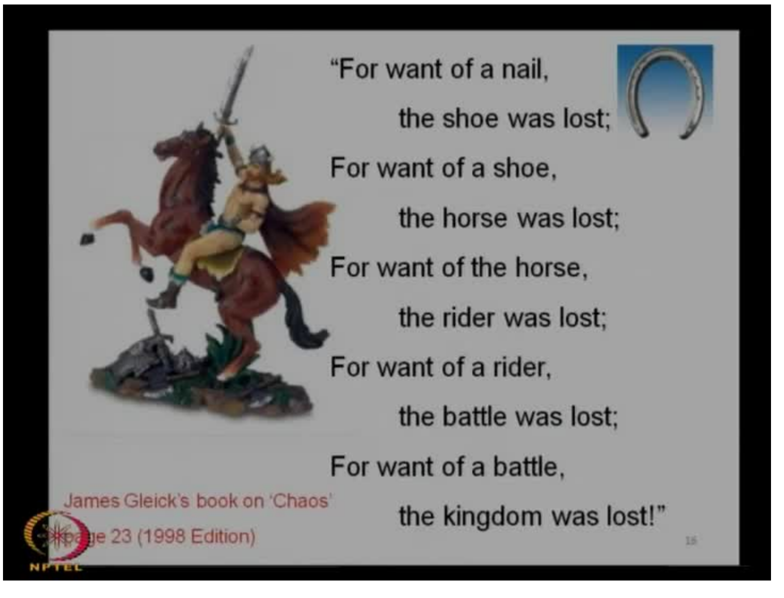

So, one has to be very careful about this, in number crunching. This is very nicely illustrated in a small lyric which you will find in the book by James Gleick. He talks about horse shoe. You know what a horse shoe is. So, what is going to happen if the horse shoe is not there? Means, and this is a very nice, very beautiful poem that, for want of a nail, the horses have this shoe nail, 'for want of a nail the shoe was lost; for want of a shoe, the horse was lost; for want of the horse, the rider was lost; for want of a rider, the battle was lost; and for want of a battle; the kingdom was lost!' Everything, just because you did not have the nail; it is again a butterfly effect. You begin with something very small and all you did not have was the nail at the right time and what you end up losing is the kingdom itself. This is there in James Gleick's book on chaos; is a wonderful book to read and it has another edition; so you will find this in Gleick's book.

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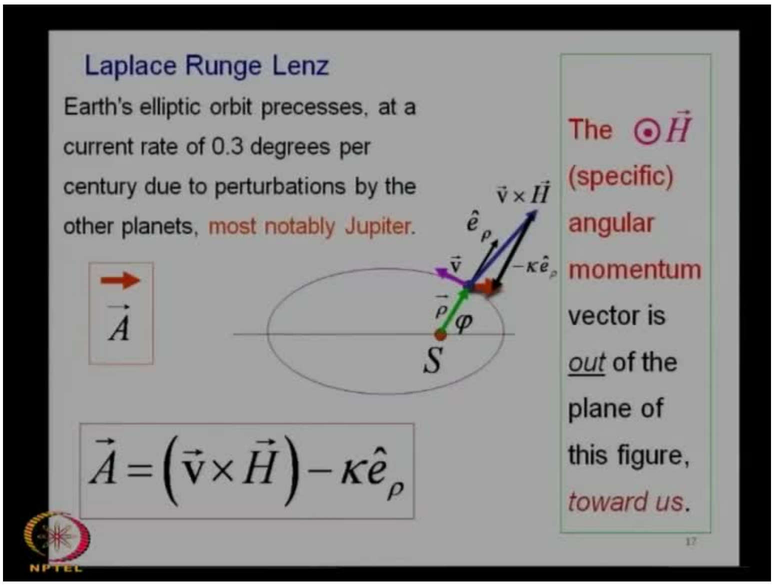

We were talking about the predictability of the solar system. Now, we do know that the earth goes around the sun in this elliptic orbit; this is the velocity of the earth, this is the radial position vector, this is the unit vector in that direction (Refer Slide Time: 39:30). Now, we have discussed this in our unit 4 I think where we studied the Kepler problem. We defined a vector that you have the angular momentum vector which is coming out of the plane of this and then form this angular momentum vector. We construct the v cross h; v cross h is a angular momentum vector, the specific angular momentum vector which I have used, which is the angular momentum per unit mass.

Then you subtract from this, this minus kappa e rho then the addition of these 2 vectors (Refer Slide Time: 40:13), the v cross h and minus kappa e rho gives you the Laplace-Runge-Lenz vector. We have discussed this at some length in our unit 4. Now, what we learn from this is that the earth's orbit is stable and if we go by this consideration, the answer to the question posed by the king of Sweden is that the solar system is stable if this vector is conserved.

But then, we also learned that this vector is conserved for a strictly 1 over r potential if, the whole solar system was made up of only the sun and the earth and nothing else. But, then you have a 3-body system if you deal with the sun the earth and the moon. And it turns out that the earth's Elliptic orbit actually precesses because the Laplace-Runge-Lenz vector is not conserved. It is strictly conserved only for the purely Kepler problem of the 1 over r potential; but the presence of a third body is responsible for the Laplace-Runge-Lenz vector to be not conserved. As a result of which the earth's orbit actually precesses and its current rate is about point 3 degrees per century and it is due to perturbations by others sources of gravity in the solar system.

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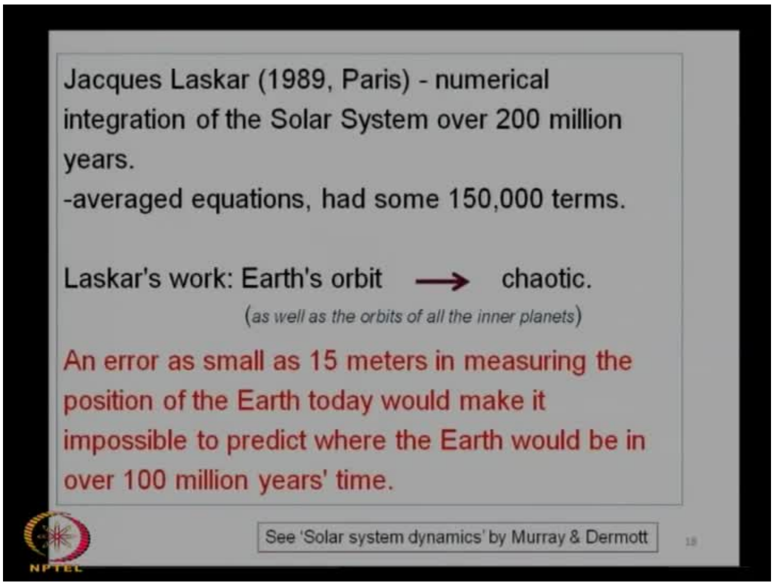

Jupiter plays the most dominant role. It is a huge planet; it just stopped of being a sun as a matter of fact. So, Jupiter is responsible for this and the earth's orbit actually precesses and then you are led again to chaos because, then it becomes impossible to predict where the earth will be. In fact, these calculations have been carried out; these have massive number crunching calculations. This Laskar published this work by numerical integration of the solar system over 200 million years now you and I are probably not bothered about it; but then, as scientists and mathematicians, we are concerned about these things. What he found is that the earth's orbit is actually chaotic as a matter of fact. If there is a small error of only 15 meters in measuring the position of the earth today it would make it impossible to predict where the earth would be after a hundred million years. Again, this is the butterfly effect. The initial condition being slightly different just by 15 meters which is not much of a distance over the huge path that the earth takes around the sun. You can get the details in this in literature, Solar system dynamics by Murray and Dermott.

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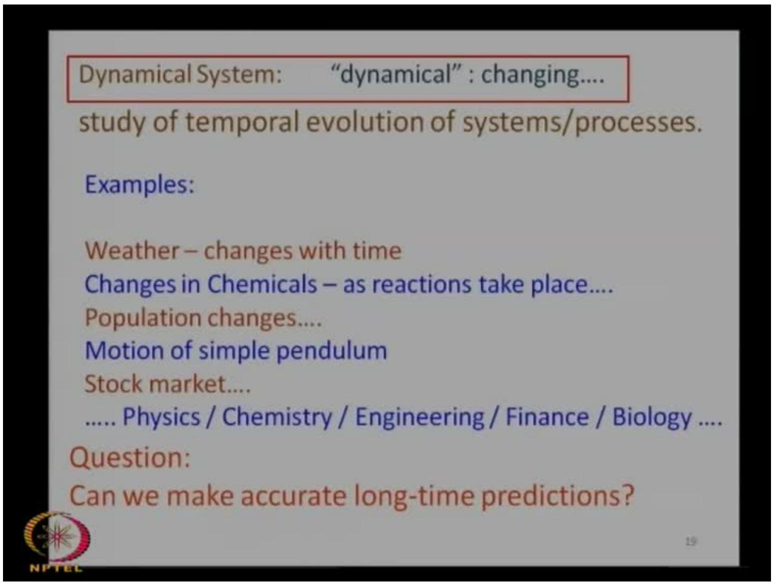

Not only the earth's orbit, the orbits of the other planets also would be chaotic and this is what the study of dynamical systems is about. It is the study of temporal evolution of systems and this is going to happen with any dynamical system which can be just the molecules of the air around you. It could be the position of the earth or anything which changes with time.

So, naturally this has consequences in predicting weather, chemical reactions, population changes, motion of a simple pendulum, stock market that will interest many of you, and the various branches of physics, chemistry, engineering, biology, finance, whatever. Our main concern is, how do we make accurate long term predictions? This is the central problem in mechanics. How do you make predictions? This is of importance and if the initial conditions are even marginally different you may get chaos under certain circumstances.

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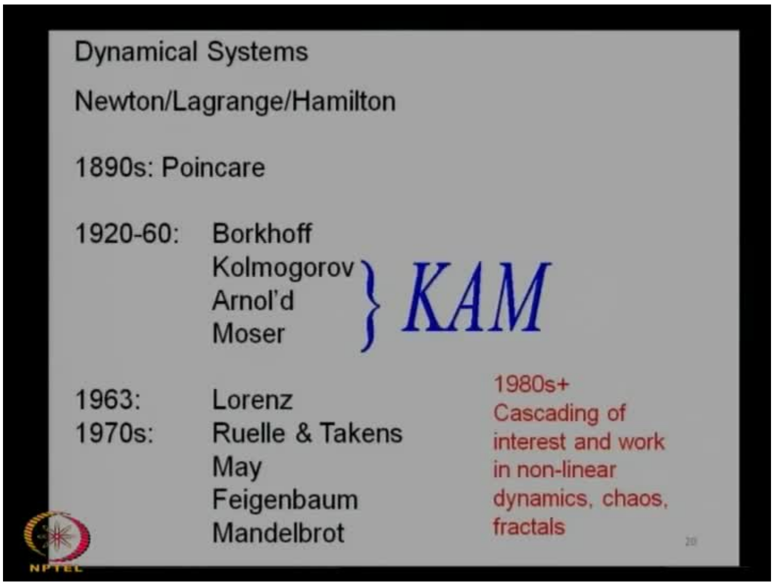

There have been brilliant contributions to the development of chaos theory; Borkhoff, Kolmogorov, Arnol'd, Moser. This is a very famous theorem known after them. This is known as the KAM theorem and in the last 30 40 years, there has been a lot of interest in this field, in non-linear dynamics, chaos and fractals. I will be introducing you to some of these details in this and the next few classes.

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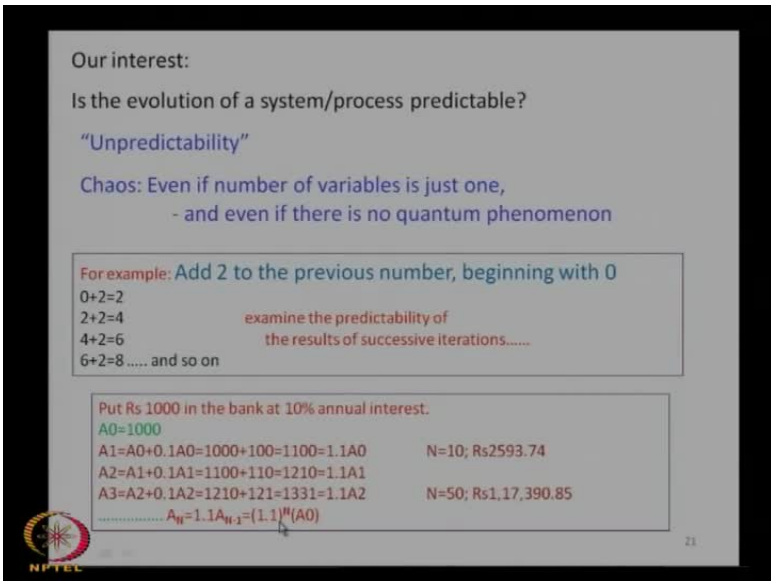

The interesting thing about it is that there is a certain degree of unpredictability which is a matter of concern for us. What is interesting is that this unpredictability is involved even when the number of variables is only 1 and also when we are not talking about any quantum phenomenon. Within quantum theory one knows that you must deal with statistical interpretation from the very beginning. You cannot avoid statistical mechanics in quantum theory; it has nothing to do with the large number of particles that one has to deal with in classical mechanics. You would do averaging and generate statistical laws because you are too tired to follow the dynamics of each individual particle. But, it is not because you cannot; you can in principle; you can because each particle is distinguishable. You can number it and the averaging processes which are involved rely on the distinguishability of particles and your ability to follow them. But, statistics enters because there is no need to follow the dynamics of every individual particle. You are not interested in the speed of every molecule of air. You are interested only in having some average property of various molecules in the air and that is what defines temperature; so that is how statistics enters classical mechanics.

Over here (Refer Slide Time: 46:40), you have even if the number of variables is only 1, even if the phenomenon that you are talking about has nothing to do with quantum theory, you are still confronted with a certain unpredictability which is coming from the sensitivity of the evolution of the dynamical system to initial conditions. If the initial condition is slightly different there are certain control parameters. So that is something that we will continue to discuss and what we have to be worried about is, how accurately can you predict the evolution of a system and let us deal with predictions of certain kinds?

Now, let us say that you takes begin with the number 0; you add 2 to it and then you predict what will be the result. You can predict this accurately; there is no problem. 0 plus 2 is 2; 2 plus 2 is 4; 4 plus 2 is 6; 6 plus 2 is 8; you can go on (Refer Slide Time: 47:43) and you are not worried about any unpredictable feature. Likewise, if you put 1000 rupees in a bank with 10 percent annual interest and then you ask how much money will you get and then the answer is given over here, that you can begin with a certain number A 0 and then you will have to multiply this by 1 point 1 to the power n when you can predict; so that is so far so good.

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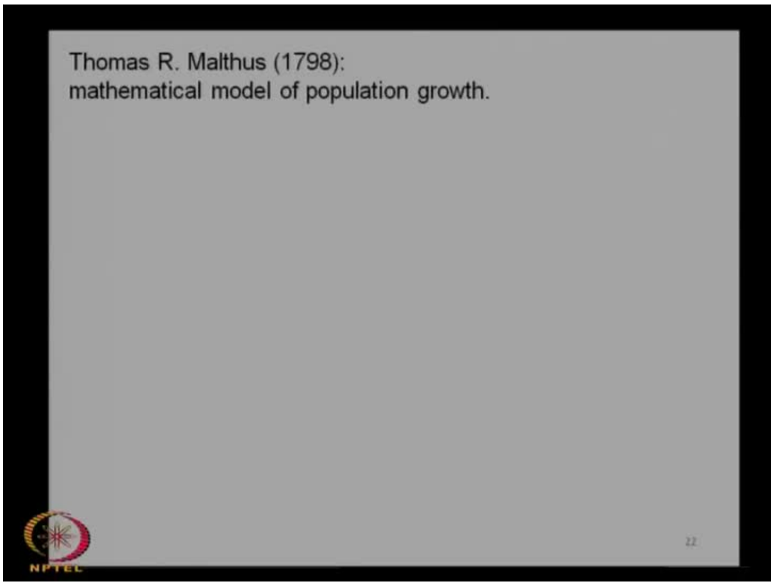

Now, can you always do that? This question in the context of biological species was addressed by Thomas Malthus. He developed a mathematical model for population growth and this is an issue of great importance to everybody; especially to people in India. We are one sixth of the world population; we are more than a billion people. There the world population is a little more than 6 billion I guess. 1 in 6 human beings is an Indian and Malthus developed the mathematical model for population growth. We will discuss this but we are coming toward the end of this class. Before I discuss this I will take a few questions and then we will take a short break and then we will go for the next class; so if there are any questions I will be happy to take.

Nothing was there that is how, that is how we take it and add.

Yes

We also mentioned that it is in other inoculate concept you have to take.

But my question is what about the other two weak matrices.

Well, they would also be because, in any planet will go along a strict Elliptical path only; if the Laplace-Runge-Lenz vector is conserved and the Laplace-Runge-Lenz vector can be conserved, only for a strict 1 over r potential. Now, a strict 1 over r potential between any 2 masses, take any 2 masses, whether it is a bottle and this mouse or 2 planets or the sun and the earth or the earth and the moon you take any 2 objects and if the universe did not have anything else other than these 2 objects, then the potential between them is g m 1 m 2 by r which is a strict 1 over r potential.

But, the presence of a third body automatically guarantees that the potential is not a strict 1 over r potential. But, that was the question posed by the king of Sweden means, in his mind the solar system of importance to him was the sun, the earth and the moon. So, it was the 3-body problem that he dealt with and he asked if this system is stable and what Poincare found is that it is not because this 3-body system is already, it is setup of 3 different objects. That ensures that the potential between any pair is not 1 over r. It cannot be so; there is no way the Laplace-Runge-Lenz vector can be conserved.

So, for any planet the motion will be chaotic as to how long it will take, excuse me, how long it will take to make measurable differences is the matter of detailed computation and very extensive number crunching will be involved. So, this computation was done by Laskar for the earth and the sun system and he found that the bigger influence really came from Jupiter. Well, understandably so, because it is a very massive object; so it has a much larger perturbation on earth compared to anything else.

So but The fact that the universe does not consist of just 2 masses which are interacting by through 1 over r potential guarantees that a Kepler Ellipse cannot really survive. It will have to précess; it will not only have to précess, there will be other chaotic consequences. We do not have to worry too much about it because it is not going to happen in our lifetime; yes.

It is not going to arrive to us for a million years because earth is too far.

Yes

I mean for all the main days will be the earth sun moon relationship.

Yes

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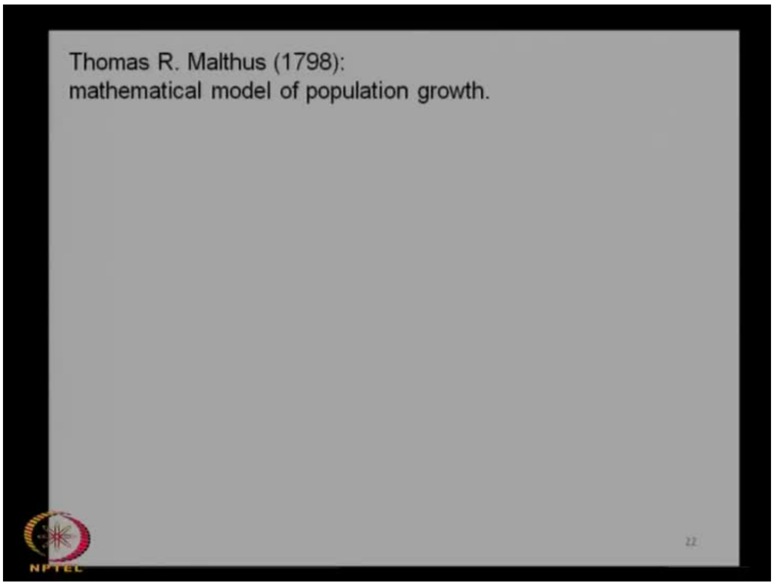

Happened we did not find any Measurable deviation

Right, means it depends on how accurately will people have been measuring things? Means, people did not carry out any measurement a million years ago. Measurements began quite recently; measurements began very recently. I believe the earliest means people have been studying the motion of planets and so on, for may be a few thousand years.

No; what I am saying is that if there is any deviation cannot we see it from the orbital and the…

I do not think that few thousand years is enough to see anything means we are talking about like a million years and what is the 5000 years in that.

It is only because the earliest observations means if you really go by some observations which may have been carried out at the time of Mahabharata which was like 5000 years ago and at that time also people did predict how, where the Jupiter will be on a certain date and so on. Then more recent astronomy developed perhaps with Galileo which is just a 500 years ago when telescopes were developed. Then you started making more detailed observations and Tycho Brahe systematically recorded the data and so on.

But that is just a 500 years ago. So, even if you go as far as 1000 years ago, means you certainly there is, there are no data available which are older than a 5000 years ago. Perhaps, the longest that I can think of is about 5 or 6 or 7 thousand years ago and certainly not older than that. So, this is too shorter time; so science is very young; science is extremely young; it is a baby in a certain sense.

But, for time being you can simply say it is we follow a night-day path from the sun.

Yes, but we can equally safely say that it cannot be forever and this is something we can say because, now we are conscious of the fact that the universe does not consist of just 2 bodies interacting through 1 over r potential and exactly what is the reason for the departure; so that there will be chaos.