Select / Special Topics in Classical Mechanics

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Module No. # 11

Lecture No. #38

Chaotic Dynamical Systems (IV)

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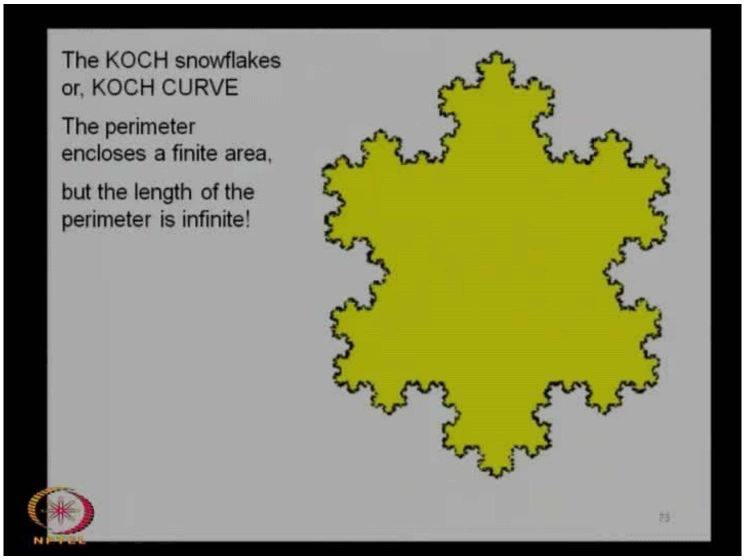

Greetings again, we will resume our discussion on fractal dimensions and in this class we will also talk about Mandelbrot sets, if we can get there. And the slide here comes from self-similarity that we started talking about. And we discussed this Koch curve, we generated it from an equilateral triangle by dividing the side, each side of the equilateral triangle into 3 parts and in the middle part we constructed an equilateral triangle stand on that and that was over here, I believe. Yes.

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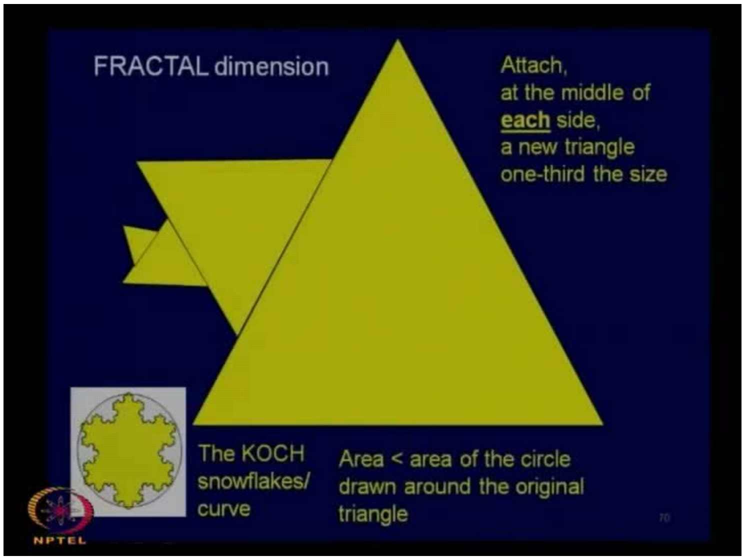

So, this was our equilateral triangle, we take the middle part of each side, construct an equilateral triangle on that, and then on each side of the smaller triangle, you do the same and you iterate; you do the same thing over and over again and again and again and again, and what comes out of this is what is called as the Koch curve. And as Milan was pointing out to us during the discussion that there are crystal growths, which take place following a self similar pattern and there are large number of natural phenomena in which you see this self-similarity.

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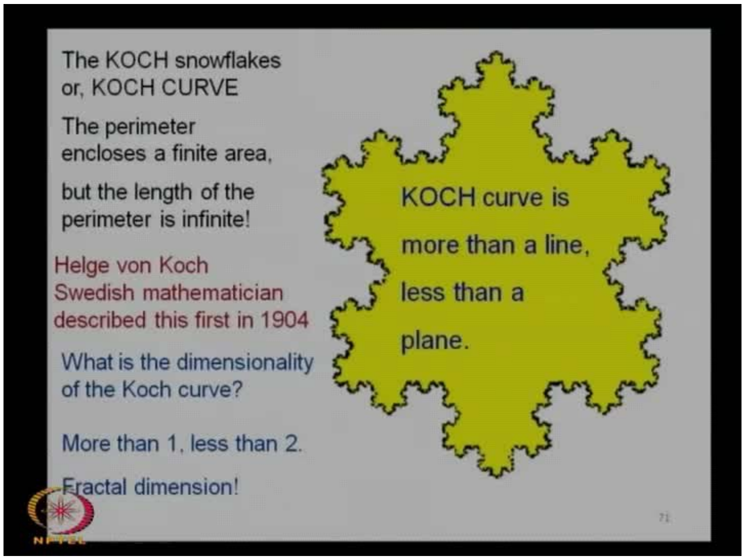

So, the dimension of this Koch curve is what we are now discussing and that brings us to this idea of the fractal dimension and it will turn out that this is more than a line less than a plane and the fractal dimension of this turns out to be between 1 and 2, as we shall define it.

And Jayanth had a question about this. So, can you what is your question Jayanth?

When we have to translate those equilateral triangles in the middle portion of the

Yes

Larger triangles

Yes

But we are having those small triangles in the other portions, as well so

I believe the confusion is arising, because of the manner in which you are looking at this resultant pattern. Let me see, if I can get this cursor on the screen, yes.

Now, you are referring to this pattern over here and you are wondering as to, how it got here at all, because you did not construct anything on this side and it is, because you are looking at this equilateral triangle as something that you began with. Now, this is because the figure that you are looking at, that is cheating our eyes and I will show you that this pattern comes essentially from the prescription that we discussed, that you take the equilateral triangle, take the middle part and keep constructing triangles on the middle base.

And there is another picture, which I have, which will make this very clear, but I need to jump a few slides and then I will resume that discussion, but yeah here it is and to show this I am going to come back and discuss this slide.

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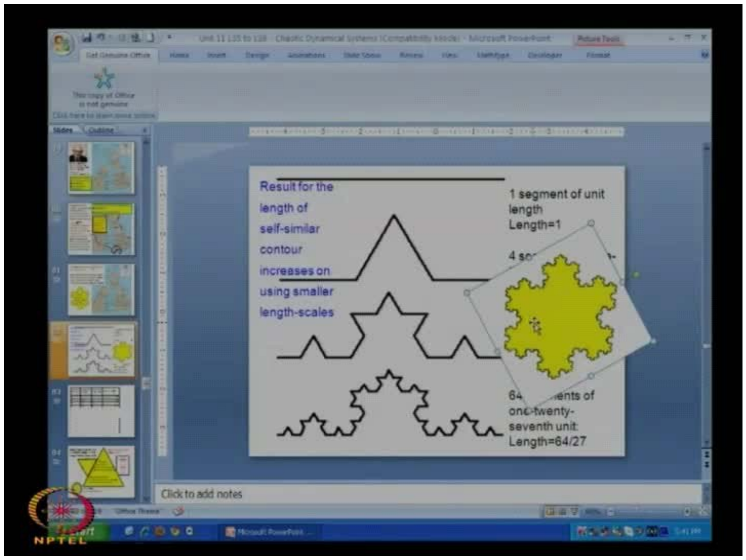

So, just ignore everything else that is of the slide for the moment, just see, how one line and think of this as the side of the triangle that you talked about. This is the middle side and on this middle side you have constructed this triangle and then on this side you construct this middle, on the middle of this you construct this, then you have this one and on this one also you construct triangle in the middle and this is how you from the first side you generate this pattern. Now, this is exactly what is coming in the Koch curve, which you see in this figure and you can see it, if I just copy this and paste it over here, here it is, so let me move it over here, so that you can see the point that we are looking at the only thing I need to do is to rotate this and bring it like this.

It is not fully rotated I guess like this, now, what you see is this pattern, which has come from exactly the prescription and this is the side that you are looking at and you will generate identical sides from the remaining two sides of the triangle. So, you are following exactly the same pattern except that, you felt cheated, because the picture has to be rotated to see the pattern that you are looking at, but it is exactly the same prescription.

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Of course, so this is what you see if you just see, means if you see the cursor you draw a line over here and what you see above this line is what you see over here. From the other two sides, you will get the remaining part of the Koch curve. So, you consider all the three sides, you will generate this closed perimeter that is exactly what is going to come. So, let me take this half and go back to our original slide and continue our discussion over here and let me resume our discussion on the definition of the Hausdorff-Besicovitch dimension.

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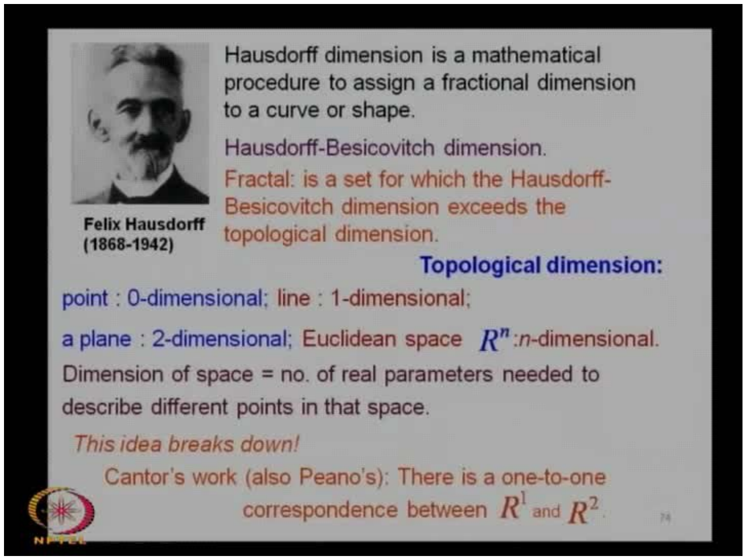

So, this is a picture of Felix Hausdorff and this idea is different from that of a topological dimension. We started discussing this idea that, if you talk about the Euclidean dimensions, the topological Euclidean dimensions you have a point, which has got is 0 dimensions, then you have got a line which is a 1-dimensional object, a plane, which is a 2-dimensional object and this way you can have a Euclidean Rn space and what you mean by the dimensionality of the space, is the number of real parameters that you must specify to characterize to give complete information about a point in that space.

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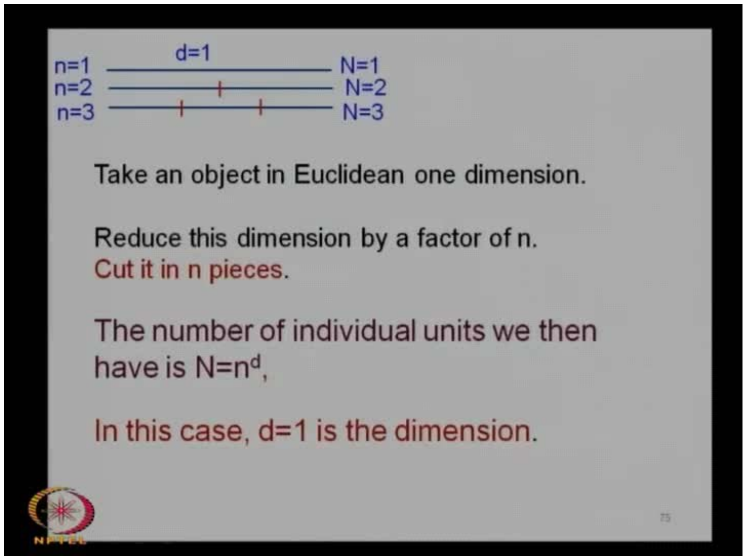

So, what happens is that if you take one line and divides it into two parts, then you get two pieces, which are identical to each other; if you divide it into three parts, you get three pieces, which are identical to each other. So, essentially, what you are doing is you reduce this dimension by a factor n, you cut it into n number of pieces and the number of individual units - the number of pieces that you get is given by small n to the power d, where d is the dimension.

In this case, d is equal to 1, so capital N is equal to small n, but their meanings are different; capital N is the number of individual units that you get the upper case N; the small n is the factor with which you are dividing the side. For a 1- dimensional object, the two are equal, because capital N is equal to n to the power - small n to the power 1.

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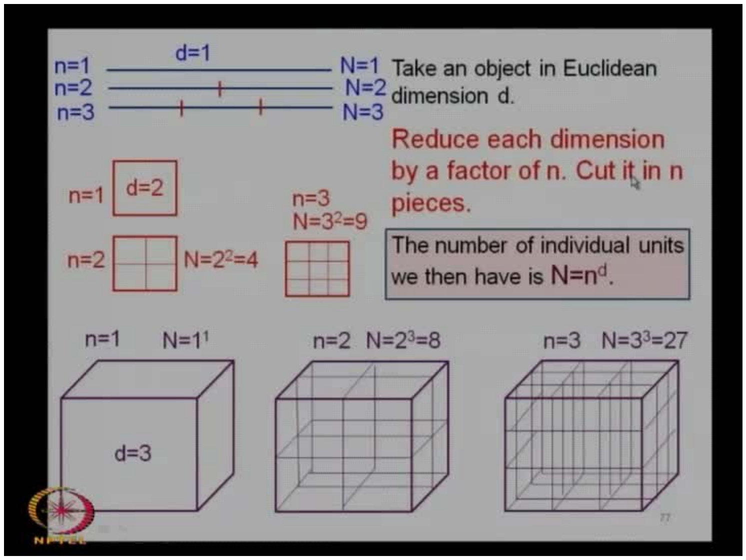

Now, this will become clear when you consider a 2-dimensional object, so let us do that. So, you have a 2-dimensional object, you have a square, then you divide each side into two parts; so you divide this side into two parts and this side into two parts, how many number of pieces do you get? You get four pieces.

You divided each side into two parts, but the number of pieces you get is different. So, capital N is now 4, little n is 2, little n is the number of times you divide each edge, but for d equal to 1, it was the same; for d equal to 2, it is already different. If you divide it into three parts, so small n is equal to 3 and you divide this edge into three parts and this edge also into three parts, then the number of identical self similar pieces that you get is 3 into 3, which is 9.

Let us do it with a cube; so you take a cube this is the 3-dimensional object; so d is equal to 3. You divide each side into one part, which is the same as the part that you began with, you have one piece; now, you divide it each side into two parts, so you divide this into two parts; so you factor it over here; you divide this part into two parts, so you have got the factor over here and you divide this side also into two parts, so you have got a middle point over here. And the number of pieces that you get, the number of self similar pieces that you get is 2 to the power 3, which is equal to 8. Now, if you do this, if you divide each edge into three parts now, rather than, 2. So, you see this figure in the lower right corner over here; so here are the three parts 1, 2 and 3 this edge is also divided into three parts 1, 2 and 3, and this edge is also divided into three parts from here to here which is the first, from here to the here which is the second part, from here to the third to the corner which is the third part.

And the number of pieces that you get which are self-similar in this case will be nine at each level and there are three layers of those. So, you will get 9 into 3 or 3 cube essentially, so you get 3 into 3 to the power 3, which is 27 self-similar identical pieces.

So, now, you see that capital N and small n is no longer equal, but they have a relationship, and that relationship is given by this formula, that capital N is always equal to small n to the power d, whether you are dealing with this line or with this plane or with this cube in every case, capital N is equal to small n to the power d. The capital N, the upper case N and the lower case n are equal only for d equal to 1, but for d equal to 2 it is not the same; for d equal to 3 it is not the same and there is a relationship. There is a rigorous mathematical relationship and we can exploit this relationship, because now, you can define d to be your mathematical dimension, which is the fractal dimension from there this relation between upper case N and the lower case n.

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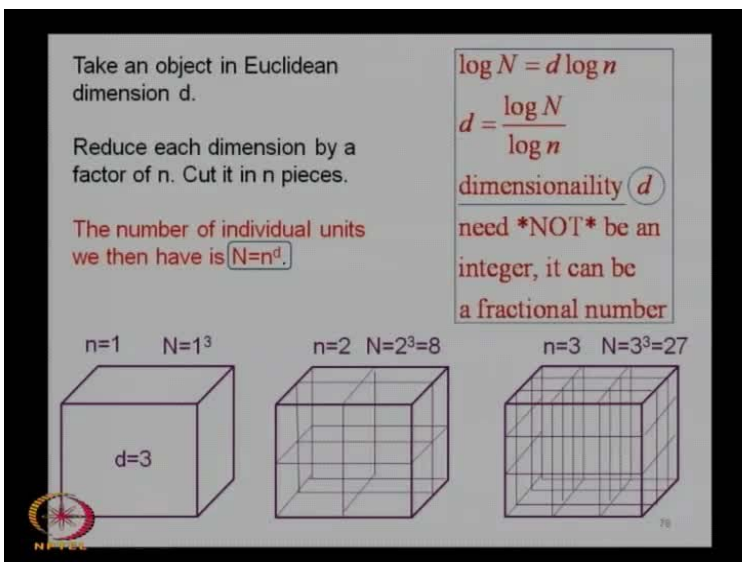

So, this is what we have done. This is the relationship that we get, that the uppercase N is equal to the lower case n to the power d and that tells us that you can define d as log N over log n, log upper case N over the logarithm of the lower case n. Because this relation is completely coming out of this general expression there is nothing else; if you take the logarithm of both sides that is what you get. And now, this is the ratio; if you now, define the dimension to be given by this ratio, where the relationship between small n and capital N is coming out of this idea of self similarity that we pursued; that we divided, we took an object and factored it into small n number of pieces.

And it is this idea, which has lead us to this. So, it is a mathematical idea that we are pursuing in this context, then depending on the kind of the objects we are dealing with, there is no reason to expect that d, which is given by a ratio of these two logarithms must be an integer it will not be in some cases, then I will show you those cases.

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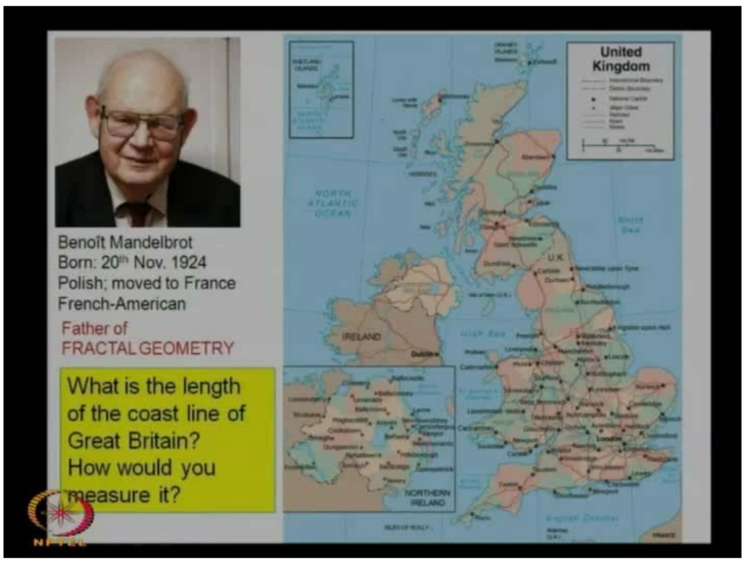

The Koch curve is already one such case; let us see how it works out. Now, this problem belongs to the area of fractal geometry and this is an important branch of chaos theory. Sometimes, Benoit Mandelbrot is called as the father of fractal geometry; He was born in 20th November 1924. What he did was to address this question as to what is the coastline of Great Britain.

So, here is a map of Great Britain and you want to determine what its coastline is? This goes back to the question that Jobin asked, as to why is the dimension more than 1 and less than 2. And what I was showing is this, if you may want to measure the length from here to here, then you take a piece of thread and run it touching the skin of this water bottle.

But then to go across this it will have to bend over these portions and get into these nooks and it cannot do it unless, it gets out of the first dimension, it has to get into the second dimension to get that; that does not make this length exactly to dimensional object, but it is somewhere between 1 and 2.

So that is a idea that you are going to run into, when you talk about these fractal dimensions, and let us consider the question that Mandelbrot raised, which is what is the length of the coastline of Great Britain? So, let us perform an experiment and ask ourselves, how we are going to measure it?

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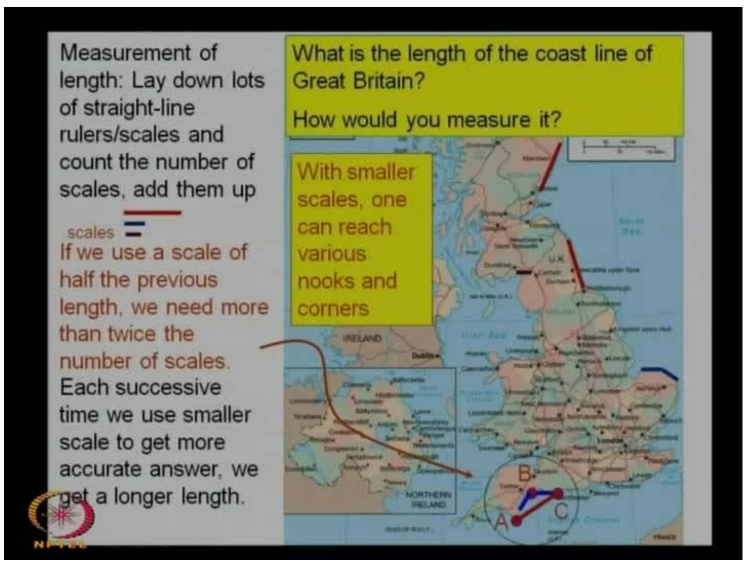

So, we are actually going to how to measure this, what we will do to measure it? We need a certain scale; we need a meter rod or may be a tape or something. right It means you want to measure some distance between one point and the other what you need is some measuring device and you take a scale, take the this straight scale and have a number of this. Now, if the length of the scale is only this much, I cannot measure any length more than this. So, if I have to measure a length more than this, what do I will have to do is, to take the scale and place it again and again and again and count the number of times I place it and when I multiplied by that number, I will get the length.

So that is a process, which is involved in measuring the coastline of Great Britain or India or any country that you like or do not like, but these are nice countries. So, let us think of a certain scale and let us say that there is a certain length that you are talking about and then you take this length, which is this, place it over this edge, place it over this edge and you try to place it over here and you know that you are going to miss out on some corners.

So, what you need is the smaller scale and when you take this, you can certainly get into some parts, but then you will still miss out on some nooks and corners. So, you take even a smaller scale; so you will have to take scales, which are smaller than before, but the number of such scales that you will need to use. If you take a scale, which is half the size of the previous scale, the number of pieces you will need will be more than 2, because now you are covering the corner which you had missed out.

So, the number of pieces you will need, if you take a scale of half the size will be more than 2, do you get it? So, what is going to happen is, this is means if you if you see this portion of the diagram, if you want to measure the distance the perimeter between the point A and point C, and you have got a scale, which has got this length. And you want you now decide that you will measure it by a scale which is half the size, so that you get more accurate answer, that the half will take you from C to B, but then from B it does not quite reach A.

So, the number of scales of half the size you will need will be more than twice the number. So that is what is going to happen and when you keep adding all of this you will end up getting a huge answer. Because the number doubles, then it will become 4 times, then it will become 8 times, then 16 where does it stop? You will get infinite length.

Now that is the suspicion, but you have to be careful, because in this case you are dealing with a physical coastline; this is not like the Koch curve which is a mathematical object. So, let us ask this question, let us deal with it a little more rigorously, that when you start covering these nooks and corners you can get smaller scales and you need more of them.

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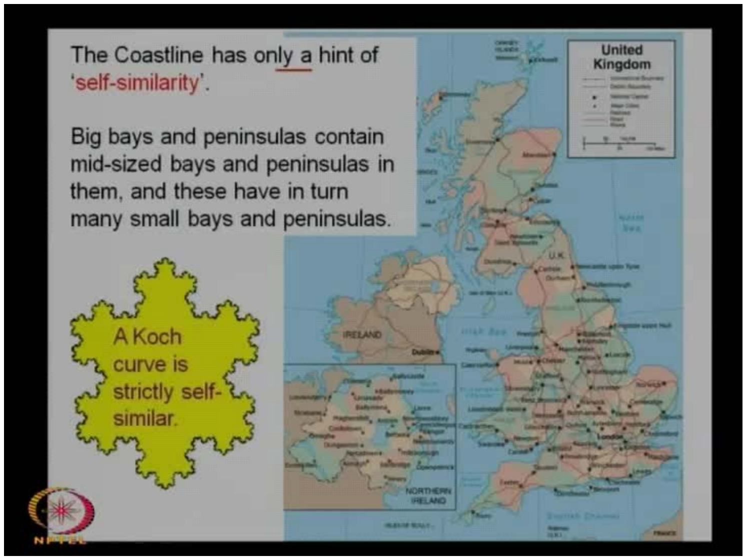

But the essential point over here is that when you talk about your coastline, if you were to look at it from an aeroplane or from a satellite, which is high up or you get a certain pattern or you go closer still, then you start seeing the details. And the closer you get, you see more detail in every detail, because each small part will have some nooks and corners. So, there is a resemblance of self similarity, but it will not be exact, because this is a physical coastline and like the Koch curve, which is a mathematical object it will be exactly self similar. So, there is a difference in the two; so Koch curve is strictly self-similar; a coastline is only approximately self-similar. You will see detail.

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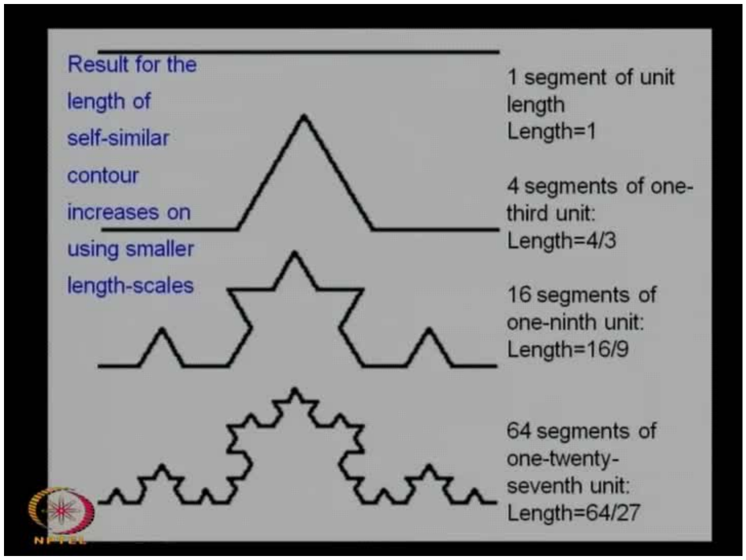

So, you will see you know, bays and peninsulas coming again and again, but it is qualitatively similar, but not exactly mathematically self-similar. Now, let us start counting the number of pieces we get. So, we begin with one segment here, this is how we generated the Koch curve; we get one segment. Once you divide it into three parts and then construct a self similar object namely, an equilateral triangle, the self-similar object that we are talking about is the equilateral triangle. You construct this equilateral triangle on the middle part, the number of segments that you get is four - one, two, three and 4.

Now, you take this segment; raise an equilateral triangle on the middle part right. So, what was 1 over here, you get four pieces - one, two, three and four, but you have done it for all the other sides. So, the total number of segments that you get is 16 segments of one-ninth unit, this was one-third, this is one-ninth; you do it further, you will get 64 segments of one- twenty- seventh unit that you began with.

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Iteration Number	Segment Length	Number of segments	Curve Length
1	1	1	1.00
2	1/3	4	1.33
3	1/9	16	1.77
4	1/27	64	2.37
5	1/81	256	3.16
6	1/243	1024	4.21
...	...	...	...
10	1/19683	262144	13.31
25	1/2.82e+11	2.81e+14	996.62
50	1/2.39e+23	3.17e+29	1324335.72
100	1/1.71e+47	4.02e+59	2338486807656.00

So, now let us make a table. So, the iteration number, you have got the segment length, which becomes smaller by a factor of 3 every times. So, the first time you do this, you have got a segment length of unity, next time it is one-third, next time it is one-ninth, next time it is one-third of that; so from 1 to the next, it becomes smaller by a factor of 3. So, one-third, then one-ninth, then one-twenty-seventh and so on, what happens to the number of segments? Number of segments will be 1, then it increases to 4, then it increases to 16, it increases to 64 we saw it in the previous picture.

What about the curve length? Now, your curve length is measured as number of segments times the unit length; so this will be the measure of the perimeter. So, whatever you measured as 1 unit now becomes 1.33, now it becomes 1.77, now it becomes 2.37 and you see that it is getting bigger and bigger.

You continue 5, 6, 7, 8, 9, 10, 11, 12 go on and on and on and on and you get to 10 and 25 and 50 and 100 and see all large numbers you are getting. So, what you have measured as 1 unit of length is blowing to infinity. What you measured as 1 unit of length is blowing up to infinity and that happens, because the segment length becomes smaller and smaller by a certain factor. This is 1 divided by 10 to the 47, the number of segments goes to 10 to the 59 and this is just the iteration number 100 and you sure can count beyond to the 100.

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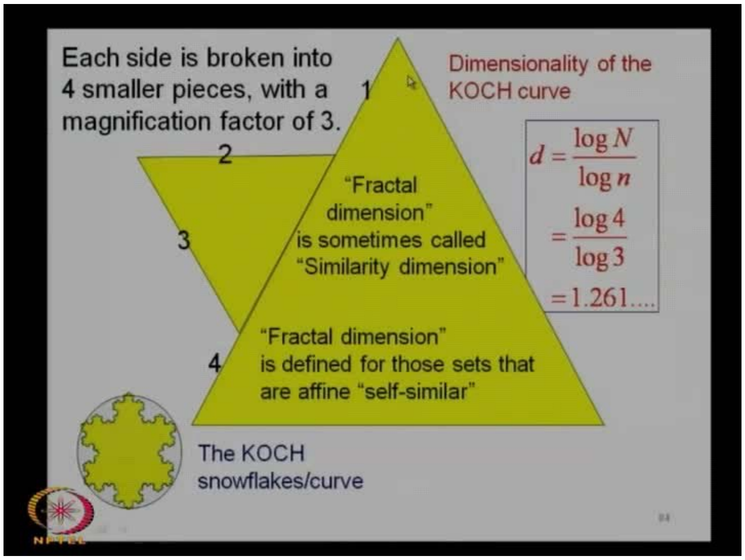

The next number Jayanth is 101. So, we know how to do that and this will just go on and on and you will get infinite length. So, the primary process of division generating self similarity is following this pattern that each side is broken into four small pieces; so this is 1 2 3 and 4.

This is also called as the similarity dimension or the fractal dimension. And if we now find ask ourselves, what is the dimensionality of the Koch curve, using the same idea that we did in defining the fractal dimension as the ratio of the logarithms of n upper case N and the lower case n that is how we defined it.

So, if we do it now for the Koch curve, you have log 4 over log 3, because you divided into three parts, you get four parts; the number of pieces becomes 4. So, the upper case N is 4 and this is the result of generating a self-similar pattern in which each side is divided into three parts. So, the ratio of log N to log small n is given by log 4 to log 3, which is 1.26, and it is a fraction.

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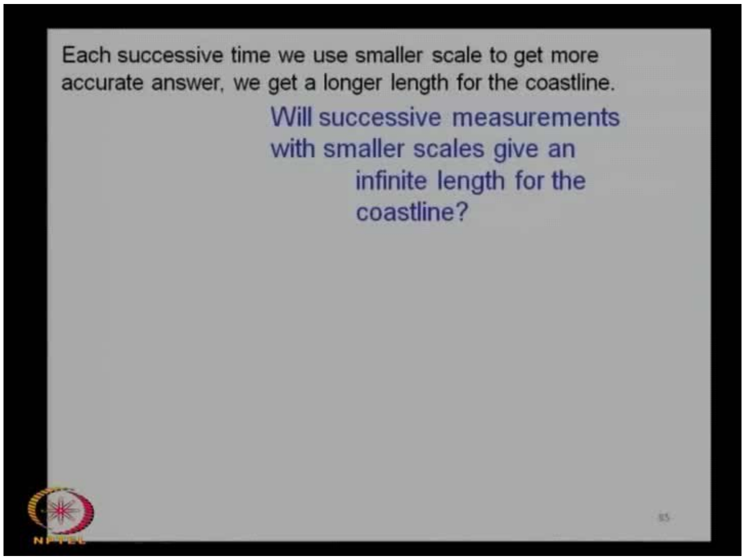

It is a completely mathematical idea; it is a purely mathematical idea, but a very meaningful. It is in the context of, how we define the dimension it makes perfect sense. So, it is a great idea. So, every time we make an additional, we use a smaller scale to get a more accurate answer, we get a longer length of the coastline.

Now, if we keep doing this on a real coastline or Great Britain or India or any country, where you like or love or whatever, if you do this for any country will successive measurements with smaller scales give you an infinite length?

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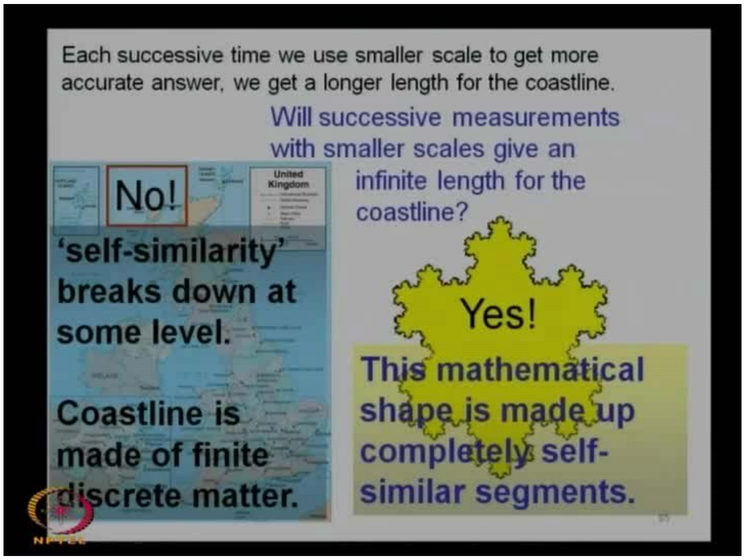

Now, think of India, are you willing to suspect that its perimeter is infinite? It does not seem to be so. What about Great Britain? Answer cannot be any different. So, for Great Britain or India the answer of course is no. For a Koch curve, it is yes. The reason of course is that the coastline is made of finite discrete matter, it is a physical boundary made of sand and mud and whatever. So, each grain is going to have a certain size, which is finite and when you stack it all together and add it up together, there is no way you can get infinity, because there is finite amount of granular particles along the coastline.

So, there diameters or whatever are going to be to add up to generate the coastline and there is no way that finite amount of matter can give you an infinite length. But a Koch curve, there is no stopping, because you have got a mathematical triangle, equilateral triangle, you have broken into three parts, you get four pieces.

And on each base, each one-third base in the middle, you generate a self similar triangle and you keep doing it and there is absolutely no stopping, you can do it a 100 times, 100 billion times or a 100 billion times and there is absolutely no stopping. So, for the Koch curve the answer is yes; for a physical coastline the answer is no.

So, this is completely self-similar, the coastline has got a resemblance of self-similarity, but it is not exact. But then there are many physical objects in nature like the crystal growths that Milan mentioned. There are large numbers of physical processes in which there is self-similarity, where it provides an excellent, at least an approximation to what you are dealing with and then you can apply these mathematical ideas, which are really very fascinating.

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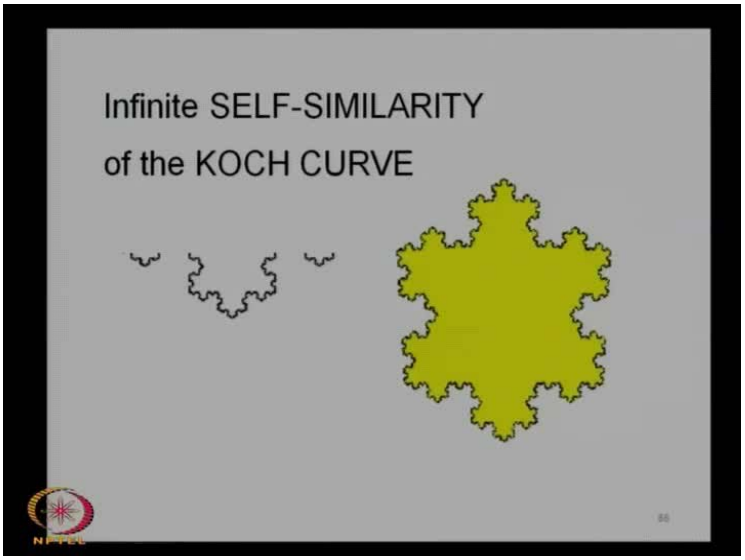

This is the infinite self-similarity of the Koch curve, there is no stopping. You can you zoom on it, go to smaller pieces and you will keep getting these additional length if you were to measure it, because each side no matter how small it is, you take a small length, which is like 10 to the minus 81, you can still divide it into three pieces.

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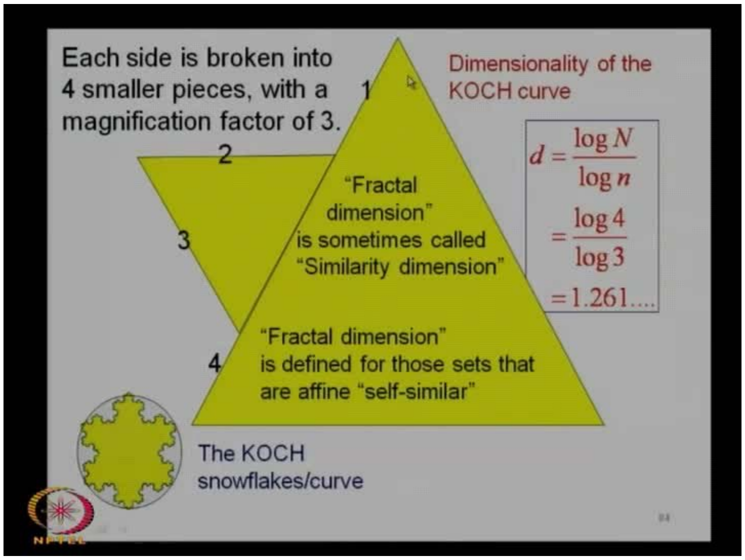

Then you take a smaller thing, which is 10 to the power minus 363 or whatever, you can still divide it into three pieces and keep doing this. So, this is the idea for fractal dimension. For the Koch curve this fractal dimension is log 4 over log 3, as you can see, this is 1.261, as it turns out to be…

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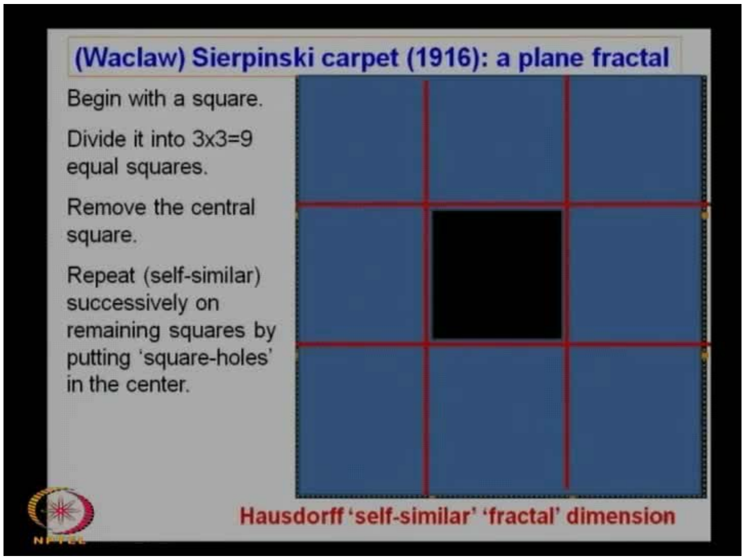

Now, let us consider some other cases. This is a nice example, this is called as a Sierpinski carpet. What you do is, to begin with a square, divide it into 3 into 3 equal squares. So let us do that. Now, some of you are going to stop listening to the lecture and start playing tic tac toe. So, what you have done here is to begin with the square, divide it into three parts, each side into three parts. Now, take the central part and remove it; so this piece is gone. How many self-similar parts are here now left with? – 8. Now, each of these 8 parts is a square and a square is what you began with. So, that is now your entity that you will carry over the process of self-similar constructions.

Each of these, you can again divide into 3 into 3, 9 parts. Take the middle part, remove it you will again be left with 8 parts in each of these cells and you can keep doing this on and on, this is called as the Sierpinski carpet. So, this is how it is going to look like after you will repeat this self-similarity process of removing the central square. And I have referred to it as making square holes through which is a very odd term, because holes are not squares and squares are not holes. But I believe you get the idea, what you are doing is, you are removing the central square.

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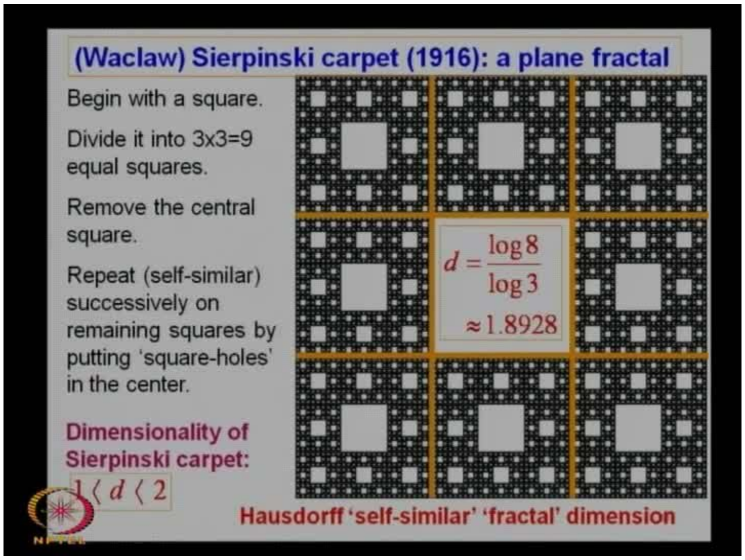

Now, if you do this, the pattern that emerges is this, as you can see very clearly and every time you do this, you will be left with eight self-similar objects on dividing each side by it three parts. So, the fractal dimension of the Sierpinski carpet will be logarithm of 8 divided by logarithm of 3, it is more than 1 less than 2, it is 1.89. This is the Hausdorff dimension or the fractal dimension. This is the Sierpinski carpet; now, let us do it in this.

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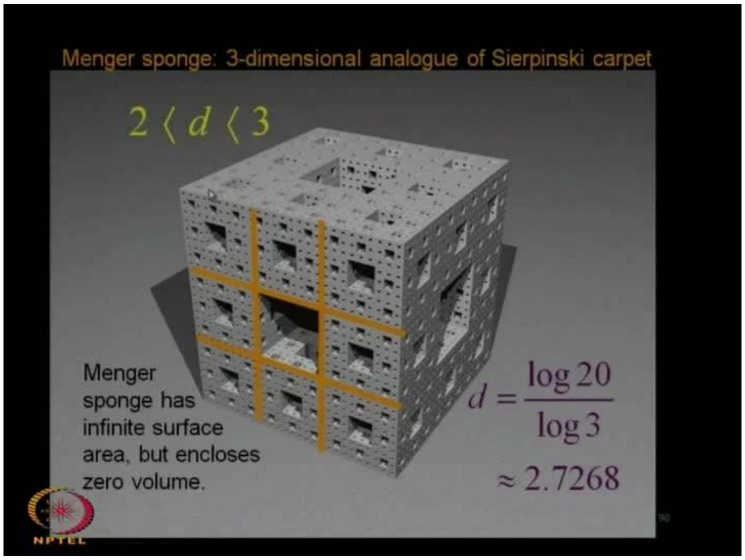

Can you figure out what will be the fractal dimension of this? Now, what you have done? This is the 3-dimensional analogue of the Sierpinski carpet. So, what you have done is, you scoop out this from here, it goes all the way it up to the bottom from here, you scoop out a square hole if you like again.

How do you, is there another word for it? I do not know how to say that, but you scoop out from this a square portion all the way, which runs from the front face up to the back space, from the top face to the bottom face, and from this side to the other side. How many pieces will you be left with? No, you will have, where is the cursor, Look at this corner, you will have one over here, second over here, and third over here, that is the 1, which survives. So, you have 3 over here, 3 over here, 3 under this corner, and 3 under this corner, so that is 12. Then you have got 1 over here, and 1 over here, that is 14; then 1 over here, and the other over there, that is 16; then you have another 2 pairs like this 1, this, and this, and this 1, and the other 1 behind it, so you get 20.

So, the fractal dimension of this will be logarithm of 20 divided by logarithm of 3, this is more than 2 and less than 3, it is 2.72. So, this is called as the Menger sponge and it has got infinite surface, but encloses 0 volume, because you can keep scooping out and what you are going to be left with? If you keep scooping out again and again nonstop, and there is no stopping. So, you will be left with 0 volume, but if you look at the area, if you keep adding it, because you are adding smaller areas, you are using smaller scales, smaller units of measuring areas, just the way you did for the Koch curve, that is going to add up to infinity.

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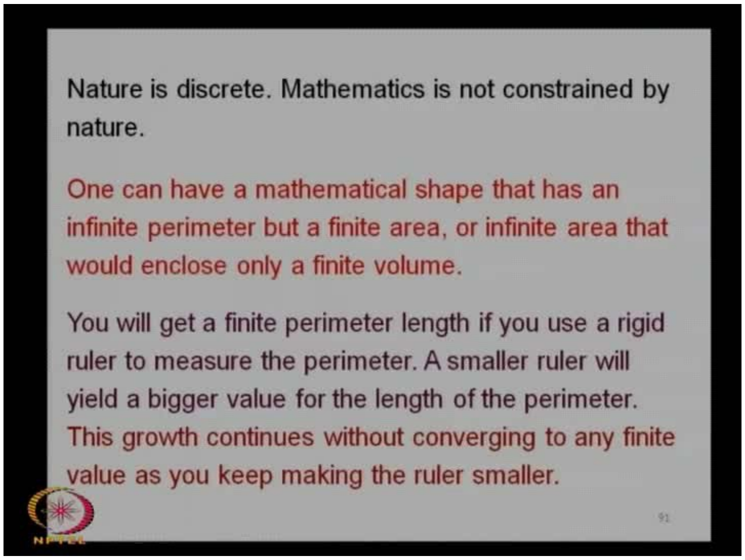

The fractal dimensions are very interesting mathematical objects. Nature, of course, is discrete. So, we found that the coastline of any country: Great Britain, India, United States, China, which ever on any continent, anywhere, that will be finite, because nature is discrete and mathematics is not constrained by nature, it is coming out of your understanding of basic mathematical ideas. And you can have an infinite perimeter, but a finite area or infinite area that would enclose only a finite volume. So, the Koch curve is an example of the first, and this Menger thing sponge an example of the latter.

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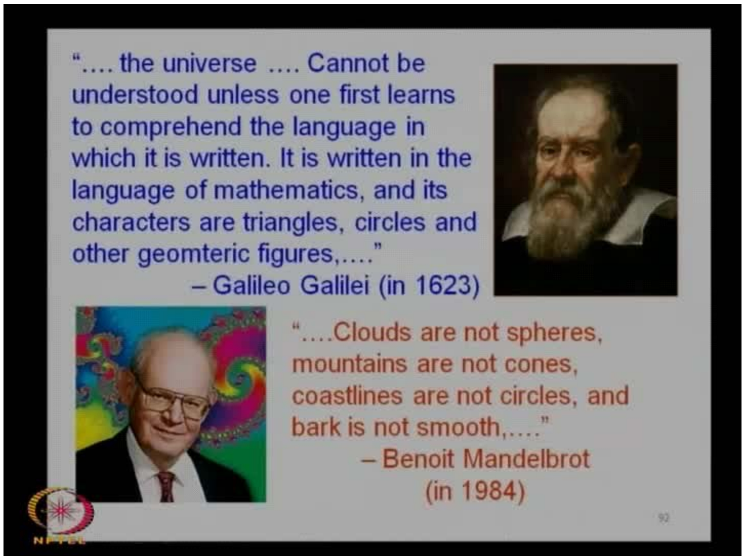

So, this is the idea of self-similarity; essentially, it is an iterative process, and this is where Mandelbrot differs from Galileo. Galileo was very intrigue by geometry and he pointed out that- the universe cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics and he was thinking of mathematics in terms of, you know geometries and characters like the triangles, the circles, and geometric figures and so on. This was a great idea and it worked very well in Galileo's time and quite until now. What Mandelbrot tells us is that "clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth". So, you have to introduce more ideas.

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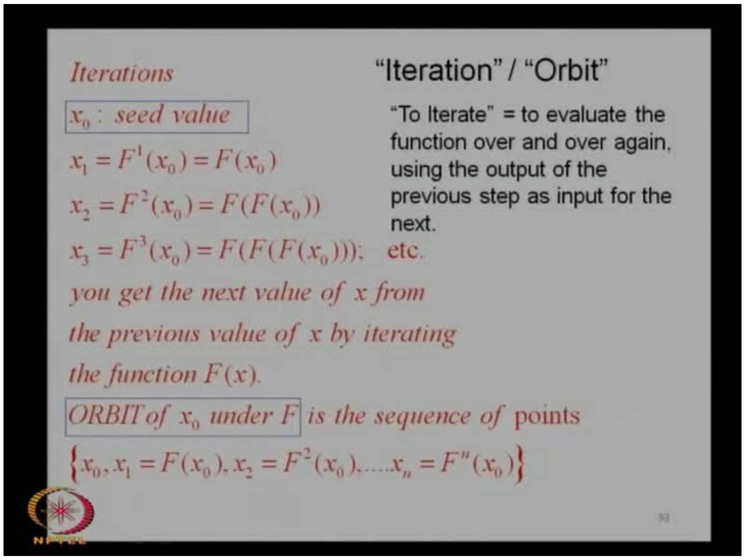

Mandelbrot is therefore called as the father of fractal geometry, because he brought out these ideas and applied them beautifully in mathematics and physics, and various other branches of science. So, we are talking about an iterative process and we will talk about a mathematical iteration now.

What we do, is to construct a function of a certain argument. So, this x, is a function of this argument, the function is defined by this upper case F, and we are going to carry out iterations on this. So, we take begin with a seed value and for this seed value we find through a function, the result of the function will be, which we call as x 1.

And hen we use this as the seed value and from it, we get the next value. So, what we have for x 2 is function of the, function of the seed value, this is the second value; the third value will be the function of the, function of the, function of the seed value. So, this is an iterative sphere step - in a certain sense it is like an orbit. You do the same thing again and again, and this sequence of values that you will get, as a result of this, you will generate a sequence x 0, x 1, x 2, x 3, x 4, and so on. So, when you stack all of these together, put them in a set. So, here is a set, what you see in the bottom of this figure, you have got x 0 over here, x 1 is a function of x 0, x 2 is function of the function of x 0, and so on. And X n is the function of the, function of the, function of the, function of the done, n times of the seed value. So, you have this whole sequence and this is the orbit of x 0 under F, according to our definition. So this is called as the orbit.

So, orbit the idea is quite simple to understand, the terminology is almost self-explanatory. If you take the simplest example of an orbit, the earth going around the sun, then it is one point of the earth's position, then the next point, then the next point, when it keeps doing the same thing, which is to move under the gravity of the sun.

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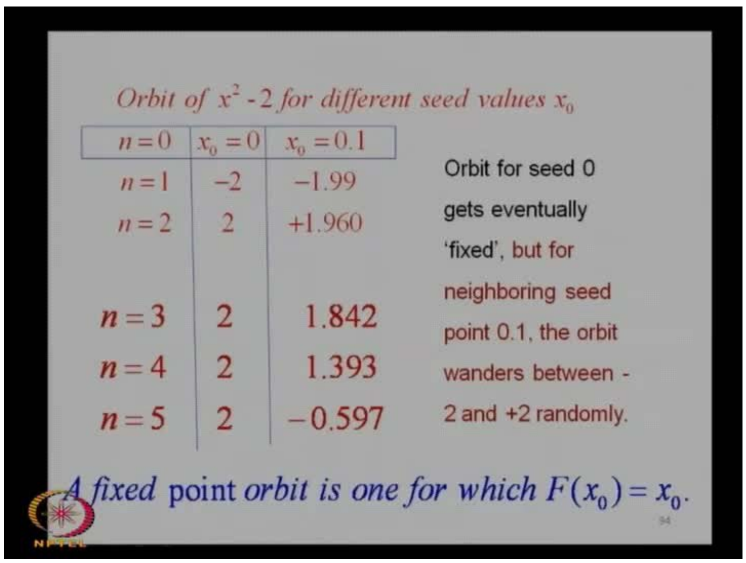

So, the sequence of points through which the earth moves, is what generates the orbit. So, here, this is the mathematical orbit and it is the sequence of this set, which is defined by this function. So, let us take a few examples. We will consider the orbit of x square minus 2, a function of x, which is defined as x square minus 2. x square minus 2 is the function of x.

We ask, what is the orbit of this function for a particular seed value? We can choose that seed value to be 0, we can choose it to be 0.1, we can choose the seed value to be 0.2, or anything. To discuss this point I have considered 2 examples over here, one is the seed value of 0 and the other is the seed value of 0.1. So, in the first iteration, when the seed value 0, x square minus 2 is equal to 0 minus 2. And the next one, you take the square of this, which is the square of minus 2, so you get 4, subtract 2 from it, and you get 2. So, the orbit has moved from 0 to minus 2 to 2. If the seed was different, and it is only mildly different, mind you. If this was 0, it is not that this is too far from, it is 0.1. You construct the square of this and subtract from it 2, you get minus 1 point 9 9, then you square this, subtract from it 2, you get 1.96. If you keep doing this for n equal to 3 4 5 and so on, this one settles down to the value 2, whereas this one does it. So, this orbit of x square minus 2 sort of, settles down to a fixed point, but just the neighboring seed value 0.1 is not too far of from 0.

If it is money in units of rupees or dollars, you will ignore both of them neither helps much with inflation, even a 100 rupees do not mean much, but 0.1 certainly does not. And if you take just a neighboring value, so you will see the sensitivity to the initial condition, the initial condition is only slightly different, but the orbit is very different.

In the first case, when the seed was x 0 equal to 0, the orbit settles down to a fixed point. In the other case, when the seed value is only slightly different, at different initial condition, not too far of from the previous initial condition, the orbit oscillates rather randomly between plus 2 and minus 2, look at these numbers you do not see any pattern in that, it is quite irregular, 1.99, 1.96 1.84, 1.39, minus 0.597. It will always remain within the confines of plus 2 and minus 2, but within this, it will be very random.

So, I believe, we are again coming close to this class. So, we will stop here and we will carry forward this discussion on what an orbit is, and that will lead us to what a Mandelbrot settles. You will see some very beautiful pictures, they are amazing, but that is for the next class. For now, I will be happy to take a few questions.

Sir, is there any relation between fractal dimension and curve length?

And

Curve length because those values are pretty close?

Which

Which you showed as before?

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Iteration Number	Segment Length	Number of segments	Curve Length
1	1	1	1.00
2	1/3	4	1.33
3	1/9	16	1.77
4	1/27	64	2.37
5	1/81	256	3.16
6	1/243	1024	4.21
...	...	...	...
10	1/19683	262144	13.31
25	1/2.82e+11	2.81e+14	996.62
50	1/2.39e+23	3.17e+29	1324335.72
100	1/1.71e+47	4.02e+59	2338486807656.00

Which figure or which slide are you are referring to? It means you want me to go back somewhere?

No, I need to understand your question.

Here

Yes sir

Here

This value seems to be pretty close to the small d value, so is there any relation between these 2?

Which value is similar to small d, I do not understand your question?

Fractal dimension sir.

Yes.

N equal to small (( )) to the power e sir.

Are there any relation between curve length and then small d sir?

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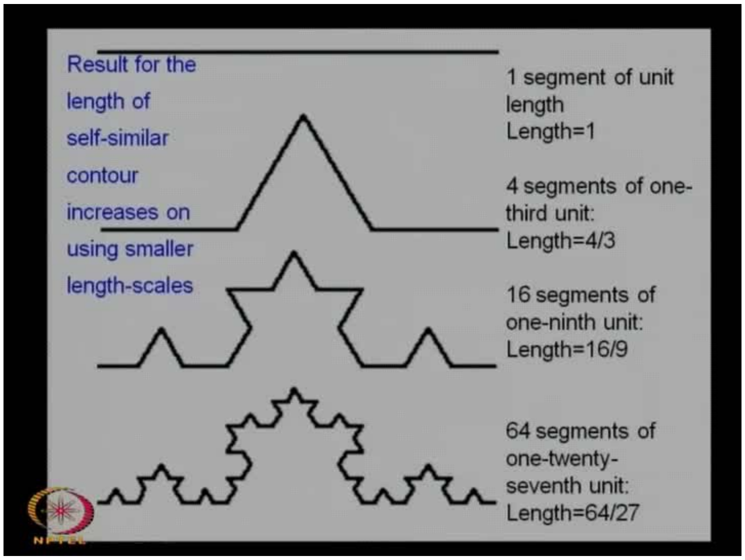

Of course there is, there is, certainly yes. If you go back here, this is what is generating the length, that you take one piece, divide it into three parts, on the middle part you construct a self-similar object having done it, you get four segments of one-third the size.

When you do it further, iterate, you get 16 segments, each of length, which is one-third into one-third, which is one-ninth. So, the length will be 16 times one-ninth. So, the number, which gives you the length, which is 16 by 9, is directly connected to the 4 by 3. Every time you are multiplying the previous number by four-thirds. So, you begin with 1, then you get 4 by 3, then you get 16 by 9, then you get 64 by 27, you are multiplying by 4 by 3.

Now, this is what you are doing in this, table.

So, it is directly connected to that, it is you know there is an exact relationship that you have to, scaling it by that factor, ok any other question?

Sir, while scooping out that tube, the dimension was reduced from 3 to 2 point, something sir. Why not above this, the other 1 sir?

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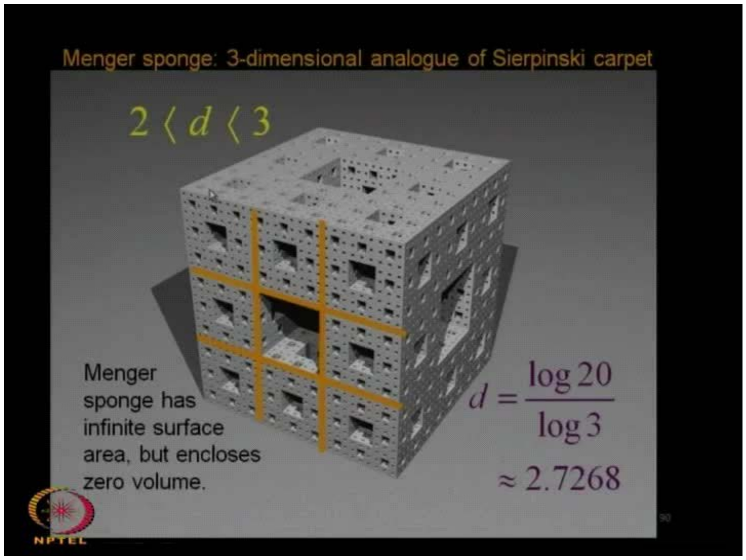

The dimension, the fractal dimension reduce from 3 to 2 point, something.

Sir.

Right

Why not above 3?

Well, how will it be more than 3? There is no way; it will be larger than that, because what you are doing is you are taking a piece, divide it into smaller number of parts and then, you count the number of parts that you are left with.

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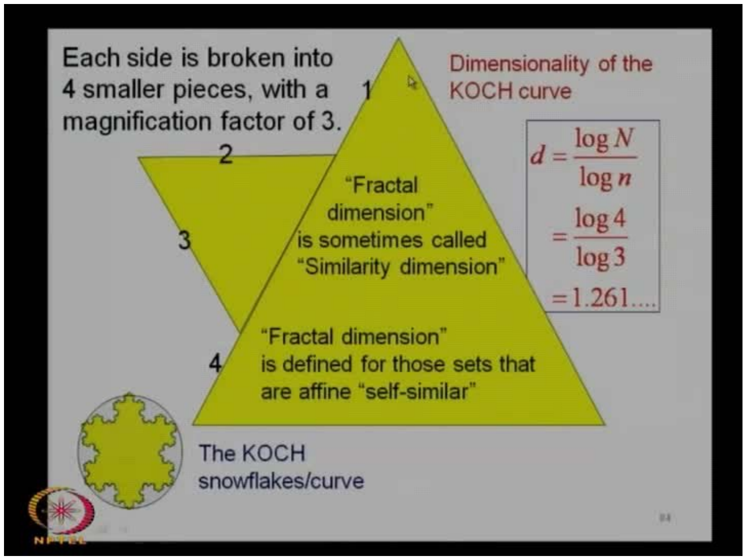

So, the number of parts that you will be left with, when you begin with 1 will be more than 1. So, if you go to this relation, this one log N by log n capital N will always be either equal to small n or larger than that, it cannot be smaller than that.

So, thank you very much, and we will continue from this point in the next class.