Orientation of Fibers

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Lecture No. # 01

Fibers and Yarns: Terms, Definitions and Relations

Let us start today's lecture, today's lecture is have the team fiber orientation. You all know that the fiber orientation is very very important phenomenon in the textile practice.

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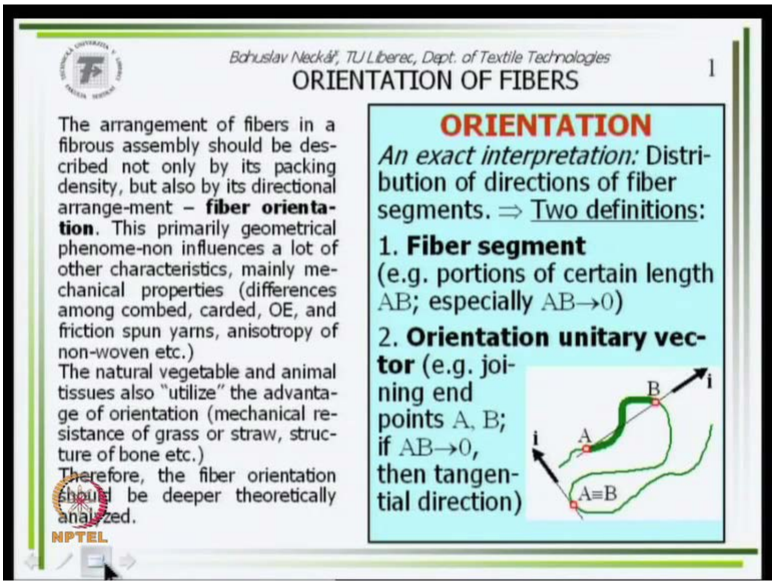

So that, we want to to to analyze this program more deeper. When we say orientation we need to say, what we mean under this term? We must say something about the fiber segment, which we use for orientation and then about the definition of direction; how to define the orientation vector? We can speak about very short fiber fiber segments extremely short like here, then the direction is our problem it is is evidently tangent of this small fiber segment. In other case, when we use this green green segment having a longer plans, we usually use the line from end points located to end points A and B. Yes.

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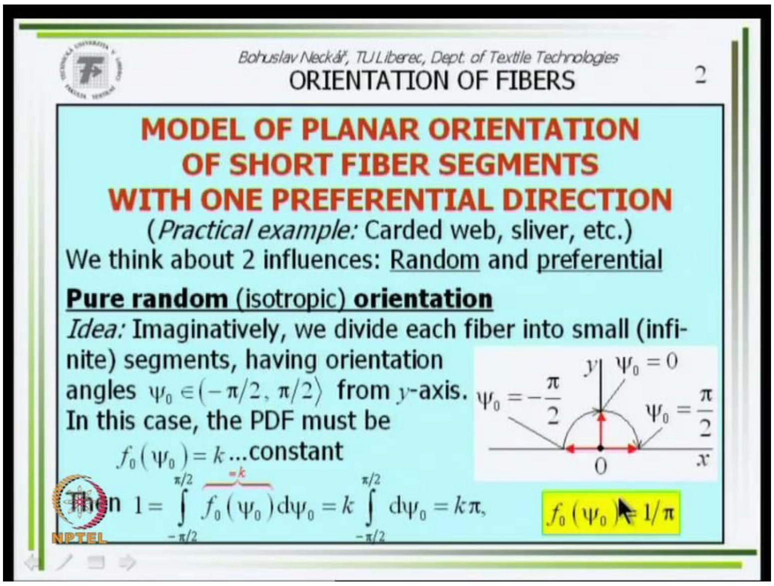

We were to speak about the easiest case; it means we will speak about planar orientation of fibers. It is often a case; for example, of web and yarder typical extract structures. Let we have some coordinate x and y and then we want to measure the directions from minus pi by 2 to 0, 2 plus pi by 2 understandable. So, from minus 90 degree to 0 to plus 90 90 degree, the easiest case is if our probability density function represents pure pure random it means isotropic structure. So, that in each case the direction is in the same probability, how must be this probability density function evidently must be constant is not it.

If it is constant, we can write we can write that such function, this function we call f 0 psi 0 is this our staring probability the density function, this integral from this PDF f 0 psi 0 d psi 0 from minus pi by t 2 plus by 2 must be equal 1. As every time integral from probability, each probability density functions, but because this function is constant we carry this one. So, this one and from this we obtained that the such probability density function is constant and equal 1 pi by 1 pi 1 by pi. Normal way, normal way this isotropic structure is not real why.

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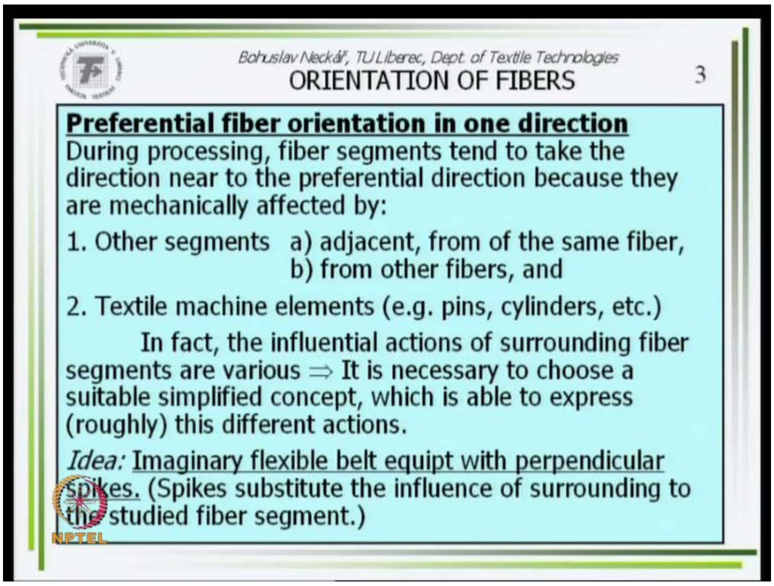

Usually, no every thing, but usually one direction is preferred. In the example of (( )) it is usually to longitudinal direction. Why is why the one direction is preferred because first sorry first other segments connected our segment. How to say it? Kick to this segment through the process because oriented it more to to preferential direction. So, it can be other segments or adjacent from the same fiber or from the other fibers and also of course, the textile machine elements different pins cylinders and so on and so on.

In the reality in the reality, we obtained some structure, which preferential direction. The real physical mechanism of this process is very vey complicated it is very difficult to to describe it.

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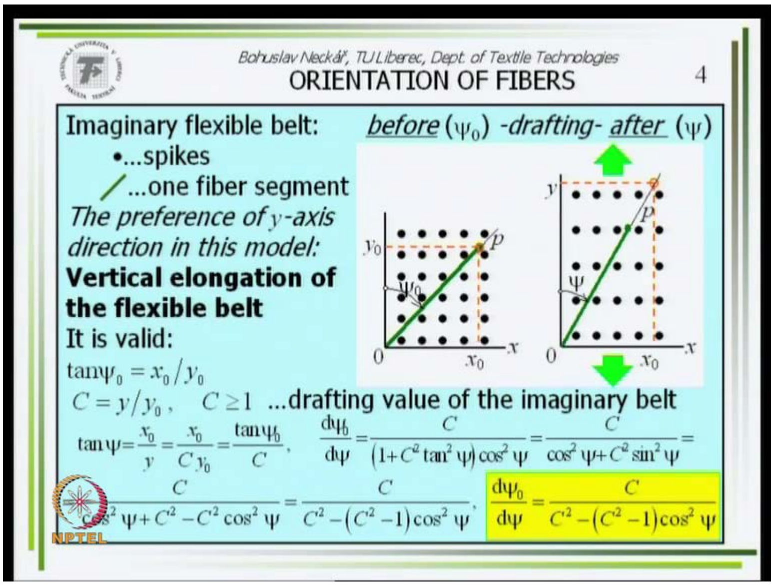

So that, we need to use some some thing, which is more easier easier. Therefore, we start listen idea, let us imagine like an imaginary flexible belt equipped with this perpendicular spikes, such spikes substitute the influence of surrounding of our our fiber segment. This is shown here, let us imagine such, imaginary flexible belt having some spikes some spikes like like flicker that here. And this substitute influence of surrounding of our green fiber.

On the end, our short fiber segment sometimes I says, fiber what I mean a very short fiber segment, which is possible intemperate as straight on the end of our fiber segment, let us take some some choke some orange choke and make on our imaginary belt and orange point. Our fiber segment is lying on the is lying on this straight line P, evidently end point of fiber is is coordinates x 0 y 0. So, z tangent psi 0 is x 0 by y 0 evidently. And now let us elongate let us elongate our imaginary flexible belt. Please our belt is imaginary and not real. And using this moving the x x coordinate. I must say the the fiber fiber segment is a slip between the slipping between the amount of the the spikes, but it stay (( )) on our straight line P.

So, that after our elongation, after our moving the situation is alike on our picture here, x coordinates still be same y coordinate it is higher in the traditional a spinning technology. We know the quantity and drafting value, which is ratio in our case it means y by y 0. Let us use this quantity as a degree of elongation of our imaginary belt, well if C is equals to zero then if C is equal to zero sorry sorry, if C is equal to 1, then it is without without elongation, higher it is, so high is high is the elongation. Now, from the from the picture, we see that the tangent psi, hang up psi is here, must be x 0 by y, but y from this equation is C times y 0. So, that is this one, x 0 by y 0 is tangent, psi 0 so that this one. Now we can make very differentiate, we can differentiate this equation and after few steps you can own to to derive it, we obtain we obtain the such equation is here. This this of three arranging in final, we obtain this equation to be important for us. Well, it was the discussion about one fiber, now about all fibers. The starting probability density function is the starting situation was an isotropic structure probability density function f 0 psi 0.

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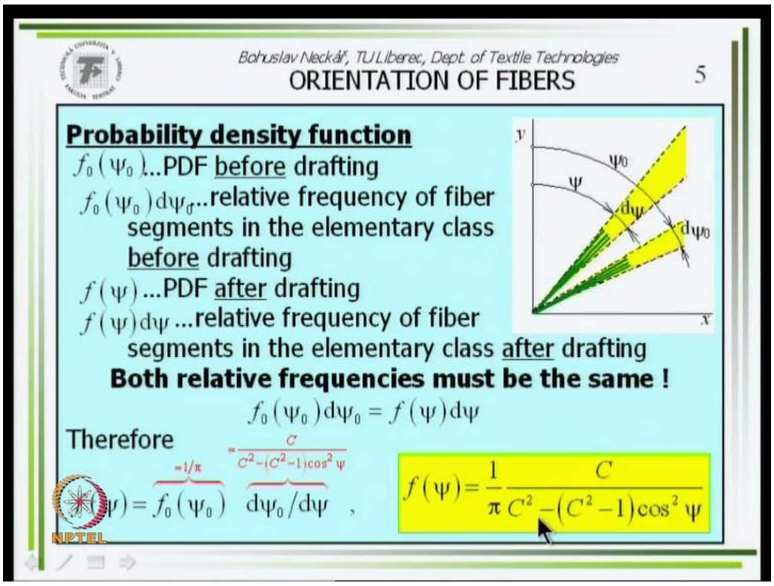

You know, that the f 0 psi 0 times d psi 0 probability density function times differential quantity have some logical sense. It represents the reality frequency, but a little abnormal relative frequency, the relative frequency related to a elemental class interval; class interval, which is very very short, differentially short.

So, we can imagine such elemental class, the relative frequency of such class is every time probability density function times differential quantity. We will use this logic logical moment more times in our lecture. So, it is f 0 psi 0 time d psi 0, relative frequency of fiber segments in the elementary class before drafting. After drafting a new probability density function is coming. It is a probability density function f psi, this also (( )) a subscribe. So, the f psi d psi is relative frequency of fiber segments in the elementary after our drafting of our imaginary belt, but both such relative frequencies must be same. Because the fibers inside of inside of a starting of a staring angular elemental class must be same then after our elongation of our belt, imaginary belt is not it.

So, therefore, it must vary, f 0 psi 0 d psi 0 relative frequency before drafting must be same that f psi d psi means the same after drafting. Using this equation, after substitution of our known expressions, do you think the equation for probability density function of fiber short segments after after using some drafting, it means after preferential of one may be longitudinal direction. I will see, how it is graphically, how is this graphically on.

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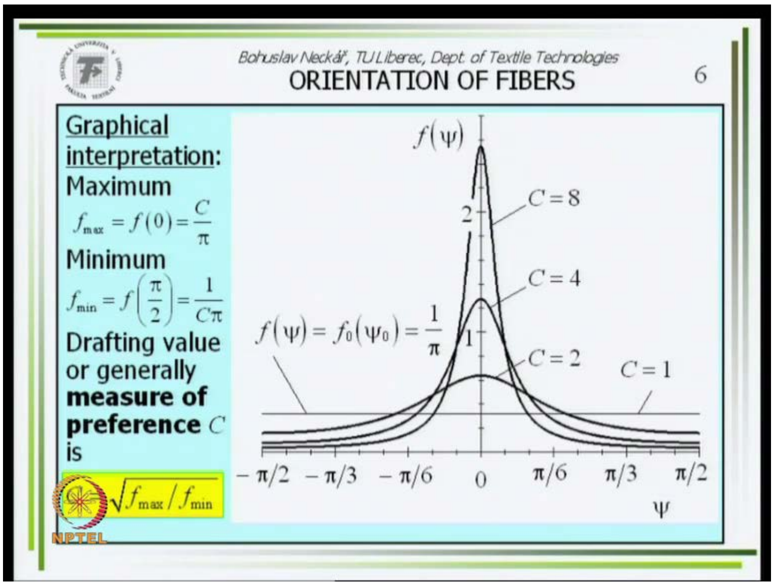

So, more are the fiber concentrated round our in our imagine the imagine longitudinal longitudinal direction.

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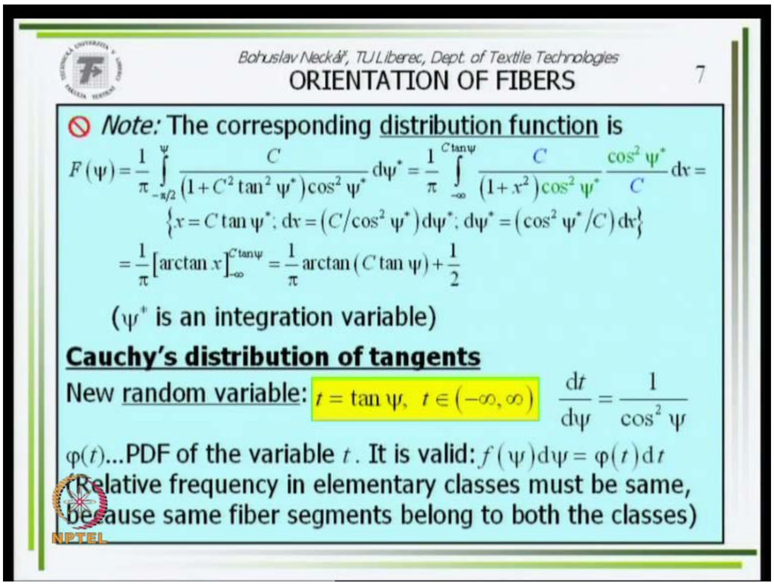

Well, now only for your information, I do not want to in detail to explain it. When you need to the to have some distribution functions is here is derivation of this. We can say in the moment that, we are that we can be very proud to obtain quite new quite new probability density function. We we can published it in the special journal of applied the theory of probability sorry.

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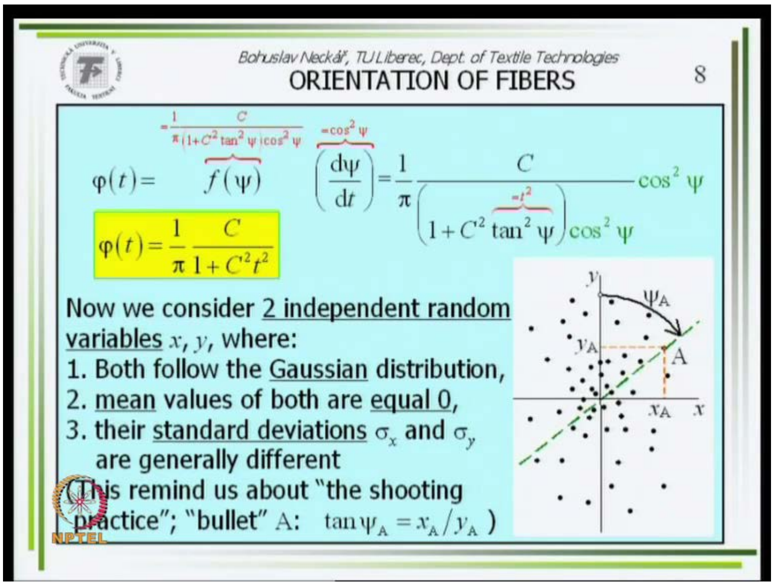

Now, this this pictures show that, how as a tangents of our angle f psi have so called Gaussian distribution. So that, it is this is a one of a known distribution from theory of probability.

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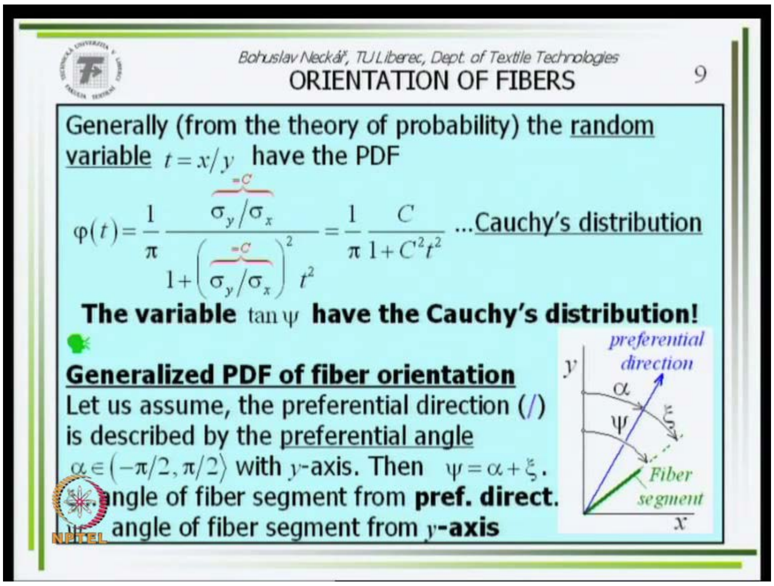

Even even you want, you can study it in more details. Sometimes it is coming or often it is coming the the following program to this moment, our preferential direction was same than the y axis vertical axis.

But you sometimes preferential direction is an order, we have y axis here, x axis here and let us imagine the preferential direction is the the direction of our blue arrow, which have 2 y axis some angle alpha. So, we can write the angle psi to y axis is now are the then the angle psi to preferential distribution. Our earlier equation is now valid to the angle to preferential distribution to angle psi and our angle psi is another and its valid from this easier from this very trivial picture that psi is alpha plus psi.

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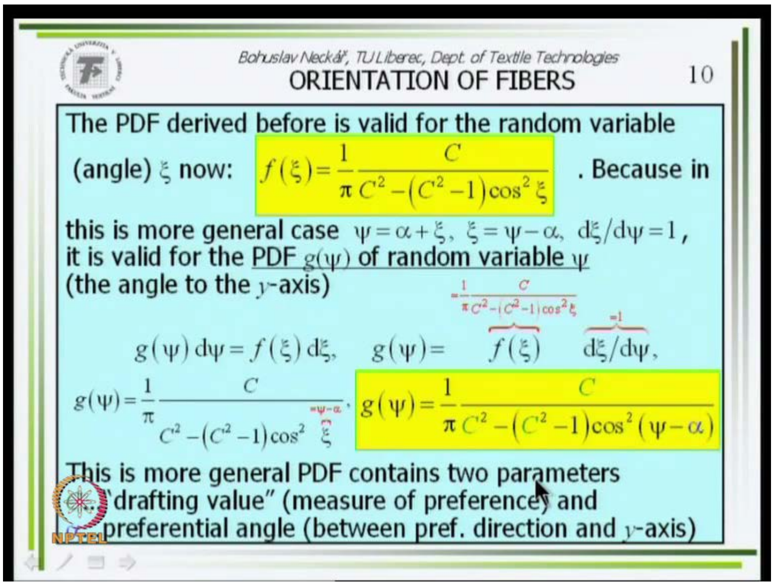

Yes. So, I said our earlier equation is valid Also, but angle psi and towards valid for now for probability density function. I call it now, as a new symbol, under the new symbol g psi, it is angle to to y axis now obliged that it through preferential direction. It must be it must be our own function, but this angle psi using it we obtained final probability density function g psi, which is given by such expression having 2 parameters, our known parameters C. If parameters is represents the generally say the intensity of the preference of this are the things direction and parameter alpha, which is angle between 2 directions y, y axis direction and preferential direction.

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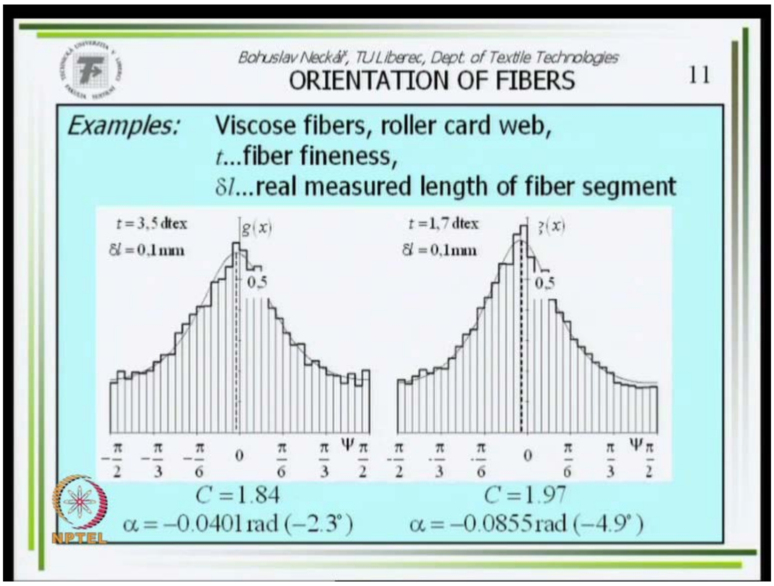

Well it was our theory, now is the question and how it is in the in practice? We analyzed using thrice the fiber technique for you may be known known experimental method. We analyze the curves, the of fibers in a fiber vamp, it was research for non woven textiles and we we take lot of points from this fiber to computed and we reconstruct whole curve of fiber in computer then we divide our fiber to very small, very short parts. It in this picture it was 0.1 millimeter and measured or calculate angles of orientation and then we construct the histogram of this this this distribution.

On this two pictures are in first it is a fiber viscose fibers finite 3.5 and the second one 0.7 decitex and there are this such such histograms obtain experimentally. When we use our curve and there are if we using some sophistical regression or something. So, two parameter C and alpha, we obtained parameter C rough 1.84 alpha may be 2.3 degree minus 2.3 degree and the second case 1.97 and the alpha is minus (( )) 4.5 degree, may be that is an alpha was the result of the experiment are or our our mistakes definitions of opposite longitudinal direction is very very small, but you can see that the comparison bring a good results, good correlation of this experiments and theory, theoretical result.

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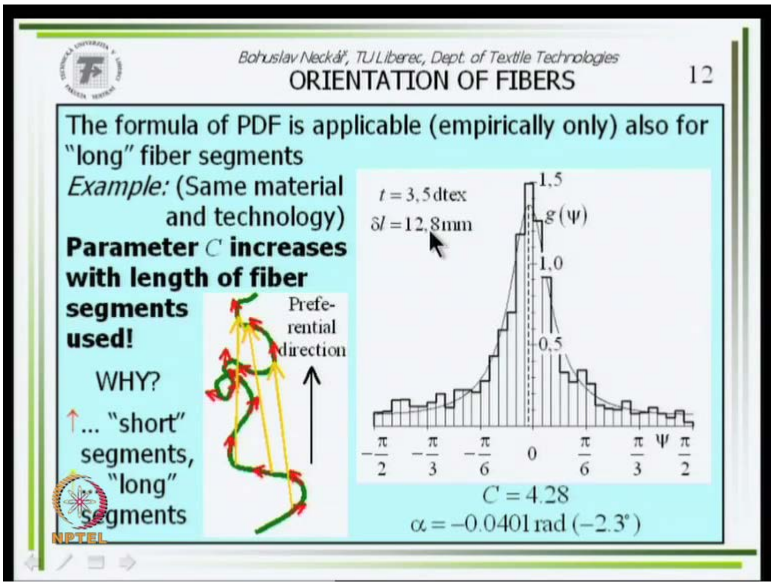

We applied our equation for long longer fiber segments through in this case it is 12.8 millimeter, in this moment it is it is empirical application because we derive it for very very short fiber segments. So, is it this (( )) distribution, you can see that the result is also acceptable is also acceptable, but what is interesting the C is quite harder, C is (( )) then for the question is why? Why it is, how it is possible? It was the same structure, we evaluated the for a longer fiber segments, how it is possible? It is diagrammatically show this picture. Let us imagine a fiber green fiber on our picture having given given given shape, in macro in macro trend we can say it is due to vertical direction. In a shorts segments it has lot of loops and waves and so on and so on. Long fiber segments are the direction of long fiber segments are shown through yellow arrows here.

You can see that they are near to the longitudinal position, but the short fiber segments, the set of short fiber segments is straight through a short erect arrows and you can see that it is not so. Therefore, C for in (( )) web C value for long fiber segments is much more is much more, something is wrong? The the distribution of red of red arrows is quite other than the distribution of the yellow arrows. Well, and it is answer why the C is increasing from refer to the value over for.

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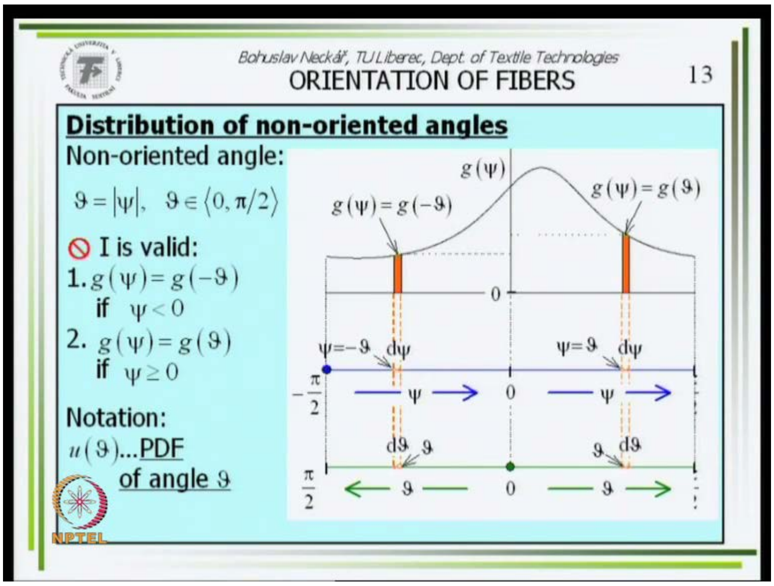

Sometimes, it is necessary to define probability density function for a non-oriented angle what I mean? Let us imagine some fiber segment, our angle psi is or to your, from your side plus 30 degree or minus 30 degree. There are 2 different angle side plus 30 degree minus 30 degree is not it? But sometimes I want to know, how is the distribution of angles between fiber and our y axis independent of them if it is on the right hand side or on your left hand side? So, I need to understand, how it is probability density function of an angle of an angle theta, which is absolute value of our angle psi.

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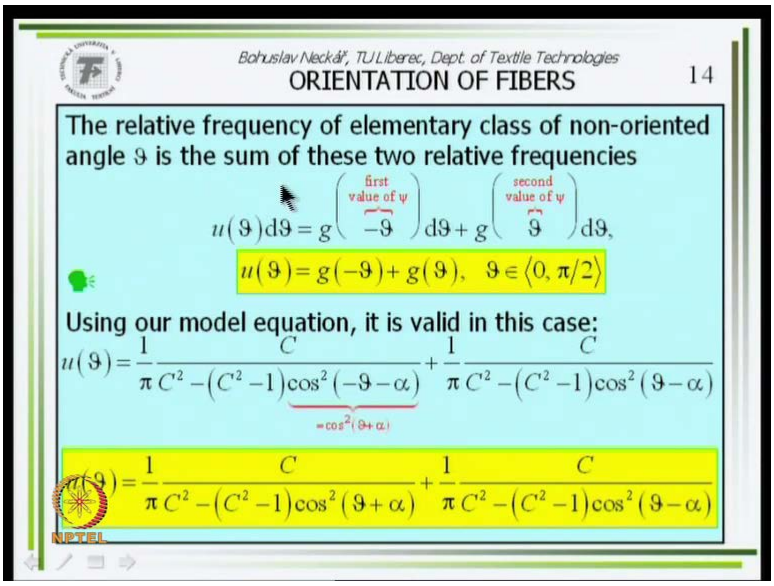

I think you can home more think about his picture, it is also intuitively evident that this probability density function must be some of two our g functions. One is in angle minus theta because theta is every time positive, theta is 30. So, for example, 30 degree then then minus theta is minus 30 degree and the angle of psi minus plus probability density in plus here. So, that u theta is g minus theta plus g plus theta, which is evident of intuitively using our equation we obtained the final equation in our model as shown.

Well, so, we that is all for probability density function of orientation of fibers same direct segment, segments in fiber assembly, but we often we often cut our our fiber assembly or by preparation, by practice in microscopic microscopic of textile structure, it is real resection or some imaginary sections by by, when when we use a breaking machine then the jaw have some some line, which is something like imaginary sections is not it? So, section is very often, the question is; how is the orientation of fiber the direction of distribution of fibers? But, only this these fiber sequents, which are cut it, which was cut it, only this one.

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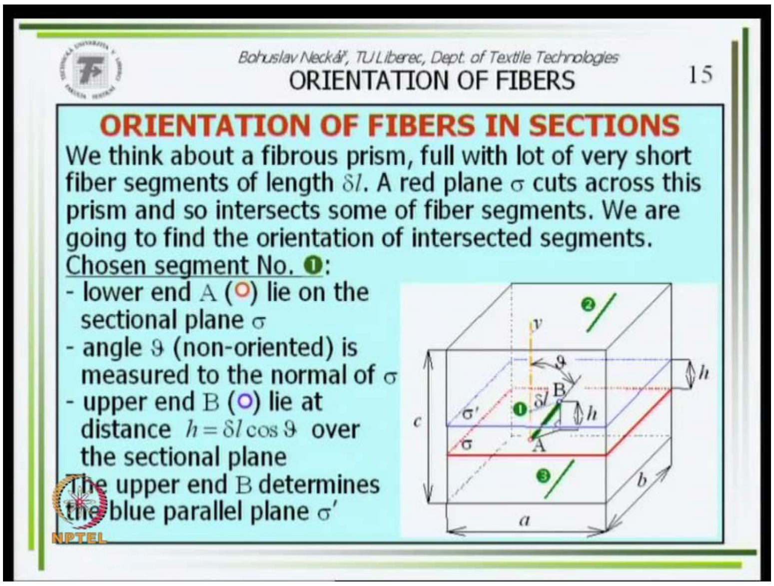

Let us think about his programmed, let us imagine some books like this here full of fibers, full of fibers. These books have the dimension a, b, c and sectional plan is direct plan sigma, this one. The fiber portion fiber segment number 1, which is shown here is a special segment, which have its its end points a, immediately in the cutting plan alpha then the second thing of such fiber segment is lying in an height h over our our sectional plan sectional plan sigma, such segment have the lines delta l, the angle theta is here and then h is delta times casino theta, it is evident for own picture got a same picture is here.

How is it? Now, let us think about a fibers having angle theta only, having angle theta no other. There are lot of fiber in side of our books, lot of fibers no no all fiber source cut it, is not it? Some are the fiber having angle theta like fiber number 2 or fiber number 3, we do not know, they are all over or under our sectional plan. The question is, which of fibers, how is the probability that fiber having angle theta will be cut it? Well, let us imagine the parallel plan to our sectional red sectional plan sigma, this blue which is lying on the distant h from sectional plan.

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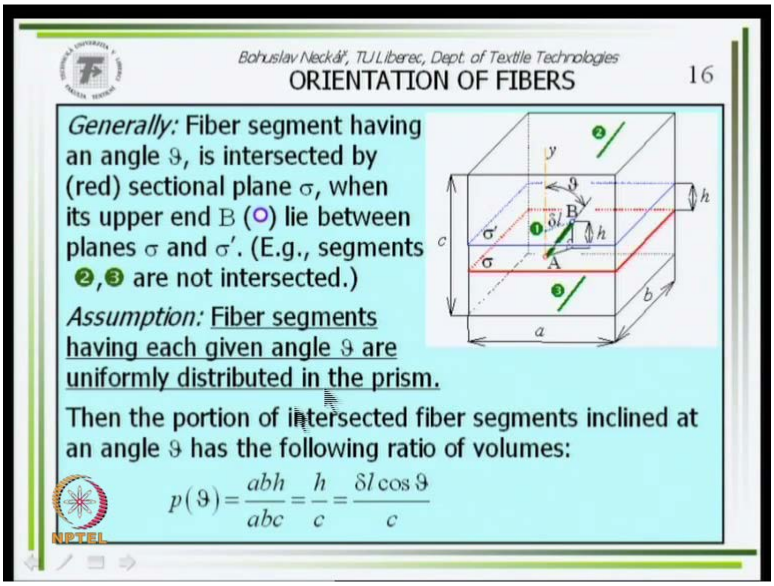

So, that point b of our fiber number 1, fibers fiber fiber segment I shall say, this is laying on our blue blue plan sigma dash. And now, how must be the position of fiber segment, when such segment shall be cut it, evidently the upper point b of such fiber segment must lay between these two plan, sectional plan, sigma and our blue plan sigma dash. I think this evident. So, how is the probability of this situation? Using so called geometrical definition of probability, we can say probability of such section is given by a ratio of 2 volumes, volume between our red and blue plans by total volume of our box of course, if fiber, if the fibers are hologram distributed in our in our books. Now in the case in which all the fiber having angle theta are laying near to this this corner only, by hologram distribution. So, the probability of section of our section of our our our fiber, segment is a b h by a b c after the arranging, we obtain this this expression.

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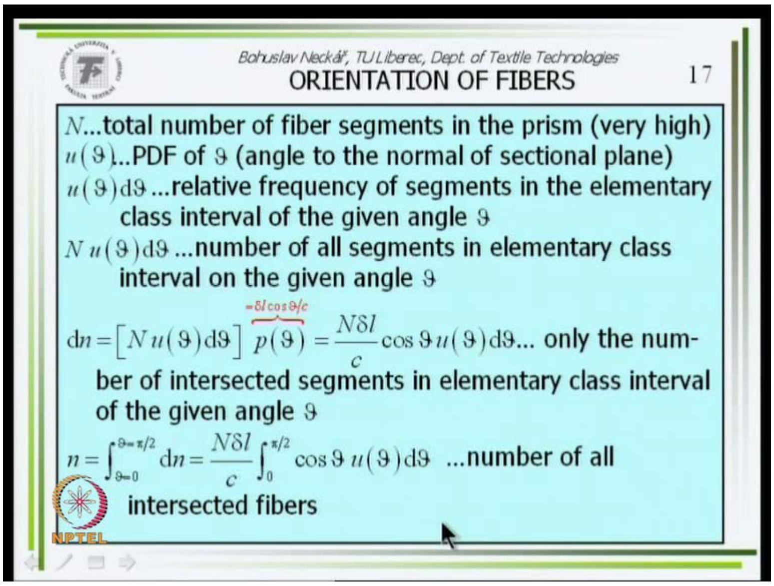

Well, now let us think about fibers having angle theta, all fibers with all angle theta together, all fibers in our books are capital n and its number is very high. Just imagine it; u theta is probability density function of angle theta. So, that u theta, t theta, this u theta t theta have some logical sense, what is it? Is a really yes, is relative frequency of segment in the elementary class, interval of the given angle theta is not it? What is it, n times u theta d theta?

A relative frequency time's total number, it is number of cases in class knows it from also from laboratory and so on. So, that n times u theta d theta is numbered of all segments elementary class interval on the given angle theta and now no all this fibers was sectioned through our sectional of plan, but also some of them; how is it the number of a cut it fibers. It is total numbers times probability, is not it? Times probability, this time this, using our equation (( )) is here. It is number of sectioned fiber segments from the group of fibers having angle theta.

But a total number of a cut a fiber segments is higher because not only our angle theta exist in our box also are there angles theta are there well. So, what is total number of cut it fiber segments, it must be integral. May be you know that, the symbol of integral is from known mathematician, which start it with symbol s, s like latent some sumac and because the s was similar to other a x y and so on. Then he use longer and longer s through today's symbol of integral.

So, integral is sum, that is some type of summation is not it? So, in three must sum our result over all directions. So, that you have obtained this here than this here, it is number of all intersected fibers valiantly.

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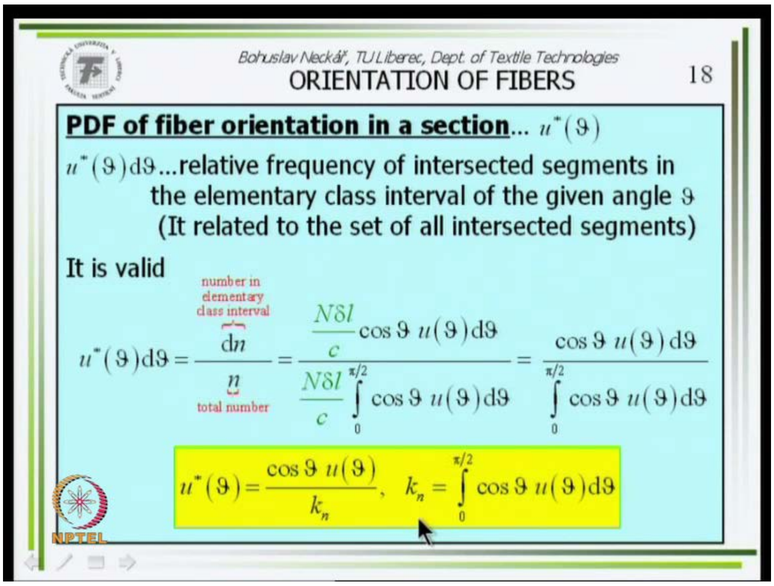

And now probability density fiber, the probability density function of of such fibers, let us call this probability density function u star theta, u star theta then u star theta d theta is relative frequency for the intersected segments in the elementary class interval of the given angle theta. Is not it? Well, (( )) u theta u star theta d theta, relative frequency of such fibers, but in other way we can say the relative frequency is in class. It is a number of fibers in class or elemental class d n to all fibers to all sectioned fibers n, using expressions after small arising obtain these equation or calling this then I mean this then under symbol k n, we obtained this formula.

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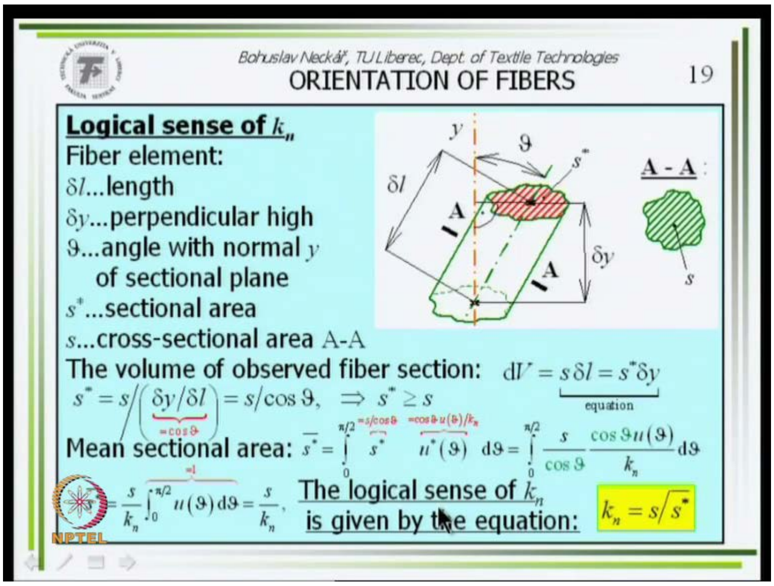

In the moment k and its something is some symbol no more, but I want to show you that this is k and f have some logical sense. Which one, let us imagine a very short fiber segment lines d l, having a section area s star section in a positive cross section perpendicular to fiber axis, which is here. And it is s from our earlier lectures here, lines of this segment is delta l, the height of such segments perpendicular to cutting line is the delta y, this fiber segment have angle theta. Well, we can write, evidently we can write the volume of such fiber segment it it can be might, it can be used to equivalent method, I can say you know it from high school. One is one is cross section area s times lengths delta l.

The second is sectional area red shaded times a height delta y, both gave, both give the same result, both here both the expression at from the from this equivalency, we obtained that s s star this area, s star is this so s by casino theta.

Well, evidently section area is larger than the cross section area in this case. Well, and now to our symbol, what it the mean sectional area star bar as a symbol for a mean, what is it now? As each means, the u star times relative frequency means probability density function time, differential quantity and integral. Using this, you obtain, we obtained this formula here. After rearranging this one and forth is here this is integral from probability density function must be equal to 1. So that, it is s by k n resulting equation is that k n is s by s star bar, it means cross sectional area of fiber by the mean divided by the mean value of sectional area in our section from all fibers.

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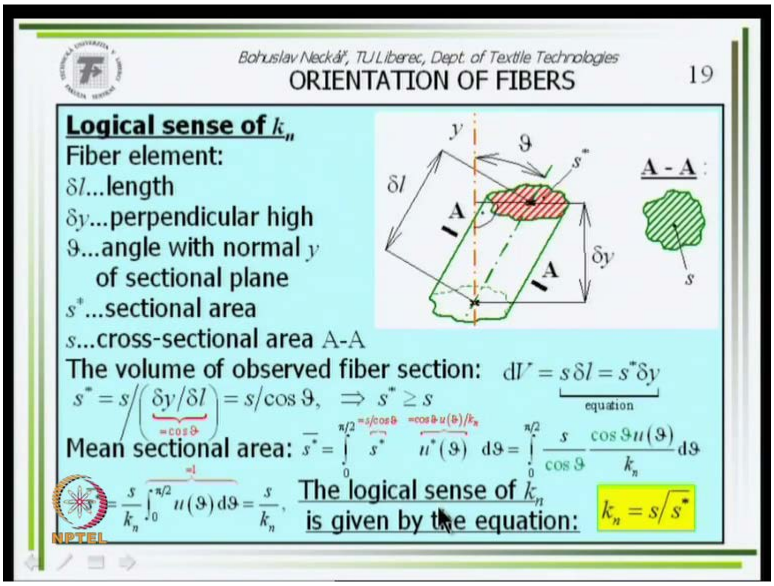

So, this is the logical sense of earlier defined k n coefficient. It to be very important for yarns, now about the applications of our model, it is the equations which we which we need it. Let us solve the problem, how is the distribution of direction, we have some, let us imagine, we have some for example, warp black black tin fibers like in our picture here. We make some section in planar case, the section is ready to section line, this thick red this thick red line is our section. We defined a y axis perpendicular to to sections. So, this is this is y axis and purple was x axis of course, exist some preferential directions by giving by blue arrow. And it is interesting for us to, there are if the probability density function of the short fiber segments, which are which which are cut it to the sectional line.

So, the distribution of this green arrows, is not it? But, this position of such green arrows, which characterized the the directions in sections of short fiber segments, which was set. Well, how is the strategy of our work? I said sectional plan is reduce now to sectional line, u theta, we derived it in our model is this here is from earlier, slight u theta u star theta be there are in this equation where k n is this here.

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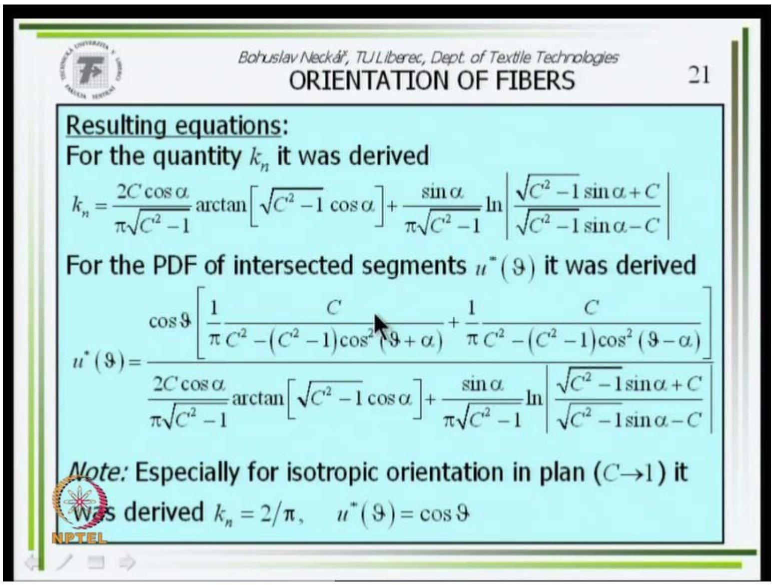

So, on the on the position of u theta here and here, we need to use this here and make this integration, which is not to not to short, formally now too much easy. And when you want to check all the way by integration and you can use this this (( )) and the result you such value of k, it is possible to do analytically and to obtain this equation for k n and then this this equation for probability density function of orientation of fiber sequence in cutting line. Now, it is that if that k n is by 2 2 by pi, if for for isotropic structure, these two are valid.

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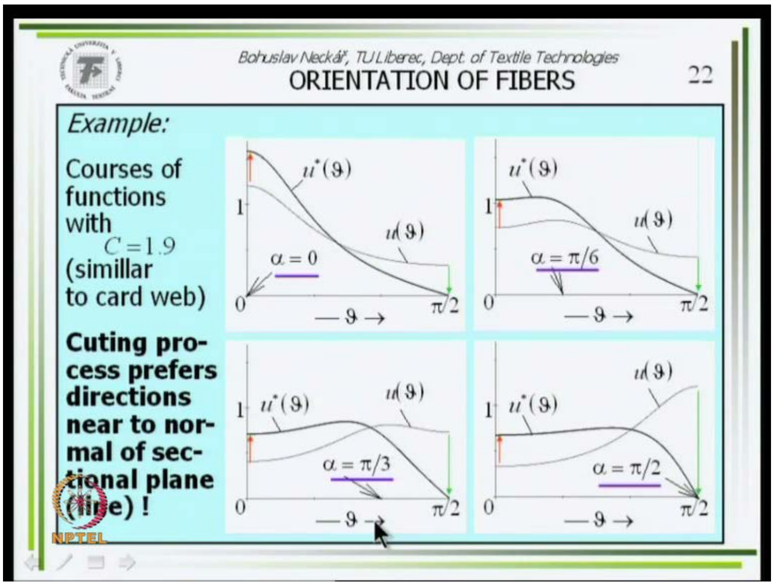

This is the graphical interpretation for our last equations, you can see that there are two different, that there are differences between probability density function of orientation of fiber segments in whole fiber assembly u theta and u star theta probability density function of distributional of fibers in cutting line. You can see that the in this four pictures, it is for alpha equals zero. So, longitudinal we cut our web perpendicular to longitudinal direction, this is for this is for pi by 6, 30 degree, 60 degree, 90 degree. In this example, C equal 1.9 was use typical value for for web card (( )), we can see that a cutting process preference directions need to normal of sectional plan perpendicular to sectional plan. You can see every time, here it is, increasing u star is increasing, in the ratio to u and the opposite side by 90 degree is decreasing is going to 0 also here by alpha equal 30 degree also here as well as here.

Well, now now let us let us speak about a number of fibers in a section, numbers of fibers. Let us imagine an something like metal plate on which you have a very thin slot, very long length C and very thin delta x delta delta h that I slot like here.

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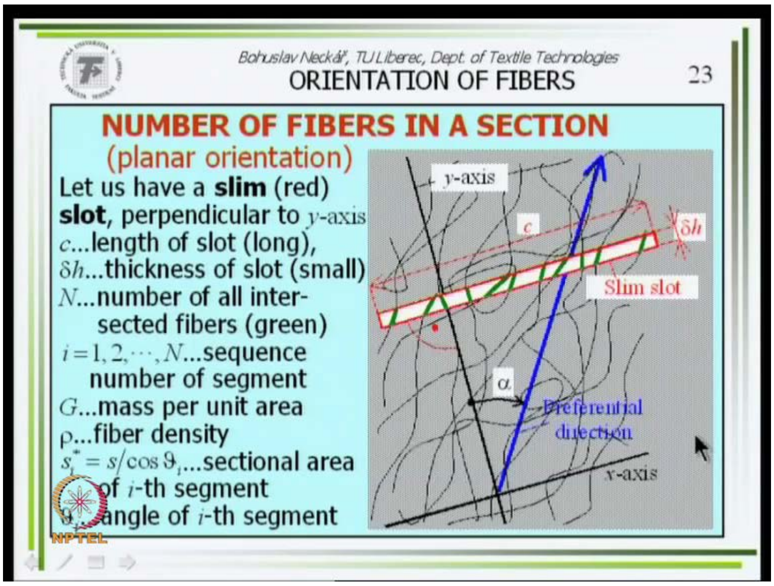

So, that you are not see another fiber, you will see only a fiber segment lying in our very short slot; see, the length of the slot delta a thickness of slot and its number of all intersected fibers, intersected because I can take use my knife and make this this section rarely is not it here? And number of all intersection fibers, i is subscribe for fibers, from 1 this green fibers segments from 1 to end, g is mass per unit area known as a area rate in industry raw fiber density s, s i star is the sectional area after one fiber for example, if i th fiber is this one or this. Here and it is as by casino of corresponding annual theta i. How is the mass of i th fiber segment i th fiber segment have it a mass cross section are a times perpendicular height, its volume times rho mass density, this is the mass per 1 fiber. Mass per fibers in our slot is some of them.

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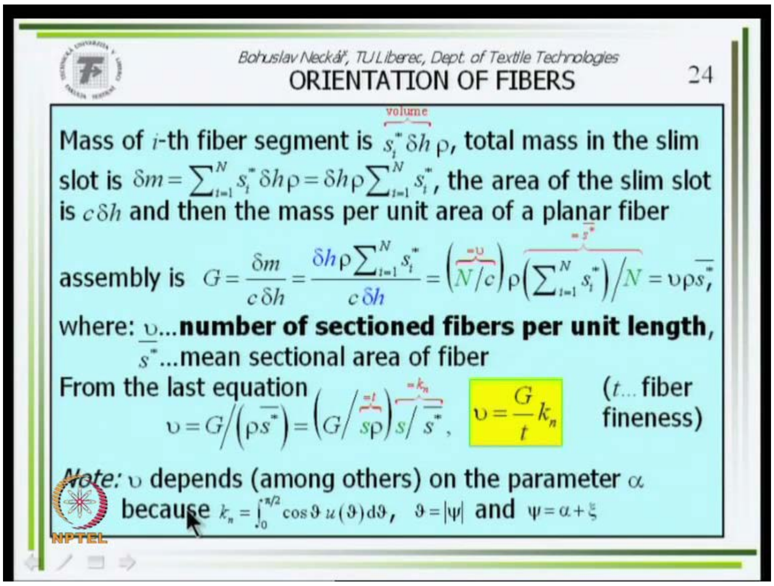

So, it is it is delta m which is some of this because delta h as well as rho constant, it is possible to write it in such form, is not it? The total area of slim slot is C times delta h and then the mass per unit area of a planar fiber assembly is g, which is mass by area by, for delta and here this is the expression C times delta h same here, delta h is here as well as in the denominator.

So, that I can write its black, this black relation, but nevertheless I can multiply and divide by n total number of fibers in our slot. It is possible, how is the sense of our the mathematical structures now? What is it n by C, number of our earlier green fibers by c, it is number of fibers per unit per unit length of our section and our slot, is not it? And total number by C rho, what is this here? Sum of all fiber sections, divided by number of fibers.

Now, it is mean value, it is s star bar is not it? Mean value of sectional of sectional plan. So, that nu number of section fibers per unit plan and s bar star means sectional area of fiber they are symbols. The nu the nu is g by rho times s star bar nu times rho times s star bar. So, this from this nu to this here. So, we can divide multiply and divide by s, we obtained this structures, but t is s times rho, it is from lecture from first lecture.

You know that from our first lecture that t is s time rho; fiber fineness is fiber cross section specific mass, this is this 1 and s by s star bar it was k n. So, that we can write the nu number of sectioned fiber per unit lengths is fiber mass per unit area by fiber fineness times the coefficient k. Then all depends of course, of alpha it means to to the angle in which we cut in relation to preferential angle, why it is it is commented area?

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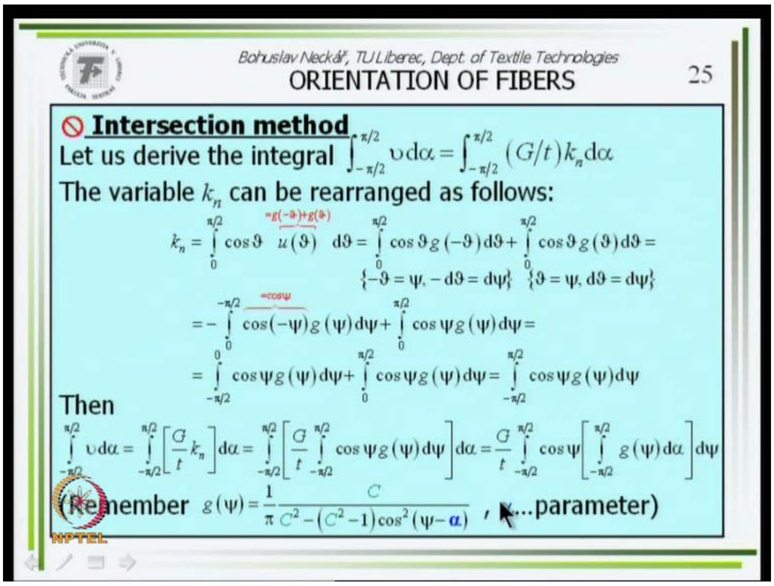

The intersection method is one method, which is possible to use for experimental evaluation of distribution of fiber directions. It it is special method, which based on our equations and it is possible to do it in laboratory also it is not too easier I must say.

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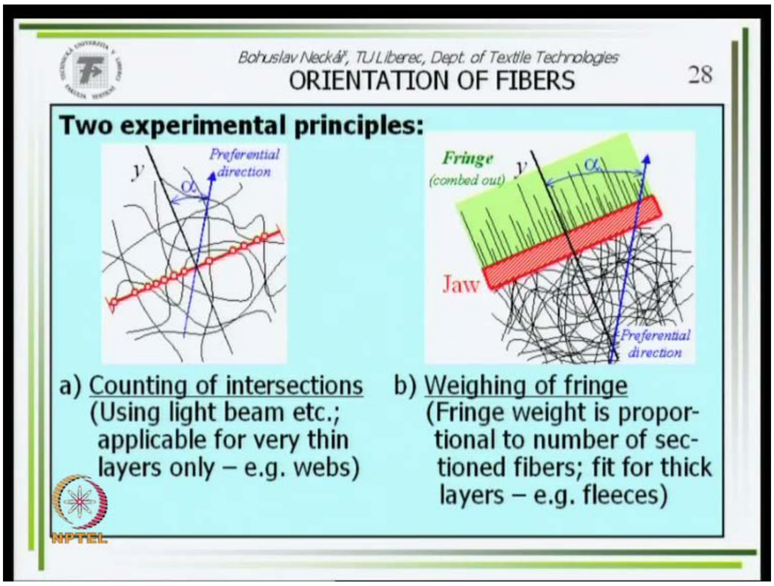

When you want, you can study this way in my lectures. We were not comment it today, I only want to say, how is the experimental background of this such method, you need to measure number of of cross sections fibers may be optically or something so. In different directions in our web, when you know, when you have the set of experimental obtained, where use for different alpha. You can from this set of experimental, from experimental data to evaluate u theta probability density function of oriental of directions of our fiber segments.

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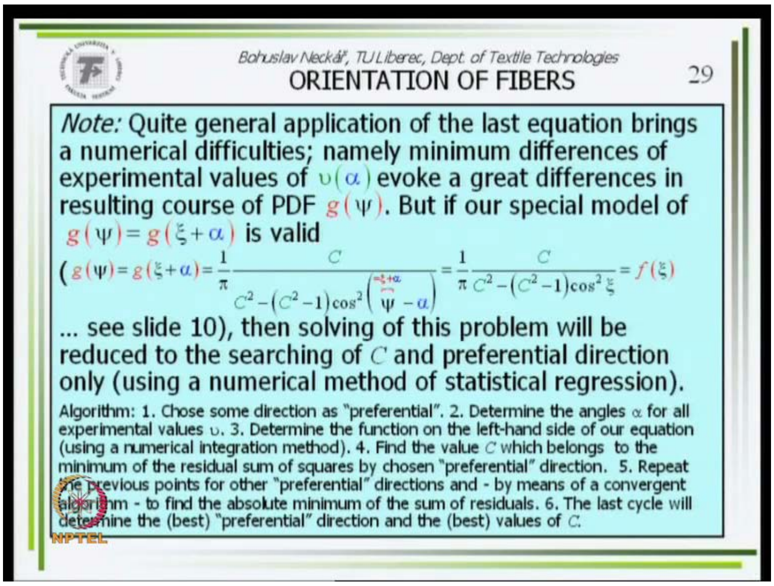

It is this here and then it is also an example, which documented correspondence our model and experimental research from another author is are well.

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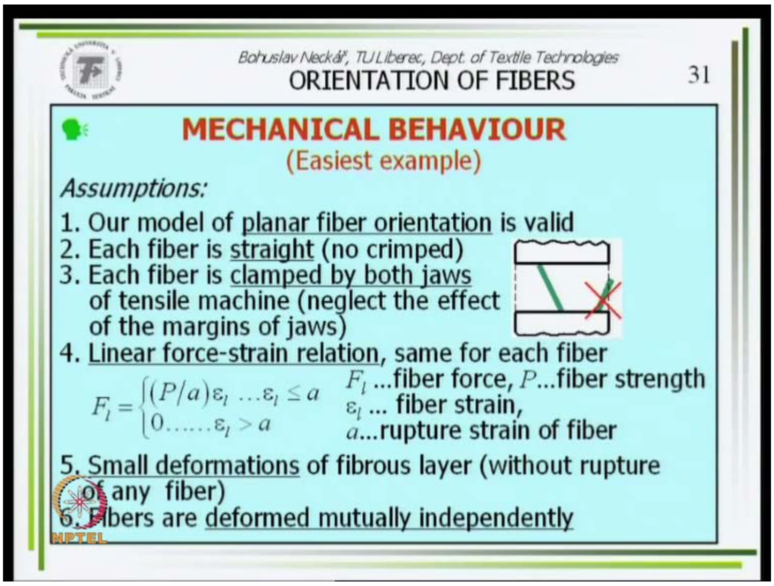

Now, stop of this lecture and in the following lecture, we will continue and where is how to apply our knowledge about fiber orientation to a mechanical behavior. So, thank you for your attention.

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