Select/Special Topics in Classical Mechanics
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Module No. # 02
Lecture No. # 09

Oscillators Resonances Waves (iii)

Greetings and welcome to the 9th lecture in this course. This is continuation of unit 2 on oscillators.

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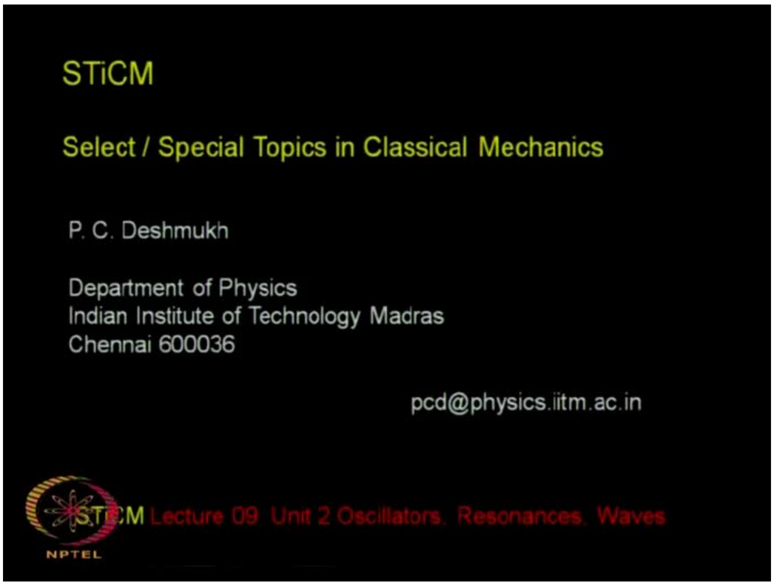

We have already had a discussion on oscillators. We studied the simple harmonic oscillator, the free oscillator which oscillates at a certain frequency when it is not subjected to any other impediments such as friction, or to any other additional external force, so which is oscillating under routs - its own intrinsic parameters like the restoring force, which a spring has.

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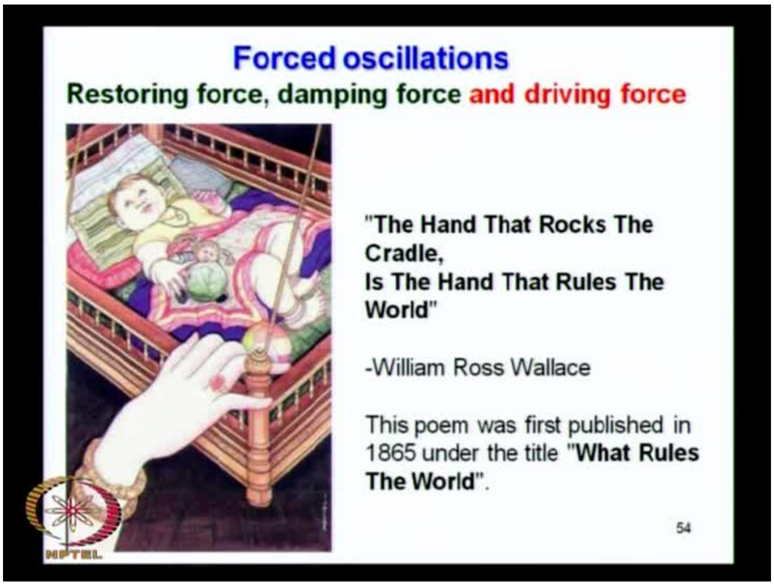

Today, we will subject this oscillator to an external force and we will talk about forced oscillations. This is the classic picture of, coming right after the mother's day I think this the wonderful picture to look at, with Yashoda rocking the cradle of Krishna and it is a tribute to the mothers; that the hand that rocks the cradle is the hand that rules the world; this is a wonderful poem that some of you may have read, written by William ross Wallace.

Now, here, you have an oscillator which would oscillate under its own internal parameters, but then, there is also some friction at this support; It means other than the fact that you know the air and the atmosphere in which it is rocking would also impede the free movement, but more than anything else, it is the friction at the supports which will cause the damping. So, this is the damped oscillator. Then, addition to that, there is an external force which can be applied; just once in a way you just tap it or you tap it at a regular frequency. We will consider such forces which are periodic.

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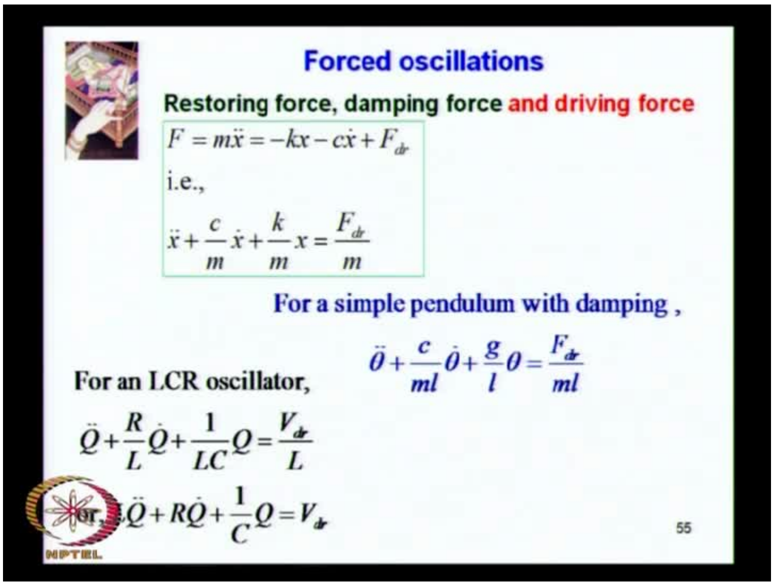

So, we will setup an equation of motion in which the net mass time acceleration for the object which is mx double dot for a one dimensional oscillator is equal to the restoring force which is minus kx. Then there is a damping term which is minus cx dot; this is the term that we discussed yesterday. In addition, there is a driving force F dr. So, your equation of motion is this: F mx double dot is equal to minus kx minus cx dot plus F, driving force.

So, in the free oscillating, you have only the first term. Then in the damped oscillator, you have the first and the second term and then this is the forced damped oscillator. You can divide it by the inertia and you get the equation for the acceleration. This would be the case for any kind of damped driven oscillator. It does not have to be just a mass spring oscillator. It can be the simple pendulum with damping which is subjected to a periodic force or it can be an electrical oscillator in which it is the resistance which introduces damping.

A pure LC circuit will have no damping; damping is caused by the resistance. So, these are the electromechanical analogs and you can have an exactly identical set of equations whether you are dealing with the mass spring oscillator or a simple pendulum in a gravitational field, or in electrical circuit with in inductance, capacitance, and a resistance. Then, you have a source for the electrical voltage which is an alternating electrical voltage. So, you can have a complete electrical analog of a mass spring oscillator.

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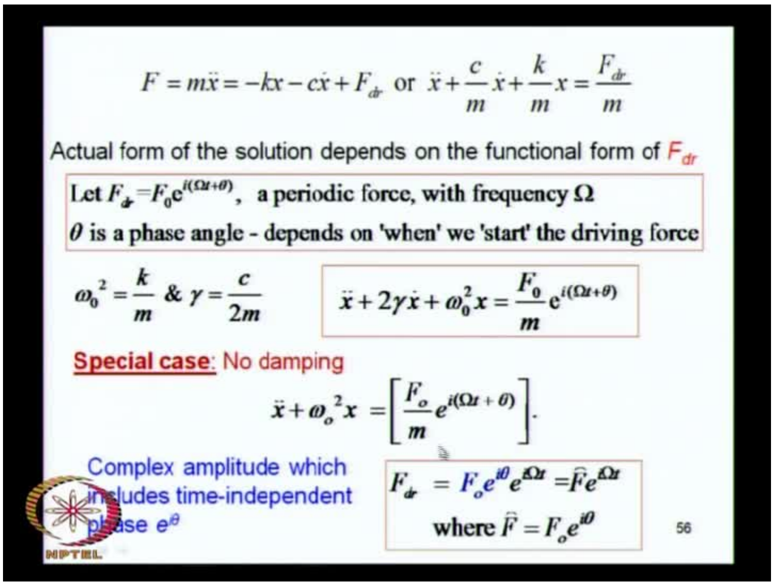

So, we need to look at the solutions for this equation. The actual form of the solution will actually depend on what kind of driving force you are applying; of course, it depends on what kind of the damping force exists in the system. The damping force - I already discussed it in our previous class that the damping force minus cx dot, is not a term which comes from first principles. But it is some approximation to the total net effect of the unspecified degrees of freedom that you have not taken into account in setting up the equation of motions. So, it is the net result of all of that, but we will stay with that form for the damping force which is minus c times x dot.

Now, we have to worry about what form of the driving force are we going to consider and like I mentioned, we will be considering a periodic damping force; this periodic driving force. So, this driven driving force F dr has got a periodicity which is indicated by this omega. Now, this omega looks like the omega one uses to represent the ohm, as it is used to write the electrical resistance. So, it is that omega; this is the upper case of omega that I am using here.

Actually, there are three frequencies that we have been talking about: one is the intrinsic natural frequency of the oscillator which is omega 0; then, there is the frequency omega which is the frequency of the damped oscillator, which is different from the natural frequency; if you remember, it is the square root of omega 0 square minus gamma square where gamma is the damping coefficient; now, there is the third frequency for which I need a different symbol. So, I am using omega again, but an upper case omega, as is used in the ohm. So, this upper case omega is the frequency of the driving force. So, this represents the periodicity with which the cradle is rocked if you have that picture in your mind. So, this is the frequency of the driving force.

In addition, there could be an angle theta because mind you, if this t goes to 0, this phase will be e to the power i theta (Refer Slide Time: 07:06). So, it will insert some kind of phase and depending on when you actually start the periodic force, means you can start rocking the cradle when it is moving away from you or when it is coming towards you, or any time in between. So, depending on that, there will be a certain phase consideration and that is taken care of by this angle theta. So, these are the intrinsic parameters.

Omega 0 square - this is the natural frequency. This is determined completely by the intrinsic parameters of the system; k belongs to the system; so, does the inertia. These are the internal properties of the system you are talking about all the corresponding electromechanical analogs. Gamma is the damping coefficient. Now, this is some net packaging of the unspecified degrees of freedom which we have chosen to have this particular form. In terms of gamma in omega 0 square, this equation for the acceleration turns out to be x double dot plus twice gamma x dot plus omega 0 square x. Then, you have got the driving term on the right hand side. So, this is the differential equation for which we must find the solutions.

Now, if there were no damping, if gamma were 0, and this would be an ideal oscillator which experiences no damping, no friction, but it is subjected to an external periodic force. So, you can always develop an approximation; all special cases in which you can ignore damping. Notice that, if you look at this particular term in which there is no damping, then the driving force has got an amplitude; means, if this term e to the power i omega t represents the periodicity, then the corresponding amplitude contains a phase factor; along with this phase factor, the amplitude itself which is F 0 e to the power i theta becomes a complex number.

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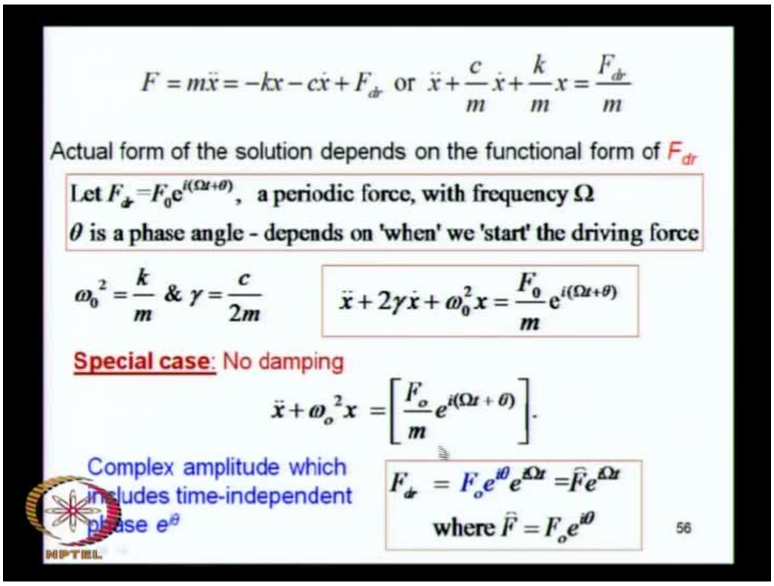

So, the manner in which we are using this algebra of complex numbers, the amplitude is complex. What it is doing is, it is allowing us to deal with this phase factor. It is not that the maximum amplitude itself has got any imaginary part; it is keeping track of the phase; means I just want to emphasis this point again, which we already discussed in our previous class; when we deal with complex numbers that is nothing imaginary that we are confronted with. We are dealing with physical observations; we are interested in the physical state of a system which is given by its position, by its velocity. We are going to see how this position and velocity changes with time, track the evolution of the system in the phase space, or whatever be the form in which you solving the differential equations.

Then, what you observe is a real number; is a real parameter. It is a real physical observable and this is just a mathematical tool which allows you to deal with two real numbers at the same time or keep track of some factors like the phases and so on. So, the algebra complex numbers is very useful in dealing with pairs of real numbers.

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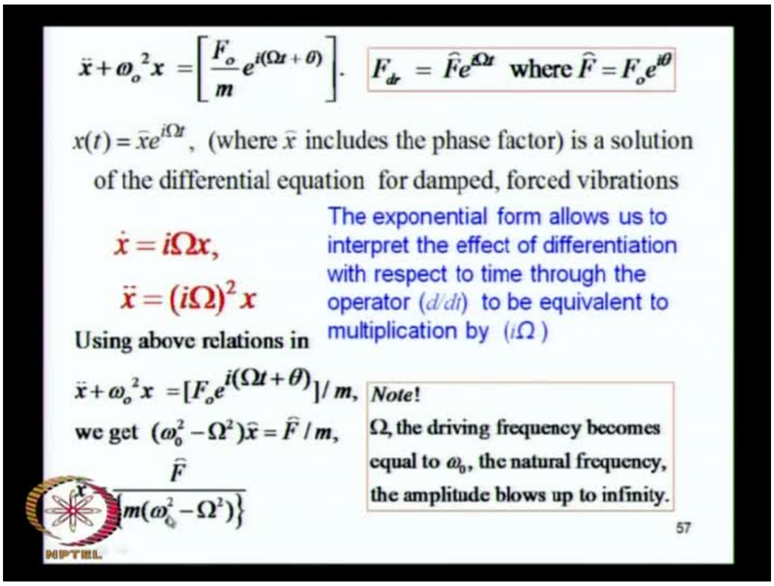

So, corresponding to this, we have a solution which will include a complex number, which will include the phase factor and this will be a solution of the differential equation for the damped and forced vibration. So, the algebra is very straightforward.

The reason to use the exponential form which I mentioned earlier as well is that, whenever you take differential equations and you deal with the velocity which is x dot, we deal with the acceleration which is x double dot. So, you have to differentiate with respect to time once and twice. Every time you differentiate with the operated d by dt, the result is a multiplication by the factor i omega. So, the process of carrying out derivatives is a very straightforward one, when you deal with this particular complex form or exponential form of the function that you are working with. So, here you go. The first dot d by dt of x which is x dot, this is i omega time x; when you do it twice, it will be multiplication by i omega twice. So, that gives you an i omega square x.

Now, this differential equation translates to an algebraic form and then you can get for it solution, a form in which you have a complex amplitude, in which the phase theta is included. If you look at this form, you might expect that when the denominator goes to 0, the amplitude will go to infinity. Does it mean that the physical pendulum will actually go away to infinity? Is that what you going to expect? Now, this really does not happen because damping is always present. So, do not worry too much about it. This is the mathematical form we dealt with the special case, in which we presumed that there is no damping; but damping is always present. That is one factor to keep track of. The other thing is that, as the system stretches away from the equilibrium point, it could cross its elastic limit. When it stretches beyond this certain point, then it is going to snap and then it is not that goes to any arbitrary distance away from the equilibrium. The restoring force will retain the form of minus kx, that it will bring it back to the equilibrium; no. It will at some point snap. So, it is not that you have oscillations of infinite amplitude so that that is not physical and it does not happen.

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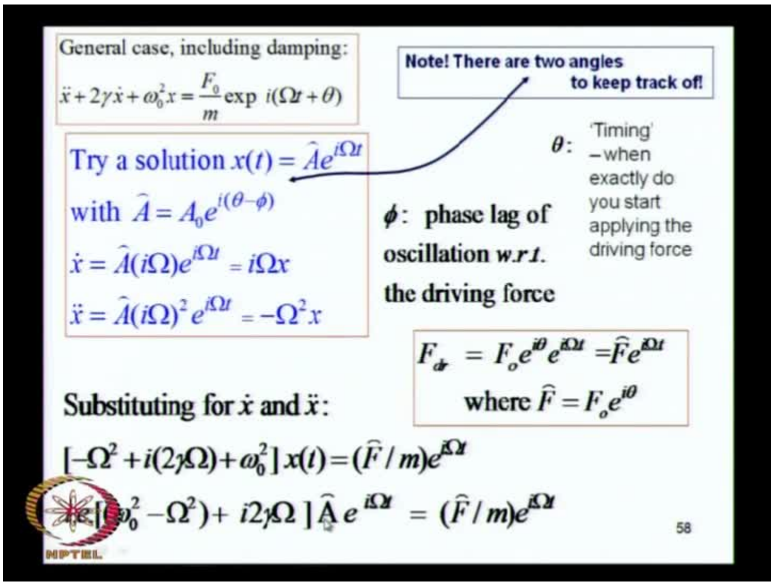

So, this is our differential equation now, inclusive of damping. This is the term that we had ignored in the part of the discussion we just had. So, we will now insert damping gamma. So, the complete differential equation has got three terms in the left hand side. We shall try a solution in this form with a complex amplitude and we can use the same trick that, when you take the first time derivative, you must multiply the function x by i omega; when you take the second time derivative, you must multiply by i omega twice. So, i square will give you minus 1. So, you get a minus omega square x in this second derivative term.

There are two angles to keep track of: theta and phi. What theta does is to keep track of just exactly when you start applying the driving force. Then, phi will give you a measure of it further phase lag with respect to the driving force because it cannot be anticipated or it cannot be assumed that the oscillation will be completely in phase with that of the driving force. So, these phase lags have to be kept track of. So, there could be an additional phase factor and these two angles, we will follow them rather closely in our discussion.

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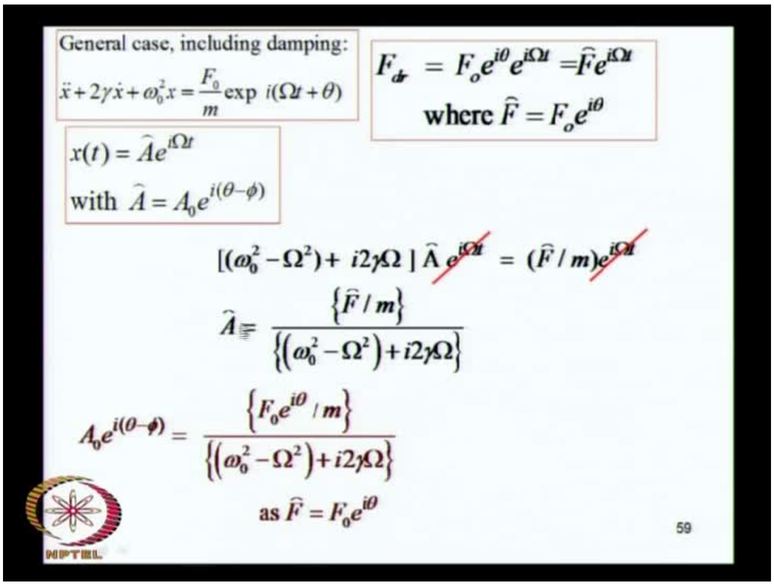

So, here you go. You now have this form of the driving force and in this differential equation; you substitute for x dot and x double dot that you have already found over here and you get an algebraic equation. Then, we have to solve this algebraic equation so that we can understand what is the nature of the solutions.

So, this is the general case we are now discussing in which damping is kept track of. We cancel out the common factors e to the i omega t and we get a solution for the amplitude. This amplitude, now we know depends not just on the difference between these two frequency terms, but the damping also controls it. Now, this is the point which I had hinted earlier; the damping is always present. Just when the upper case omega becomes equal to omega 0 square is not going to go to infinity because generally speaking, damping is present; we hadt dealt only with the special case earlier.

So now this is the complex amplitude and I have now written it explicitly. This is written with A, with this symbol on top of A. So, wherever I have a complex amplitude, either for this driving force or for the amplitude of oscillation, I have used this symbol which looks like a hat it is called as a carat. So, this is A carat this is F carat or sometimes in its called as F hat or A hat, and this has the phase which is included in this and this got the two angles theta and phi on the left side; on the right side, there is this e to the i theta. So, this is the complex amplitude in which the phase angles find an explicit expression.

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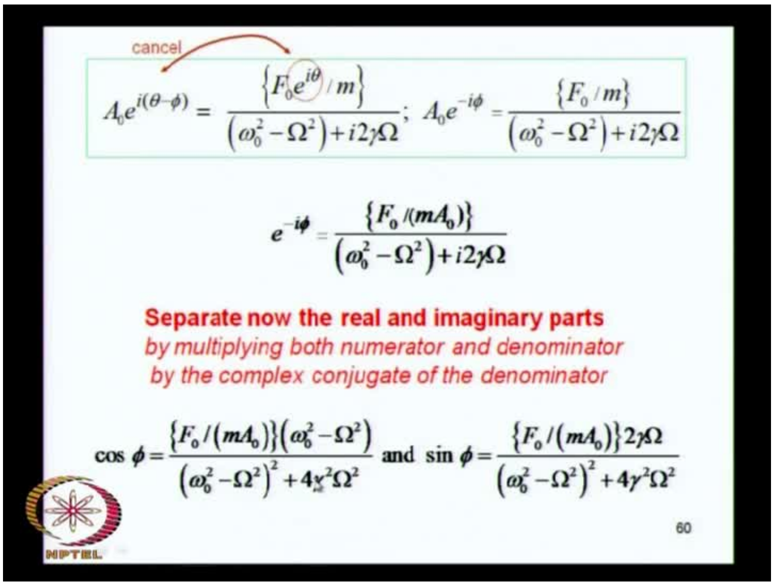

There is an e to the power i theta on both sides of this equation. So, we can cancel this out and then we get A 0 e to the power minus i phi because e to the power i theta on the left hand side cancels e to the power i theta on the right hand side and we have A 0 e to the power minus i phi equal to this term on the right. So, we are now getting some specific form for the amplitude function.

Now, writing this, we want to determine what is this angle phi because this is the phase that we wanted to keep track of. We can get the phase by bringing this A 0 to the other side of the equation and we get an expression for cosine phi and the sin phi. If you take the ratio, you can get the tangent of phi and its tangent will give you the angle phi itself.

So, we separate the real part in the imaginary part. So, the left hand side is cosine phi minus i times sin phi. So, this side also you have to separate into a real part and an imaginary part, but here, you have got a complex number in the denominator; say if you multiply this right hand side by the complex conjugate of this and divide it also by the complex conjugate of it, which is effectively multiplying the right hand side by unity and no problem there.

You can always multiply a factor by any factor by 1 and leave its value unchanged, but you resolve this 1 into a ratio of the complex conjugate of the denominator in the numerator and the same in the denominator, and then you can clearly separate the real part and the imaginary part. Now, once you do that, you equate the corresponding real parts so that the cosine phi becomes equal to this quantity over here. Notice that the denominator is just a product of this number with its complex conjugate. So, you get the real part which is equal to cosine phi and then you get the imaginary part which is equal to this part over here. So, you have equated the real parts and the imaginary parts. Now, from the ratio of sin phi to cosine phi, you can find by taking that inverse tangent, you get the angle phi self which is this ratio and this depends, of course, on damping

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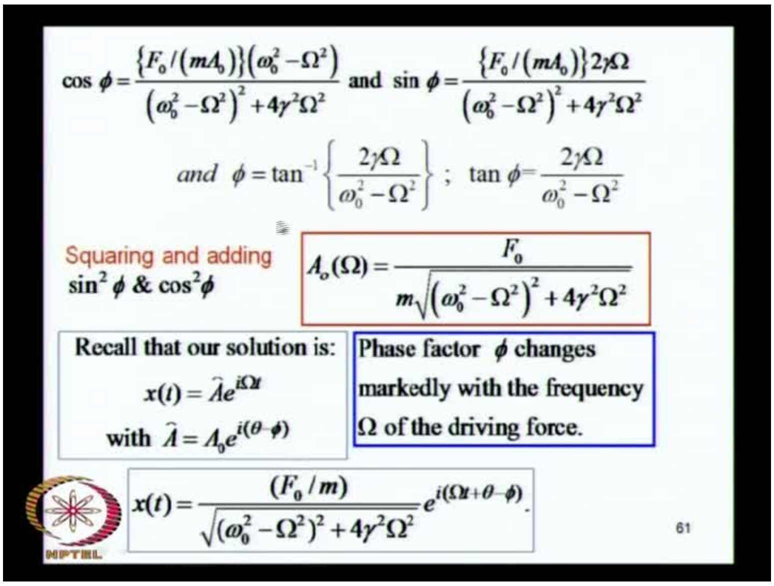

So, it is this damping which is responsible for the phase lag between the response of the damped oscillator to the periodic oscillations. If gamma was 0, this would go to 0. So, it is because of this damping that there is an additional phase that steps into your analysis.

You can also do a squaring and adding because that allows you to determine what this A 0 is. Because cos square phi plus sin square phi is always equal to unity and you take advantage of that; then phi gets eliminated from this and you get an expression for A 0. Notice that this amplitude will depend on the frequency of the driving force. So, if you apply this driving force, you think of the Yashoda rocking this cradle and if she rocks it at a different frequency, this amplitude is going to change. So, this A 0 is dependent on omega. So, you write it explicitly as the function of this omega.

This phase factor of phi also changes not just it is sensitive, not just to this gamma, but also to this frequency omega. So, the phase factor changes markedly with the frequency of the driving force and its very cause is the presents of damping in the first place. So this is now your complete solution in terms of the periodic application of the driving force for a damped driven oscillator. This is often called as a steady state solution. I will explain the term shortly.

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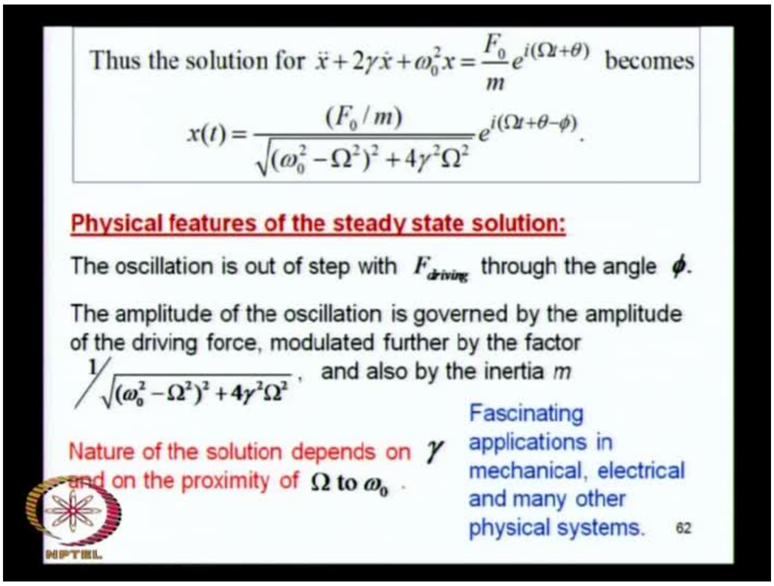

This is the solution for the differential equation. This is a steady state solution. In addition to this, there will be another solution; I will get to that.

This oscillation is not in phase with driving force because of damping. It is out of step. The difference to the phase is through the angle phi and the amplitude is governed by this factor 1 over square root of this quantity that you see which is omega 0 square minus this other omega square and then there this damping term over here (Refer Slide Time: 22:50); of course, the inertia of the system also has to play a role over here. So, that will participate in the expression for the amplitude.

So, the other thing which is going to influence the nature of the solution is this first term under the square root sign, over here (Refer Slide Time: 23:16). Depending on how close the driving frequency is closed to the intrinsic natural frequency of the oscillation, you will expect some very special features to manifest themselves. This will lead us to the phenomenon of resonances and physical system.

You can already expect that because as this upper case omega the frequency of the driving force becomes exactly equal to omega 0 which is the natural frequency, this term would completely vanish and that will give very special forms to the nature of the solutions. This has very important and very fascinating applications in a large number of physical systems, whether mechanical, electrical or whatever; but a large number of physical phenomena are influenced, including quantum phenomena.

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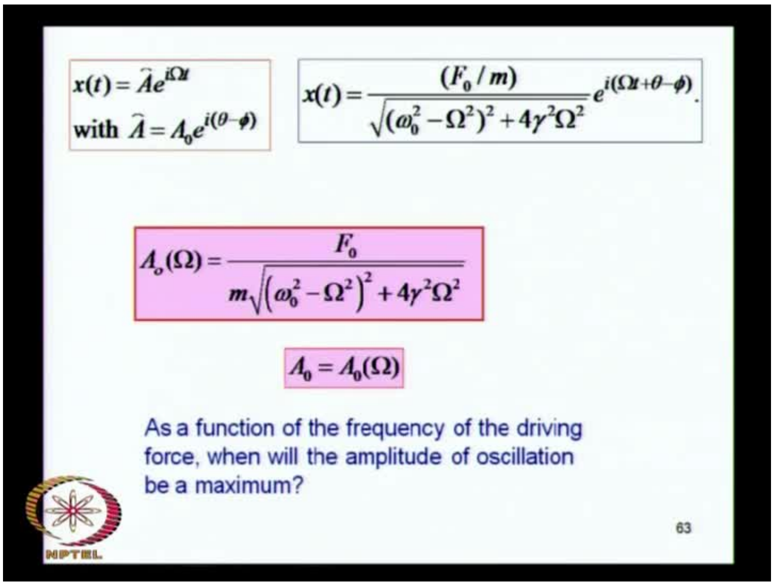

So, this is the general solution. Here, all the expressions are combined together with the emphasis here that the amplitude itself will depend on the frequency of the driving force. We can actually ask this question that if this amplitude is going to depend on the frequency of the periodic force, can I control the frequency of the periodic force because you know you can rock the cradle once every second, twice every second, 5 times every second or 7 times every 2 seconds? You know it can have different kind of frequencies, and depending on what frequency, with what frequency you are applying a periodic force, the amplitude is going to change. So, if you are seeking maximum amplitude, what should be the frequency that should be used for the driving force so that the amplitude will be a maximum; now, that is the question that we shall now take on.

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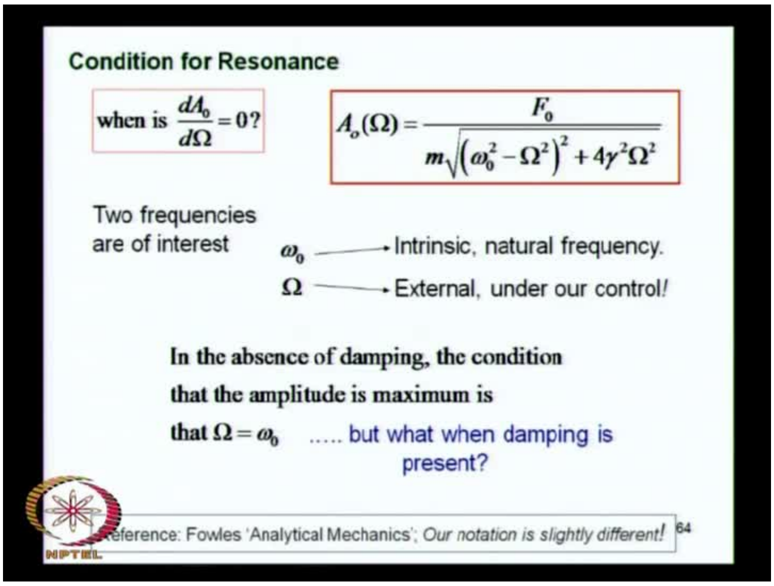

As the function of the frequency of the driving force, when will the amplitude of oscillation be a maximum? So, how would you answer this? Is there a quick guess? What you must do is find out when this derivative with respect to frequency vanish because for any function to be a maximum, its derivative with respect to the independent parameter goes to 0.

So, you take the function A 0 as the function of omega, take its derivative with respect to omega set this derivative to be equal to 0, and then analyze the consequence of this condition. Because the condition for a minimum or a maximum is that the derivative of the function must vanish. Of course, if you are interested in whether it is a maximum or a minimum, you should also look at the second derivative and so on, but here, we are interested in this particular condition.

So, we keep track of the two frequencies: the intrinsic natural frequency and the frequency of the driving force. The intrinsic frequency belongs to the system; it is determine by the spring constant k and the inertia m for the mass spring oscillator. The external frequency, of course, is something that we can control because that is in our hands. We can decide how often we want to apply the periodic force. So, if gamma was 0, then of course, this condition is that this omega must be equal to omega 0. That is quite straightforward because that is when the denominator vanishes. The gamma is already set equal to 0 as a special case. So, that is quite straightforward; it does not make quite mean that the amplitude, you are going to see an oscillation of infinite amplitude; that as we have discussed earlier. Thus, the result that we get when damping is present; we must look at this particular term carefully and find dA by d omega, set it equal to 0 and then examine the consequences.

Some of you have asked me for references; here is a good reference for this part of the discussion; there is a very nice book by Fowles called Analytical Mechanics. But the notation that we have used is slightly different. So, you cannot use the notation from Fowles directly, but Fowles is a good source; the (()) physics course, volume 1 is also a good course. You find discussion in some of the other books as well, but Fowles is a good source of this.

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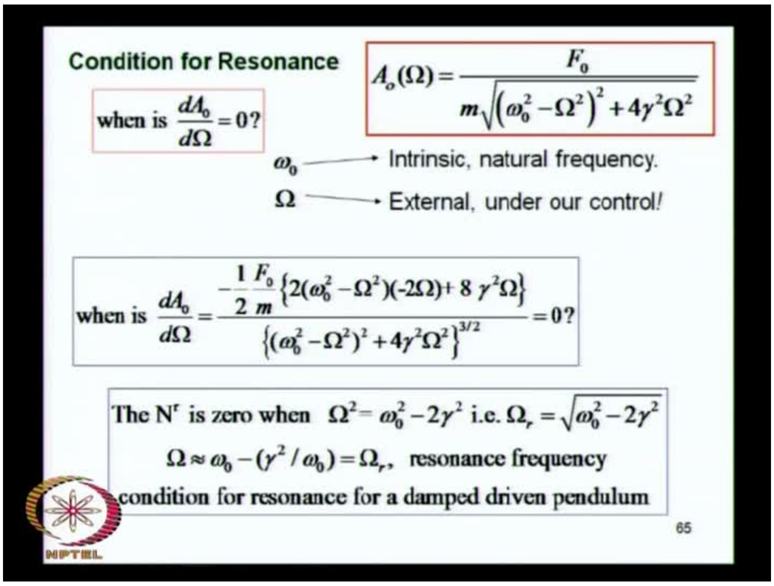

So, anyhow, the condition for resonance which is when we are going to have the maximum amplitude is dA by d omega equal to 0. So, this is where you take the derivative of the right hand side with respect to omega. So, you just do it carefully because you have got a denominator, omega appears in the denominator and that too twice; once over here and once over here (Refer Slide Time: 28:48).

So, if you take this derivative carefully term by term, you get this particular condition and in this condition, you ask - when will this ratio go to 0? Obviously when the numerator goes to 0; this is what would happen when omega square is equal to omega 0 square minus twice gamma square. So, this is the condition which emerges; that omega square which is the square of the driving force is equal to this difference - omega 0 square minus twice gamma square; or, in other words, the corresponding frequency which is called as a resonance frequency and I will write it with the subscript r. This resonance frequency will be given by this square root of this term which is omega 0 square minus twice gamma square.

Now, this is like omega 0 square minus twice gamma square to the power 1 half. If you expand this term, then you can develop an approximation, retain the highest order terms, and then you get this omega which is the square root of this quantity, which is approximated to omega 0 minus gamma square over omega 0 square. So, that is just straightforward expansion and which have factored out omega 0 square and then expanded the term omega 0 square minus twice gamma square to the power half.

So, this is the resonance frequency and it gives the condition under which the damped driven oscillator will have maximum amplitude oscillation.

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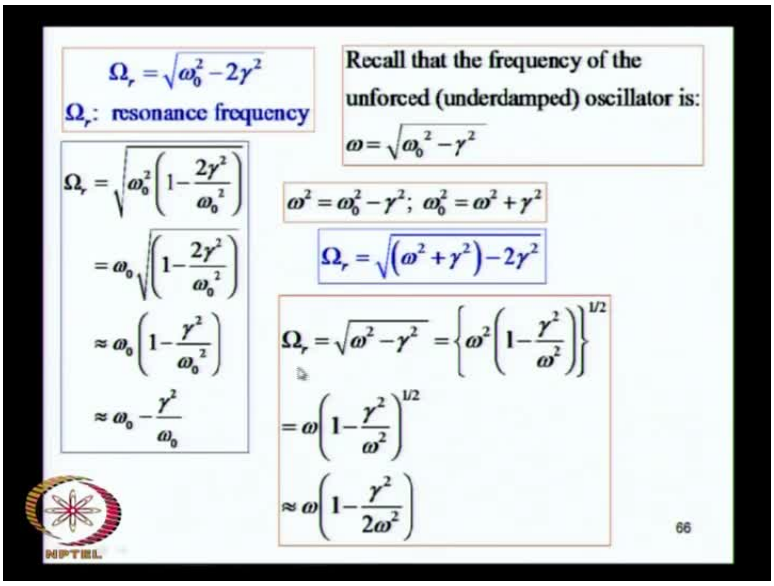

Now, if you remember, this frequency is different from the frequency omega. This is the frequency of what we had called is the under damped oscillator. This was end the class at we had yesterday, in which we found that the damped oscillator under goes in the case of what we called as the under damped oscillator. It is not periodic motion because amplitude diminishes exponentially, but then, it is a periodic motion because the zeros are periodic. It goes to the 0; it crosses the equilibrium periodically at the specific time interval; that time interval is governed by a frequency and twice phi times 2 phi times are frequency is the circular frequency which is omega, which is slightly less than the intrinsic natural frequency. The difference is controlled by the damping coefficient, gamma.

So, there are these two frequencies that we already talked about yesterday and there is this frequency of the driving force which we can control. If you set this frequency of the driving force omega, set it equal to what we will now call as a resonance frequency, which is equal to this square root of omega 0 square minus twice gamma square. So, it is different; obviously from this omega, which is why we needed three different symbols for omega for the frequency, but these are three different frequencies that we are talking about. We can now develop is for approximation to this, which makes of a discussion a little simpler.

So, in this square root I factor out omega 0 square. So, the remaining term is 1 minus twice gamma square divided by omega 0 square; the square root of the omega 0 square gives me omega 0, outside the square root factor. Then, under the square root factor I have got 1 minus twice gamma square over omega 0 square. This is nothing, but this bracket to the power 1 half which I can expand and retain the leading terms and I immediately get and approximation in which I must take half of this factor twice gamma square by omega 0 square, which is nothing but gamma square by omega 0 square itself. So this omega 0 minus gamma square by omega 0 gives us an excellent approximation to the resonance frequency.

So, here are our results. This is just the frequency of the under damped oscillator. This is the square of this term and you can swap the term; so, omega 0 square. You can take gamma square on the other side; so, omega 0 square is equal to omega square plus gamma square. You have omega r which is this, which is omega r minus gamma square over omega 0.

So, you combine these results and you get omega r to be given by the square root of omega square plus gamma square which is actually this omega 0 square, which is you can see it clearly from this form here (Refer Slide Time: 34:15 to 34:25). Also, this omega 0 square is replaced by omega square by gamma square in this term and you have got this minus twice gamma square which is coming over here. Then, if you remove this bracket, you can see that you are left with plus gamma square minus twice gamma square. So, you will get only one factor of gamma square to be diminished from omega square.

So, anyhow, you have got this omega r and if you approximate this square root factor by the usual trick that we have been playing, then you have this form for the resonance frequency; in terms of the damping, the frequency of the under damped oscillator. So, this is the resonance frequency in terms of the damping coefficient and the natural frequency of the oscillator omega 0. Here is an expression for the resonance frequency in terms of the damping coefficient and the frequency of the under damped oscillator. So, you can write it in either one or the other form.

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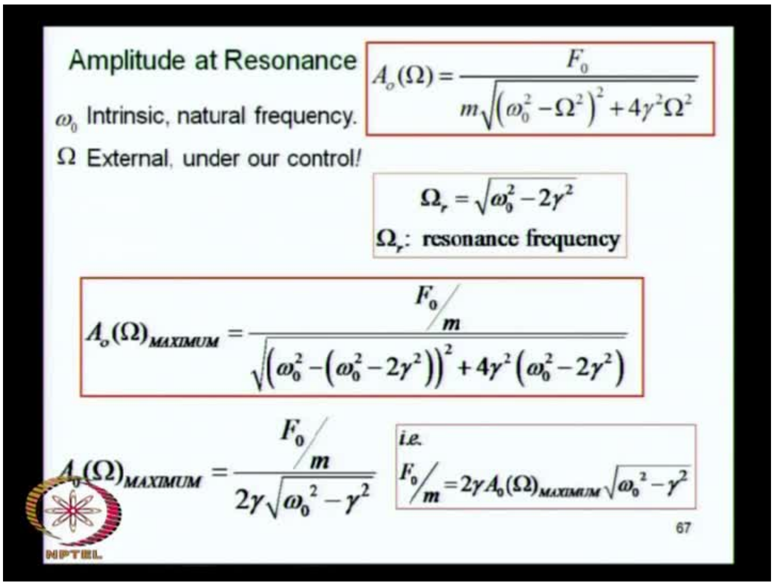

Now, what is the value at the resonance frequency? What is the value of the amplitude at the resonance frequency? We already know that the amplitude changes with frequency. We are now asking what will be this value when the frequency of the driving force omega is equal to the resonance frequency. So, all you have to do is to substitute this factor omega by the expression for the resonance frequency and you get the expression for the maximum amplitude.

So, you do not have to write down all these sayings in your notebooks. I think it is important to follow the discussion and then try to regenerate these results yourself. So, do not worry if you are not able to write fast enough, as I speak through these slides, but the important thing is to follow the discussion and then try to work out these relations from first principle because there is nothing in it that you cannot do; all we started out with is the differential equation for the free oscillator; then we plugged in a damping term; then we plugged in at driving term; then we keep track of all the phase factors; we recognize the fact that the amplitude of oscillation will be dependent on the frequency of the driving term; we ask under what condition will it be a maximum? So, we set dA by d omega equal to 0, solve it for that particular condition, and up come the answer; nothing else that we have done in this entire analysis.

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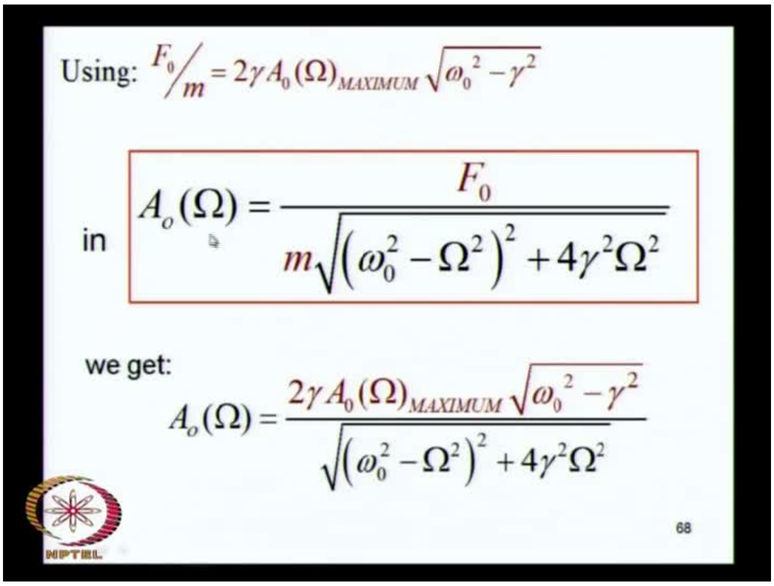

So, this is the maximum amplitude. So, there is no need to write every term down or you know check every term. It is something for you to sit down and catch up, and derive for yourself.

You can also turn this expression around and write the ratio F 0 over m in terms of these maximum amplitudes. So, instead of writing our parameters in terms of the amplitude have 0, we can write it in terms of the maximum amplitude of the oscillation itself. If you do that, then this A 0 omega becomes expressible in terms of the maximum amplitude which is written as A 0 omega at the maximum value. So, it is just a straightforward substitution of the amplitude function at the resonance frequency.

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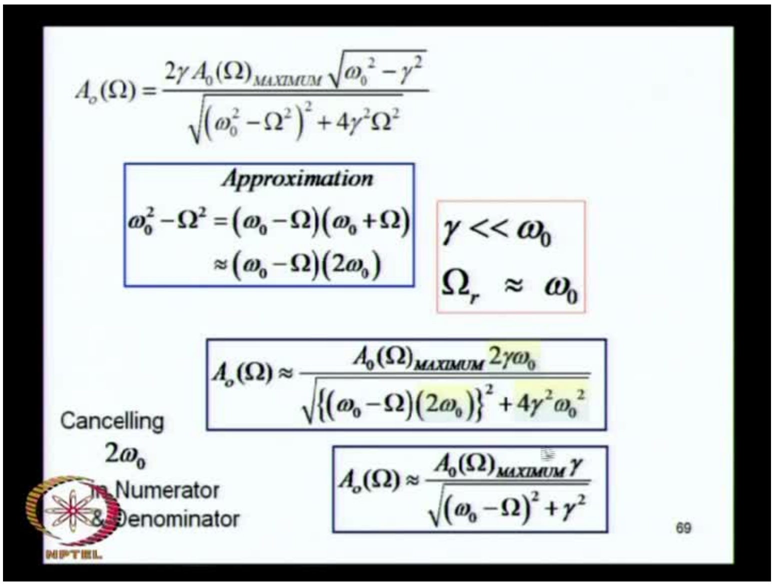

Now, we will develop further approximation because we are working in the region which is close to the intrinsic frequency of the oscillator. So, this upper case omega is nearly equal to omega 0. Now, if you look at this term, omega 0 square minus omega square - this is exact, for any difference in the squares like a square minus b square the factors are a plus b and a minus b; that is exact. But in the next step, we introduce an approximation. we set omega nearly equal to omega 0, which becomes twice omega 0, but you do not do it in the first factor.

Now, does that worry anybody is to why is that that you are putting uppercase omega equal to omega 0 in the sum, but, not in the difference? That is always how approximations are made, that it is the sum of omega 0 plus omega which is not so different from twice omega 0. That difference which is coming is being made in a large number. So, you are neglecting a small number compared to a large number.

So, whenever you ignore any difference, you do not ignore it because of its absolute value, but only in comparison to some other parameter. So, this approximation is completely valid. You can in fact replace omega 0 plus omega by twice omega 0 because the difference between them is a small number compared to a large number. You cannot do that when you are dealing with that little tiny difference between omega 0 minus omega itself. So, this approximation is perfectly valid

We will deal further with such cases when the damping is small because in physical systems that you work with like either LCR circuits and so on, or in other mechanical oscillators, you often try to minimize damping. So, gamma is quite often pretty much smaller compared to omega 0. Then, that makes this resonance frequency not too far from the natural frequency of the oscillator itself. Under these approximations, you can substitute for this term, this approximation which is omega 0 minus omega square and twice omega 0 square which is coming over here (Refer Slide Time: 41:11).

For the second term, this omega square is very nearly equal to the square of this omega 0, which is the natural frequency. So, here again you do not have to write down everything that you see on the screen, but what is important is that you keep track of how approximations are made; that you make approximations when you are setting some term to be nearly equal to another, then you must ensure that the difference that you are ignoring is ignorable, and it can be ignorable only in comparison to a large number; not otherwise.

So, now you have got twice omega 0 in the numerator; you got the square of twice omega 0 in this term; you got a squared of twice omega 0 in this term; everything is coming under the square root sign; so, twice omega 0 can be happily struck off from the numerator and then the denominator; then, the result takes the slightly simpler form in which the needless twice omega 0 is not written because it just cancels in the numerator and the denominator.

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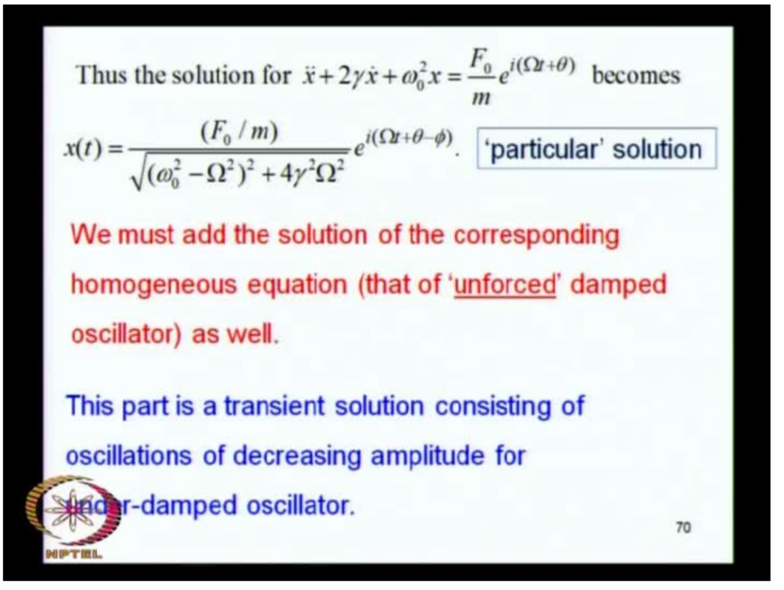

We now have a solution, but this is at best, a particular solution. This is not the general solution. The general solution will have one more factor. This is how you solve any differential, second order differential equation. There will be additional solution and that will come from the solution to the homogeneous part of the differential equation. The homogeneous part, is of course what we have already dealt with; it is the case of the under damped oscillator because that is the under damped oscillator in which there was no driving term. This is the discussion we had in yesterday's class. If you have a unforced oscillator, you get essentially the homogeneous equation and that will give you the transient solution because the amplitude is damped. So, you will add to this particular solution, the transient part. So, this particular solution is the steady state part because it is oscillatory.

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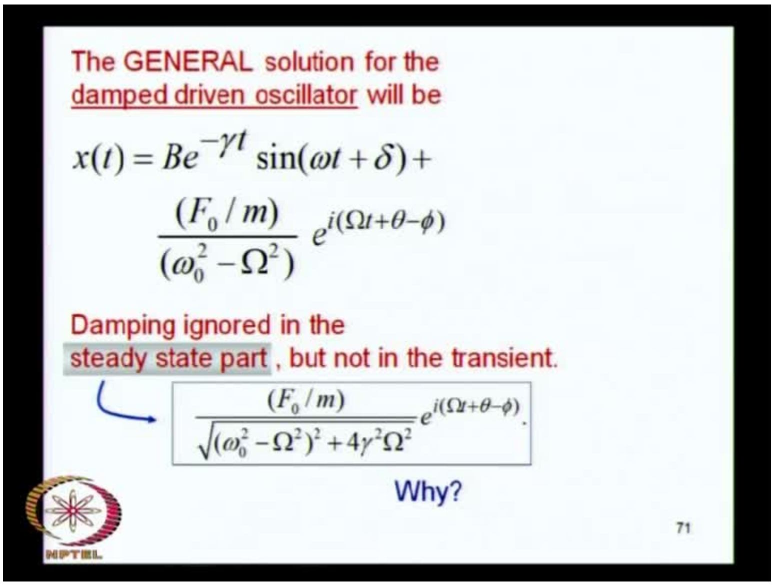

The other part which is the solution to the homogeneous equation is the transient part because it is going to die out in a finite amount of time. It hass got the a diminishing amplitude of the under damped oscillator. It is going to die out in a certain amount of time. So, that is the transient solution.

The general solution now is superposition of this transient plus the steady state. In the steady state, I have ignored the omega part, but not over here because the contribution of omega to this whole term is not going to be very significant, whereas over here to the transient. It is critical because it is actually going to cause the overall amplitude. You remember, the envelope of the under damped oscillation which became dominatingly small. You have also talked about the logarithmic decrement factor so that every successive cycle, the amplitude of the under damped oscillator diminishes. So, you must keep track of the damping term in the transient.

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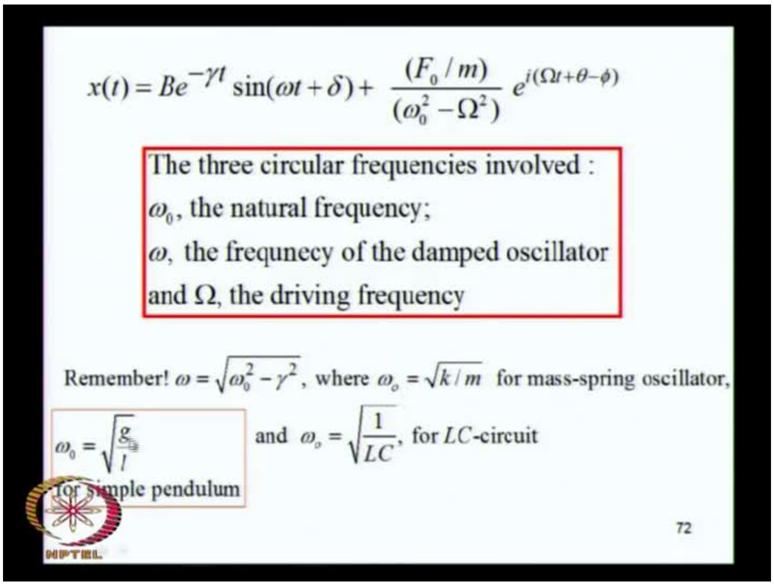

Now, this is just to remind you that there are three frequencies that we are talking about: One is the natural frequency omega 0; then there this frequency omega of the damped oscillator; there is this frequency of the driving term and you can write this for whatever oscillator you have in mind; the mass spring oscillator in which omega 0 is root k over m, or the simple pendulum in which the natural frequency or root g over l, or the electrical oscillator in which the natural frequency is 1 over root LC.

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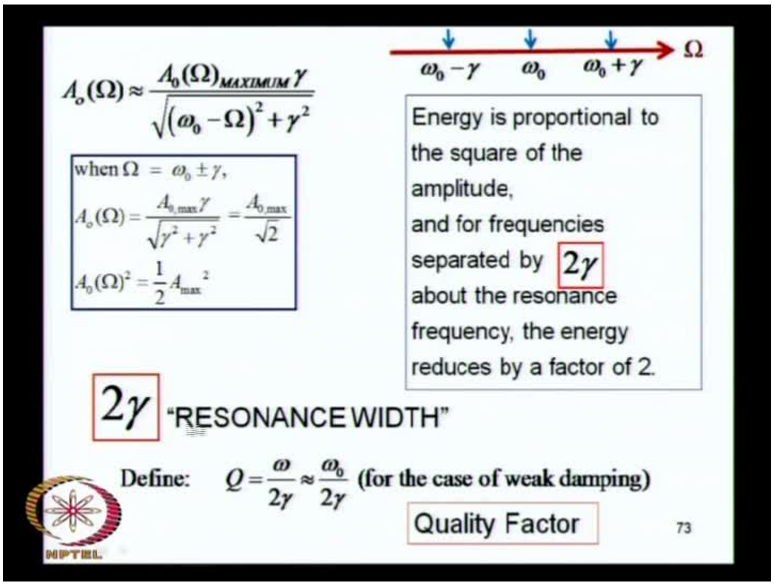

So, depending on any kind of oscillator, you can develop the corresponding expressions. So, in one go in setting up just one differential equation, you can solve very many different kinds of problems in physics. So, that is the advantage of developing these electromechanical analogs. You can extend it to many other branches; may be we markup fluctuations, but you have to set the corresponding analogs.

What will be the driving term over there? I think Bill Gates buying something or selling something, or periodically over something of this kind Microsoft or whatever, but any way, you can do this analysis in principle for any kind of oscillatory behavior for any physical parameter. So, there is a certain degree of freedom which has an oscillatory behavior and if there is damping present, if there is the periodic force which is applied on it, then you can solve, you can take this algebra that we have done and just replace the corresponding terms and you have the solution. You do not have to deal with them separately. So, that is the advantage of doing this.

Now, if you look at the behavior near the resonance frequency, so omega 0, the natural frequency is the resonant frequency. This is where it is in the vicinity of this, that the amplitude would be maximum. We ask that if you go away from this frequency either to larger frequencies or to lesser frequencies, the amplitude would diminish, but if you see that if you go not too far, but go far through at distance in units of frequency determined by the damping coefficient, you have got omega 0 plus gamma on the higher frequency side and omega 0 minus gamma on the lower frequency side.

So, you are looking at a frequency range from the lower end to the upper end which is made up of twice the damping coefficient, twice gamma. So, that is the range that you are looking at. If you stay within this range, then you can immediately see that close to this frequency - to the resonance frequency within this range, the amplitude will be in this range at the upper end; at the lower end, it will diminish, but it will not become lesser than by a factor of 1 over square root 2; which means that, if you look at the corresponding intensity and the intensity goes as square of the amplitude, the intensity will diminish away from the resonant frequency, but not by a factor of any more than a half.

So, it will still remain of the same order of magnitude. It will be diminished not by a factor which is larger than half. So, in this width of twice omega, the energy which goes as a square of the amplitude, the energy will reduce at the most by a factor of 2 and this is sometimes called as a resonance width. So, the resonance is not always very sharp because the amplitude will increase significantly, not just at the exact frequency of resonance, but also somewhat away from it. How far away? Well, if you remain within the damping coefficient gamma in units of frequency on either side of the resonance, you are guaranteed that your energy will not reduce by anything more than half, although the maximum energy transfer will be at the resonance frequency.

So, this is sometimes called the ratio of the frequency itself to twice gamma, which is the resonance width. This ratio is sometimes called as a quality factor. It is a measure of how sharp the resonance is, because twice gamma, the smaller it is the larger will be the quality factor.

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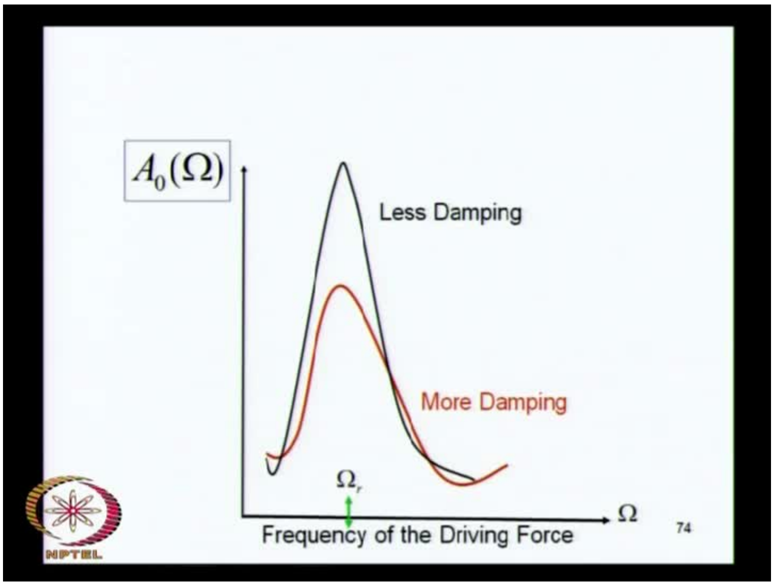

So, this is sometimes called as a quality factor and you can see that if you have less damping, then the amplitude function has got a sharper profile compared to that when you have more damping when it tends to get flattened out. So, this has several important applications in mechanical and electrical circuits.

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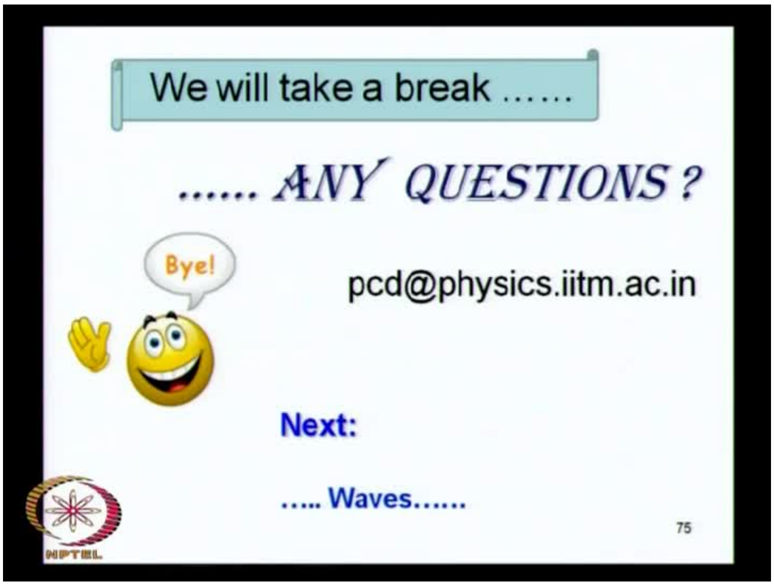

So, I will take a break over here. If there are any questions, I will be happy to answer. Otherwise, we can certainly resume the discussion after the break.