Select/Special Topics in Classical Mechanics

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Module No. # 12

Lecture No. # 40

The Scope and Limitations of Classical Mechanics

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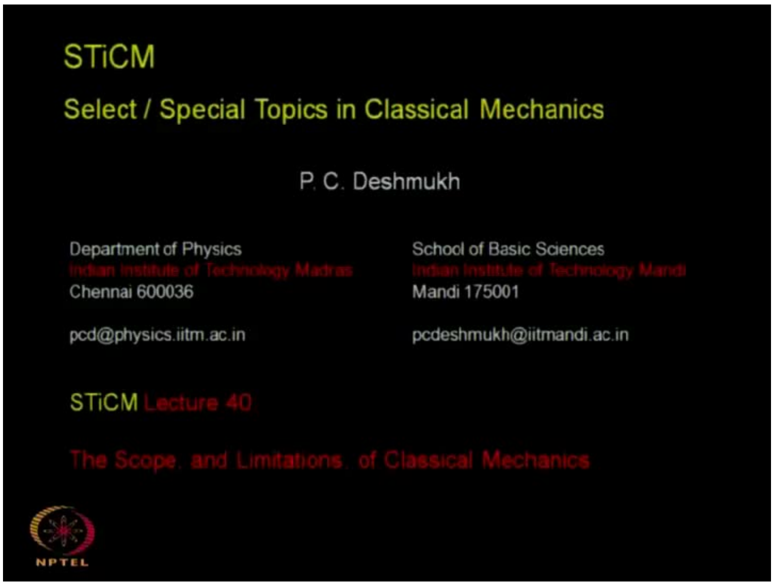

Greetings. We have now come to the end of this course. I planned this as a Select Topics in Classical Mechanics, and then there was suggestion, that it should be called as Special Topics in Classical Mechanics. And I did not so much worry about the title of the course. I thought it is convenient to call it as Select slash Special - use the letter ‘S’ as a common feature and then call it as STiCM, and you can read it as Special Topics or Select Topics, and we have certainly not been able to cover a whole lot of ground. But yes, it is my hope that we acquainted ourselves with some essential elements of classical mechanics.

And before we say a final good bye at the end of this class, I will like to, you know, summarize the scope of classical mechanics, and also the limitations of classical mechanics, and also give a hint of what is it that we are going to have to do go beyond classical mechanics.

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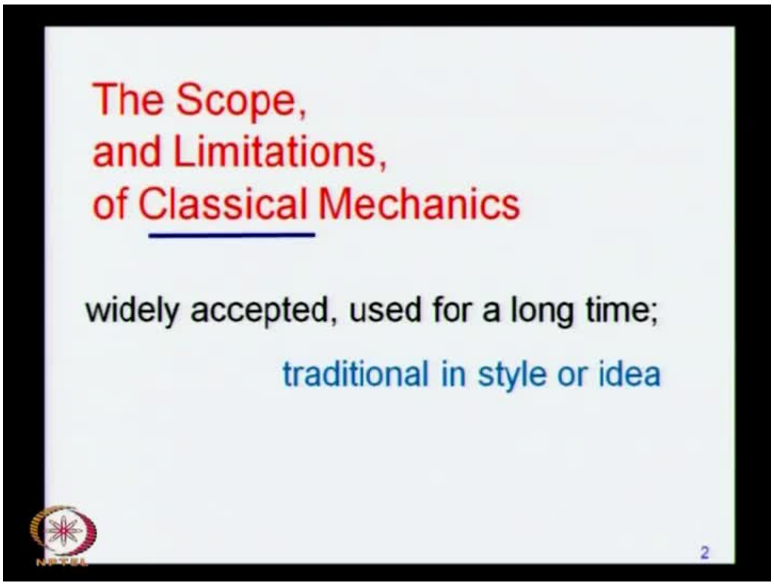

So, when you think of a classical subject, what is it that comes to our mind? Classical means something which is widely accepted, something which has been there for a long time; something which is traditional, whose values has been respected, it has stood the test of time; it may be improvised, it could be changed, but it has got a very robust and solid foundation, which has stood the test of time, for a very long passage of time; it is not like a hurriedly given Nobel prize, which sometimes gets hotly debated; it is not like something which has got only a transient value, but a value which stays, and the meaning to me becomes most clear, when I remind myself of what other classical things I know of.

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And I have to think a classical music, and my taste of classical music grew by listening to a large number of musicians, and amongst those who I have listened most, perhaps for the maximum length of time over my life; I was a child, and I started listening to Pandit Bhimsen Joshi - great artist, and many of you would have heard him, I am sure. And if you see, if you listen to some of his recordings, which are very old like 10 years, 20 years, 30 years, you will still enjoy them, and these recordings have also stood, the test of time.

So essentially what classical terms suggest to us, that it is something which remains solid, remains attractive, it sustains; it has a sustaining, enduring appeal to mankind, and a scheme of mechanics, which has such a characteristic feature, is what classical mechanics is.

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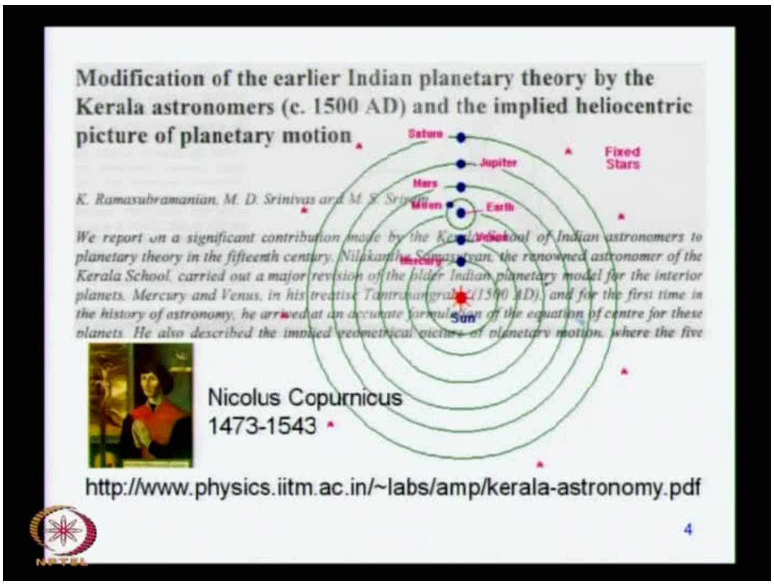

Now, this is not very young. It really has its routes, which go back to 100 of years perhaps even 1000s, but not very many 1000s. I had mentioned this is in one of our introductory units that the Kerala astronomers were quite conversant with the heliocentric frame of reference and the beginnings of classical mechanics actually are from this period.

And then, of course, in more recent context, Copurnicus was one of the first European physicist who recognized that the sun is at the center of the planetary system, contrary to what appears to the human eye - we see that sun rising in the east, and going over the skies, set in the west - but then, it is us, who are really going round the sun and not the other way round which is of course a point of perception, but that is a matter of detail we have discussed that in considerable detail.

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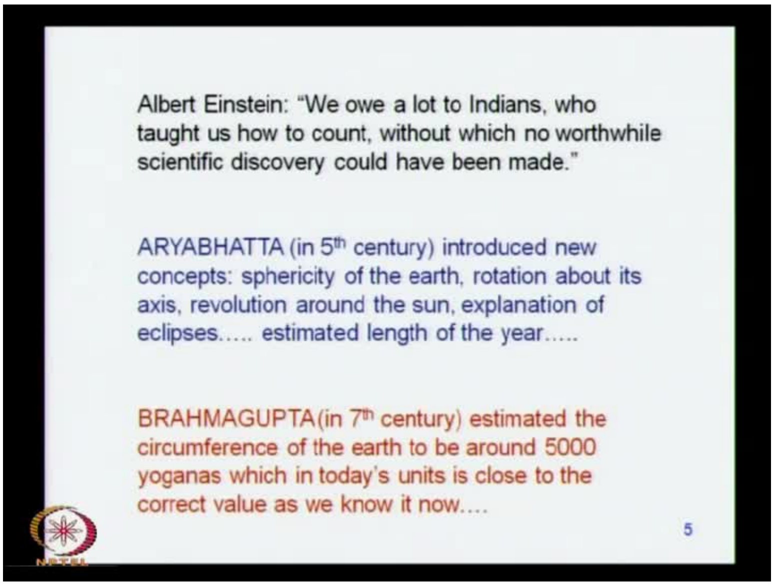

Now these were the beginnings of classical mechanics in some sense, and the Indians made very important contribution to that. Albert Einstein himself had pointed out that without the contributions of the Indian scientist, no worthwhile scientific discovery actually could have been made, because of the very seminal contributions they made. Aryabatta's work, Brahmagupta, you know these are just some of the very many names that we can reflect on.

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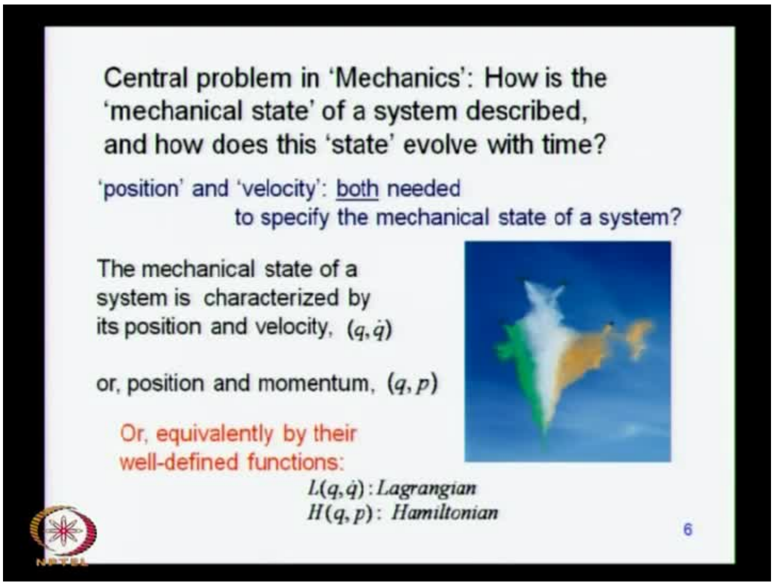

But then, the central problem in classical mechanics, as I had pointed out, is to answer this question - as to how exactly do you describe the mechanical state of a system, and how does this system evolve in time? And this is the central problem.

And the answer to this, and what we now call as classical mechanics, is that the system is characterized by two parameters - a system having a 1 degree of freedom is characterized by two parameters: the position and the velocity, or equivalently by position and momentum; it does not matter which pair you take. You can always go from one formulation to the other, but this is the essence. You need two parameters - the position and velocity, or the position and momentum - or else you can also do it by defining functions of the position and velocity, or a function of position and momentum, which is the same thing, in a certain sense. If these functions are well defined and you can define a state of a system also by the Lagrangian or the Hamiltonian. So, this is the basic issue in classical mechanics.

That the first thing is, that you define the state of a system, and then describe how it evolves with time, how is its dynamics, how does it temporal evolution - and this is the central problem in mechanics.

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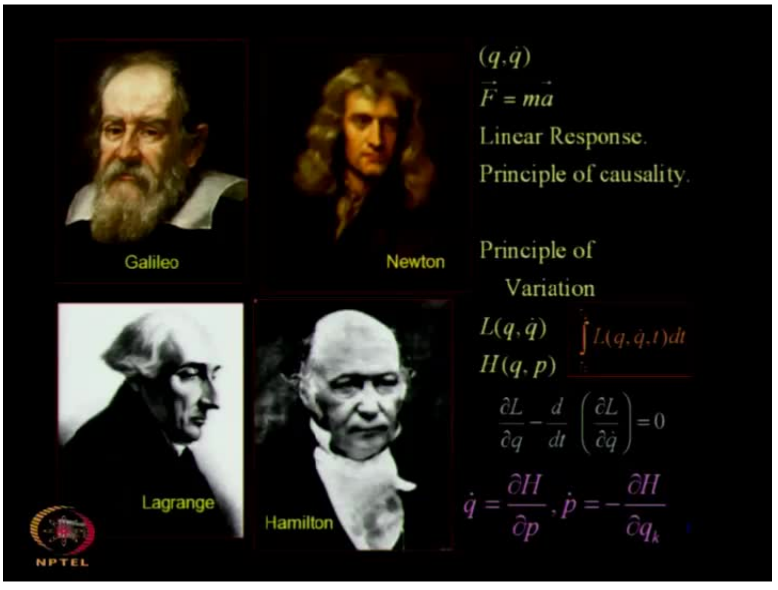

This question in modern times is answered very rigorously by the works of Galileo, who told us how to recognize an inertial frame of reference; by Newton, who explained to us what is it that causes departure from equilibrium. And he identified that cause, that stimulus, as the force to which the result, namely the acceleration is proportional, the proportionality being inertia or instead of invoking the cause-effect relationship in Newtonian scheme, it can also be - this problem of how does the system evolve with time - can also be addressed by invoking the principle of variation that this system evolves in such a manner, as would generate an action integral which would be an extremum, so that any change of that action would vanish at an extremum; it is often called as the principle of least action, but essentially it is an extremum; it could be a minimum; it could also be a maximum; in general, it is stationary, so that any variant of that action would vanish if you take a neighboring path in the configuration space or the phase space.

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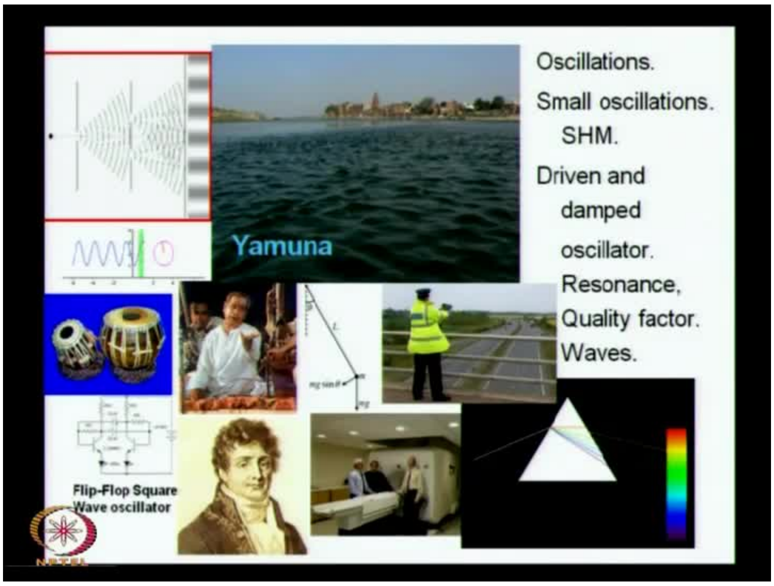

And these were developed by Hamilton and Lagrange, and these are able to explain a large number of phenomena, the physics and the dynamics near points of equilibrium is very nicely explained in terms of simple harmonic oscillations, because close to an equilibrium, close to a point of stable equilibrium, you always have simple harmonic oscillations; you have a linear harmonic oscillator. And that explains a large number of phenomena in physics and mechanics, and different branches of science including optics, wave phenomena, and the phenomenology of electromagnetic radiation, Young's double slit experiment, dispersion, even negative dispersion, many of these things, you know, are understood in terms of these oscillations. The basic science on which all these things are built is contained in the domain of classical mechanics, which is why it is so powerful and so widely applicable.

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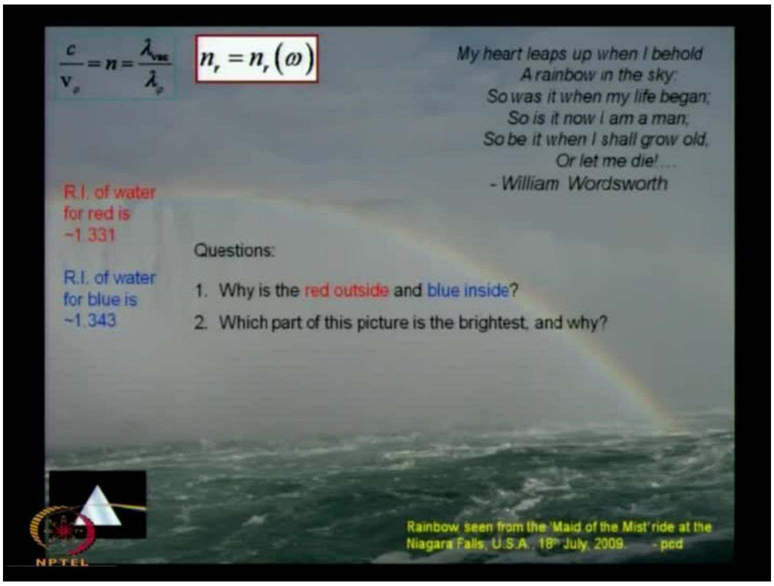

It explains reflections, total internal reflections, they are kind that of experiences that we have when we look at a rainbow, for example; all of these phenomena are explained by invoking the mathematics and the analysis that we generate, we develop from very simple phenomenology of the simple harmonic oscillators and so on; and all this you know the basic science, the basic physics which is involved, comes from classical mechanics.

So the scope of classical mechanics is really very vast. It answers a good number of questions for us. It tells us what a refractive index is and then we study various conservation principles.

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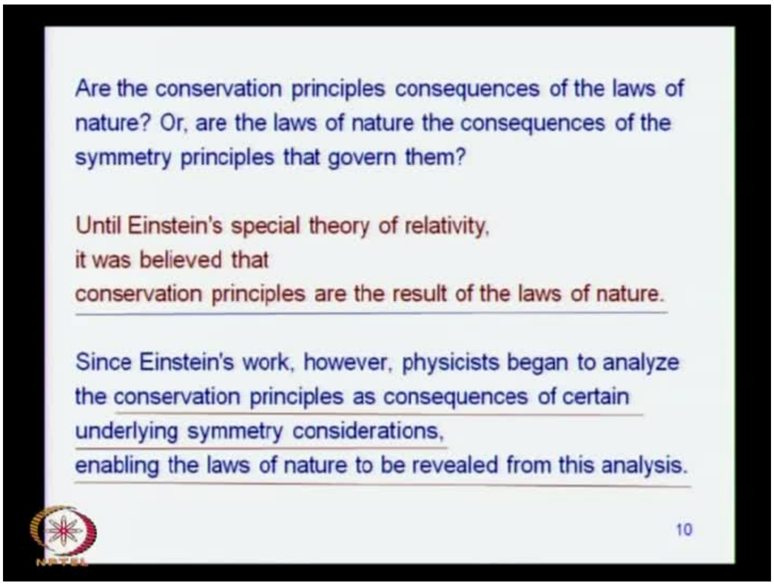

Now, this is a very exciting area, because we do know that if in a process certain properties are not going to be conserved, then those processes cannot take place. So conservation of energy, for example, is a necessary condition - it may not be a sufficient condition, but it is a necessary condition; in addition to energy, it may be necessary to have some other physical properties, which are conserved. So this is also something that you study in classical mechanics.

And quite often, you learn about the equation of motion and you also learn about the conservation laws. And you can ask this question - do you get the conservation principles from the equation of motion? And we discussed in one of our units, that if you begin with the equation of motion, and play with it, do some simple mathematics, you can actually get the expression for conservation of angular momentum; you can get the expression for conservation of energy by simply playing with the F equal to ma equation and doing simple mathematics with that.

So without putting any new ideas, by doing elementary mathematics, you can get, you begin with F equal to ma and you can get the principle of conservation of energy; you can get the principle of conservation of angular momentum. In fact, you can also get the conservation of the Laplace Runge Lenz vector, just by playing with the equation of motions; so you get many conservation laws out of it.

Now the conservation principle, you can ask - is it a result of the equation of motion? Or is it the equation of motion which tells you what a law of physics is? If that is related, and if it can be brought out from a conservation principle, and governed by some symmetry principles, which are connected to the conservation laws. So, this is a different question, which is a very fascinating question and you can answer this even within the framework of classical mechanics. We discussed in some detail how the Newton's law can be interpreted in this context , the Laplace Runge Lenz vector can be interpreted in this context, and this change in the way a physicist thinks about these issues, began with Einstein's theory of relativity, you know, and until which, it was believed that it is a conservation principles, which are the results of the laws of nature, rather than the other way around.

After Einstein's work, it became possible to see that conservation principles are consequences of certain underlying symmetry considerations, and from this, you can actually deduce the laws of nature as a revelation.

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This is a very beautiful approach and we saw that this can be illustrated by using very simple ideas from Newtonian mechanics - Newton's third law, for example - and this has now become a corner stone of modern Physics. In Noether's theorem, that there is an intimate connection between symmetry and conservation laws, and how laws of nature can be deduced by studying this relationship between symmetry and conservation laws.

So this was… This approach began with Einstein, elucidated very nicely by Eugene Wigner and mathematically put in a very compact and rigorous form by Emmily Noether. This, of course, has implications beyond classical mechanics, but you can see that it has got a place even in classical mechanics; so it comes very much within the scope of classical mechanics; but then you can also extend it beyond classical mechanics.

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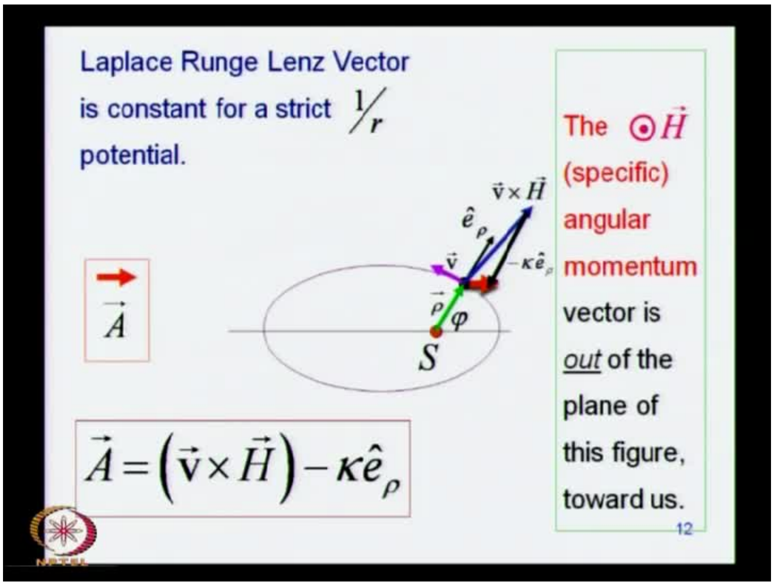

We could show that, this not only in the context of conservation of energy, conservation of angle of momentum, but also the constancy of the Kepler ellipse through the constancy of the Laplace Runge Lenz vector.

Now, this of course, we learnt that this could be explained only if the potential between the two masses is governed by the 1 over r law; so that was a necessary feature. The force that the 1 over r potential generates, is given by its negative gradient, so that will go as 1 over r square, and that becomes a necessary feature for this constant c 2 hold; this again falls very much within the domain of classical mechanics. So, you have a Laplace Runge Lenz vector which is directed from the focus to the perigee. And this is its constancy is what keeps the ellipse fixed in a two body problem, in which the interaction goes as 1 over r, is the same with an electron proton system of the whole quantum theory, which is the Neils Bohr’s atomic model.

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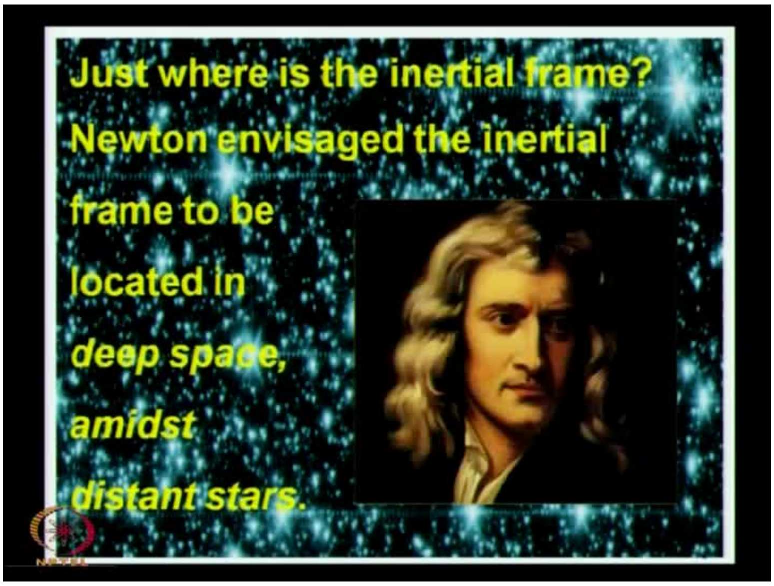

Now, all these requires the identification of an inertial frame of reference, which Newton envisaged as one, which is located in deep space.

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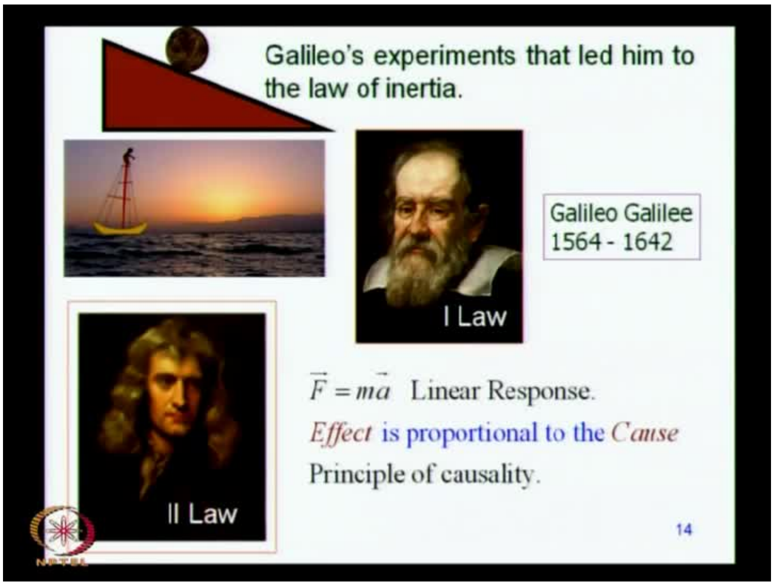

But this was very beautifully elucidated before Newton by Galileo who performed a large number of experiments, like dropping objects from the top of the mast of a ship. And what he found is, that the laws of motion remain invariant, if you observe them in a certain frame of reference, but also in another frame of reference which moves at a constant velocity, with respect to that. Now this is the essence of Galilean relativity in classical mechanics. So this relationship, F equal to ma, is sustained in any two inertial frames of references. The same physical law operates in an inertial frame of references and this is the essence of Galilean relativity in classical mechanics.

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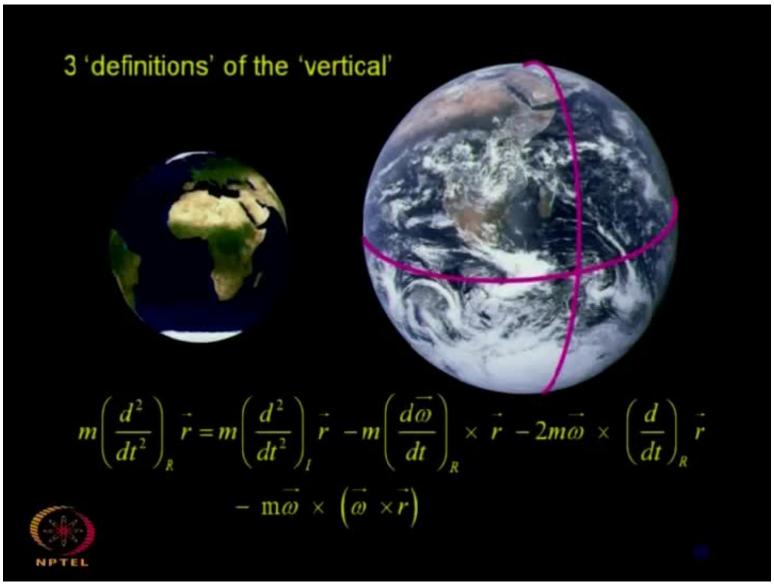

Nevertheless, when you carry out observations, in accelerated frames of references, and we know that on earth, we are already in an accelerated frame of reference, since we are rotating about its about the axis through the earth, other than the fact that we are going round the sun once in a year, in that elliptical orbit, and then of course, the whole solar system is going; so there are all kind of corrections that one must bring in, if you want rewrite the equation of motion as a cause effect kind of thing, but now you will have to invoke causes which really do not exist.

So these are not physical causes, but you write mathematical terms which have got the form of a cause in Newton's law - in Newton second law - and then reinterpret mechanics using these pseudo forces, that is a reason they are called as pseudo forces and all these belong to the domain of classical mechanics.

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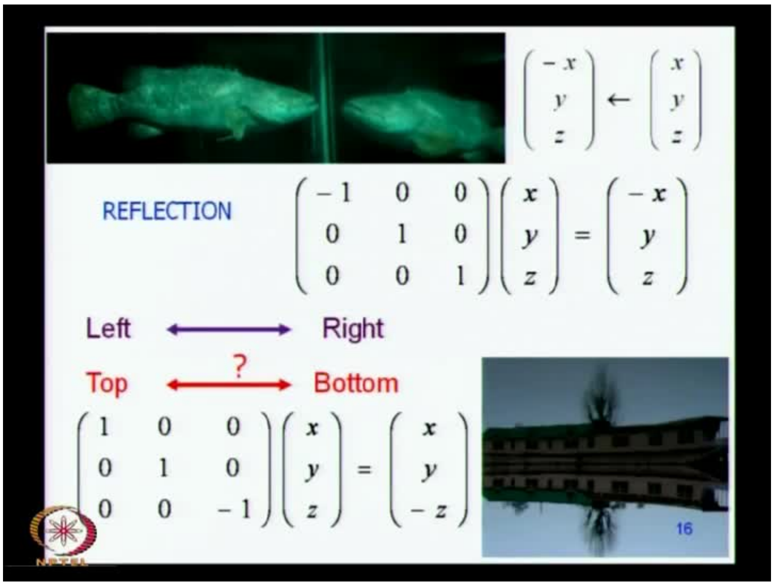

So you have got all kinds of terms; you have got the leap-second correction; you have got the centrifugal term; you have got the Coriolis term, etcetera. And it has got very exciting applications in our experience, because motion around us is observed by us in this rotating frame of reference and we want to understand it. If we want to understand it invoking the cause effect relationship, then we have no choice, but to invoke the pseudo forces if we want an accurate description.

Then there are certain symmetry considerations - the left going to right, right going to left, and the top not going to bottom, or bottom not going to the top. And this brings us to transformations of x, y, z, in one way or another, and the way these transformations take place is different if the transformation is because of the rotation of a coordinate system, or by an inversion of the coordinate system or a reflection of the coordinate system, then the transformation laws are different and these are mathematically, rigorously explained by looking at the corresponding transformation matrices, and you then have a rotation matrix or a reflection matrix, whose determinant is either plus 1 or minus 1 and depending on this, you develop very rigorous formalisms of carry out transformations.

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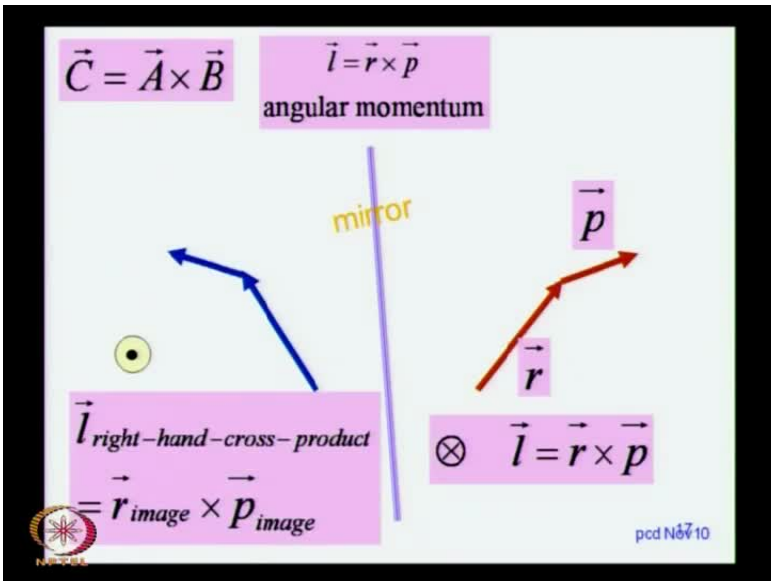

It also explains various things like, you know, there are different kinds of vectors. If you have a position vector, and a momentum vector, and if you construct the cross product of the two to generate an angular momentum vector, then how the angular momentum vector would appear under the reflection, is not the way the position vector would appear under reflection. So there are different kinds of vectors, and all this study of detailed transformation laws, which govern the rotations and deflections of different physical quantities, which are represented by vectors, all these again belongs to the domain of classical mechanics.

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The essence of Galilean relativity is that a game which can be played on a playground - a contest between Brett Lee and Sachin Tendulkar, if you like - could be played just as effectively on a huge play field, carried by a truck, as long as the truck is moving at a constant velocity, with respect to the previous frame of reference. So there is complete equivalence. This is the essence of general relativity. What it does, is it tells us how velocities are added in Galilean relativity. Because if the truck is moving, the velocity of the ball thrown by Brett Lee, which is like 160 kilometers an hour, there about is it right? I believe it is, 140-41 he bowls quite regularly, and I suspect that he has been at 160, but you might want to check the records for that.

But then, if you see it in a different frame of reference, and if you see it in a frame of reference, you could add or subtract to the ball's velocity, and you could make the velocity of the ball look either more or less by simply subtracting or adding your own velocity, to the velocity of the ball.

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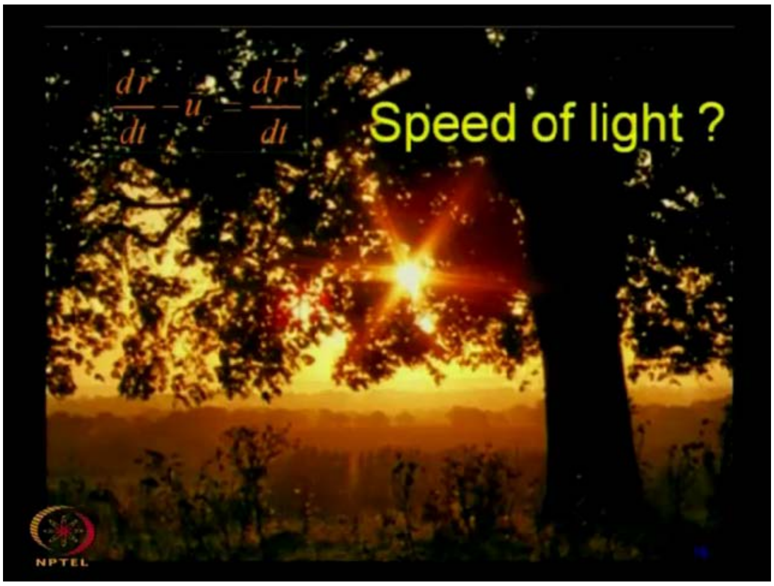

Now, this is how velocities are added in Galilean relativity. But can you do that if the velocity that you are looking at is not the velocity of a cricket ball, but the velocity of light. Then you cannot add the velocities using this kind of formalism.

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And then, you must enter a different domain, which also belongs to this domain of classical mechanics, which is the theory of relativity, which really came from Einstein, when he studied the laws of electrodynamics in some details. And these laws of electro dynamics were basically empirical laws developed by Coulomb, Ampere, Gauss, and Faraday.

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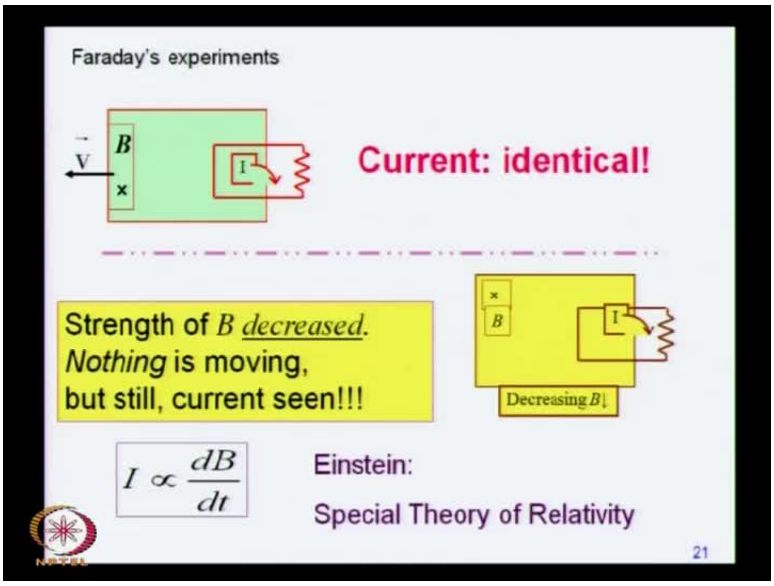

And then, there was also this Faraday Lenz law, that you have a circuit, you have an electrical circuit, you put it on a magnetic field, you drag this circuit, you would have no source of… you have no battery, in the circuit, but you just drag it in a magnetic field, and what you find is that a current set up and you explain it happily using the Lorentz force law; that is not very difficult to see. What is very difficult to see is that if you do not drag the electric circuit, but you drag the magnetic field, then result is identical, and then it becomes even more intriguing, when you realize that if you move neither, but just change the magnetic field, even then a current is set up.

So Einstein was very troubled by this equivalence. And the study of the symmetry, which is involved over here, is what really led him to formulate the special theory of relativity. So this whole area of electro dynamics, Maxwell's equations inclusive, belongs to the domain of classical mechanics; and, of course, also the special theory of relativity.

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Now, these are the major contributors - Galileo provided the essential thinking of what equivalent frames of references are. But what happens when you are carrying out observations on light, is that you have to revise these ideas, because what is equivalent in two different frames of references when we are moving it constant velocity with each other, is not the perception of space and time intervals, but the speed of light itself.

Now, Einstein had the intuition to figure this out by analyzing, the Faraday Lenz law in electrodynamics. In fact, Feynman points out in his lectures, that, there is hardly any or perhaps he goes as far as saying that there is no other place in physics, where a phenomenology has two different explanations, which are so different from each other. But the phenomenology is the same - that you have got a current, which is set up in that circuit, whether or not you move the circuit or you keep the circuit or move the magnetic field. The result is the same, but the explanations are very different.

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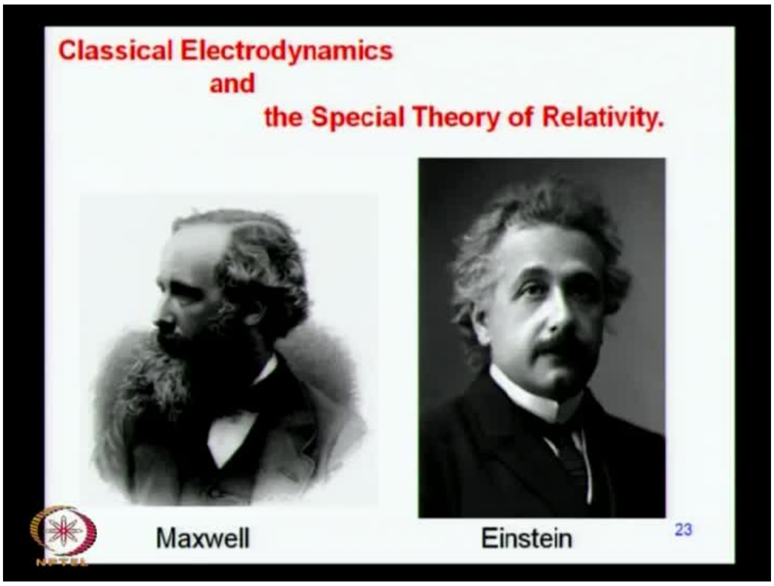

So it is the study of classical electrodynamics, which led Einstein to the special theory of relativity.

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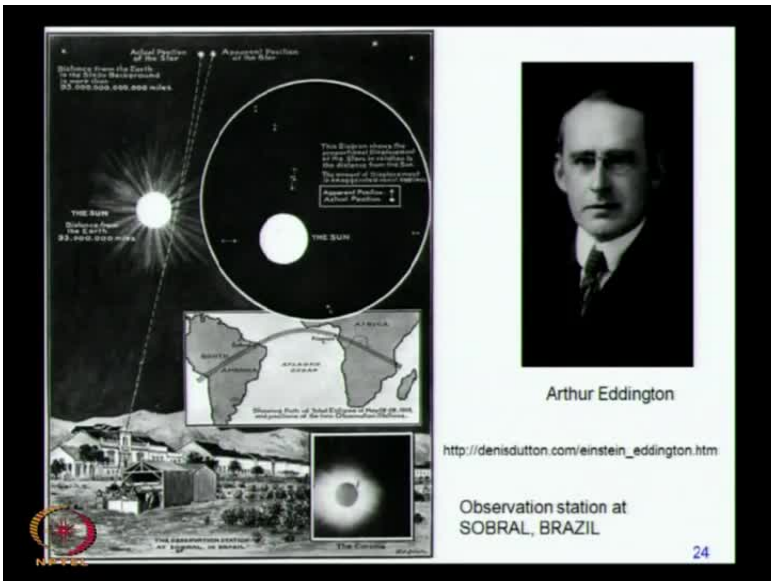

And subsequently also to the general theory of relativity, which goes beyond the special theory of relativity, so we did not have much chance to deal with the general theory of relativity, but it has got very amazing consequences, it could be verified. Einstein predictions could be verified by Eddington's experiments, which were carried out during a solar eclipse.

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And this has got very important applications in our lives. Cell phones work, of course, these are devices, which make use of quantum theory as well, because there are chips and there are the physics is governed by quantum mechanics, and that part of quantum mechanics is certainly, quantum mechanics is certainly, beyond the scope of classical mechanics, but quantum mechanics alone is not enough to explain how the cell phones work. You also need the theory of relativity, and you need not just the special theory of relativity, you also need the general theory of relativity. And the special theory of relativity certainly belongs to classical mechanics; it is intimately linked to Maxwell's theory, the laws of electrodynamics.

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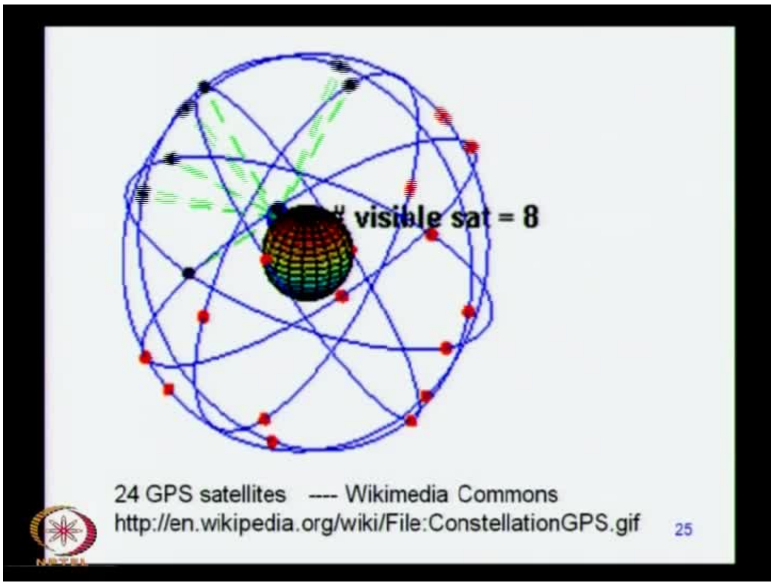

The laws of electro dynamics and all of these, the communication that we have, there are the satellites, there are global positioning system which works through all the satellites, all these work, and certainly these devices make use of quantum gadgets, but they also depend heavily on the consequences of various phenomenology that we learn from classical mechanics, in particular, the theory of relativity.

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Then, what is contained in classical mechanics, are some very rigorous mathematical formulations of various phenomenon, which are commonly observed in our human experience. We see water are flowing, we see fluid mechanics, we see fluid dynamics, and to explain all of these things rigorously, we see the planes flying and again very important properties of fluid dynamics are involved in this. And this requires very rigorous mathematical formulations, because you need to understand not just the qualitative behavior of various physical properties, but also their quantitative behavior, how these properties are connected with each other, how our potentials connected to the fields, how do you get the fields from the potentials, and how do you get potentials from the fields. And is it just a matter of mathematical transformation or is there something deeper in it? So there are plenty of very fascinating questions which come up and these are answered using very rigorous mathematics developed by people like Gauss, Stokes, Thomson, and so on.

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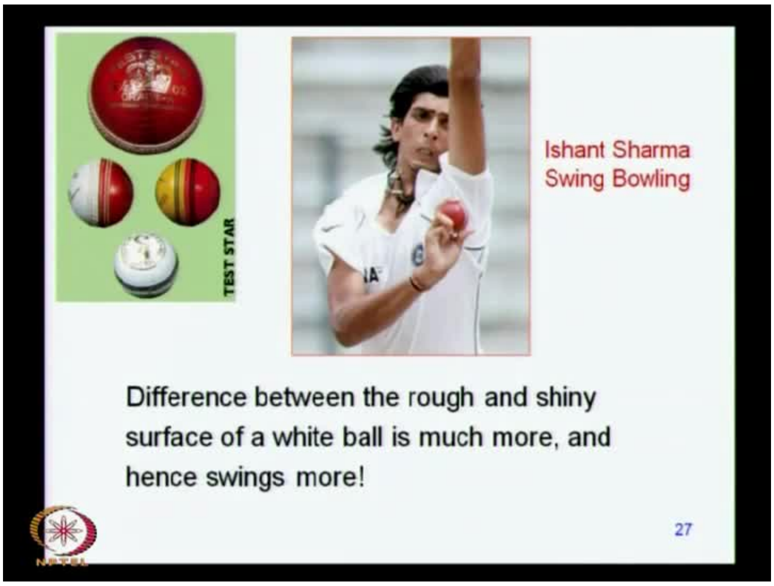

In fact, Thomson is one, who developed the Stokes theorem, but it is popularly known as the Stokes theorem, and this explains fluid dynamics; it explains things about how water flows, how vorticity is explained, why you would use a white ball in a 2020 cricket tournament.

And these are of course issues of interest to us. And to be able to explain this, to be able to explain the vector fields, the velocity fields, you need to understand both of the divergence as well as the curve of the vector field. You must know the divergence and the curve, both. Knowing just one or the other is not enough. And you see these getting into the equations of motion for the fluid, it becomes the manifest in when you study the Bernoulli's principle in some detail, and it is involved in understanding all of these features around us. Now this again belongs to classical mechanics.

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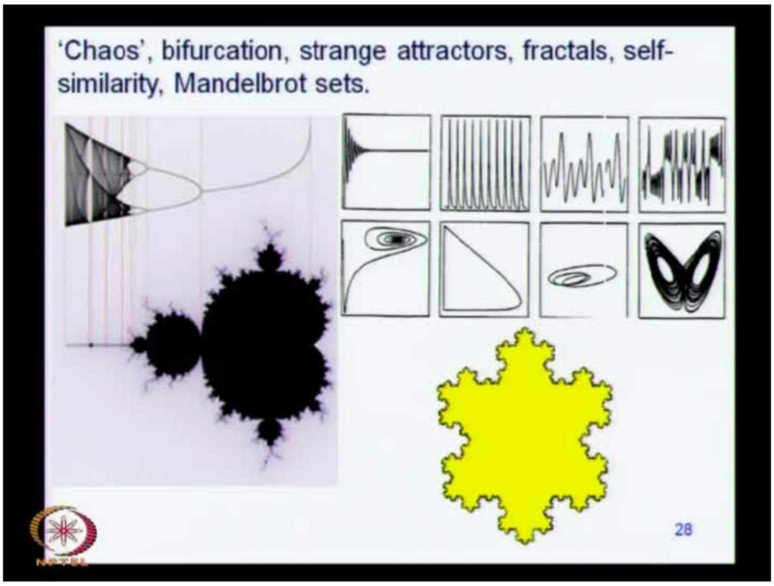

And then as we studied in our last unit, the study of chaos, bifurcations, attractors, strange attractors, fractals, self-similarity, Mandelbrot sets, all of these things again belong to classical mechanics. So classical mechanics has got a huge scope.

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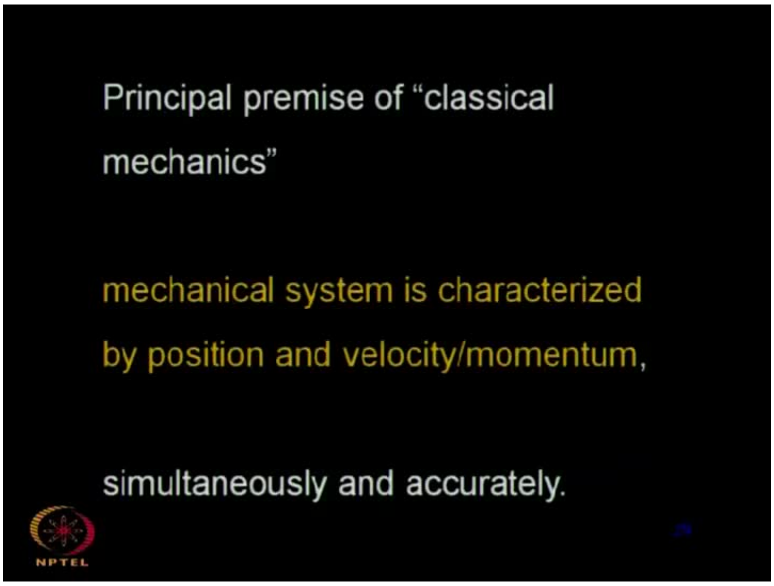

The principle premise of classical mechanics is this contention, that the mechanical system - and this is what I said at the very beginning of today's discussion - that a system is characterized by position and velocity, or position and momentum, and this must be known together. Knowing one or the other is not enough; you must know that together.

And at some point we need to test this assumption - is this possible at all? You perform a measurement, the way to know it, is to perform a measurement, is there any other way of knowing it? You have to perform a measurement or you cannot perform a measurement, without a probe. If you can make the interaction between the probe, and what it is measuring weak, and of course, you look for a weak interaction, because if it is a strong interaction, then you will disturb the target. Then the target will acquire properties, which are not the properties you wanted to investigate, because the perturbation between the interaction and the target, would change the properties of the target. So you make the interaction weak. Or you make it weaker still, so that you get a more accurate description of the target? Can you make it 0? If you make a 0, then you get no information about the target. So there is no choice, you have to have an interaction and you cannot avoid disturbing the target.

Now if you disturb the target, and then, you seek more information about the target, but then, this would be seeking more information about the target, which has been disturbed by a previous measurement. So it is not guaranteed that, after you do this measurement of the second property, you come back and repeat the first property, you will recover the first result. In particular, when you carry out these measurements and position and momentum, you go ahead and make a measurement of momentum of an object, you need these two to describe the state of the system, and now we are talking about the limitation of classical mechanics.

You want to perform a measurement of the momentum of the particle, moment of an object, and you must seek this information, because these are the essential characteristic features of a mechanical system. The premise whether it is a Newtonian mechanics or Lagrangian mechanics or Hamiltonian mechanics is that the system is characterized by the position and the momentum, or some function there of, it could be the Lagrangian, it could be the Hamiltonian, it must have information about the position and the momentum, and both of these parameters must be simultaneously specified.

So you measure one, make the interaction as weak as you like, but having measured it you cannot make the interaction 0, and when you perform this measurement, you will get some result, which you will tabulate as the momentum of that object. Now, you want to measure the momentum, so you go ahead and perform a measurement, which will give you information about the position of that object. Momentum, you have already recorded in the first measurement. Now you measure the position of the object; you perform an experiment to find the position. Yes, you will get an answer, it is not that you cannot get an answer; of course, you will get an answer; so you get their answer you record that.

Now would you like to check if the answer you got for momentum is reproducible? If you measure the momentum again, now what is going to happen? Is that, if you repeat the experiment which will give you information about the momentum, you get an answer which is completely different from the previous experiment. So you cannot believe in your measurement. The reason this is happening is because of measurements of position and momentum are not compatible. It does not mean that no two measurements are compatible; certain measurements are and some are not. Those which are not compatible, are the ones which are involved in the Heisenberg's principle of uncertainty.

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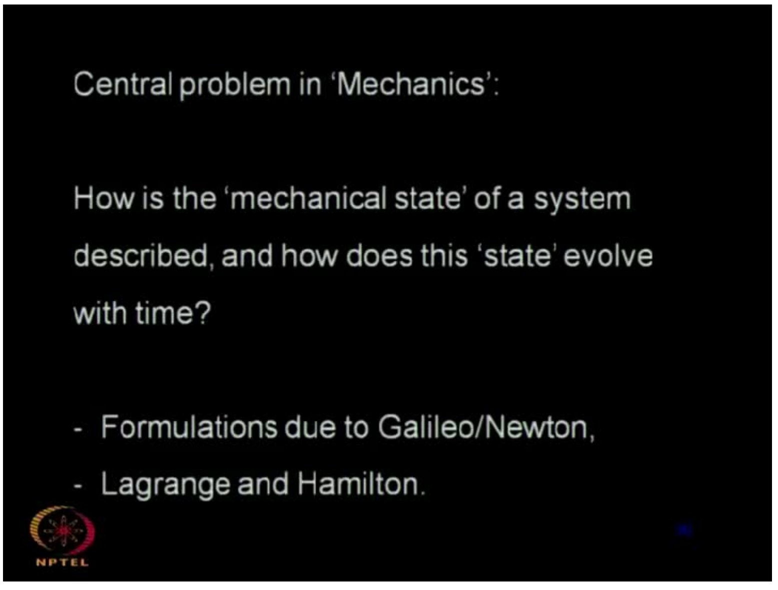

So this is central problem in mechanics, in which you require position and momentum whether it is the formulation of Newton, Isaac Newton or the Galileo Newton formulation or the Lagrangian formulation or the Hamiltonian formulation - all of these would break down, if you cannot measure the position and momentum accurately.

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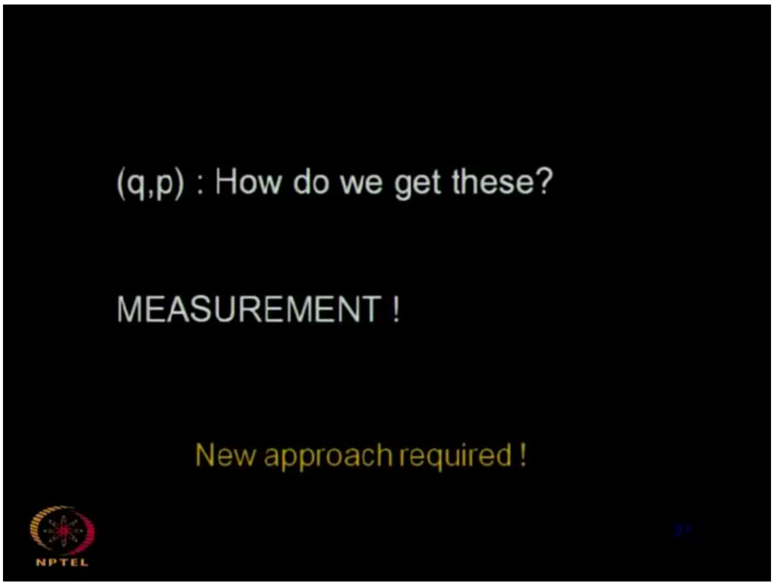

Now, it is important to know that these measurements are not compatible, and if they are not compatible, then you cannot specify the state of a system by position and momentum, because a point in a phase space, then loses its essential characteristics feature, that it can be specified at all. Because you cannot get position and momentum simultaneously, so you need a new approach all together.

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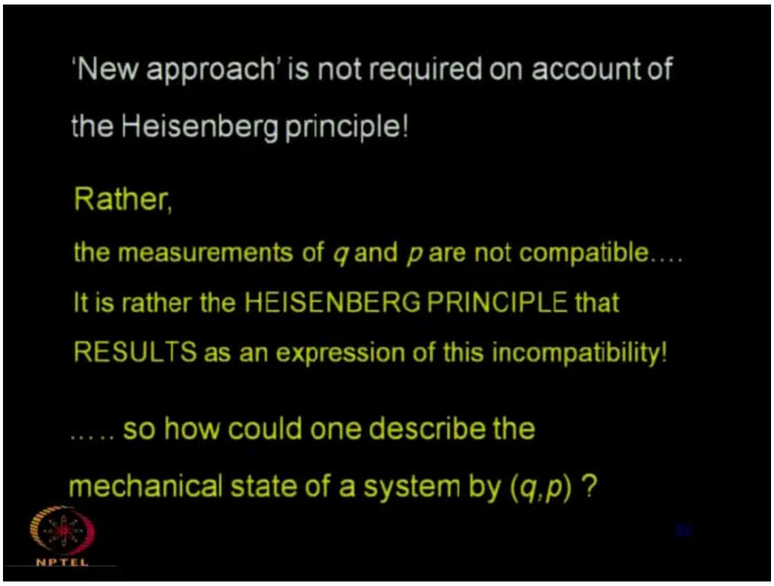

Now, one refers to this as the Heisenberg's principle, and in introductory courses, students often make a mistake that you cannot get simultaneous information about position and momentum, because of the Heisenberg's principle, which is very silly statement to make. It is just the other way round. The Heisenberg principle is a very quantitative expression of the fact that you cannot make accurate measurements of position and momentum simultaneously. So, that you cannot make these measurements is not because of the Heisenberg's principle, but it is the Heisenberg principle because you cannot make the simultaneous measurement; so you still have a point, but it is important to state it.

So we have to abandon this idea, that you can describe the state of a system by a position and momentum, and this takes us beyond classical mechanics. This is where you meet the limits of classical mechanics. So, classical mechanics has got a huge scope, no doubt about it. But if you seek detailed answers to such questions, you hit the limit and you must go beyond it. You have to abandon the idea that the system can be described by a position and momentum; you must look for something else; what you are going to look for? It has to be something else. Can you think of something? We know it has to be something else and this is the beginning of quantum theory. What else you find to describe the state of a system is the state vector; that, this is what describes the state of a system, but the basic problem in mechanics remains the same. How do you characterize the state of a system? Classical mechanics answers by saying position and momentum.

The next question is how does it evolve in time? It evolves by writing the expressions for q dot and p dot, dq by dt, dp by dt. These are the rate equations; these are the Hamilton's equations. So, the problem remains the same in quantum mechanics, that you are now abandon in the description of the state of the system by qp, you are describing it by a state vector. Now you ask - how does the system evolve in time? How does the state vector evolve in time? What is the time derivative of the state vector? Is the same question. And this is answered by the Schrodinger equation.

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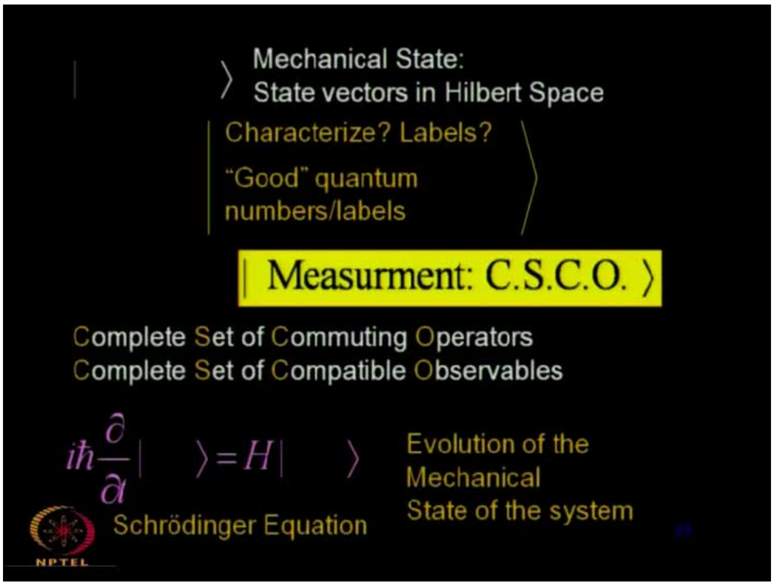

The Schrodinger equation tells you the rate at which the state vector evolves with time. Then, of course, there are various initial conditions and other things, boundary conditions and so on, which have to be specified; you then need labels to characterize these state vectors, which you call as good quantum numbers, because we just agreed that it should be described by a state vector. But then, how do you characterize the state vector? How do you designate these state vectors? And, these of course, can be designated only by measurable properties of the system. So measurement is fundamental to physics at all levels in classical mechanics as well as quantum mechanics. And you have to perform measurements, and the only measurements which will give you a consistent set of parameters to characterize the system, all the measurements which are compatible with each other, and these measurements in quantum mechanics are represented by commuting operators.

So you get, you seek maximum information about these parameters, and you can get it by looking for the complete set of commuting operators. Now you get into a different domain of physics, which is the quantum mechanics, and the time evolution of the system, which is a rate equation again, the time derivative of the state vector is what you are looking for, this is given by the Schrodinger equation.

Sir, whatever the good quantum numbers that depend upon the choice, whenever, suppose, in atomic physics we will do some when the interaction is stronger than given effect, and the good quantum numbers changes, then how can I know, how can you differentiate them whether it is good or bad?

Well, a good quantum number is what you can measure. Now depending on the strength of the magnetic field, now you are talking about a more complex situation, I would be happy to comment on that. What happens is that there are various couplings which are involved; an electron has got an intrinsic angular momentum, which is this spin, it also has that the orbital angular quantum number, and depending on the strength of the magnetic field - if the strength of the magnetic field is large, then the spin and the orbital angular momentum couples to the external magnetic field more strongly than the way it couples to each other. So the relative strength of these couplings is what tells you, what a good quantum number will be. Because, if one phenomenon is stronger, then the corresponding labels are the only ones that you can measure; you can only get the Eigen state of the strongest terms. Otherwise, you get a super-position and a system which is at a mixed state, you can never say that it is in this state or the other. So the quantum numbers of the constituent terms which go into the super-position, do not remain good quantum numbers any more.

So the moment the system has to be written as the super-position of two states which have got different quantum numbers, then it does not remain a good quantum number. But if you perform a measurement, the system collapses into that Eigen state, into an Eigen state of that measurement, and you repeat that measurement, you can always recover the same value, then it is a good quantum number.

Then it will be good quantum number.

Yes, because when you perform a measurement, the system collapses into an Eigen state of that measurement, and having collapsed into an Eigen state, repeated measurements will give you the same results; but if it does not collapse into an Eigen state, but as a result of the measurement it is described by a super-position of states, then the next time you perform a measurement, it could fall in one component or the other component with different probabilities, which are given by the square of the amplitudes of the coefficients in the super-position. So neither of the components gives you the good quantum number.

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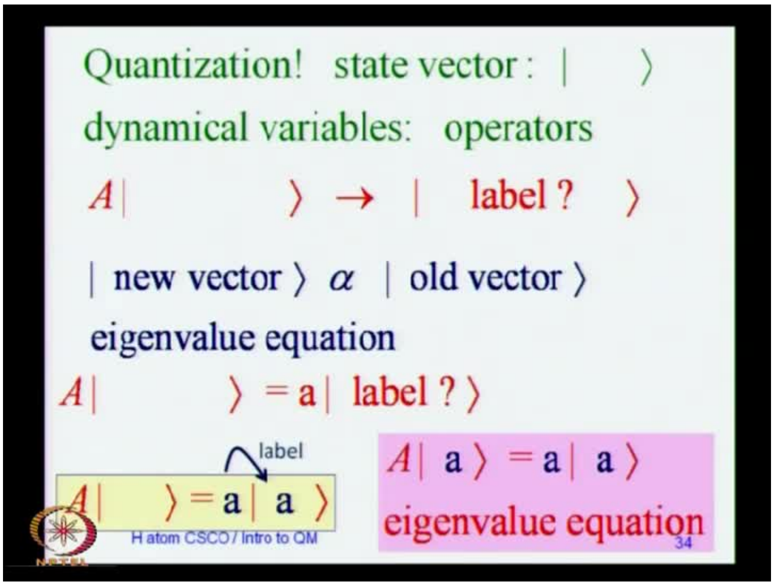

So this is quantization that you have to describe the state of a system by state vectors and dynamical variables as operators, and then, you carry out these measurements, and from the results of these measurements, you get the Eigen values; the result of the measurement is what manifests in an Eigen value equation and what you will need to do is to use this Eigen value as a good quantum number. This is what gives you a good quantum number; it is a measurement. The Eigen value equation, in fact, may mix a measurement process, that when you perform the measurement of a certain property, the system collapses into an Eigen state. And this, the result of that measurement is what goes as an Eigen value, and this Eigen value can then be used to label the state vector, and that is why it becomes a good quantum label.

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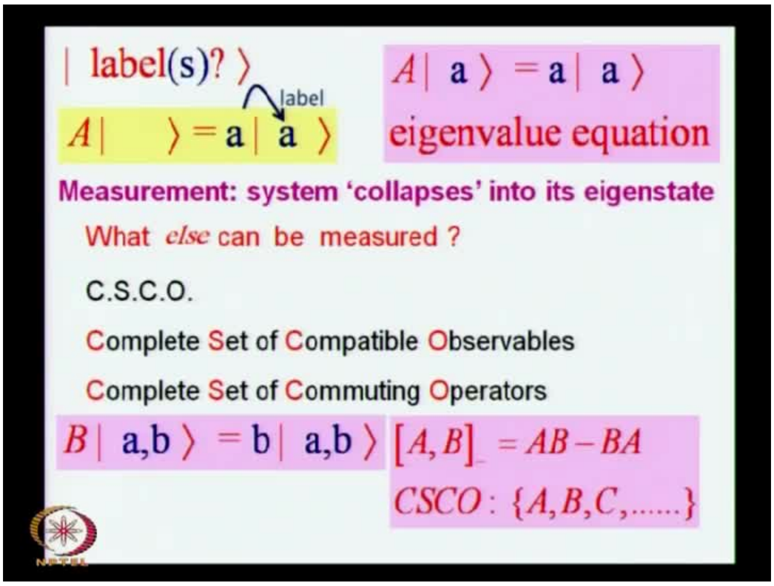

You want to characterize the state of the system as best as you can. So you need more than one label, because you want to maximize the information you can get about the system, which is why you look for a complete set of compatible observables or complete set of commuting operators, and you look for additional measurements, not just the measurement of some property called A, but another property called B, but when you perform a measurement of B, then the system must continue to remain in the Eigen state of A, only then are A and B compatible. So that is a small exercise in quantum mechanics, that you can diagonalize two operators in the same basis only if they are coming to.

Sir giving A avoiding on small A operator you giving in the A operator there with a Eigen value small a, but if it is something different, if it is suppose c or d ,which that it is a number apart from A, then it is it can also be integrated as an Eigen value equation.

It is an Eigen value equation, but the question you are asking is what is going to happen if you measure A; now you measure B, now come back and measure A. Do you get the previous value of A? If you do, only then A and B are compatible, which is just the order in which you carrying out the measurements. That is the principle of commutation, that whether you do a measurement of A first and B next or B first and A next. So, only commuting operators will give you good quantum numbers. If they do not commute, then you cannot get good quantum numbers out of those Eigen values.

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So this is how you get complete set of commuting operators and this is the domain of quantum mechanics developed by Schrodinger, Heisenberg, but of course, many others. And I, in my mind, the contribution of Niels Bohr is amongst the tallest, but for different reasons in that though these are some of the questions which are to be addressed in a different course.

And that pretty much concludes our discussion on Select Topics or Special Topics in Classical Mechanics. If there are any further questions, I will be very happy to take them. Otherwise, goodbye for this one last time, and thank you all for participating in this, specially for asking questions and you are quite welcome to send me your questions, send it to me by email if you like. And I am notorious for replying, so if you send an email, you will get a reply. So do not ask a question in the hope that any way I am not going to read it; I certainly will reply.

It will be a good idea to write something on the subject line, that it is a question on classical mechanics or quantum mechanics, because if I do not recognize the subject nor the ID, then most likely I will end up deleting the email, because I get scores of emails every day as everybody else does, and many of them get deleted, just because I do not recognize either the sender or the subject line, and I would not know the email IDs, of most of you, who will be sending me these emails. So make sure that you write something on the subject line, which connects that it to the physics question, as then I will immediately identify, that it is an email, which I want to read.