Select/Special Topics in Classical Mechanics

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Module No. #02

Lecture No. # 08

Oscillators Resonances, Waves (ii)

Greetings again. We started discussing about the oscillators. We discussed what the simple harmonic oscillator is and we pretended that the only force which is acting on the oscillator is a restoring force, which is proportional to the displacement.

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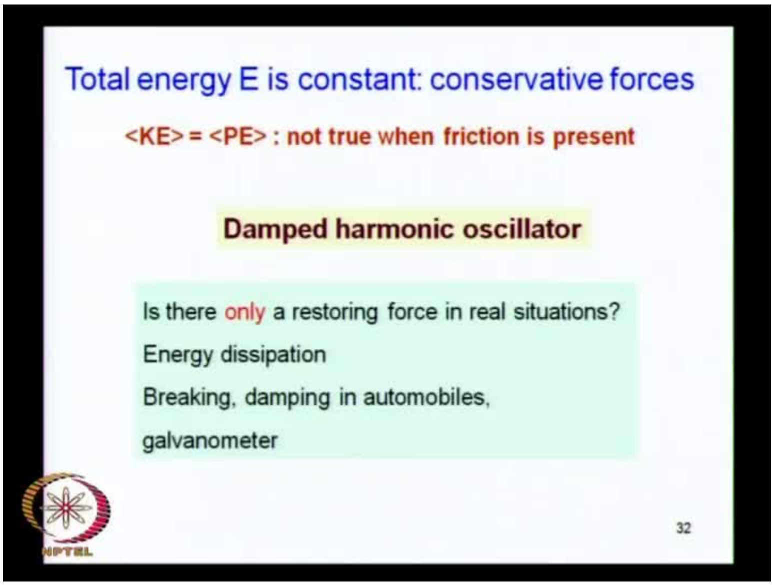

In other words, we ignored damping, because quite often, there are other forces which act. And in such situations when the system is treated as an ideal system, then the system is conservative - the total energy remains unchanged; the average kinetic energy is equal to the average potential energy, but this is not true when damping is present.

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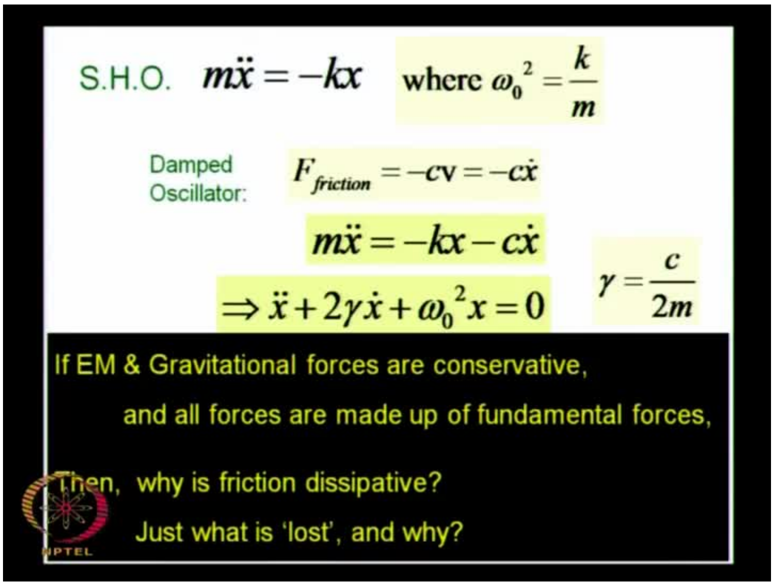

Now, in this class, we will discuss the damped harmonic and we will discuss what is damping. In real situations, damping is not always a nuisance, because quite often you need it , like if you are driving fast and you want to stop, you do need brakes. So, it is not always malice; it is not always a bad thing. You need damping in various devices - in galvanometers for example. And one really needs to understand what this damping is and how it works.

So, this is our equation of motion for a simple harmonic oscillator. And this sample harmonic oscillator means - We have written the spring constant and the inertia, but it could be an LC circuit or it could be any other physical phenomenon which displace displays simple harmonic oscillations in which the potential is a quadratic function of the independent degree of freedom.

Now, in a damped oscillator, we consider such cases in which in addition to this restoring force which is minus kx - in addition to this, there is some other force which we had not earlier taken into account. And this force - a very common form that is used for this force is one which is proportional to the velocity and given as minus c times the velocity or this is a very common expression used for the damping force.

The equation of motion will contain the restoring force minus kx and it will have this additional term. There is a position dependent force and there is a velocity dependent force. And it is for this reason that very often one is led to believe that if a force is velocity dependent, it is not conservative. Now, that is not quite correct; that is not how a conservative force or a non conservative force is identified; because, there are some velocity dependent forces which can in fact be conservative - the electromagnetic force is one such example; because you have got the v cross b force coming in the Lorentz force and it is of course a conservative force.

So, the criterion of a conservative force is not that it is independent of velocity, but that the line integral that it generates is independent of the path. So that is the criterion that should be used and not anything to do with just the velocity; that means - a velocity dependent force, in this particular case, is a non conservative force, but not always. So anyhow, we consider here damping to be represented by this minus cx dot term and this is the equation of motion that you need to solve.

So, you arrange, rearrange these three terms, bring all of them to one side and divide by the mass. So you get acceleration plus twice gamma x dot gamma is this ratio c over 2 m; because, I have divided every term by m plus omega 0 square; because, this is the relation between k and omega 0. So, that relation I have used. This is the differential equation of motion that we now have to solve.

Now, one must ask at this point before we go any further - that ok, what is meant by conservative force? Real forces, in nature, are made up of fundamental forces and then what is it that is dissipative? Because, when you talk about dissipation, when you talk about friction, when you talk about a force being non conservative, when you talk about energy being lost, what is the cause of this loss? What is being lost and why?

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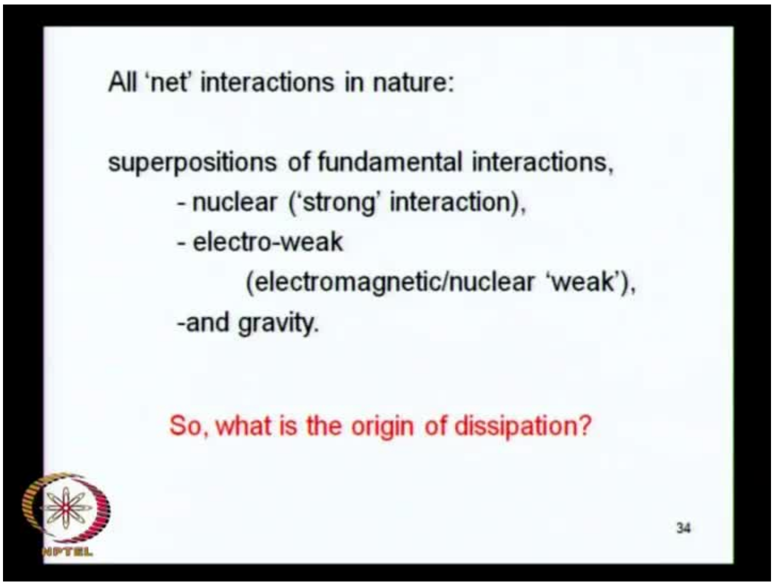

Now, the net interaction between any two objects in nature is some superposition of the fundamental interactions. And the fundamental interactions are nuclear, strong and weak, which we really do not have to worry about in day to day life and most of physical phenomena. The electromagnetic interaction of course, yes. Or gravity, One can also talk at a certain level - the electro weak unification; but it does not matter – means - whether you talk about the unified field theory or the grand unification or whatever; in any case, that is some superposition of the fundamental forces of which the nuclear forces do not contribute anything to most of the physical phenomena that we observe in nature.

The gravitational interaction of course, is conservative; so is the electromagnetic interaction. Because the electromagnetic interaction, as I said, which is represented by the Lorentz force, which is charge times e plus v cross B. At the v cross B term, is also conservative interaction; because it generates a path integral for the work done which is independent of path.

So, if all the fundamental interactions are conservative, the better; because, you and I do not create or destroy energy. So that they do not bring build temples for you and me – right? We do not create energy; we do not destroy energy. All the fundamental interactions are conservative and any net interaction is a superposition of these fundamental interactions. So, how can there be any dissipation at all? How can there be any thing that you might call as energy non conservation?

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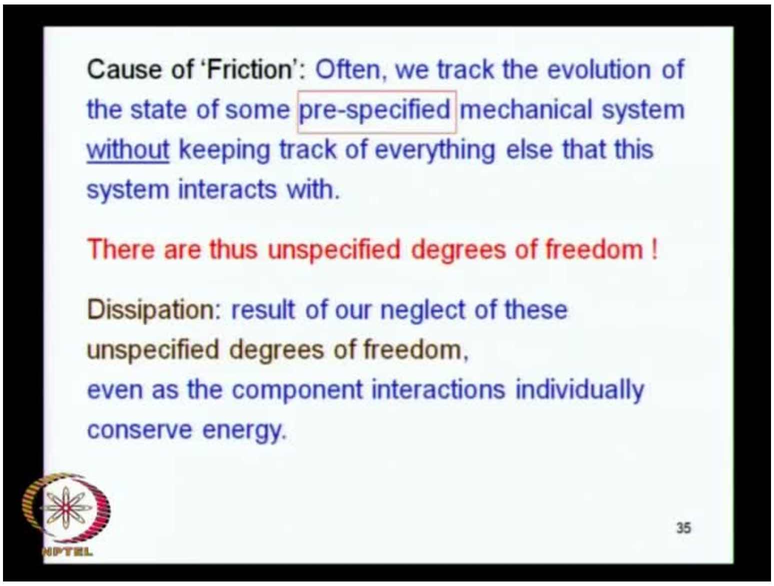

So, what is non conservation of energy? So, what is an origin of dissipation is the question that we must really address. Whatever I like to point out is that, this comes only because of the manner in which we set up the equation of motion – means, ultimately when you solve a mechanical problem, what you do is - you identify some system – that ok, I am going to solve the mechanical problem for a certain system. There is a certain mechanical system - it could be this; I look at it. Alright and say that I want to study the mechanical state of this system and how this state evolves with time.

This mechanical state of the system is described by its position and velocity. These two parameters are required. And how these two parameters will change with time is what I want to study. When I discover that, I would have solved the mechanical problem – right? So, define my mechanical system to be this; nothing else. But this system is on the table and when I trace the mechanical evolution of this system - if I am dragging it along the table. right then I have not included the table in my analysis, but the system is interacting with the table. So, the whole issue is centered around this particular - you know - identity of the system for which you are solving the mechanical problem.

If the mechanical problem is being solved for this system and nothing else and you do not consider anything else, then this is my pre-specified mechanical system. And I am not keeping track of everything else that this system is interacting with.

There are these unspecified degrees of freedom and if I keep track of all the degrees of freedom and take all the interactions into account, I will not have to deal with non conservation.

Now, if I set up the equation of motion for this system alone without keeping track of these other degrees of freedom, but I still want to take their effect into account, what I am going to do is to bundle up the effects of the interactions of this system with everything else. So, I am going to study the response of this to a primary force which could be the push of by my hand - that is the primary force and that is the only interaction I am going to be concerned with. And I am going to ignore the interaction of this object with everything else - with the table and so on. But I still want to bundle up those effects into my analysis - not in terms of the pair interactions between every particle of this object and every particle of the table and the air and the atmosphere and so on and so forth.

Then I can do it in some approximate manner by pretending that the net result of all of this is an additional force, which is minus C times of velocity. Why is it minus c times the velocity and why not minus c times velocity square? Yes, there may be some features which go like that. It is not coming from first principles.

So, in a large number of cases, minus c times v turns out to be a good approximation. It is not going to be so in every situation; it is not coming from any fundamental principle and it works in a good number of cases and that is the kind of situation that we are going to work with. So, this is what causes dissipation; because, we have not kept track of all the details. And having ignored the details, we are then condemned to an approximation; we are not now condemned do an analysis only in an approximate manner; we are not going to get exact solutions. We will therefore lose some of the energy of the system; because, this the energy of the system is not that of this piece alone; there is the energy of the other unspecified degrees of freedom which we are not keeping track of. So, some energy will be lost to the other unspecified degrees of freedom and this is the cause of dissipation. It comes from essentially the unspecified degrees of freedom. This is the origin of friction in real situation - in real physical situation. Friction is not a fundamental force; fundamental forces are only gravity, electromagnetic forces, nuclear strong and nuclear weak – that is; nothing like a friction force.

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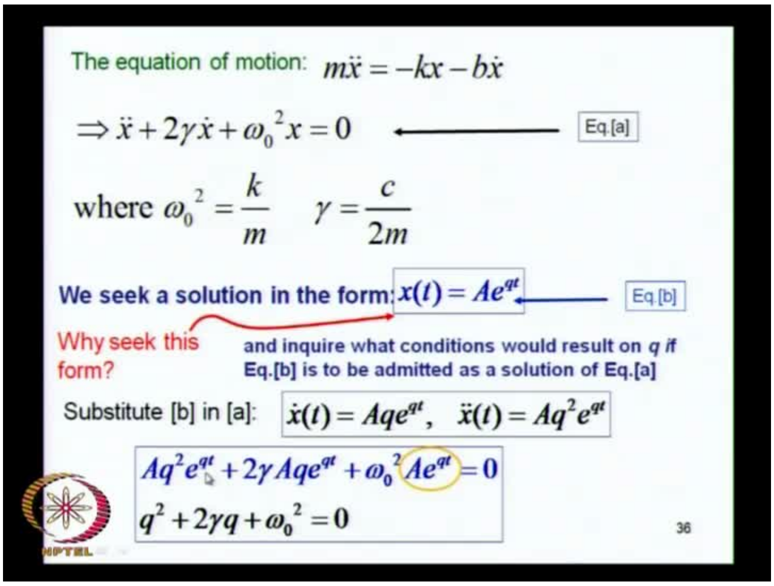

Now, this is now our equation of motion. Omega 0 and gamma are connected to the intrinsic properties of the system. c is coming from our approximation c or b. I think I have used b over here and c over here - sorry about that; it is the same thing and What we do is - we seek a solution to this differential equation in this form. Now, note the expression here. We seek a solution in this form which is A e; to the e to the power qt, where q is independent degree of freedom. And then we ask the question - what conditions would result on q, which is what we are trying to determine; because, the description of q and q dot, as time evolves, is what we are after. That is our mechanical problem - right?

We are asking - what conditions will result on q and q dot if such a form is admitted as a solution? Now, why do we seek this form? There is a good reason to do that. One is that if you look at the form e to the q t, then the function e to the q t is already like e to the plus or minus i omega t. Then we have met for the simple harmonic oscillators. So we know that the solution is going to be different, but we expect it to be somewhat similar. So we expect a solution which will have some similar form and that is similarity is happily accommodated in this. And that is part of the reason that you seek a solution in this form.

The other reason is - you can also see from the form of this solution that you can meet - you can expect very easily to accommodate a situation in which, as t goes to infinity, x will go to 0 and the system will come down - be arrested in its equilibrium position, which is what you intuitively you expect the system to do in the presence of damping. So there are good reasons for why you seek a solution in this form. And then, of course, we want to look for the most general solution and we do know that we are dealing with a second order differential equation; it should have two linearly independent solutions; there should be two and not more than two constants and these will have to be determined from certain initial conditions.

This obviously is not the most general solution. It does not even have two constants right. So this only gives us the form and not the actual solution. Now, using this form, what do we get? What we do is to take the derivative. The first derivative x dot and the second derivative x double dot. When you do that and you plug this first derivative and the second derivative back at this equation, you get Aq square e to the qt - these exponential functions are so easy to take derivatives off and then you get this Aq square e to the qt from this term. And then you get twice gamma Aq e to the power qt from this term and omega 0 square x, which is Ae to the qt. And then Ae to the qt is common to all the three terms; so you can actually strike it out; because the right hand side is 0. You can divide the whole equation by Ae to the power qt; strike it out. And then you are left with a quadratic equation in q and this is a consequence of the second order differential equation for x.

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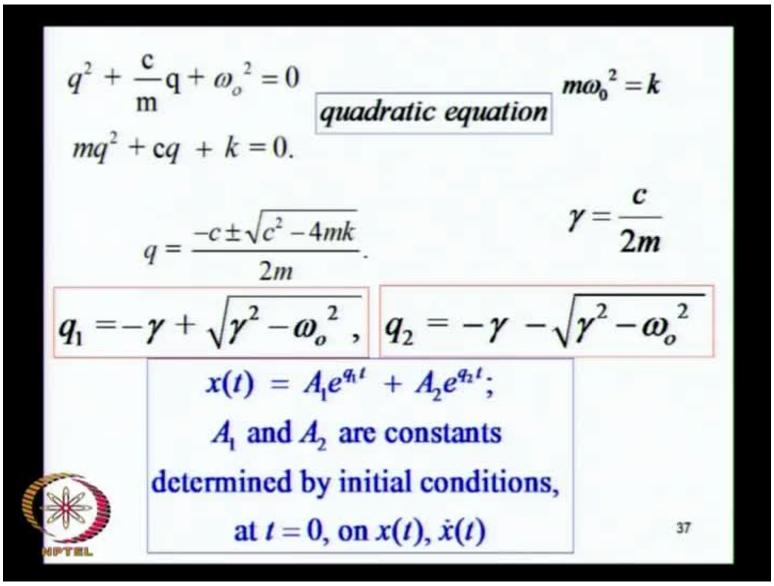

From differential calculus, we have reduced the problem to algebra. Iinstead of dealing with a first order derivative and a second order derivative and having a second order differential equation, all we have to do is a quadratic equation in q which we know how to solve. It has got two roots.

This is the quadratic equation for q; it has got two roots. These are the two roots and if you take, a few admit both; because both are valid solutions to the quadratic equation. Then you have the general solution, which is A 1 e to the power q 1 t and A 2 e to the power q 2 t. So, this becomes your general solutions. Now, it has got two constants; these two are independent and you get a general solution.

You can get A 1 and A 2. These two unknowns are to be determined. But they can be determined by putting - by finding what is x at t is equal to 0 and x dot at t equal to 0. So, if you plug in the values of x and x dot at t equal to 0, you can determine the two unknowns and your problem is solved.

So, essentially, the whole problem of a damped oscillator is now done. It is only the details that we have to worry about. And these details are quite interesting; because the details depends on the nature of these two roots; because these two roots have got values - one is minus gamma plus this square root factor and the second is minus gamma minus this square root factor.

So, the nature of the solution will depend on how this gamma, which is the damping constant, compares with this intrinsic natural frequency omega 0. If gamma is equal to omega 0, then this square root factor will vanish. And gamma the relation between gamma and omega 0 can be one of equality and the only other two possibilities are that gamma can be either greater than omega 0 or gamma can be less than omega 0. Once you admit these three possibilities - gamma greater than omega 0, gamma equal to omega 0 and gamma less than omega 0, you would have covered all the possibilities. And these are the details which govern the dynamics of a damped oscillators.

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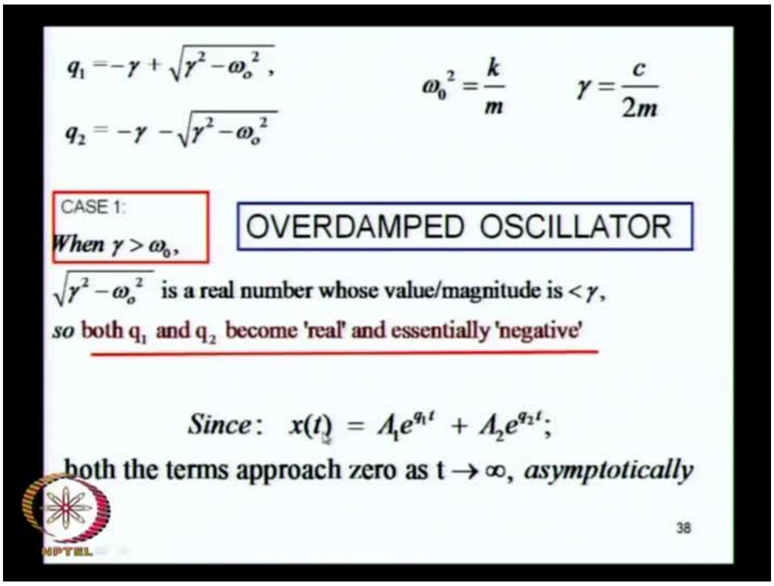

So, we will now consider the case when gamma is greater than omega 0. This case gives us a damped oscillator which is in fact, called as an over damped oscillator and I will explain why it is called as an over damped oscillator.

So, let us consider this case - the first case is when gamma is greater than omega 0 and in this case, square root of gamma square minus omega square – means - depending on the proportions of gamma square and omega 0, the factor under the square root sign can be either 0 or positive or negative. So, in this case, the square root will be a real number whose magnitude must be less than gamma; because it is a square root of gamma square from which is certain positive quantity is diminished.

Omega 0 square will have to be positive; omega 0 also is positive - of course. So, from gamma square the omega 0 square is diminished and therefore, this real number must have a value which is less than gamma.

And what you are doing is - you have minus gamma in both the roots to which you are either adding or subtracting in number whose magnitude is less than gamma. If you subtract, this becomes even more negative; if you add, it still remains negative; because, you are adding a number whose magnitude is less than gamma. So, no matter what both the roots q 1 and q 2 become essentially negative - both the roots - neither of the root has a chance of being positive.

Now, what did it means is that, if both q 1 and q 2 are negative, contribution from either of these terms - the first term or the second term can allow x to go to 0 only as t goes to infinity. At no finite time can x ever become 0. In other words, this oscillator can never hit the equilibrium in finite time. It will need infinite time to come back to 0.

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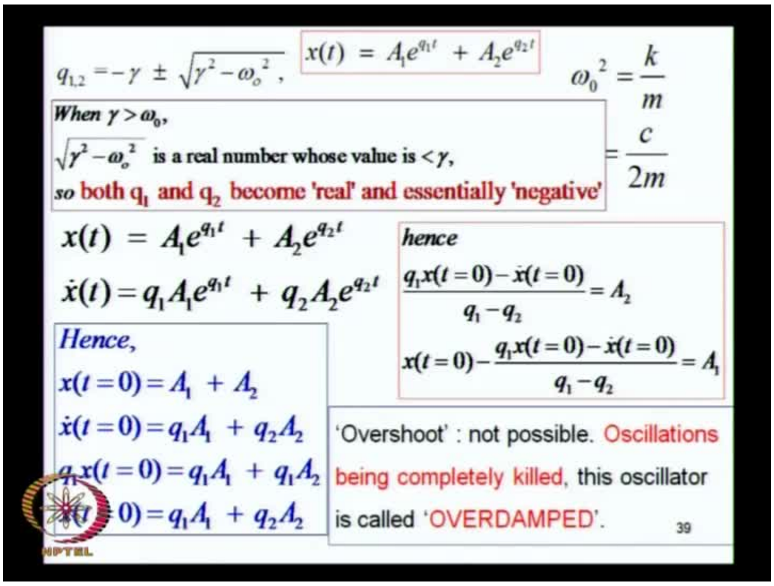

So, this is the case we are dealing with. And these are the two solutions - you have the position and you have the velocity. If you take the initial conditions for position, you get A 1 plus A 2. If you take the initial condition for velocity, you get q 1 A 1 plus q 2 A 2. And from these, you have two equations and two unknowns - A 1 and A 2. You can solve them algebraically and get the two unknowns - A 1 and A 2 and you have your complete solution.

So, you get A 2 as the difference of this numerator and q 1 minus q 2. And then you get A 1 from this equation by subtracting from x at t equal to 0 the value of A 2. So you can get both A 1 and A 2 and your problem is essentially solved.

Now, obviously, an overshoot is not possible; because the system can come to the equilibrium and it cannot cross that equilibrium point - that is called as the overshooting which is to get the equilibrium point and then should be on it; that cannot happen; because, that is going to happen only in infinite time. So, the oscillation is completely killed. If it cannot cross the equilibrium point, the question of it coming back - so it is really not an oscillator at all; because there is no oscillation.

So, it really does not oscillate; the oscillation is completely killed, but the equation of motion is basically governed by the dynamics of the equation of motion for the oscillator which is damped - subject to the conditions that we are dealing with. Such an oscillator is called as over damped oscillator. So, that is the origin of the word over damped. This type of an oscillator is called as over damped oscillator.

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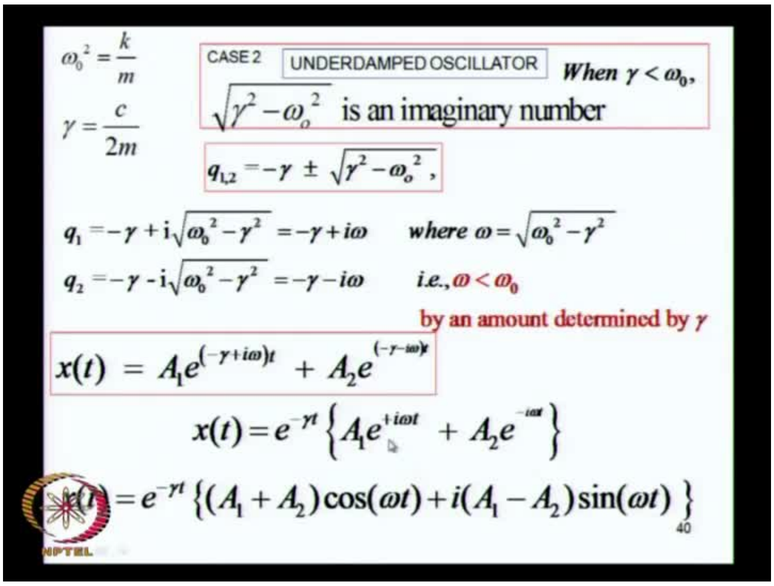

Now, we will deal with the case when gamma is less than omega 0. These are the three possibilities that we agreed to discuss - omega greater than omega 0 which is when we get the over damped oscillator. Now, we have the omega gamma less than omega 0 and when gamma is less than omega 0, gamma square minus omega 0 square will be a negative quantity and we have its square root. So, this square root of a negative quantity must be an imaginary number and this imaginary number is what we write as i omega.

So, the two roots which are minus gamma plus or minus this imaginary number - the two roots are now written as q 1 is equal to minus gamma plus i omega and the other root is minus gamma minus i omega - so these are the two roots.

This omega, of course, has the dimensions of frequency and this omega - its value is determined by the difference between the damping coefficient gamma and the natural frequency omega; the difference between these two, when you take the difference between their squares and then take the square root, you get omega. So, this is determined by the properties of the natural frequency omega 0 and also the damping coefficient gamma.

Now, these are this is the complete general solution, which is A 1 e to the power 1 root times t and A 2 e to the power the second root times t; the second root is minus gamma minus i times omega. So, this is now your general solution to the oscillator, when gamma is less then omega 0 and I still have to explain, why it is called as an under damped oscillator; I have used the term which we will understand soon enough .

So, let us write this solution. Now, e to the minus gamma t is a common factor to both of these. So, I factor it out; the remaining part of the solution is this and you can write this remaining part of the solution in terms of the cosine and the sine function, instead of the exponential function, because cos e to the i theta is just cos theta plus i sin theta.

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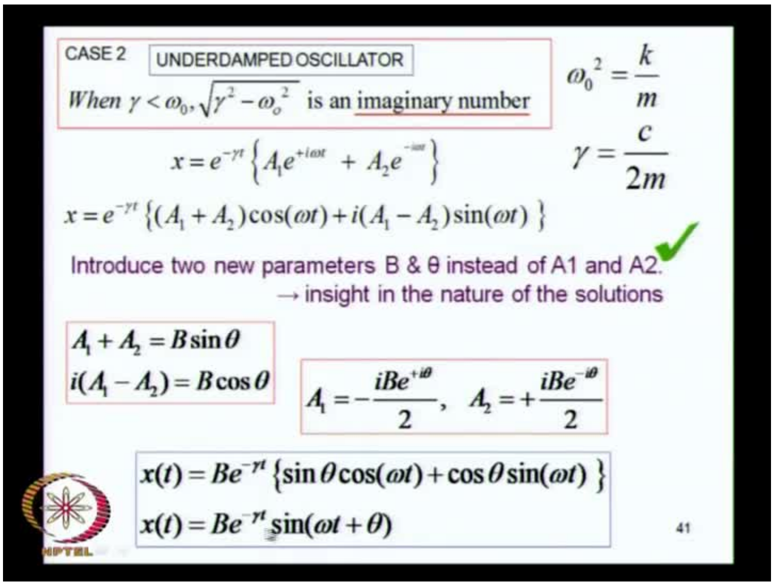

So, you can write these exponential functions in terms of the cosine and the sine functions. This is the nice way of looking at it; because when you write it as such - you find that you, can transform this form in which you have written the same solutions in terms of the cosine and the sine functions. By introducing two parameters B and theta instead of A 1 and A 2. As long as, we do not introduce additional parameters.

So, these two parameters b and theta, instead of A 1 and A 2; not in addition to A 1 and A 2, but instead of A 1 and A 2. So, B and theta are such parameters, which you can determine in terms of A 1 and A 2. If you do so, then nature of the solution becomes very easy to interpret and it enables you develop some insight into the form of the solution.

So, that is the advantage in introducing these two parameters. So, this is how you introduce these parameters - you define A 1 plus A 2 as B sin theta. And the difference A 1 minus A 2 times i as B cos theta. And that is these are the relationships between A 1, A 2 and B, and theta. In terms of this, the solution x theta becomes B e to the minus gamma t times sine omega t plus theta.

Now, this is a general solution in which there are two parameters B and theta; whereas in this form the two parameters were A 1 and A 2. So, this is the general solution there are two parameters but this form is very easy to interpret because you immediately see that this is the sinusoidal motion just like it is for a simple harmonic oscillator.

So, you can compare it with an ordinary simple harmonic oscillator. We know that this is a damped harmonic oscillator. So, it will have some behavior similar to as simple to harmonic oscillator and you get a solution in the same form, which is an oscillatory sinusoidal function and then there is a phase shift theta and there is a peculiar feature here; that there is an amplitude here which is not constant as it is in the case of a free oscillator but an amplitude, which has got this e to the minus gamma t factor. So, the amplitude is going to progressively diminish with time and that is the effect of damping. Because gamma is not 0; if you put gamma equal to 0 damping will vanish and e to the 0 will be equal to 1.

So, e to the minus gamma t is the damping consequence on the amplitude and here you have a simple harmonic oscillator whose amplitude diminishes exponentially as time progresses.

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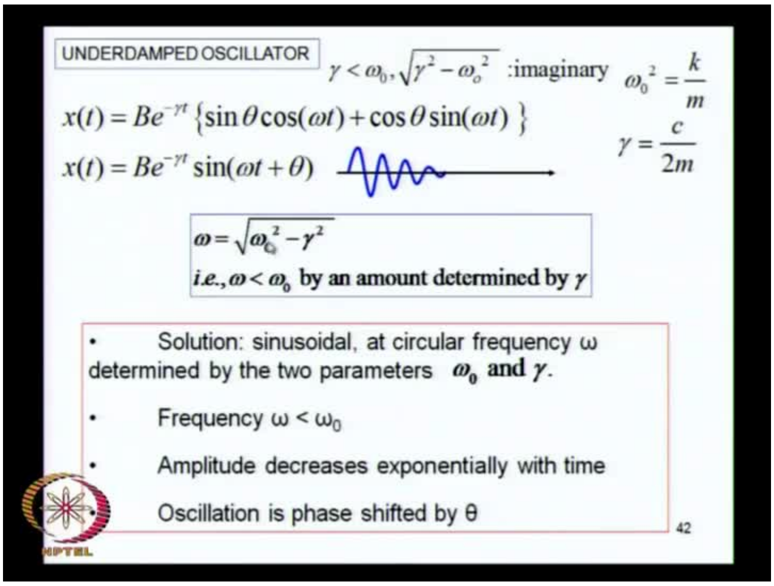

So, here you see that this amplitude of this oscillatory motion diminishes and if you look at the envelope envelope it will go down exponentially. So, this is the general function this is oscillatory it oscillates at a frequency omega which is determined by omega 0 and gamma.

So, the periodicity of this oscillation is governed by the natural frequency as we expect but also by the damping, which has been inserted; so omega 0 square minus gamma square is responsible for omega not being equal to omega 0. So, this oscillator will have oscillations but not at the original natural frequency of omega 0 but at a slightly different frequency.

Motion is sinusoidal at a frequency omega, which is less than omega 0. Why less because, you are taking the square root of omega 0 square but diminished by a factor gamma square. So, omega will have to be less then omega 0. So, the frequency is less than the natural frequency, the amplitude decreases exponentially with time and there is a phase shift because the argument is not just omega t but phase shifted by the factor theta.

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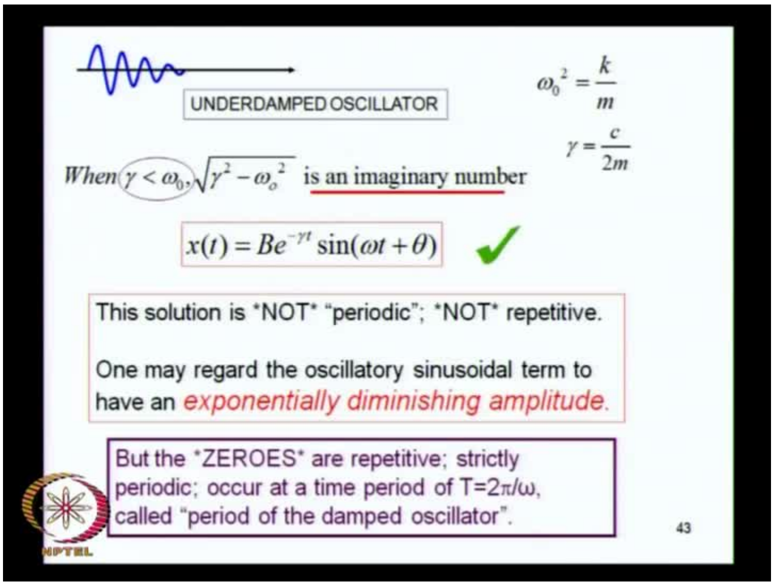

So, this is our overall picture, this solution is not really periodic; the motion is not completely periodic. What is a periodic motion? Motion is periodic if it exactly repeats itself; now this cannot repeat itself because amplitude is diminishing what was true from here to here is not true from here to the next point. So, there is some sort of periodicity but not complete.

Because the amplitudes are not periodic the zeroes are it goes to the 0 periodically that is exactly periodic. So, omega is the circular frequency, which is exact and this zeroes will repeat exactly at that frequency. So, many times per unit, per unit time. So, the zeroes are repetitive but the amplitude is not the total motion is not periodic amplitude diminishes exponentially and this omega is called as a period of the damped oscillator.

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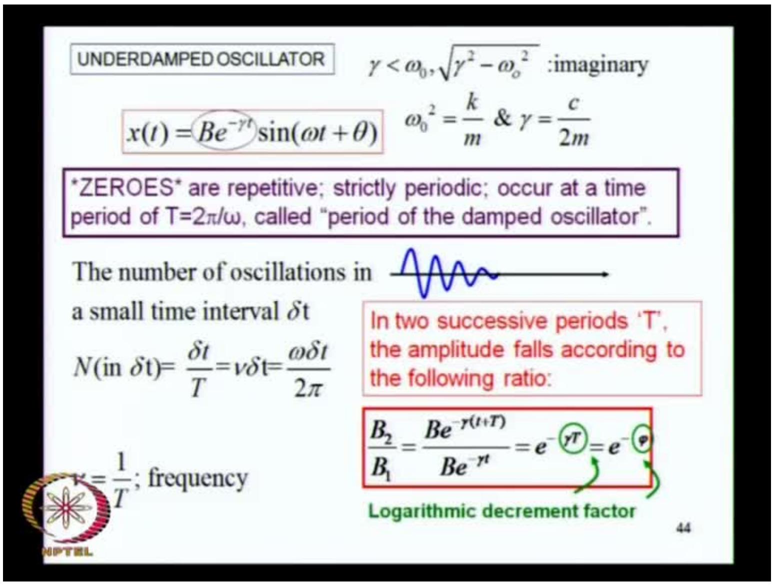

So, this is called as under damped oscillator because motion is damped but it is not damped the way the over damped oscillation was so as oppose to this. This is called as under damped oscillator.

So, now you can ask, what is the number of oscillations in a small interval of time delta t because you can see that there is an oscillation it goes up remains positive, it goes down remains negative; becomes positive again. So, the oscillator is actually swinging on both the positive on the left positive and the negative sides of the equilibrium points. So, there is in fact an oscillation at a certain frequency that frequency is 1 over T and the number of times it will oscillate in a time delta t is simply the ratio delta t divided by the periodic time 1 over T is the frequency nu and this nu is nothing by the circle of frequency divided by 2 pi. So, this is the number of times you will have oscillations in a time interval delta t.

So, you can get that very easily; you can also ask over two successive periods, you go from here to the next point and from here to the next point and over this period, you see that the amplitude has fallen and you can ask by what factor it has fallen. This is the corresponding factor because this amplitude, which is B 2 B times this factor at the next cycle divided by the corresponding factor in the previous cycle is what I call it as B 2 over B 1 and this is just B times e to the power minus gamma t but here the argument is t plus 1 period where as here it is just minus gamma t. So, this ratio is nothing but e 2 the power minus gamma T. So, this is you know since this is an exponential function this is called as a logarithmic decrement factor.

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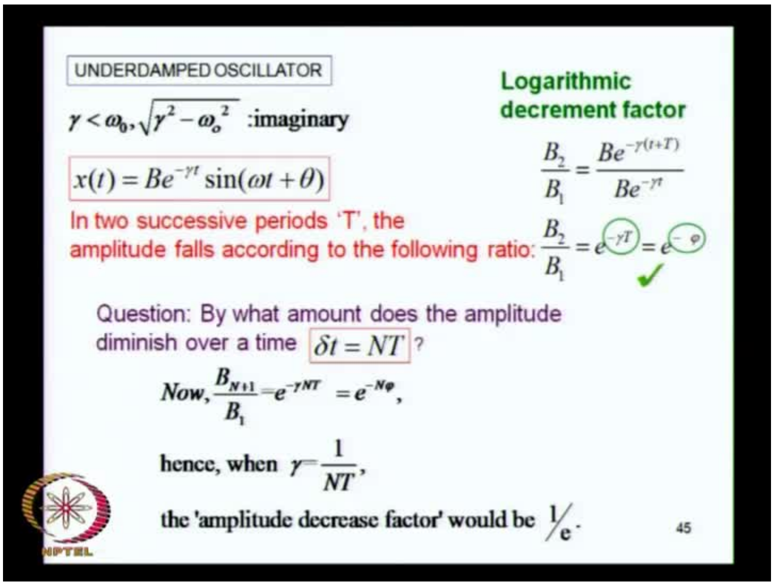

So, this is the factor through which the amplitude falls over successive periods and you know the amplitude decrease factor would be 1 over e because if you let gamma equal to 1 over NT if gamma is equal to 1 over NT then this successive ratio it will go as 1 over e. So this is the factor.

V over here is imaginary right

What is it

Its complex e is complex

It could be its when you plug in the initial conditions and get the exact values your final numbers will come out to be exactly real because you are looking at real quantities.

You have to put in you have to put in initial conditions we are not determining the value of B.

B is dependent on A 1 and A 2 but you have not put the condition that A 1 minus A 2 is 0 is A 1 minus A 2 is the coefficient of the imaginary part.

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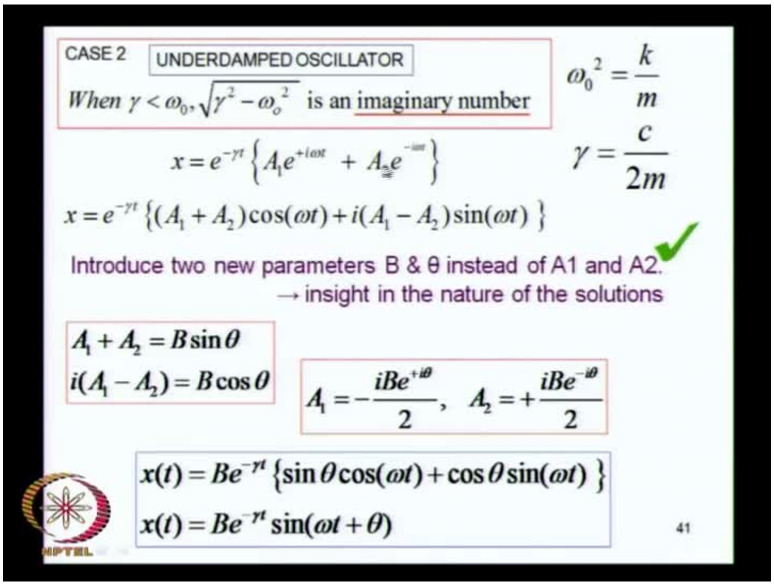

It does not matter here means if you look at this. Nobody were said that A 1 and A 2 are always real. You have a general solution right, you have a general solution A 1 and A 2. When you put in all the initial conditions and the initial condition on x, and the initial condition on velocity both will be in terms of real numbers. There will be a certain displacement, if it is a displacement in position it will be so many millimeters or centimeters or whatever it is some length parameter and there will be some velocity which will have the dimension of length over time.

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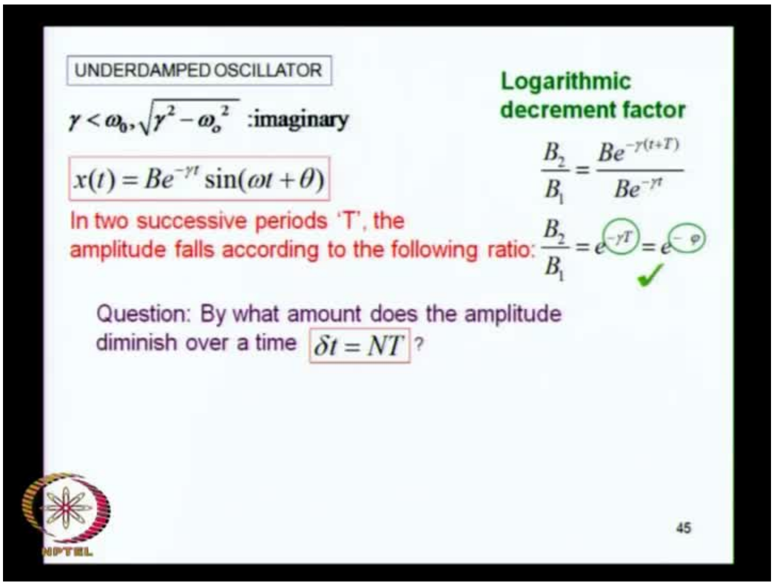

So, your final answers whether you deal with A 1 or A 2 or B or theta they will all turn out the(). So, that is not an issue the only reason why you use complex numbers is because when you express it allows you to write your solution instead of sinusoidal functions as e to the power i omega t kind of thing i omega or minus i omega t and that is a very convenient form because when you take the derivative, you get the same function, so no matter whether you take the derivative once or twice all you have to do is to take the corresponding powers of the multiplier.

So, converting differential equations to algebraic equations becomes very simple when you deal with exponential functions. Essentially, the only thing you are doing is you are dealing with two real numbers together.

So, whenever you work with complex numbers there is nothing imaginary about the physics or complex number, only gives you the power to deal with two real numbers at the same time; one is what you call it as a real part. The other is what you call as imaginary part, but the imaginary part is as real as real part. So, there is nothing imaginary about the imaginary part. It is just imaginary part of the complex number by itself, it is a real number. B is a real number in A plus i v right, so that is not an issue here.

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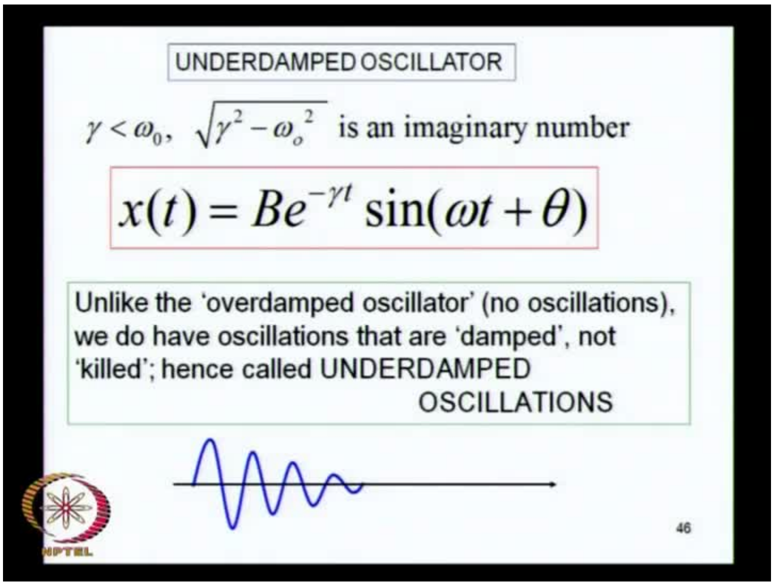

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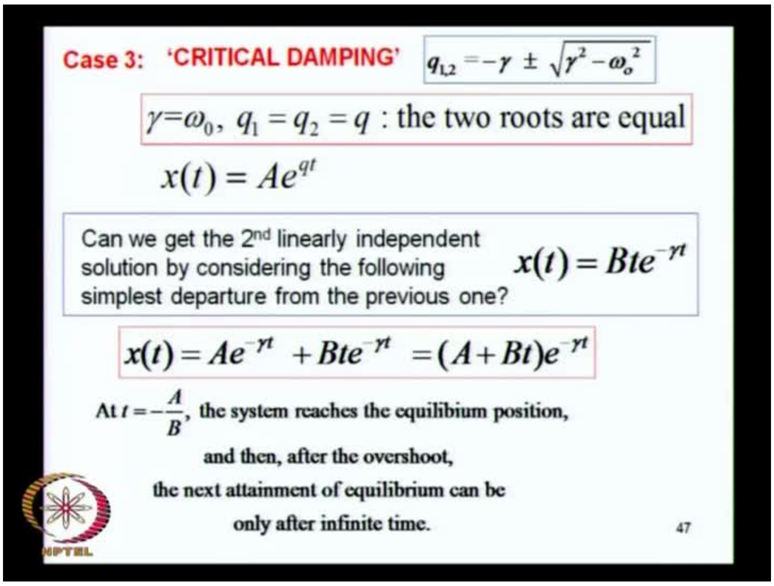

So, this is your under damped oscillation because the oscillations are damped and not completely killed as was the case in the over damped oscillators that is the reason this is called as the under damped oscillations. It has you know two unknowns. So, it is the solutions is completely general second order differential equation must have you know two unknowns and now, we consider the only remaining case, which is called as critical damping because the only case that we have not consider is when gamma is equal to omega 0. We consider gamma greater than omega 0, we consider gamma less than omega 0 and now we are going to deal with the case, when gamma square is equal to omega 0 square gamma being equal to omega 0 the two roots q 1 and q 2 will be exactly equal because the differed by this quantity.

And since the difference between the gamma square and omega 0 square has vanished the two roots become equal and you really get only one solution and how can that be, you need two solutions, you have a second order differential equation. So, there must be another solution and you have to look for it; you also know that this other solution must be linearly independent of this.

So, what you can do is look for this simplest linearly independent solution by considering this least departure from the previous one. So, it is the previous function was e to the qt - if the previous one was e to the qt - if you multiplied by some function of t and this arbitrary function of t could be some polynomial function in t, you take only the first term in t to the 1. So t to the power 1 and e to the power minus gamma t plus Bt e to the minus gamma t; if you construct the superposition of these two linearly independent functions; you get a general solution.

Because this form Ae to the qt cannot give you the complete solution. So, you recover the complete solution by adding to the first term Ae to the minus gamma t this simplest departure from this term which is Bt to the minus gamma t and now you factor out e to the minus gamma t as common and this is your solution to what is called as a critical damping case and you will see what, why it is critical damping because you see that when t is equal to minus A or B; when t is equal to minus A or B, this factor goes to 0 and this system really reaches an equilibrium and after this it can return to equilibrium only when time goes to infinity .So, this system can cross the 0 but only once, so this is also not quiet an oscillator because it does not keep oscillating across the equilibrium point every now and then as one expects from the term oscillations.

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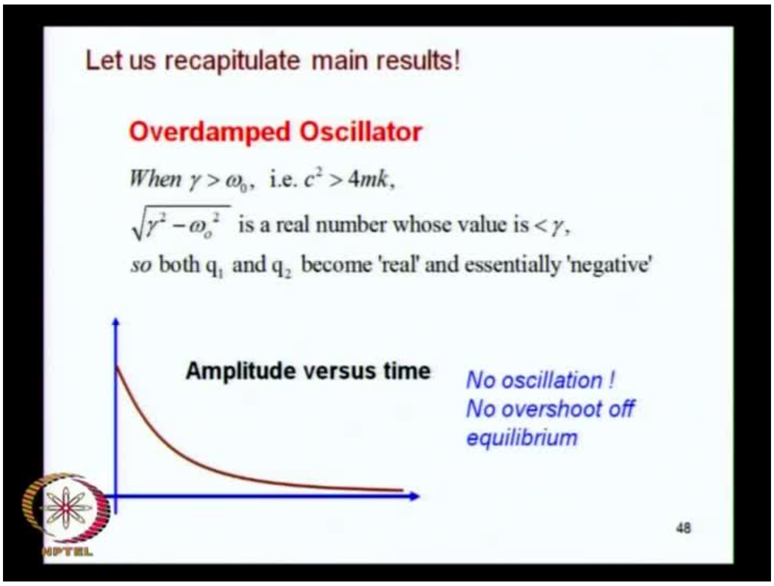

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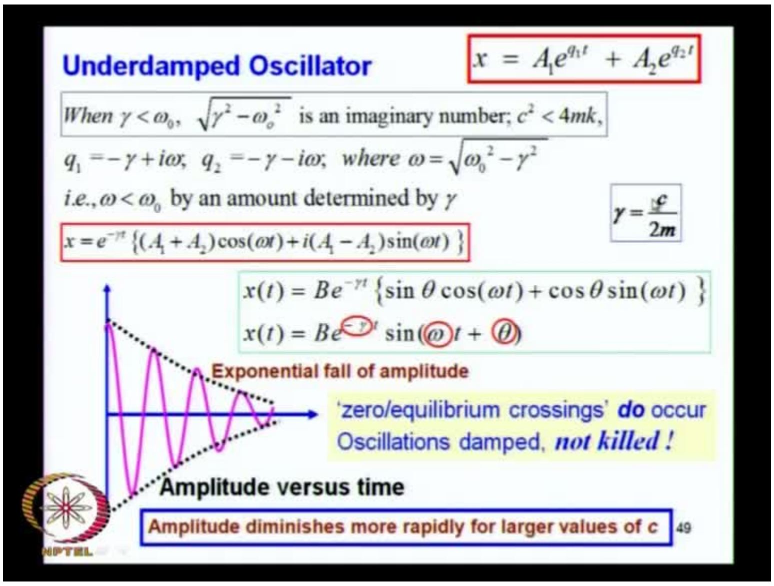

But an overshoot is possible, it does cross the equilibrium point but the next time it can come back to the equilibrium is only asymptotically over infinite period of time. So the amplitude verses time function you know there will be no overshoot of the equilibrium you can now have this case of under damped oscillation as we have seen this is the under damped oscillation in which the amplitude diminishes this is the earlier case that we have discussed . We have also seen with no damping at all you do not have any decay in the damping at all.

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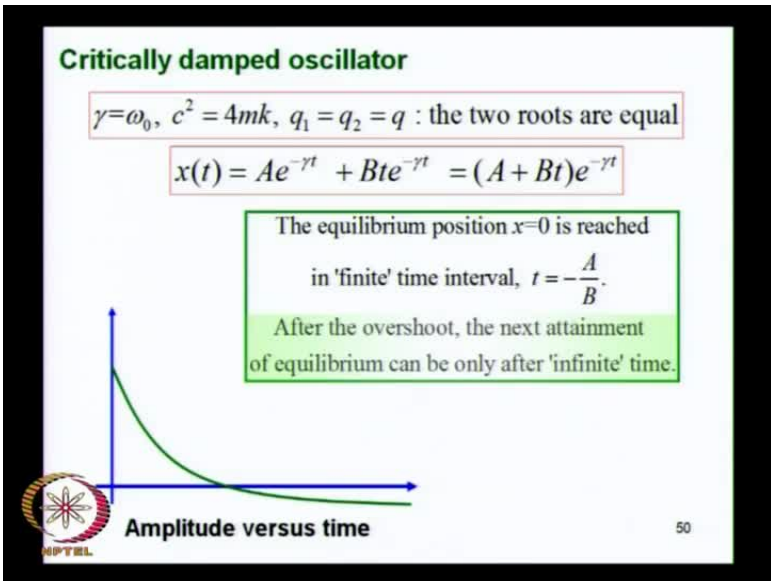

Notice that there are these factors to keep track of; there is a damping coefficient over here there is a frequency omega over here which is different from the natural frequency which is reduced by the damping term which is c over 2m c over 2m c is the ad hoc damping coefficient that we plugged into the equation of motion. In the critical damping case we had one overshoot, but no oscillation the next time it would come to 0 will be only after infinite time but there will be one finite time which is minus A or B when it can actually go to 0. So, one over shoot is possible but no real oscillations in that sense of the term.

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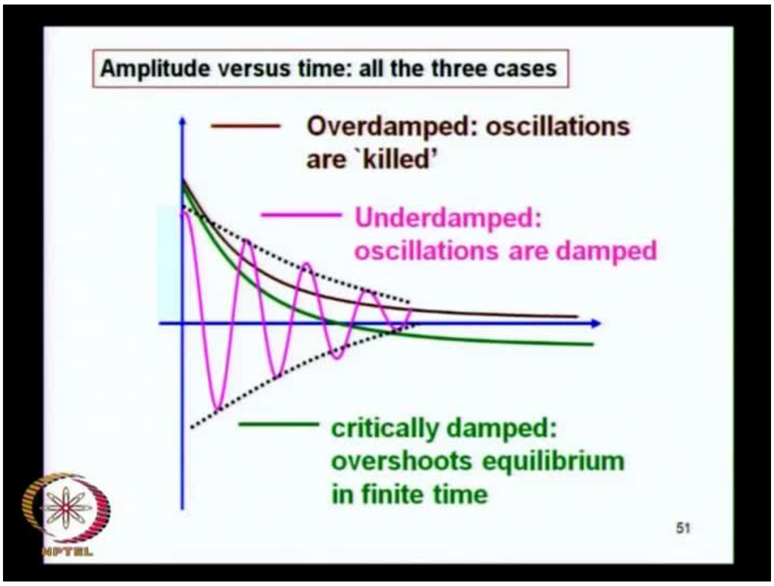

So, the next attainment of equilibrium will be only after infinite time; so, these are the various cases to put them all together. These are the various profiles that you see if you plot amplitude as a function of time for all the three cases. So, you have got the over damped oscillator which really would approach equilibrium point only as t goes to infinity.

You have the under damped oscillator, which does go through oscillation in which the amplitude is not periodic but the zeroes are periodic, you know what is meant any zeroes of a function: zeroes of the function are the points of the independent argument at which the value of the function goes to 0.

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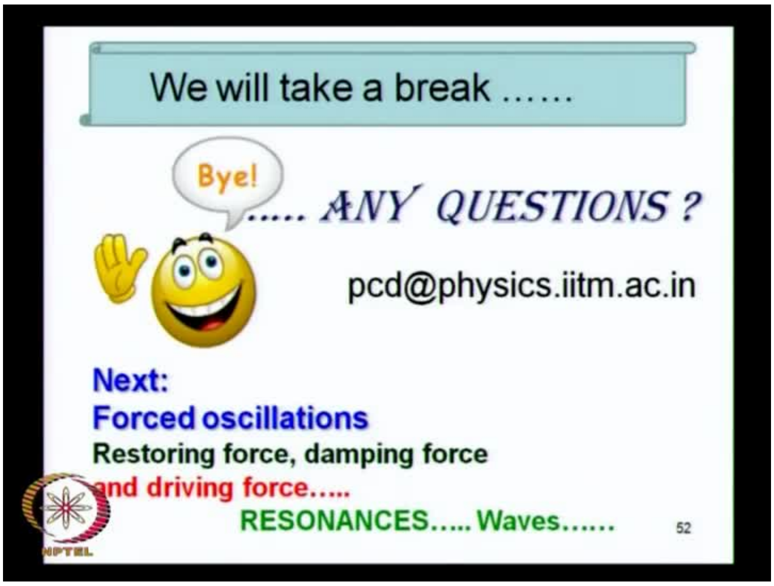

So, the zeroes of the function are periodic and then you have got the critically damped oscillator in which one overshoot is possible but the next time it all if it all there is any that the oscillator will come to the equilibrium point will be after infinite time. So we will take a break here if there any questions I will be happy to take those, otherwise in the next class we will now subject .This oscillator which has got its own intrinsic natural frequency, which is determined by the inertia and the spring constant all the corresponding electromechanical analogs whatever they be.

So, the dynamics would be governed by the intrinsic parameters of the system plus the damping that maybe there and that damping is coming from everything that you have ignored. These are the unspecified degrees of freedom that you have ignored.

And then in addition to this if you subject it to some additional external force, so there is not just a restoring force, which is intrinsic to the system not just the unspecified degrees of freedom which you had ignored but in addition to this now you subjected to a known external force a periodic force. So, this is what gives us a forced damped oscillator.

So, in our next class we will discuss forced oscillations in which there will be an external driving force we will consider an external force, which has its own periodicity and the periodicity of this external force need not be equal to the periodicity of the intrinsic system. In other words, the frequency of the applied force need not be equal to the frequency of the intrinsic natural frequency of the oscillator; there is another frequency that we have met which is the square root of omega 0 square minus gamma square, which is the frequency of the zeroes of the damped oscillator.

The frequency of the damped oscillator is not equal to the natural frequency its less than that. So, they are going to be three frequencies that we shall be talking about one is the intrinsic frequency omega 0 , second is the frequency of the damped oscillator, which is omega which is less than omega 0, which is square root of omega 0 square minus square of the damping coefficient, and then there is a third frequency we will talk about, which is the frequency of the external periodic force and then what will be the nature of motion and then depending on the phase relationships, depending on the comparisons between these three frequencies, we get very fascinating phenomena. Resonances for example and then we will also discuss wave motion, as we go along that would be in the next class.

Actually, we consider critical damping, so in that case we consider another solution; Which is another polynomial? So, any arbitrary function will not work; no we have to consider another, I mean the next step should be the solution of((…))

It should be Yes

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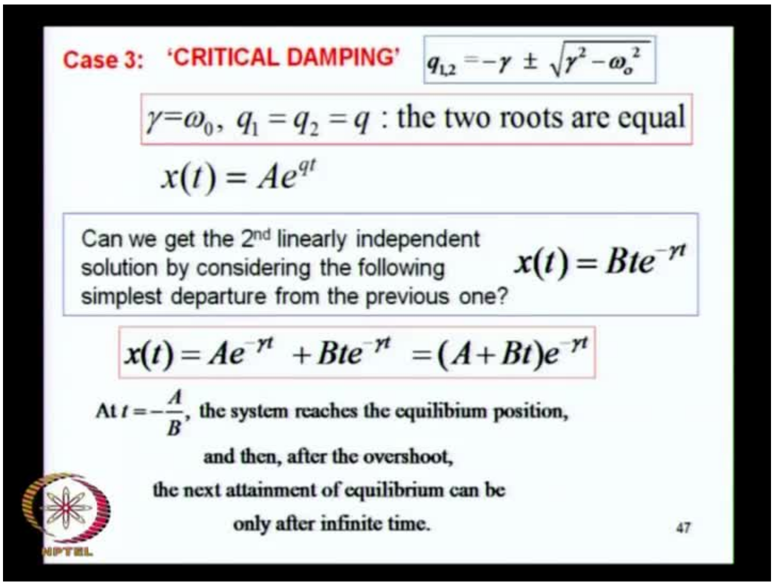

What we do let me go back to this, this is essentially a problem in calculus, you have got a differential equation and there is several methods of solving differential equations.

Mathematicians have developed these physicists use them or there are techniques which physicists have also developed to solve differential equations.

Now, one can solve this differential equation from first principles using various different methods and one method may work in one case and another method may work in another case and you have to look for the best we have solving a differential equation.

Physicist look for such ways which are physically intuitive, our requirement is that the solution must be exact and must be correct there is no compromise so far as rigorous concerned but there is no need to look for the most complicated way of getting the solution. So, we look for a way which is intuitively the easiest physically most appealing and then ask ourselves is it rigorous, if it happens to be not rigorous we must correct it, we must dispense it, and we cannot use it.

So, in this case what we do is we have a solution, we have already determined the solutions to be given by these two roots. But we have confounded with a very unique situation in which, we have two roots but both are equal. In other words, we really do not have two roots in this case and if we do not have two roots A into the qt and in this case q is equal to gamma; that will not be q is equal to minus gamma sorry A e to the minus gamma t cannot give us the most general solution it has got only one unknown, so it cannot be the complete solution to the second order differential equation it cannot be the complete general solution and that is what we are interested in. So, we ask what are the solution should be mixed. So that I get through the principle of superposition of two linearly independent functions get the most general solution to the differential equation.

It should have two unknowns and the two solutions must be linearly independent of each other, you cannot get one from the other, by simply multiplying it by a constant; you are getting one from the other by multiplying it not just by a constant B, but by a function of time . Therefore it is linearly independent.

Now, what function of time do you want to take? You can consider any arbitrary function and try it out and if you take terms with other powers of t; you can plug it in nothing wrong but then, you will get many more constants because tq will come with its own constant c t cube d t to the power of 4 e t to the power of 5; you do not want those many you want only one.

So, you take the simplest of which will give you two constants, you do not want more than two because the second order differential equation cannot have more than two unknowns. Because everything in the end must be determinable in terms of the initial position and initial velocity, you do not have any more information available about the system nor do you need; you neither have not do you need. Which is why the equation of motion is a second order deferential equation, so, you take the simplest function which is the departure from the first one and the departure cannot be just scaling by a constant it must include a function of t. So, you take the first power of t and that is what gives you A plus B t times the A to the minus gamma t.

Now, you are satisfied that you have got the complete general solution. It is not just a particular solution it is a completely general solution it consists of a superposition of two linearly independent functions. There are two unknowns and the whole physics is contained in it now that you are satisfied; that you have a rigorous solution you have solved the problem.

Any other question good question

We saw the case of over damped oscillations right sir, we say that there are no oscillations are practically speaking, we do not get any oscillations in this case then why do we call it an oscillator?

Its semantics it is a mean to call something, which does not oscillate as an oscillator you have every right to object that nevertheless there is some justification because the solution has come from the differential equation of motion for an oscillator, with a certain amount of damping it is coming from a specific condition of the nature of damping. And will you have that the combination of that condition is what prevents the system from oscillating but if you change that condition or if you take away that condition you will have an oscillator. So, if you will please, let me use the term oscillator anyway. But you are quiet right there any other question.

Very well, thank you very much and we will we will take a break here.