Fluid Dynamics for Astrophysics
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Lecture – 19
Dimensionless numbers contd: Plasma beta, Magnetic Reynolds number, Alfven Mach number, Prandl number

So hello, so we are back and we are now starting to, we will continue with our discussion of Dimensionless numbers and Fluid Dynamics. Before going on I thought I would clarify a little bit about small point that I did not quite address properly in the last bit. So, there I introduced this quantity mu was the coefficient of viscosity.

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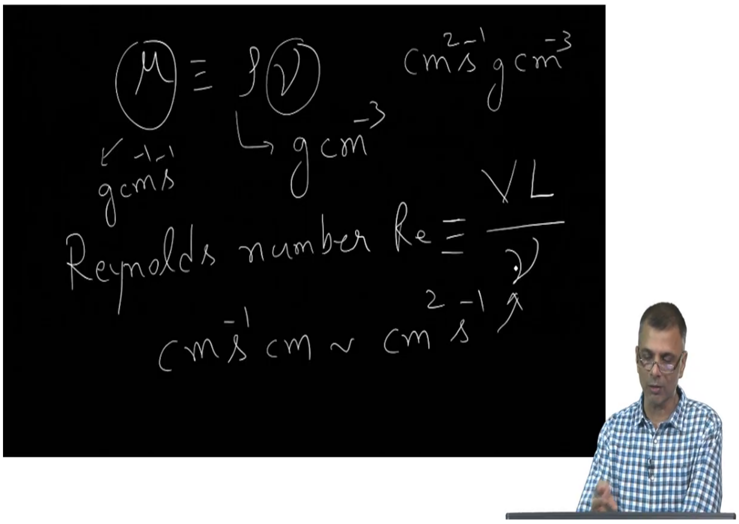

And I introduced this quantity called nu right and I was wondering about the dimensions of nu right, the dimensions of this are just grams per centimetre cubed right. So, now the way to one way to easily say this is you remember towards the end of our discussion we said that the Reynolds number which is R which is often you know given the symbol R e is defined by a characteristic velocity time times a characteristic length divided by nu. And from this we can immediately figure out what are the dimensions of nu.

Well, the dimensions of nu have to be the dimensions of V which are something like centimetre per second times the dimensions of length which are centimetre. In other words, centimetres squared per second those are the dimensions of nu simple enough. And therefore, the dimensions of mu are something like gram centimetre second right.

Because you know its nu which is centimetre squared per second times a rho which is gram per centimetre minus 3. So, therefore, this would be something like centimetre minus 1 sorry. So, this would the dimensions of this would be gram centimetre minus 1 second minus 1 those are the dimensions of mu.

So, if the terminology often confuses you both this and that are often called the coefficient of dynamic viscosity sometimes depends upon you know what the author is saying. You can simply figure it out from the dimensions, what are they talking about, are they talking about mu or are they talking about nu?

We have mostly been talking about mu, but in our discussions of Reynolds number it is more convenient to start talking about nu and which is why I introduced this nu thing right. So, moving ahead no that is not what I wanted to bring up.

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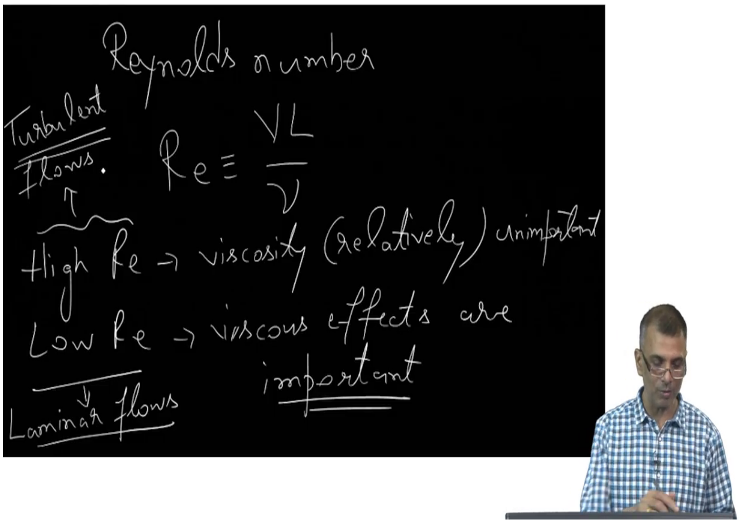

So, let us talk a little bit more about the Reynolds number which is arguably one of the most important dimensionless number for you know astrophysics and as we said. You know, as we said earlier it is given by the ratio of the inertial term to the viscous term. So, essentially this tells you whether or not you know viscosity is important that is one way of looking at it.

If the Reynolds number is high viscosity is relatively unimportant. If the Reynolds number is low, so, high Re viscosity is relatively unimportant right and vice versa. Low Re viscous effects are important, so these are the regimes.

Now, you might ask unless you give me is one thing to figure out the characteristic velocity and the characteristic length scale that those are things that I we can you know figure out fairly easily, but unless you give me the this very important transport coefficient nu.

I cannot figure out what the Reynolds number is right and you would be right. You need to know what the nu is and in astrophysics unfortunately you do not know what nu is ok. In several situations you do not know enough about the microphysics, so that you cannot make a statement about nu, this is unfortunate but true.

So, we go on and start talking about high Reynolds number flows and so on so forth without really knowing, what the transport coefficient nu is in that particular situation. Because astrophysical fluids are very very different and they are often collisionless, they are very very thin.

So, one cannot make a ready you know jump from lab situations to astrophysical situations by way of these transport coefficients by way of these very important transport coefficients. So, this is something to keep in mind.

Although we might brightly talk about high Reynolds number flows and so on so forth, we have to be careful we have to keep in mind that we really do not know, what is the origin of this high Reynolds number, ok. So, the other thing is that when at low Reynolds numbers the flows are generally laminar flows.

In other words as the you know name implies, you know the flows the streamlines of the flows are relatively regular and they are laminar. And opposite is the case for high Reynolds number, for high Reynolds numbers the flows are generally turbulent flows. In other words as your intuition would tell you know if you consider if you think about a streamline of the of a flow in a given situation you would expect it to be all tangled.

You open your kitchen sink, you open your tap in your kitchen sink with relatively you know relatively gently, of course, assuming that you know the nozzle of a tap is smooth and so on so forth. So, when you open the tap gently your V’s are relatively low and the nu is just given, whatever it is for bulk flow of water you cannot change, it is not within your control.

And there is a certain length scale say you know I mean they say the distance between you know the edge of your tap and the sink that would be a length scale. Say half of it is, but certainly not one-tenth of it and certainly not 100 times that. So, when V is low you will notice that the flow is relatively laminar is relatively well behaved ok.

So, that would be a low Reynolds number flow. However, when you turn the tap on high right, so you are increasing the speed and Reynolds number becomes large and you know from everyday experience that the flow becomes unsteady and all tangled up. You can see without resorting to very quantitative definitions, you can see from everyday experience that the flow has become turbulent right.

So, here is an everyday example of the manner in which the Reynolds number manifests itself ok. Low Reynolds number viscous effects are important that is one way of putting it, but in everyday experience ; you can see that the flow is laminar. When the velocity is relatively low so that the Reynolds number is also relatively low.

But you can increase the velocity by increasing you know by turning the tap on higher and you can see that the flow transitions to a turbulent flow right. So, here is one practical you know example of Reynolds number.

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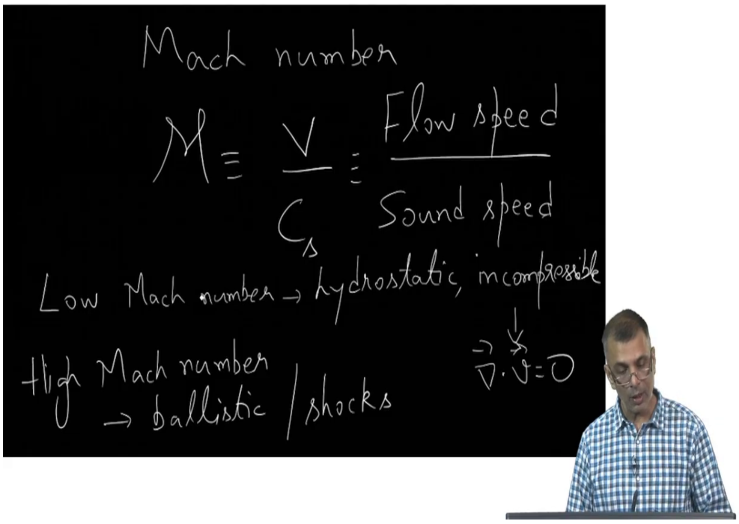

The next dimensions number we introduced was the Mach number, which generally denoted by a script M which is the ratio of I should not; I should not use this V something like this. Yeah. Ratio of flow speed or to be really correct we should say some sort of characteristic speed in the flow to sound speed.

And as we will; as we will figure out a little later the speed of sound is essentially the one characteristic speed at which small pressure disturbances travel. I clap or I am talking and the sound is reaching the microphone or if I was in a classroom the sound would reach you know the student. And there is only one characteristic speed and that depends only on the temperature of this room.

Sometimes the density, but if you are talking about you know isothermal waves it would depend only on the temperature and there is only one characteristic speed. It does not matter whether I am talking loudly or softly, whether my voice is low or high ok, whether I am a man or a woman. You know which just generally speaking it determines the pitch and tone of my voice or this frequency at which I speak, it does not matter.

The speed of sound is always the same and the speed of sound is the characteristic speed at which small pressure disturbances travel. We will illustrate this I am simply telling you the definition at this point and we will illustrate this in some more detail as we go along ok.

So, that is the speed of sound and the Mach number is the ratio of the flow speed to the sound speed ok. What can you say? I am sure you have heard of supersonic jets or supersonic flows, airplanes flying at supersonic speeds and these are characterized by a Mach number.

The airplane the plane is flying at Mach 2 or Mach 10. That essentially tells you that its flying at 10 times the sound speed the local sound speed, wherever its flying or 2 times the sound speed so on so forth. So, that is what the Mach number tells you and you recall the Mach number is essentially the ratio of the first term in the NavierStokes equation to the third term the viscous term. So, that is essentially what the Mach number is telling you and right. I beg your pardon.

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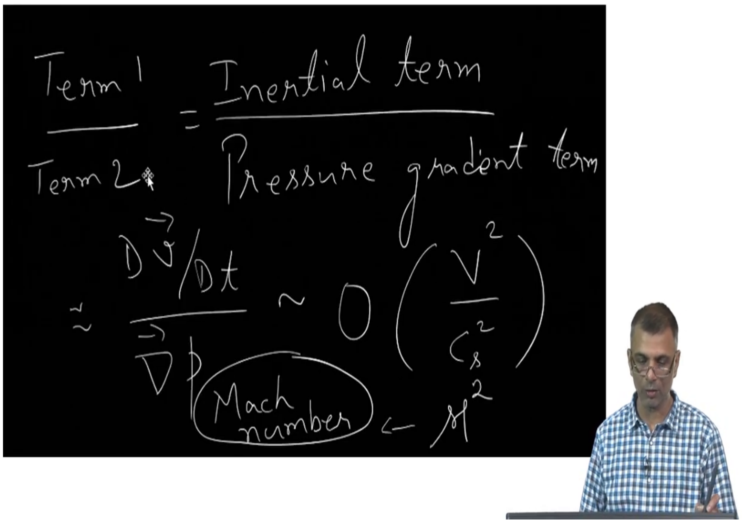

I said something wrong and here was the definition. So, the first term to the second term not the viscous term the inertial term to the pressure gradient term. This is what the Mach number is. And that turns out to be the square of the Mach number and the square of the Mach number is V square over Cs squared. So, that is what it is.

And generally low Mach number and we will justify this later. Low Mach number situations essentially means that the pressure gradient term is more important than the inertial term because that is what the ratio is. And low Mach number situations can be considered to be hydrostatic, in other words the inertial term is not that important.

So, it is the flow can be almost to a good degree depending upon how low the Mach number is of course, it can be considered to be hydrostatic even though there is a flow ok. So, here is a very concrete example of how you know one can simplify things right. A hydrostatic situation is much easier to solve for than a hydrodynamic situation.

And when can you; when can you approximate even though there is a flow, when can you approximate the flow to be hydrostatic? Well, when the Mach number is low ok. Another thing one can approximate is a another simplification that occurs in a low Mach number situation is that the flow can be considered to be incompressible, if the Mach number is low.

So, and if you recall incompressibility essentially means the divergence of v is equal to 0. Technically, this is these are what, incompressible flows. This is the definition of incompressible flow, perfectly incompressible flow. But you know the flow does not have to be perfectly incompressible, it can be incompressible to a good approximation, how good the approximation is dependent on the magnitude of the Mach number.

The Mach number is say 0.1 or 0.05 then incompressibility is a very good approximation. So, this is the practical utility of low Mach number flows can be considered hydrostatic and incompressible. On the other hand high Mach number flows. The flows are we will explain what this means a little later it can be considered to be ballistic.

In other words, the inertial term is what is more important, the pressure gradient term is not important at all. If you say, if the Mach number is 10 or 100 or something like that, so, it is almost as if the flow is like a bullet it is like a; it is like its ballistic. The pressure the gradient of pressure does not matter that much and such flows are also susceptible to the formation of discontinuities. We will talk about this in some details later on, but such flows are often susceptible to discontinuities that are called shocks.

Technically speaking, shocks are places where there is a transition from a supersonic flow, which is a high Mach number flow to a subsonic flow which is a low Mach number flow. We will see all this as we go along, but I just wanted to you know lay a couple of things out. In astrophysics we will not only be dealing with neutral fluids, we will also be dealing with magnetized fluids.

Where the fluid carries a magnetic field or the fluid is flowing in the presence of a magnetic field. So, we will discuss the subject of magneto hydrodynamics very briefly. But before that I figured in the spirit of listing out a few important dimensionless numbers, I would say the following also.

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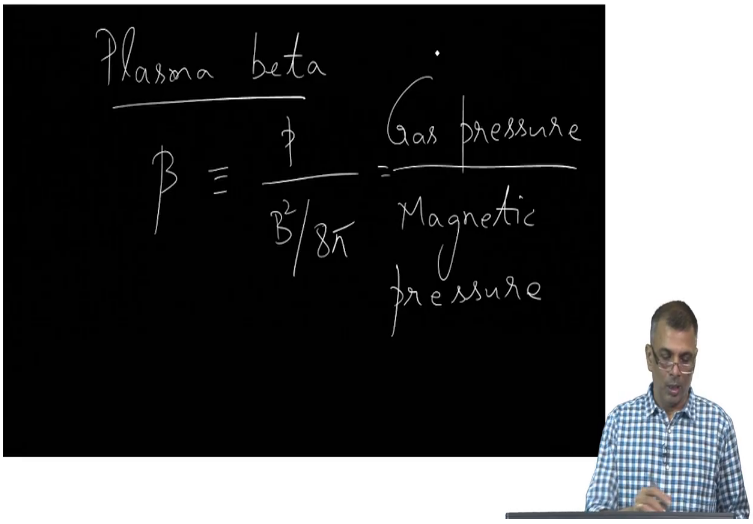

There is something called very important dimensionless number that is called the plasma beta which is defined as p over 8 pi is essentially gas pressure over magnetic pressure. So, and the gas pressure is simply your usual p equals nkt right. Magnetic pressure is something that we have not yet encountered, we will do so and I will reintroduce this when we do so, but for the time being this is yet another important.

I just wanted to list out this other important dimensionless number in magneto hydrodynamics not in hydrodynamics, but nonetheless. Obviously, when the plasma beta is low it means the magnetic pressure, it dominates over gas pressure. In other words, the dynamics of the fluid will be predominantly governed by the effects of the magnetic field.

What I what do I really mean by magnetic pressure? Roughly speaking magnetic field lines behave like rubber bands. A rubber band is a very good analogy for a magnetic field line, just like you try to stretch a rubber band this affords tension. Similarly, a magnetic field line you try to stretch a magnetic field line this even though magnetic field lines are fictitious quantities. It is very useful to think in terms of magnetic field lines, you stretch a magnetic field line and you feel a tension ok

The other thing is just like a collection of rubber bands you try to squeeze the collection of rubber bands and it there is a certain elasticity associated with that. Similarly, a bundle of magnetic field lines, you try to squeeze it and there is a certain elasticity associated with it. So, yeah I am not giving you all the details, we will encounter that as we go along.

But you know just like you know the analogy between magnetic fields and rubber bands is a very good one. Hence, you can sort of if not completely understand you can kind of begin to appreciate the concept of magnetic pressure and the ratio of particle of gas pressure to magnetic pressure is called the plasma beta.

This is yet another very important dimensionless number in magneto hydrodynamics ok. And talking about situations where magnetic fields are important we have already defined the Reynolds number. I write this down again.

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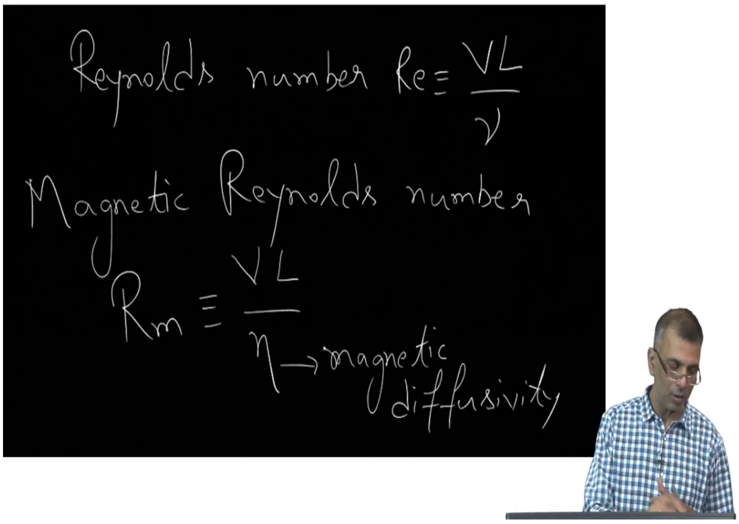

Re was VL over nu, where nu is a viscosity coefficient right. There is something called a magnetic Reynolds number also dimensionless. All of these are dimensionless right which is called which is generally denoted by Rm and this is defined by VL over eta, where eta is the magnetic. So, this you can see is the same is the same as this, magnetic diffusivity.

What this is? In order to understand magnetic diffusivity it is useful to think of the following situation. In a high conductivity medium you introduce a magnetic field, due to some reason it just stays there because of the high conductivity it just stays there ok.

In a low conductivity medium on the other hand you in other words a medium which is relatively more resistive yeah you introduce a magnetic field, it does not stay there it does not stay there for a long time, it diffuses away ok. And the coefficient of diffusivity is called the magnetic diffusion.

In practical terms what does Rm tell you? Well, Rm essentially tells you a high magnetic Reynolds number flow is one where the magnetic field is strongly coupled to the flow. There is a flow and the magnetic field is strongly coupled in other words the magnetic field goes along with the flow ok. And this is slightly different from what we talked about earlier the plasma beta ok, this is slightly different.

In this situation high magnetic Reynolds number situation is one where the magnetic field is strongly coupled, in other words the magnetic field for instance think of an analogy where the magnetic field is like a piece of wool its flowing along with the flow. And the flow takes a turn the piece of wool also does that it obeys the flow ok. It is strongly coupled and the opposite situation a row low magnetic Reynolds number situation is one where the magnetic field is not very strongly coupled to the flow.

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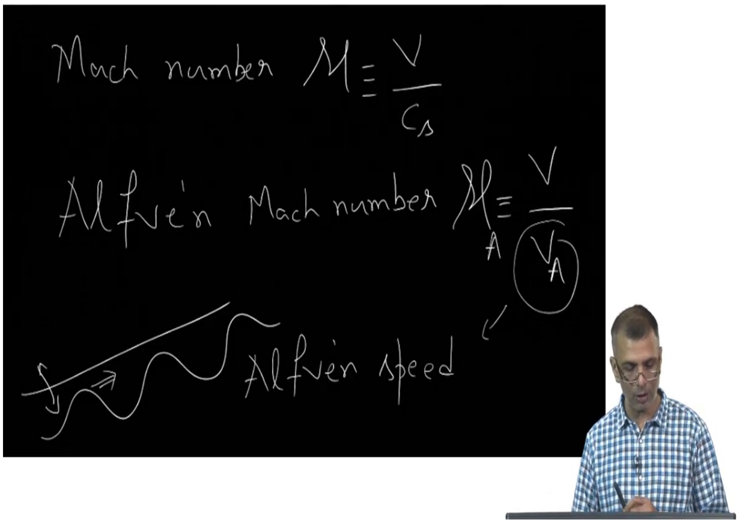

Just like we talked about the regular Mach number which you remember was the ratio of the flow velocity to the speed of sound. There is something called in a magnetic situation in a magnetohydrodynamic situation, there is something called an Alfven Mach number, which is generally.

So, you have a subscript A and this is the ratio of the flow speed to something called the Alfven speed and what is Alfven speed? I will tell you in a minute. Again we will talk about this in some more detail as we come along as we go along the course. The Alfven speed is just like the sound speed is a characteristic speed at which pressure disturbances travel in a medium small pressure disturbances are not large ones like a bomb explosion, small pressure disturbances.

Similarly, the Alfven speed is a characteristic speed at which transfers consider a magnetic field line like. So, you twang it, you introduce this kind of a transverse perturbation on it that results in the magnetic field line doing this ok. Much like a guitar string for instance and a wave travels like that and the speed at which this wave travels is the Alfven speed ok.

So, there is a characteristic speed which depends on the magnitude of the magnetic field and the density of the medium. And so, this is another just like the sound speed is a characteristic speed in a gas in a non magnetized gas.

The Alfven speed is one characteristic speed there are other characteristic speeds as well, but the alpha and speed is one characteristic speed in a magnetized gas and that is what it is. And Alfven Mach number is a ratio of the flow speed to the Alfven speed ok. In addition let me give you a laundry list of some other dimensionless numbers in fluid mechanics.

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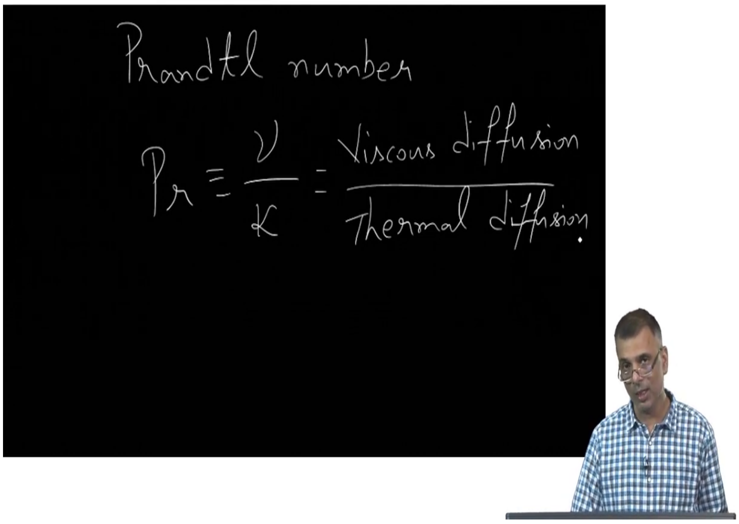

One is one important one is what is called the Prandtl number is generally denoted by Pr and it is defined by nu over kappa. Where nu is the viscosity coefficient that we have come across earlier and this is the thermal diffusion coefficient ok. So, this is essentially viscous diffusion to thermal diffusion. And as before the significance of this is simply, that if the Prandtl number is high.

Well then viscous terms are more important than thermal terms if any in whatever we have done. We have not we have never introduced a thermal diffusion term in our equations. Generally, these things are not terribly important for astrophysics these are situations like where the Prandtl number crops up or more in engineering situations.

But nonetheless it is important to keep in mind and vice versa a low Prandtl number situation is one where thermal effects diffusion due to thermal effects predominate over diffusion due to viscous effects. And so I think we will stop there. I thought I would give you know without telling you which equations these numbers arise from, simply give you the definitions and give you the physical import of these definitions just to you know emphasize the various kinds of dimensionless numbers that exist in fluid mechanics.

And I have just given you a small list, there are many many others. I urge you to go to standard fluid dynamics books or even Google dimensionless numbers and fluid dynamics you will find lots and lots. And the importance of these dimensionless numbers are, a; they are dimensionless. So, they are easy to sort of compare is the Prandtl number 1 or 2, is a Mach number 1 or 10 so on so forth yeah. So, that is one thing.

And the importance of these numbers are that they characterize the flow. For instance a low Mach number situation is one where the flow can be considered hydrostatic and the flow can be considered incompressible so on so forth. That is another you know advantage of these dimensionless numbers.

And finally, of course, from a more basic point of view it tells you which terms you can throw away. For instance, when I say you know a low Mach number situation is one which can be considered hydrostatic. So, I really do not need the inertial term at all that is what it is telling me right. I can simply solve a hydrostatic equation which is obviously, much simpler and still be reasonably accurate.

Even though there is a flow even though it is not like the situation is strictly hydrostatic my answer will be pretty good. Because I know that the Mach number is fairly low, but in order to do that I need to know this beforehand, I need to be sure that the Mach number is low and I say this with an eye on the Reynolds number which is really the one number that I care most about. The Reynolds number and the magnetic Reynolds number, these are the two numbers that I often care most about.

So, as I said earlier often in astrophysics we make the statement that these flows are very high Reynolds number flows. For instance, the solar wind or the entire solar corona is considered to be very high definitely the upper solar corona is considered to be a very high Reynolds number situation.

But, while this is generally accepted to be true we make these statements without detailed knowledge of the microphysics. Remember the Reynolds number is V times L over nu. Without we make this statement that the Reynolds number is high, without quite knowing the microphysics of nu, without quite knowing what the nu is. So, these are deficiencies that need to be kept in mind.

Thank you.