Select/Special Topics in Classical Mechanics

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Module No. # 11

Lecture No. # 37

Chaotic Dynamical Systems (iii)

Greetings, we will continue our discussion on chaotic dynamical systems. We had a little bit of discussion on bifurcations in our previous class and we started talking about chaos in general. We mention the terms attractors and also the strange attractor and we will examine this in further detail in today's class.

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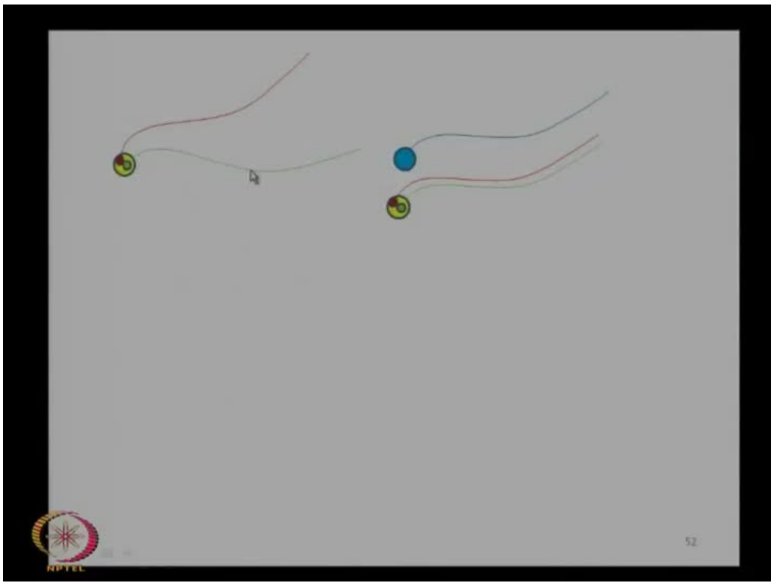

Our consideration over here is the following, as we mentioned earlier that if you have a system whose initial state is represented by this blue point which represents the initial state of this system in a certain configuration space or phase space or it is any initial condition of a set of parameters in which you are interested in it.

These parameters over a passage of time, let us say evolve; this is the temporal evolution over a passage of time. If we consider 2 initial conditions - as we see in this lower figure

- which are very close to each other. We expect that the evolution will proceed more or less along similar trajectories and these are gainer trajectories in whatever space we are talking about, it could be the configuration space or the phase space or some parameter space, which represents the temporal evolution of the system.

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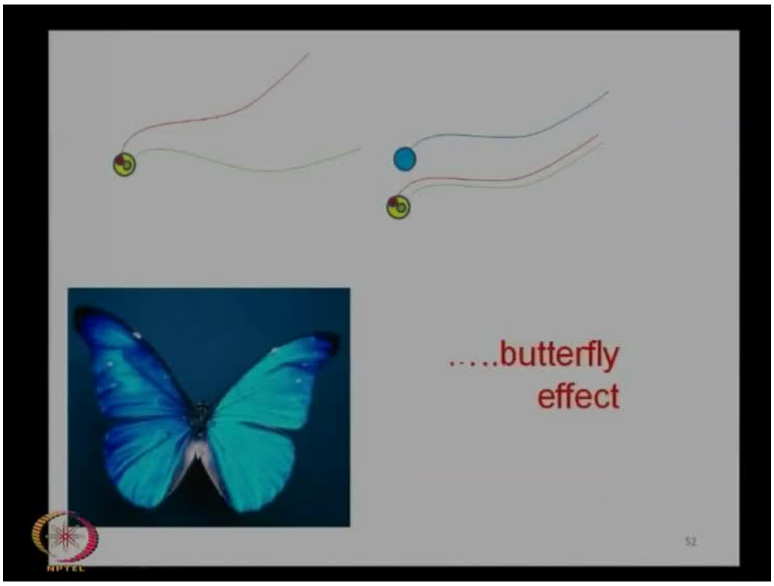

What often happens is that even if the initial conditions are very close to each other, the temporal evolution does not always go along paths which are close to each other but they start diverging. Essentially means that over a certain passage of time it will become impossible to predict what the system will evolve to after a sufficiently large time interval. This is what leads to chaos and this is what we mentioned in our previous class as the butterfly effect.

That you can have a butterfly flutter in some part of the world and then, the flutter causes a sequence of events beginning with how the molecules of air knock the other adjacent molecules of the air. How this effect cascades across from one point to the next, to the next, to the next and travels not through just you know few angstroms or centimeters or meters or kilometers, but even thousands of kilometers and it could actually causes storm in some other part of the world. So, this is the butterfly effect that is often invoked to describe the sensitivity of the temporal evolution of a system to initial conditions.

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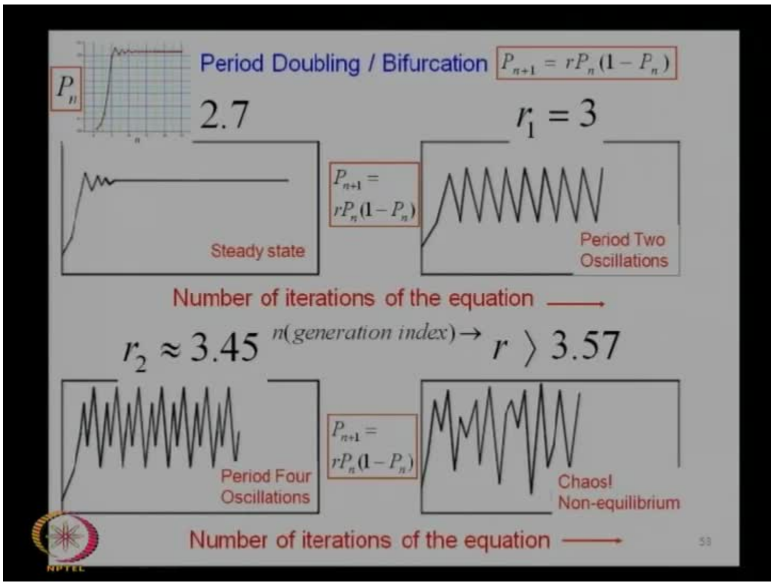

We studied in particular map, this difference equation which is a non-linear equation. You see that there is a term which is quadratic in P n and it is a difference equation - the corresponding analog in differential equation - did not lead to chaos as we saw but in this map it does. What it does is that depending on the actual value of the control parameter which is this fecundity coefficient that we refer in our previous discussion, this r if it has got a value of 2.7. Then, if you sketch this P n as a function of n, you estimate its value from one generation to the next and plot it. Then, it rises rapidly then there is a reversal slope changes and you see these oscillations; but, at some point these actually settle down to some value which we have considered as an attractor, because it settles down to that particular value and there is really no chaos at all in this particular case.

In order to examine the sensitivity of P n to this control parameter, we consider the few other values. We considered r equal to 3 and we found that after a certain number of generations P n settles down to one of the two values. So, it is not unique but you get one of the two values either one at the upper end of this curve or the one at the lower end one of the two. These two values actually alternate in what is called as period 2 oscillations and you do not get any chaos but the attractor is not a single value it has got two values.

For another value, if you keep increasing the value of r, if you go to 3.45 for example, you find that the value of P n that you get by solving this equation is not unique, but it has already gone through bifurcation. A bifurcation again the result is that you get P n to settle down to either one value or another or a third or a fourth but then, after the fourth one it comes back to the first and then it is repetitive.

So, here you have period 4 oscillations; here again the attracter is not a single value, it is one of the 4 values but then, the result of P n does not go outside the set of these four values that we are talking about. However, if the control parameter exceeds 3.57 then, there is absolutely no regularity and you simply cannot predict what peer will turn out to be. This is the bizarre behavior because you know if it is just a little bit less than 3.57 like 3.55 for example, you are not going to see this behavior, You will get period 4 oscillations predictably, but if you exceed that then you cannot make any prediction at all and then, the behavior becomes completely chaotic.

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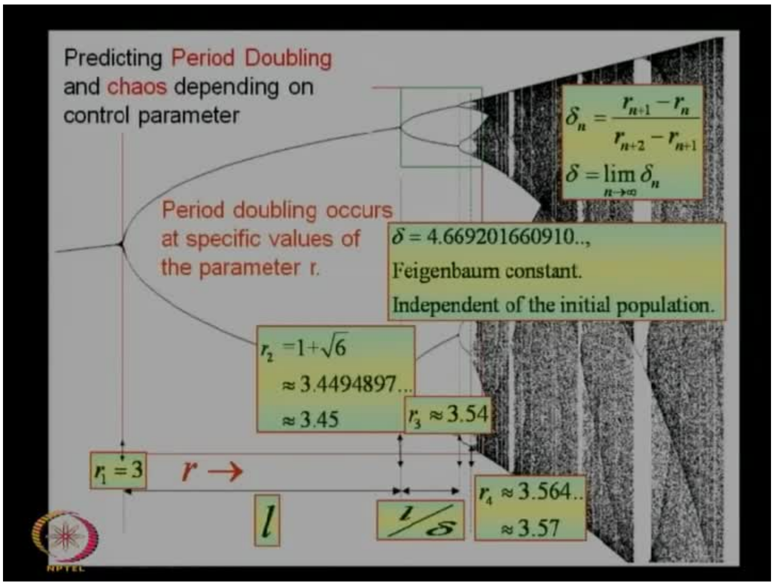

So, it is very sensitive to that value as whether it is somewhat less than this 3.57 or somewhat greater than that. This is what we have seen over here that period doubling occurs at very specific values of the parameter r, so we have seen it happen here then, it happens here, then it happens here. Here again there is a period doubling, there is a bifurcation of the upper branch, there is a bifurcation of the lower branch again, into this upper and this lower. So, this is the upper branch of this one and this is the lower branch of this branch.

Again, there is this bifurcation which keeps taking place and the first bifurcation takes place at the value of r equal to 3. The second bifurcation takes place at the value of r which is nearly 3.45 but not exact, it is 3.449 so on. Then, the third bifurcation so that instead of period 4, you will get period 8 oscillations. This you will get at a value of r which is about 3.54 and then, again at 3.56 you have this but if you exceed this then you run into the chaotic region.

Then, the dynamical system being 1 it changes over a passage of time and the behavior of the dynamical system becomes completely chaotic. We also found out that if you take the ratios of these intervals, so this is essentially the interval between these vertical lines, these vertical lines are drawn at those values of r at which the bifurcation takes place.

If you take adjacent segments and take the difference, so here in the numerator you have the difference between r n plus 1 and r n. If the denominator you have a similar difference but corresponding to the next set r n plus 2 minus r n plus 1. If you take this ratio then, this ratio as n tends to infinity settles down to a constant, this is what is called as the Feigenbaum constant. What it helps us is, it is the scaling factor and it tells us something about when you can expect chaos.

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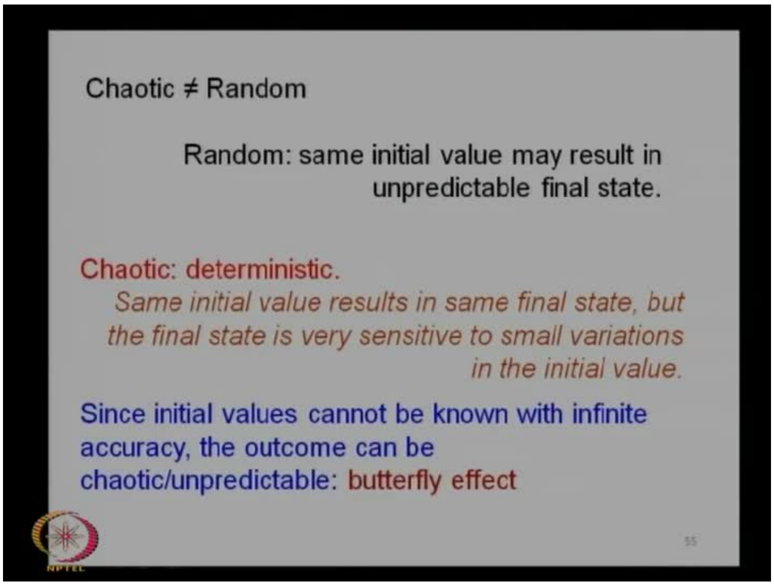

We also discuss the fact that chaotic does not mean random; random would mean that if you have an initial value then, the result will be completely unpredictable - final state. If you have a given initial value but what happens in chaos that the same initial value will always present the same final state, so that feature is not unpredictable. What is difficult is that the initial condition cannot be known with a huge amount of accuracy, you cannot know it with infinite accuracy. Therefore, if the initial condition is even slightly different then the result will become completely unpredictable and that is what leads to chaos.

So, it is quite different from being random and it is nothing to do with having a large number of parameters or anything like that. Here, we are dealing with a single parameter whose initial value if it is slightly lower than what would lead to chaos? You get regular behavior, but if it exceeds that value then you get chaotic behavior, also this has nothing to do with the quantum theory - the quantum principle of uncertainty.

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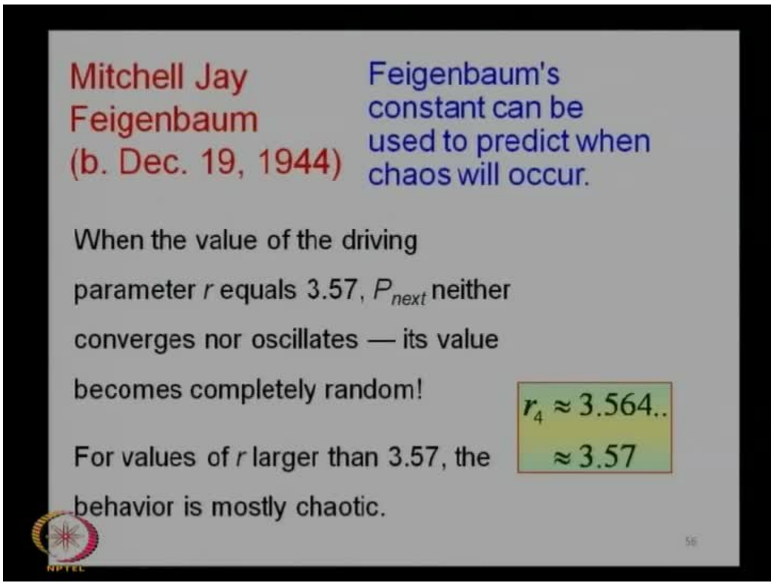

So, this is a very exciting field and it has got large number of consequences in various branches of science as well as engineering. These values need to be talked about with a high degree of precision to various places of decimals. So, let you find out what these values are from literature or you can do some arithmetic on your own and calculate these values with your own programs and discover the exact values.

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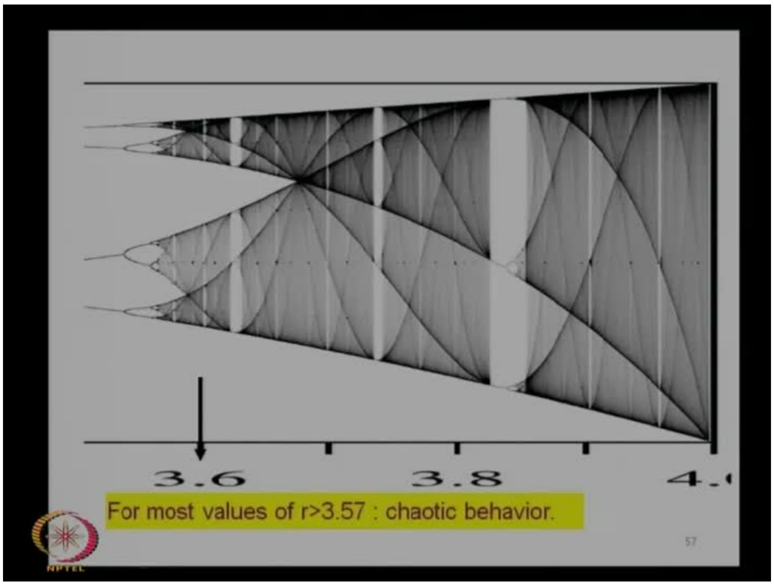

This is how the picture looks like, this is a little bit of magnification of the previous figure and it focuses on a certain region of the value of r, so on the x axis from left to right it is this value of r which is increasing from left to right. We already know that at r equal to 3 we had the first bifurcation and then, we had a number of bifurcations. So, we have one over here and another over here, but then we do see that there are certain regions, this one for example, here you have 1, 2 and 3 and there seems to be a period 3 oscillation. What is following period 3 oscillation? Is the bifurcation over here and over here and even over here.

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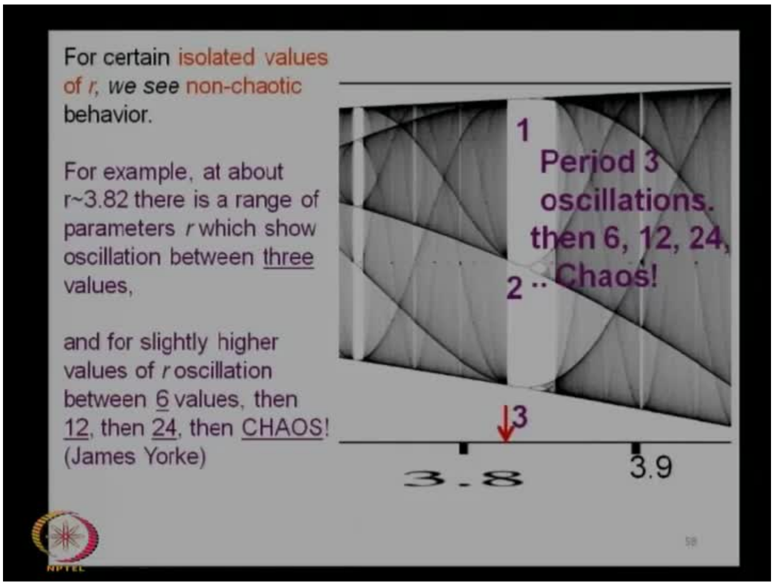

Now, we start looking at these details is to what is really going on. Now, these are certain isolated values of r and a period 3 oscillation over here is obviously not chaos because you can predict the attractor. The attractor is one of these three values which means that the system would evolve to one of those three, but you can predict that it will have to be one of these three, so it is not chaotic.

There is a certain non-chaotic behavior for certain isolated values of r and the cases of interest take place. We are looking at this particular region which is at r equal to this is almost 3.82, this is 3.8, this is 3.9 and this is about 3.82. Here, you see that at this value of r you have a range of parameters not just a unique one but from here up to here. So, r can be 3.82 and a little more than 3.82 and a little bit more than that as well and you will continue to have period 3 oscillations.

So, over a certain range of parameters of r - notice this word range over a range of these parameters - you have oscillations of period 3 and after this you have bifurcations over here then, you have period 6 oscillations and then period 12 period 24 and so on and then you get chaos again.

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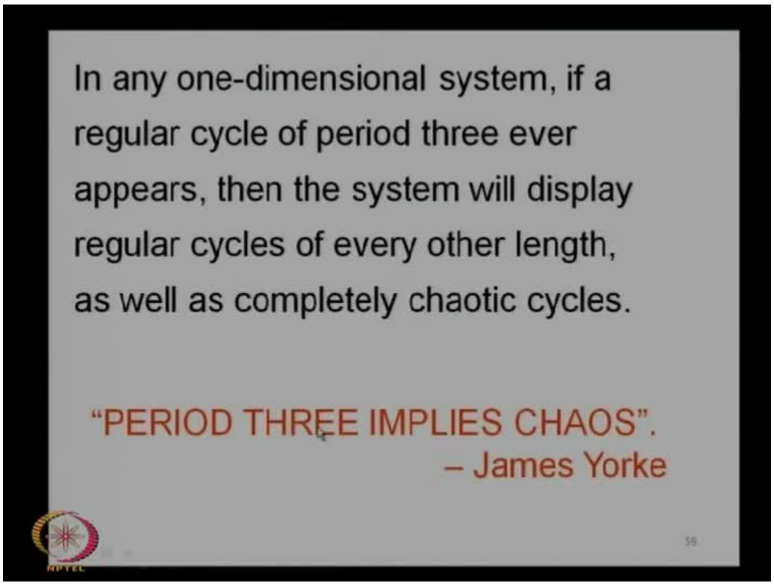

Now, this is a very famous theorem named after James Yorke, this is called as Yorke's theorem, what he found out is that period 3 implies chaos that if you have a one-dimensional system and if a regular cycle of period 3 occurs then, the system will display regular cycles of every other length as well as completely chaotic cycles. Now, this is what Yorke found out and this is known as the Yorke's theorem, this is a very famous line in chaos theory, the period 3 implies chaos, this is for one-dimensional systems.

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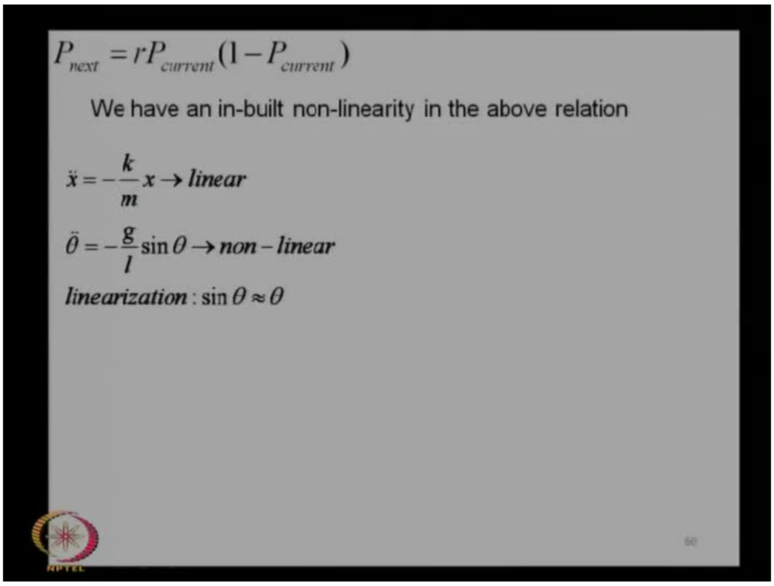

Now, let us study this in some further detail; let us examine what is this non-linearity. If you consider a differential equation like this; this is the differential equation for simple harmonic oscillator which is very familiar to all of you. You have the same differential equation for a simple pendulum, if you construct, then you have g over l sin theta - you know what theta is, the angle which the pendulum would make with the vertical line if it is suspended and set into oscillations.

If you take sin theta as it is then, you have a non-linear term because sin theta is an expansion in all powers of theta but you linearize it. If you take the lowest approximation to sin theta for theta tending to 0 for small oscillations and when you talk about small oscillations what you mean is that they are small enough for you to consider sin theta to be nearly equal to theta.

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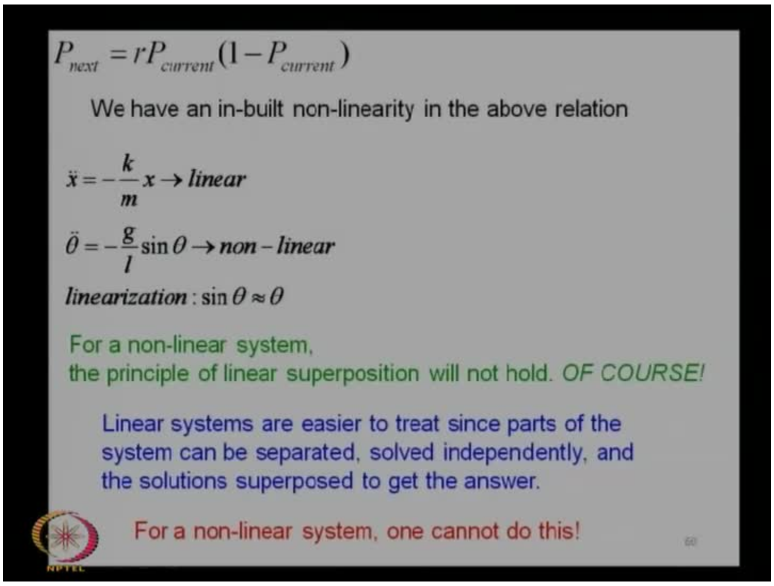

So, in the regime that you are willing to accept this approximation then you have linearization. The advantage of linear systems is that you can work with the principle of superposition for a non-linear system of course, you cannot do linear superposition.

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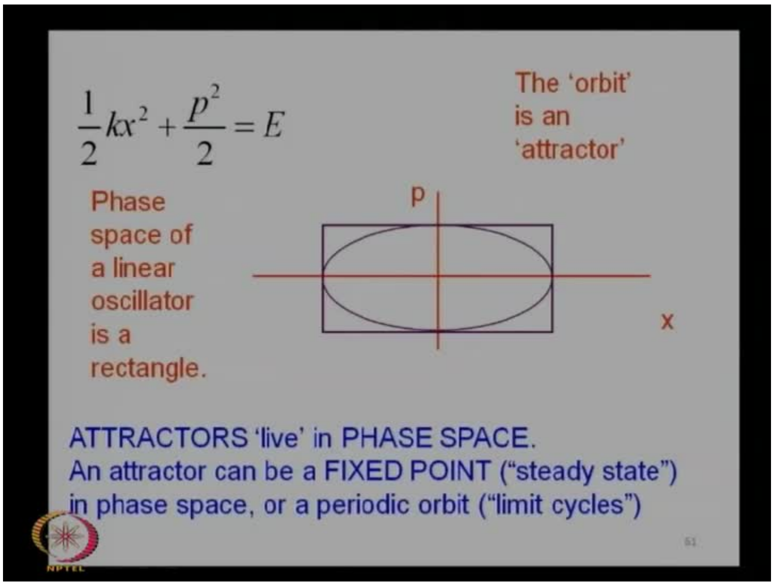

Linear systems are easier to work with and the reason is that it consist of various parts which can be dealt with independently. Then you can superpose their solutions, the way you generate these Lissajous figures for example. So, you can solve the problems independently and then, you can superpose the solutions linearly to get the solution to the full problem, but for a non-linear system you cannot do this, so let us look at the simple harmonic oscillator. Now, we are looking at the phase space diagram, the phase space consist of the position and the momentum.

If you plot p and x what you get from this solution - this is essentially the equation to an ellipse in the phase space - and you do know that x and p will both change from time to time. So, there will be a temporal evolution of both x and p; x will oscillate from this extreme to this extreme and back, so x will oscillate like this, p will change from up and down, but the description of the state of the system which is represented by the point at the phase space will always remain on this trajectory which is a closed path, which is an ellipse and the system will not ever get out of this ellipse. If the dynamics is governed by this simple harmonic linear oscillator.

What this essentially means is that the attractor of this particular system is this ellipse, it is a closed orbit and when it is a periodic orbit this attractor is called as a limit cycle. So, a limit cycle is essentially this periodic orbit and an attractor can also be a fixed point, but it can also be a periodic orbit. In this case, this orbit is the attractor which is the limit cycle.

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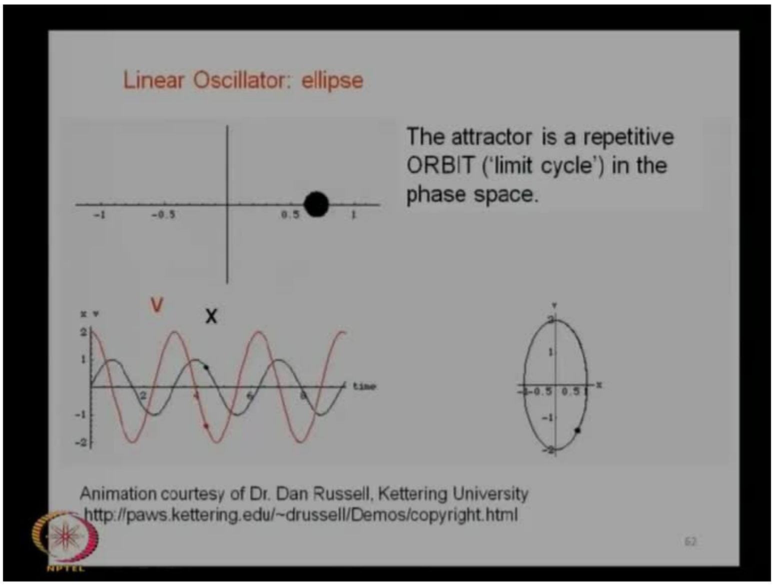

Here is the temporal evolution of V and X of a simple harmonic oscillator, this is from Dan Russell's website at the Kettering university. He has a nice diagram you can generate this yourself if you like, you are solving the equation for a simple harmonic oscillator, you have the sinusoidal solutions. The position is a sinusoidal solution which is shown by this black curve, the velocity is also an sinusoidal solution shown by the red curve.

The phase space trajectory is this ellipse as we discussed just now, so the attractor is a repetitive orbit which is called as a limit cycle in the phase space. Now, we do know that a linear harmonic oscillator is a very ideal kind of situation and for real oscillators it is very difficult to avoid damping. So, when you do have damping how will it affect the phase space diagram? Let us look at that.

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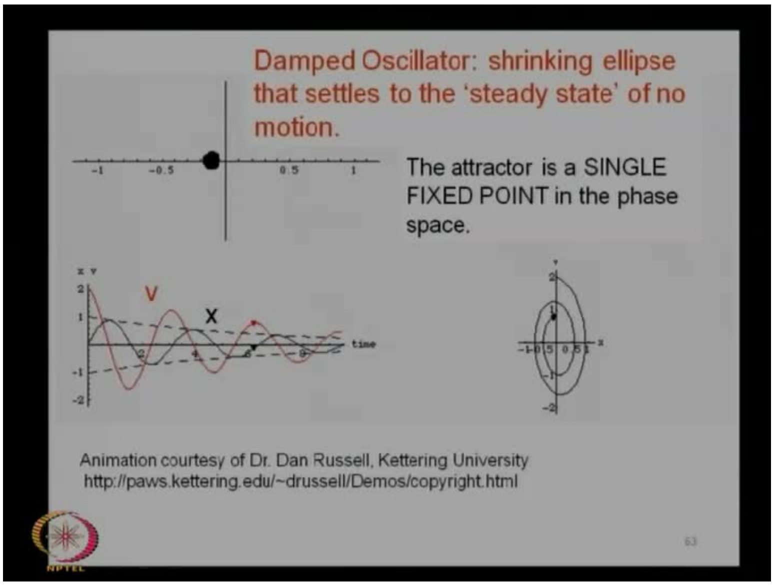

Here is the picture of the damped oscillator, you know that the amplitude of both the position and the velocity will eventually diminish. So, this is the damped oscillator and if you plot the phase space trajectory then, it will be this and this path which will spiral inward and settle down to the center. It essentially means that the attractor for this would be a point, it will be a steady state of no motion at all, the damped oscillator because after sufficiently long time it will settle down to a single point. The attractor can be a closed orbit as we saw in the previous case, it can also be a single point like a steady state.

This is the fix point in the phase space, so the attractor can have various forms, it does not have to be a closed orbit, it does not have to be a single point. In the case of period 2 oscillations, it is one of the two values. In the case of period 4 oscillations, it is one of the four values. In that context we are not talking about a point in the phase space, but we are talking about what is it that the system can evolve to, that question is answered in all of these issues.

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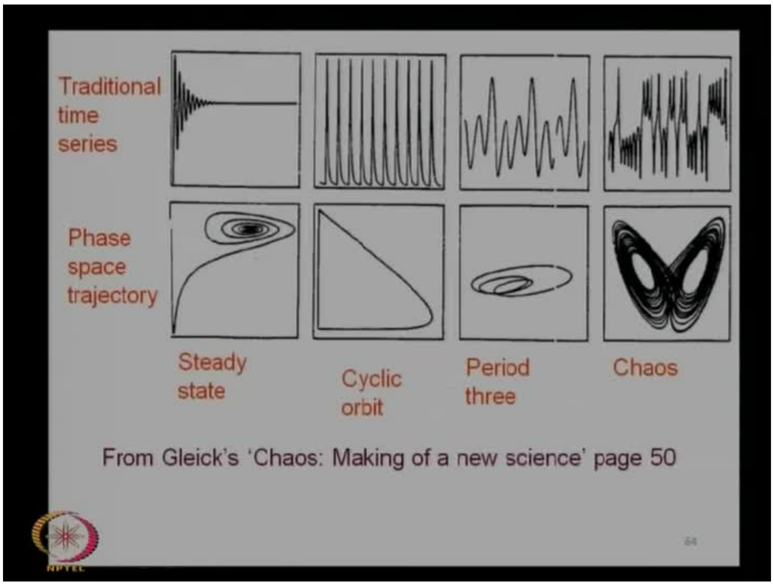

You can have attractors of various kinds and if you plot the time series which is the traditional time series and you can plot the corresponding phase space trajectories. In case of damping, you can have a steady state which is reached is the closed orbit or else in certain cases you may have cyclic orbits depending on the details of the cyclic pattern, you may have period 3 oscillations. This you could have a repetition of this pattern over and over again, no matter how many generations you examine this or you may have a temporal series of this kind and this really looks chaotic in a certain sense.

What is interesting is that if you plot the corresponding phase space trajectory you do recognize chaos nevertheless it stays within a certain domain, it does not exit this particular domain, so it is chaotic but remains in a certain domain. This is the case of a strange attractor, so I will explain this in further detail, but I hope you are beginning to see the difference between an attractor and a strange attractor, these examples again are from Gleick's book.

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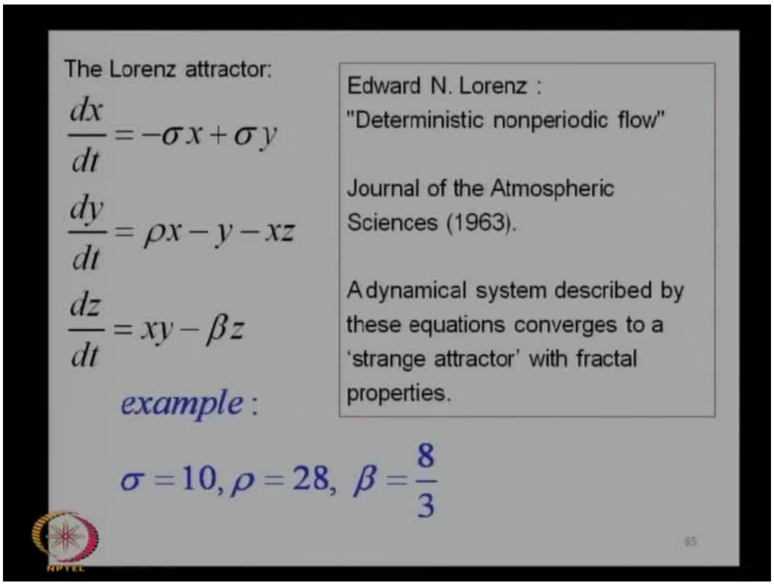

The classic example of a strange attractor or sometimes it is also called as the Lorenz attractor, because one of the best known examples of a strange attractor is the Lorenz attractor. This was studied by Edward Lorenz in the context of certain system of equations which he considered in atmospheric science and this was published in the journal of atmospheric science in 1963.

Here, he considered a dynamical system described by set of equations. Now, these equations x, y and z are coupled differential equations as you can see. This is equation for dx by dt, this is dy by dt, this is dz by dt, let us not worry about this, x, y and z are 3 physical parameters, it could be the temperature for example, it could be pressure, it could be anything.

I am not going to get into those details what I am talking about is that if you have 3 parameters x is a function of t, y is a function of t, z is a function of t these are all functions of time and the temporal evolution of these parameters is coupled. So, the differential equation the rate equation for x which is given by dx by dt is dependent not only on the value of x but also on the value of y, likewise the differential equation for y depends on x and also on y and also on z.

In physics or in science in general, you will run into all kinds of situations in which you have to deal with maybe 2 or 3 or 4 or more parameters and the rate equations for them are coupled. Then, if you construct a parameter space these been independent degrees of freedom you can plot x along the x axis, y along the y axis and z along the z axis, they do not have to be the space axis in the Cartesian frame of references. These are just 3 physical parameters x, y and z, but being independent you plot them along 3 orthogonal axis and that becomes the mathematical parameter space which describes the temporal evolution of the system.

Lorenz attractor you can solve this for different values of sigma, rho and beta. There are 3 additional parameters sigma, rho and beta and these are the classic examples which have been discussed very extensively in literature. So, you will find these examples in chaos literature in text books and so on. Let us look at the solutions for these parameters now.

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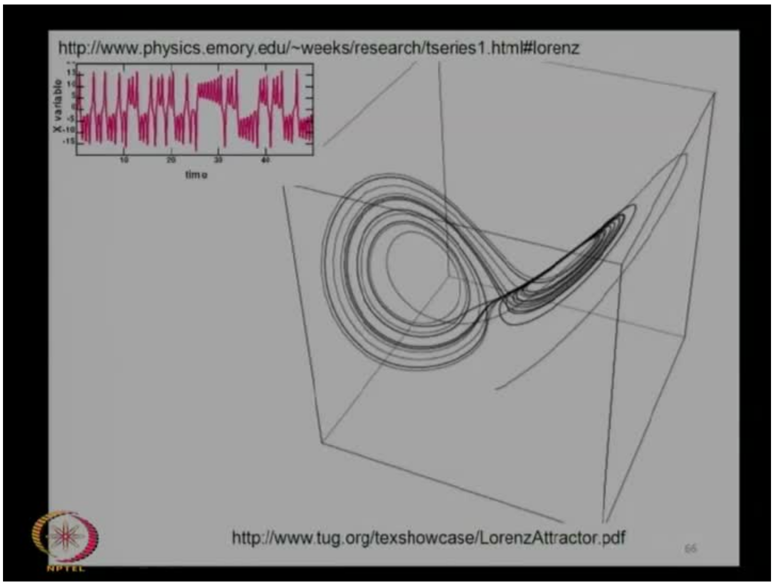

Now what happens is that if you plot x as a function time, you get this kind of chaos, you get it complete chaotic behavior. Now you plot x as a function of time along one axis, y is the function of time along another and z as the function of time along the third and at a given instant of time, you now have a point in this 3 dimensional parameter space. We describe the physical system and how this point traverses in this 3 dimensional space is now what will describe the temporal evolution of the dynamical system.

It does not have to be the phase space, it does not have to be the configuration space, it does not have to be the Euclidian space, it is a mathematical space generated by 3 degrees of freedom namely x, y and z, where these are 3 physical parameters of interest. They could be temperature, they could be pressure, they could be density, they could be viscosity, they could be just about anything; they could be population in some case.

In biology these parameters may involve populations, they may involve the birth rates, they may involve the death rates, they may involve a very complex situation in which you have two different species, one eats on the other and then there may be some survival mechanism.

This is the kind of example that we began discussing with when we discuss the biological rate equations. So, you may have populations represented by x, y and z these may be the populations of three different species or organisms. How they evolve it with time is what we are studying subject to certain values of sigma, beta and rho in the differential equation.

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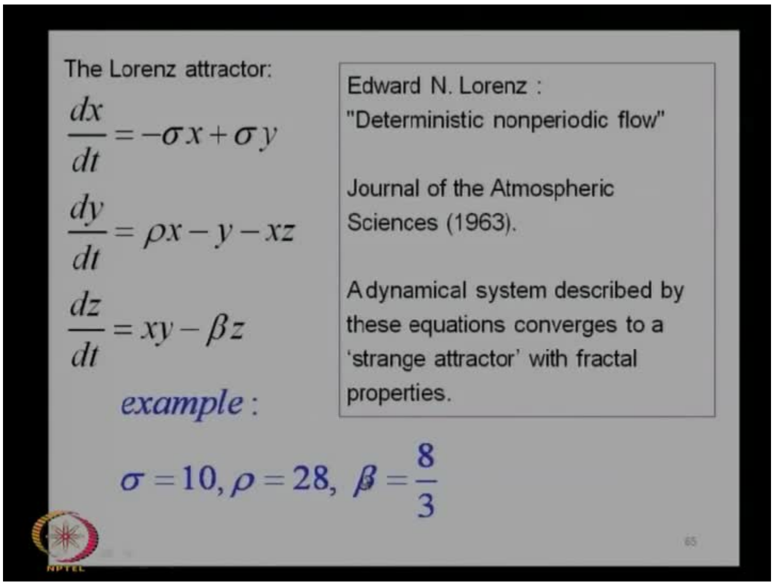

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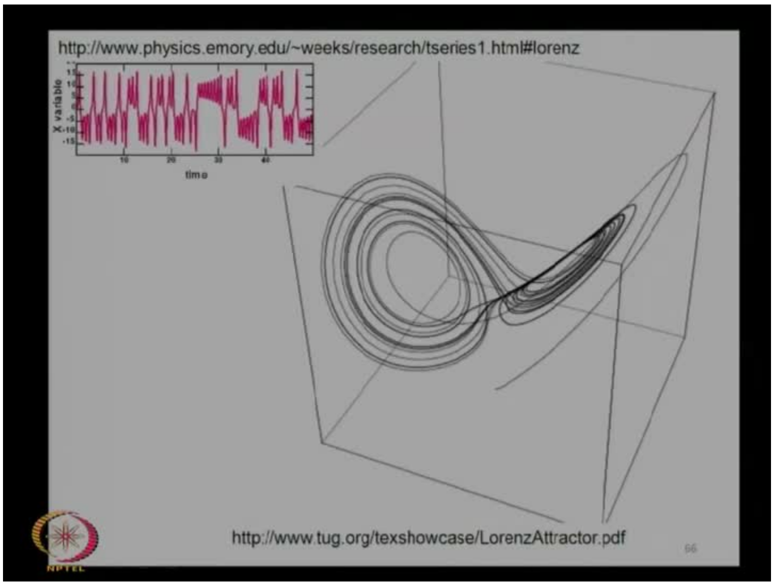

So this is the set of differential equation that we are dealing with and these are the values of sigma rho and beta for which we have plotted the temporal evolution of the dynamical system. Notice that it looks very chaotic but what is interesting is that the solution always remains within this box, it is not getting out. Where it is in the box is unpredictable, that behavior is bizarred that is extremely sensitive to the initial condition.

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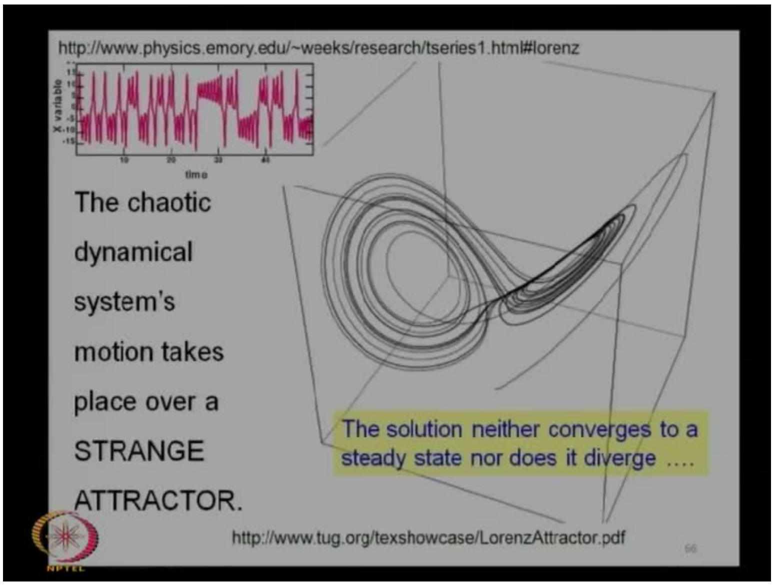

If you change the initial condition, you will get a similar pattern but not this. As a matter of that you will get a pattern, which over extended time evolution will evolve into a completely different pattern, you will see this figure in one of our next slides. You will see that as well and what you are seeing over here is that there is chaos, but the motion is in a certain space which nevertheless is confined within the box.

So there is an attractor, but now you cannot describe this attractor as the steady state like a single point or as one of the two alternative values as you have in a period 2 oscillation or one of the possible 4 values as you have in a period 4 oscillation or in damping, you would know that you have steady state. You do have an attractor but, you do not have any simple description for this.

So this is called as a strange attractor. Now this solution it does not converge to a steady state, but it does not diverge, it does not blow up to such an extent that the solution will go out of the box, it will remain in the box. So it does not completely diverge, so there is some kind of ordered are meets disorder. There is chaos but, there is some confinement of the state of the system over a passage of time. So this is what is meant by a strange attractor.

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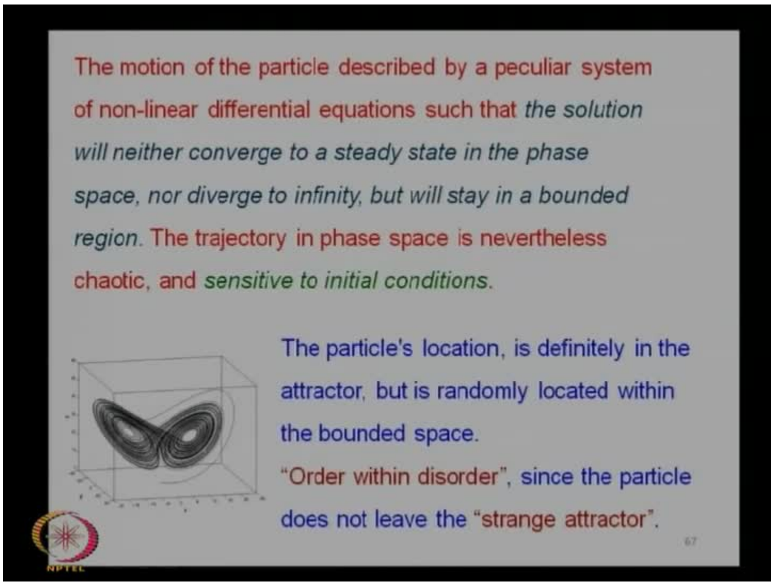

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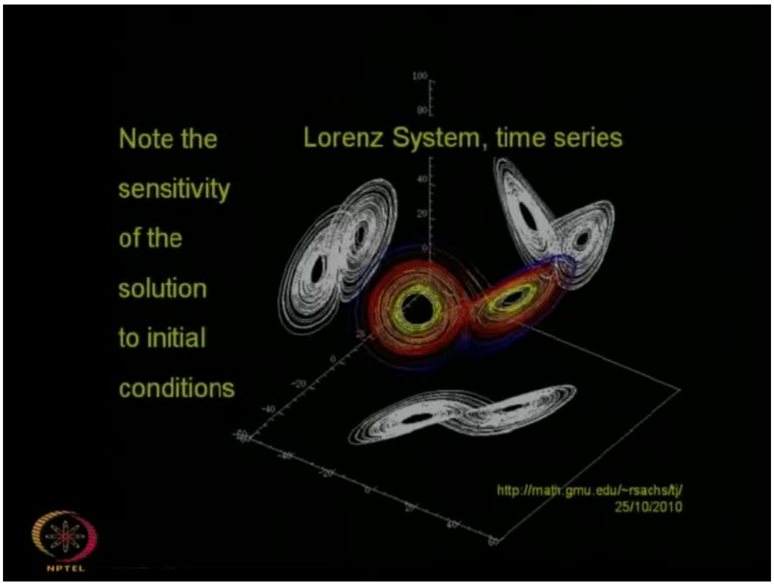

Now let us ask this question, as to how does this pattern respond to initial conditions? It is very sensitive to initial conditions, the solution does not diverge to infinity, and it remains confined to a bounded region of space. These are the characteristics of a strange attractor that the solution will be chaotic; nevertheless remain in the confines of a bounded region of the parameter space that is of interest. This picture will tell you, how it is sensitive to the initial conditions. Now just look at the central figure which has got these colors; now let me see if I can get the cursor over here yes, here it is.

So look at the central figure, it looks like a figure of 8 in a certain sense and if you look at these, there are different colors and what these different colors correspond to? They show temporal evolutions over period over a passage of time for slightly different initial condition.

If you remember this that if you have the initial conditions to be slightly different, then just after t equal to 0 after the initial state, the evolution goes together more or less but then with passage of time over large time intervals, over large time durations then the solutions diverge. Then the initial curves which would on top of each other and they being top of each other, you see only one colour inside which is this yellow but then you begin to see the other colours there is another one over here, this red and a blue and so on; you can say really see that they have branched out.

So initially it is possible to predict what the temporal evolution is. It goes more or less together, but then the branching has already taken place because the dynamics is sensitive, it is extremely sensitive to the initial condition. The differences may not show up in the earliest of times but over sufficiently large time, then these differences show up and then you start seeing these different colors.

So what you see also are these additional figures which are the projection of this on this plane (Refer Slide Time: 33:46). If this is your x axis and this is your z axis then, this figure over here is the projection of this figure on this plane; likewise this is a figure on this bottom plane and this is the projection of this figure on this side. So if this is the x axis, this is y and this is z then, the corresponding planes are the zx planes, the xy planes, and the yz planes and you see the projections of this on these planes. Now there is a remarkable sensitivity to initial conditions which is characteristic of chaos.

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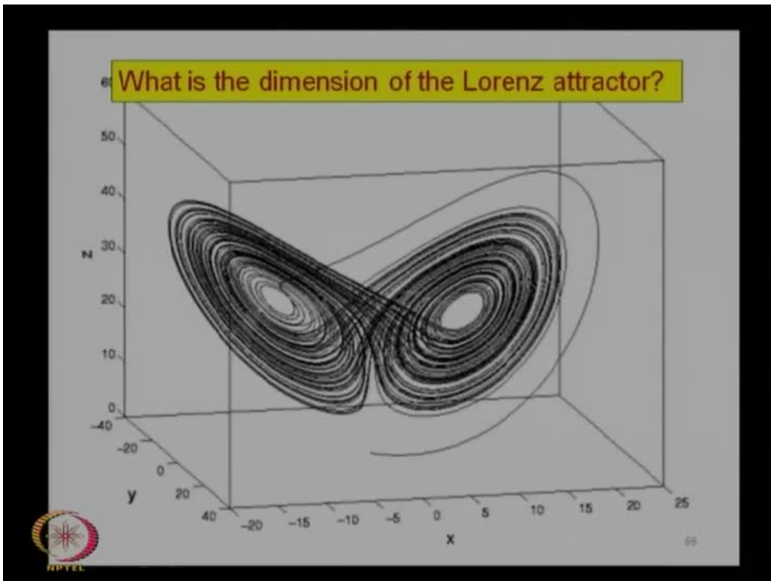

Now, let us ask this question, what is the dimension of the Lorenz attractor? We know what the Lorenz attractor, so Lorenz attractor is the strange attractor. What do you think is the dimensionality of this figure? Is it 3 or is it 2? It does look something like a 2 dimensional object but a little bit worked.

So in a certain sense it is like a 2 dimensional object, in certain sense it is like a 3 dimensional object and in some sense the dimensionality of this is somewhere between 2 and 3. Now, we get into the discussion on Fractal dimensions, because the dimension is now a fraction it is the fractional dimensions.

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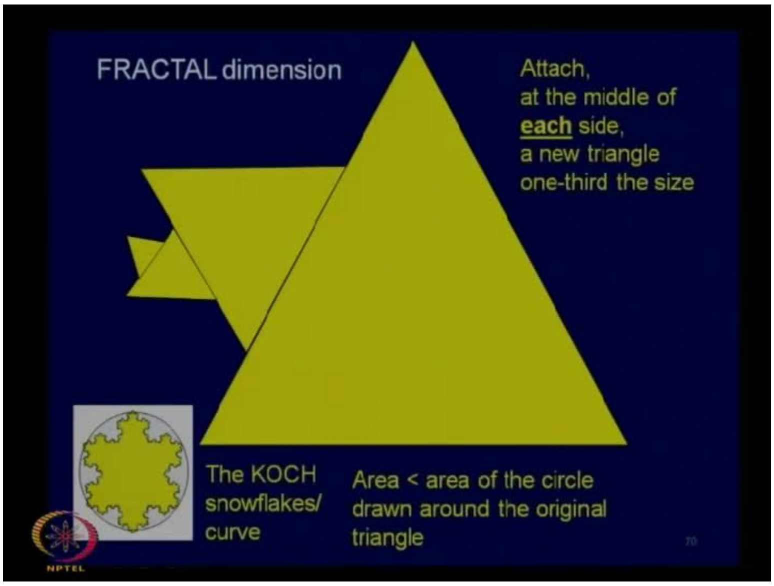

So, we will now talk about fractals. This is a fractal dimension and to introduce the idea of fractal dimension, we first consider an equilateral triangle. What we do now is we take the middle of each side, so there are three sides or what is the middle of this side? To get the middle of this side, we must divide this side into three equal parts. So, there will be one part over here, one in the middle and one at the bottom end; likewise, we can divide this side also into three equal parts one on the left, one in the middle and one on the right. This side also can be divided into three equal parts; one over here, one in the middle and one at this end.

So what we will do is at middle of each side, we raise another equilateral triangle. Now this figure is not great, because this triangle looks not exactly like an equilateral triangle. If you have a very precise vision, you might be able to notice that some of the angles are not exactly 60 degrees. Let us not get into those debates, so that is not the idea over here.

What we are talking about is the exact middle of each side, the exact one-third so that on each side of this exact middle one-third, there is the remaining one-third on both sides. So there are these three one-thirds which will add up to one side and we are talking about the exact one-third in the middle. All this middle exact one-third, we construct an exact equilateral triangle with the 3 sides being equal to each other, each angle being 60 degrees; now that is the mathematical object that we are talking about. What I have on this figure is not the mathematical object, some picture which I have inserted on the side, so that is not a mathematical object, it is a picture.

What we are talking about is the exact mathematical equilateral triangle which is raised on the middle one-third of each side of the triangle. Now you have a triangle over here, this also has got 3 sides. One is the side on which this triangle is standing that is the base and on this side, again you make divide this into 3 parts; one at the top, one in the middle and one at the bottom and on the middle you construct another equilateral triangle, can you do that?

Now again you have a triangle over here this is the small one now, but this also has got a certain length and you can divide this length into 3 parts. Again construct the middle part, again look at the middle part and on this middle part again raise an equilateral triangle. So there it goes and you can do this on and on and on and on.

What you will get is what is called as the Koch curve; this is the result that you are going to get. Because you are going to do this not only on this side, but also on this side and also on the bottom side, you will do this on all the sides looks like snowflakes; this is called as the Koch curve. What we are doing is we are exploding self-similarity, we chose to implement a certain process. The process was to divide the triangle, the side of the triangle into 3 parts and on the middle raise an equilateral triangle.

Then whatever triangle you get look at it sides and on the middle, again raise an equilateral triangle. So, here is the process and we repeat this process, no stopping the sides will get smaller and smaller. But, you can always divide any infinite decimally small length into 3 equal parts. The idea of infinity and the idea of an infinitesimal are amazing ideas in mathematics and there is no stopping you can do this endlessly.

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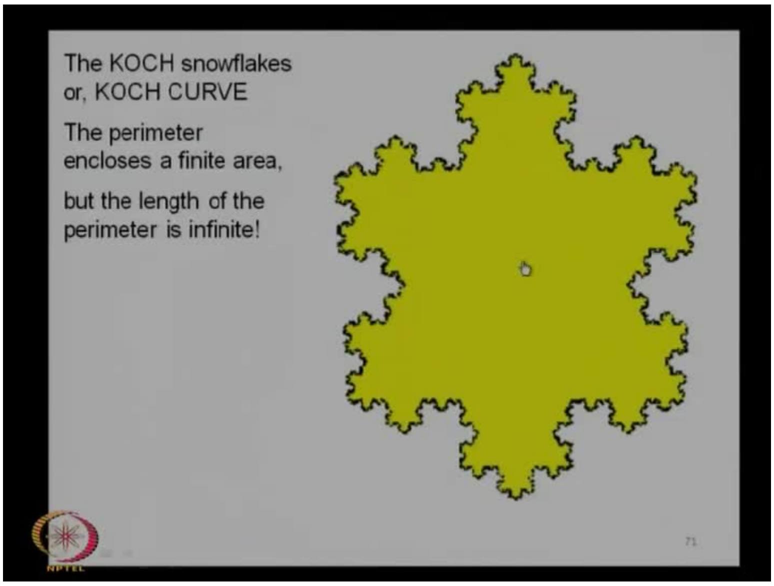

What you notice is that the area of the figure that you generate is obviously less than the area of the circle, which can be drawn around the original triangle, it has to fall in between that so the area is certainly less than that; so this is the pattern that comes out of this process, this is the Koch curve.

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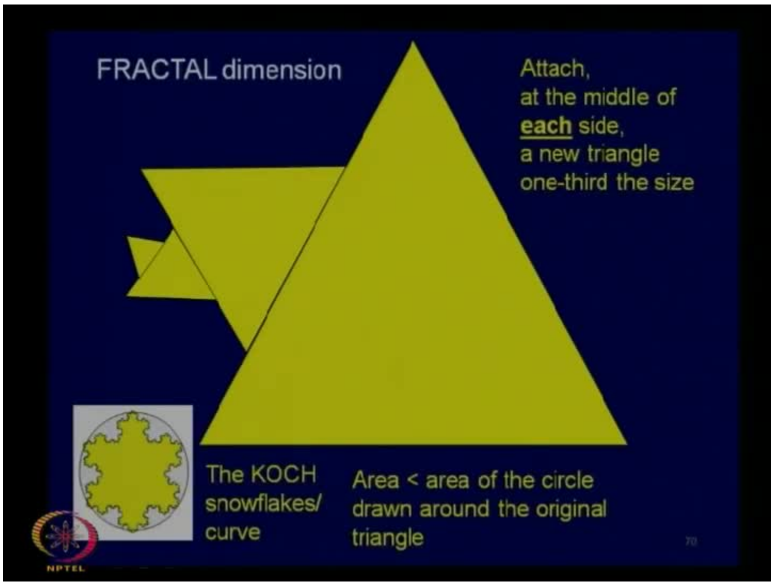

So the area of this is less than the area of the circle that we talked about, we have seen this in this picture. Here is this picture, so you obviously know that this area is finite.

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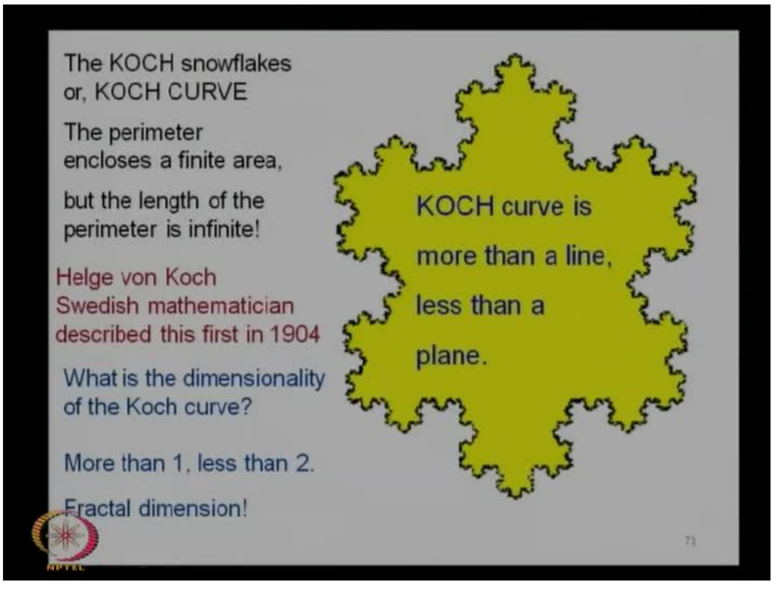

It will be some pi r square, so that is certainly a finite area, but if you now measure the perimeter of the Koch curve. How much will it be? Because every side you can keep dividing into small side and you keep measuring it but then you have to take smaller and smaller scales to measure it. You obviously cannot do it with a meter scale, you will need a smaller scale and then you we will need a scale which is smaller scalar and then you will need a scale, which is even smaller than that.

But there will always be some additional length that you will have to add and what you will get is a perimeter which is infinite. So you have got a finite area which is bounded by infinite perimeter, so very interesting object.

So this was a Swedish mathematician who describe this in 1904 more than 100 years ago. Then we ask this question what is the dimensionality of the Koch curve and just like the strange attractor, it seems to be more than 1 but less than 2. We have not define the fractal dimension is yet, so we are about to do it. It is more than a line, less than a plane.

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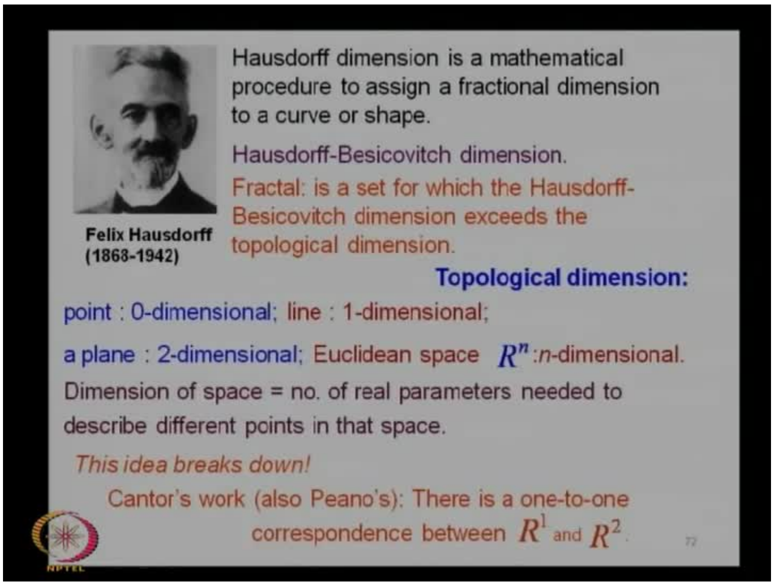

Normally when we talk about dimensions, we have in our minds the topological or Euclidean dimensions that we have common experience with. But there is another way of defining a dimension and this idea was introduce by Felix Hausdorff and this is sometimes called as the Hausdorff-Besicovitch dimension. Thanks to my Serbian collaborator O S Laura other way which I can read these names, so this is Besicovitch dimension.

We now introduce the definition of a fractal, it is the set for which the Hausdorff Besicovitch dimension exceeds the topological dimension. So we have to define these quantities now. So this is the primary definition, let us see how this works out.

Now we do know that if you consider a point, it is a 0-dimensional object. If you consider the line, it is a 1-dimensional object. Let us begin with very simple ideas. A point which is a 0-dimensional object, a line which is a 1-dimensional object, a plane which is the 2-dimensional object and typically these dimensions that we are talking about are the Euclidean topological dimensions and in general you can write this as R n space for the n-dimensional Euclidean space.

It could be 1-dimensional, it could be 2-dimensional, it could be 3-dimensional but, we can also extend this idea for an n-dimensional Euclidean space, which is the mathematical space having similar properties.

What we really mean by this by the dimensionality of this space is the number of real parameters which are needed to describe different points in that space. That is an idea which is at the back of our mind when we talk about the dimensionality of a space. What we are really talking about is that you must specify those many numbers of real parameters. So in a Euclidean 3 dimensional space, you must provide x, y and z you need these 3 parameters or if you are using spherical polar coordinates you will need the r theta and phi, but you will need 3 parameters ,which 3 is a different matter but, you will need 3.

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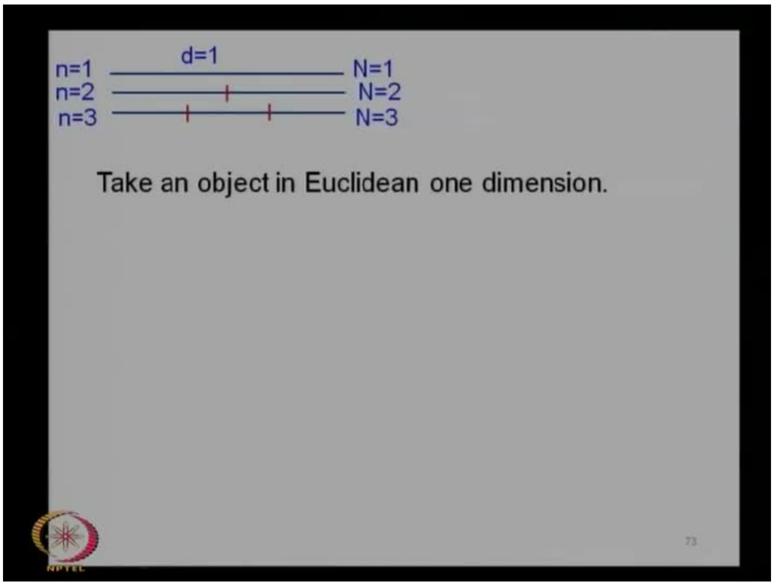

Now this idea really breaks down and there are Peano and Cantor’s theorems, there is a one-to-one correspondence between R 1 and R 2. So I will not get into the mathematical details of this, but leave it as a self-study idea for those of you who want to study this further. We will come back to the basic idea of interest was which is this idea of a fractal dimension. To that what we will do is we will begin with an object with a line, which is the one dimensional object.

So we have one dimensional Euclidean dimension, we divided into 2 parts and when we do this we get 2 parts in one dimension. You can divide it into 3 parts and then you get 3 equal parts of this kind.

So there is the self-similarity, the left part over here is completely identical to the right part. The middle part when n is equal to 3 is completely identical to the one on either side; so this is the idea of self-similarity that we are exploiting.

We are coming to the end of this class and I am going to continue the discussion from this point, but please concentrate on this that what we are doing is exploiting self similarity. But when you divide an object, the way we constructed the Koch curve you would started out with something, you generate a 3 identical parts and then you reproduce a triangle on the middle part and then you did the same with the smaller triangle.

But the smaller triangle also had 2 sides, the 3 sides - one the base on which the triangle was standing and on the other side again you constructed equilateral triangles on the middle part. You started constructing the self-similarity and what we are now doing it is this with one line that you take one line divided into 2 parts or you can divided into 3 parts, you can divided into n parts.

But when you divided into a certain number of parts, you get those many identical paths and that is the self-similarity that we are going to discuss in our next class and that will lead us to the definition of what is called as a fractal dimension of an object. So if there is any question I will be happy to take, otherwise we continue from this point in the next class.

Yes

Sir, can you move back to slide 71.

Which one

71 early one Koch of (( ))

Yes this one

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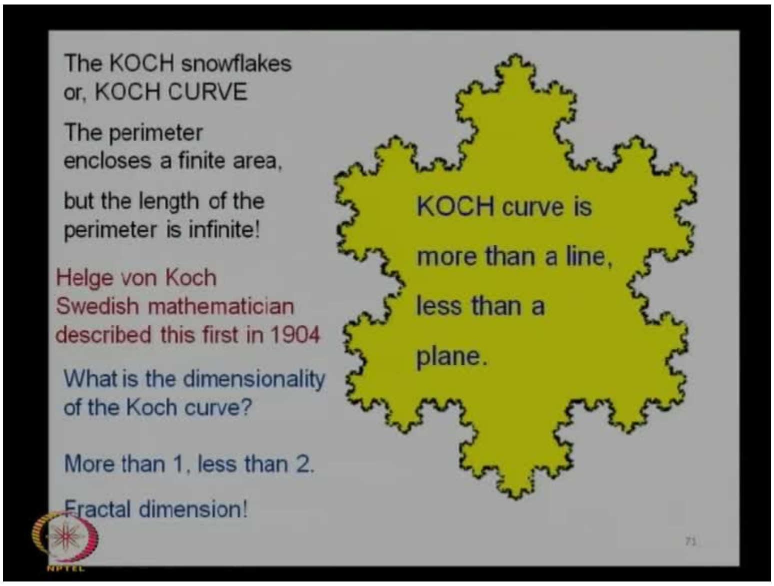

Now actually over here I could see triangles drawn in the other two-thirds of a path, in the other two-thirds, I could see triangles I mean triangles drawn.

Yes, right.

Now, how do you determine their lengths? I mean length of those triangles now we know that bigger triangles one-third.

The way you are going to measure length and this point will become quite clear in the next class when we discuss this idea because this was a very exciting question which was consider by Mandelbrot.

The question is that how do you measure the perimeter? How would you do? It means you have got the certain arbitrary shaped object the question that Mandelbrot address which we will discuss in the next class is what is the coast length of Great Britain. If I bring that map in front of you or bring the map of India in front of you and you have the Himalayas on the top and then you have got you know the Bengal on one side and Gujarat and Maharashtra on the other and then the v shape and you want to measure the actual coast length.

So, you cannot just take a meter scale or any scale which runs all the way from Trivandrum to Calcutta that will not give you the correct length. You will have to take scales which are smaller to place them into the nooks and corners, so that you get the exact coast length. You are going to have to do the exactly the same thing over here, you will have to keep inserting smaller and smaller and smaller scales, which will keep adding to the length which is how the length of the Koch curve becomes infinite. The perimeter that is exactly what makes the length infinite, but we will discuss this in some further detail in the next class.

Sir, triangle in a plane

Yes

So how does the Koch curve finally it is in the less than a plane.

Yes because finally when you talk about the perimeter itself

Ok

You are interested in the length, now your idea of a length has got one dimensionality in your mind. A length which is the distance between this point and this point, you stretch it out.

Now if you take a thread to measure this length, then what you will do is run the thread over here, it will come out a little bit over here at this edge and then you see that there is the shape of the bottle. So it will have to go inside and outside and inside and outside like this but then, the length is going to be the stretched length of the thread which is a one dimensional thing.

So there is some notion of a 1 dimension which is contained in the idea of a perimeter, on the other hand, you are quite right, you started out with the plane which is precisely what makes the dimension more than 1 and less than 2. But we will define it very rigorously in our next class.

Thank you very much.