Orientation of Fibers

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Lecture No. # 10

Mechanics of Parallel Fiber Bundles

In this lecture, we will continue our derivation of some model, which give up the possibility to better calculate packing density and diameter of the yarn.

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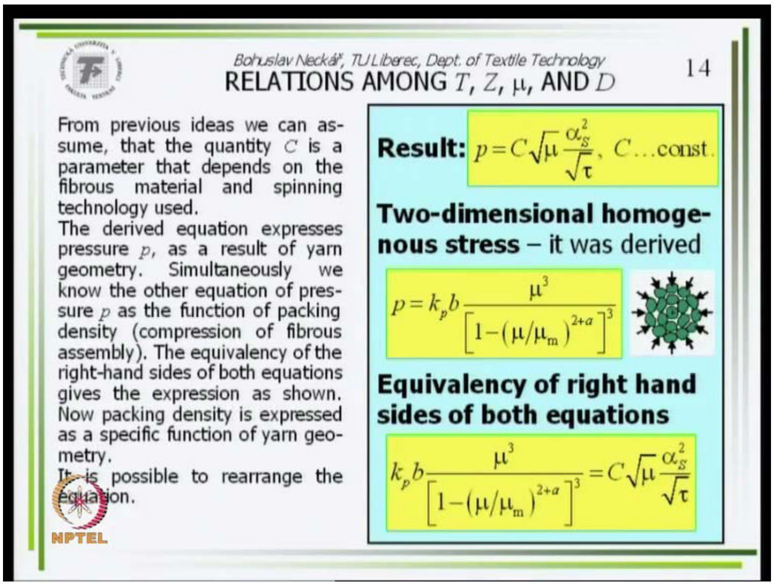

In the last lecture, we finished this equation. The pressure p which compressed fibrous material in the yarn is a function of packing… C is some constant – we assume it; mu is packing density of the yarn; alpha is aerial type of twist factor, twist coefficient; tau is relative finenesses of the yarn, so that the ratio yarn finenesses by fiber finenesses. This equation was in the last lecture, derived from geometrical relations inside of… By the pressure as a function of packing density, we know from one of earlier lecture about the compression of fibrous material. We derived the pressure – some k p times this ratio; where, mu is packing density; mu m is some maximum value of packing density; not too far from 1; a is parameter usually equal to 1.

And, we also mentioned that by solving more difficult problem of two-dimensional homogenous stress, which can be assumed like this here – comparative fiber bundle from all sides. We can obtain similar equation only some parameter b more is here (Refer Slide Time: 02:39). So, we know the pressure as a function – some parameter times this ratio based on the packing density mu. You have two equations of pressure: one equation is going out from yarn geometry; second is going out from physical model, some generalization of earlier one week model and so on. Evidently, these two right-hand sides must be equal. Therefore, based on equivalency of right-hand sides, we can write this equation.

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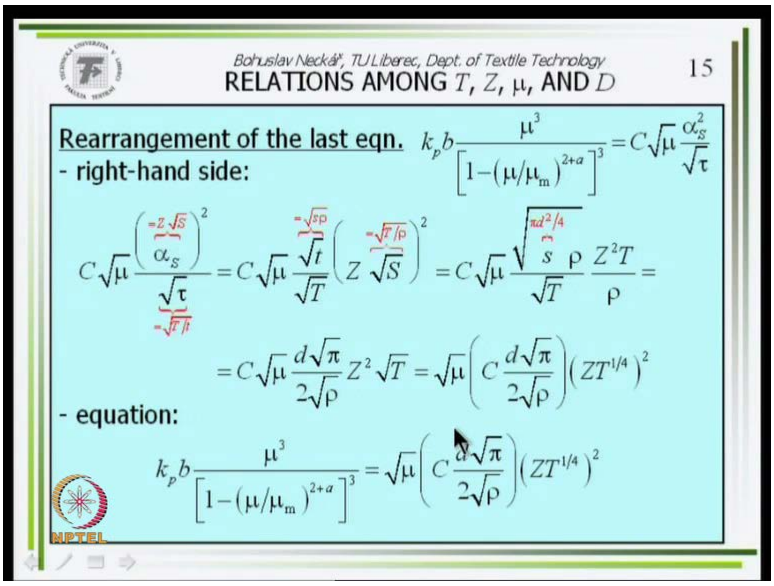

This is the same as in the last slide – this equation. Now, we make only on mechanically rearranging of such equation. For example, on the place of alpha s, we give Z times square root of S; on the place of tau, we give capital T by t, so that we obtain this equation; on the place of square root of S, we have square root of T by rho; on the place of quantity t, fiber finenesses, we have S – fiber cross section times rho. The other trivial equations, which we know usually from our lecture 1. s – fiber cross section here is pi d square by 4; where, d is fiber diameter; also, trivial n. So, we obtain right-hand side of this equation in such form. Therefore, we can write right-hand side is same is equal to… right-hand side is repeated in this (( )).

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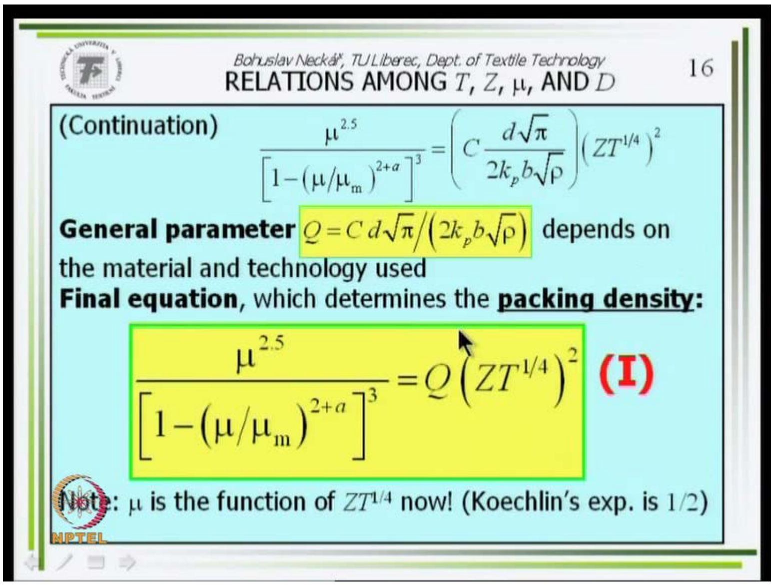

Let us continue it. We obtain this here; we can write it also in this formula; and, towards here, here is constant fiber diameter k p – b, rho, pi – different parameters characterizing the material and technology, but no twist and no fiber count, so that we can say that for given type of yarn, whole this expression represents a common parameter, Q. Then, we can write our expression in the form mu power to 2.5, because earlier was square root. Square root is also on the right hand side (Refer Slide Time: 05:45). Therefore, from mu power to 3 is now mu power to 2.5 is equal to Q, some characteristic parameter of material times Z times T power to 1 by 4 – a quarter square in Koechlin's model. On the left-hand side is packing density only as a variable; on the right-hand side is yarn count power to quarter. In opposite to Koechlin's expression, in which is square root 1 half.

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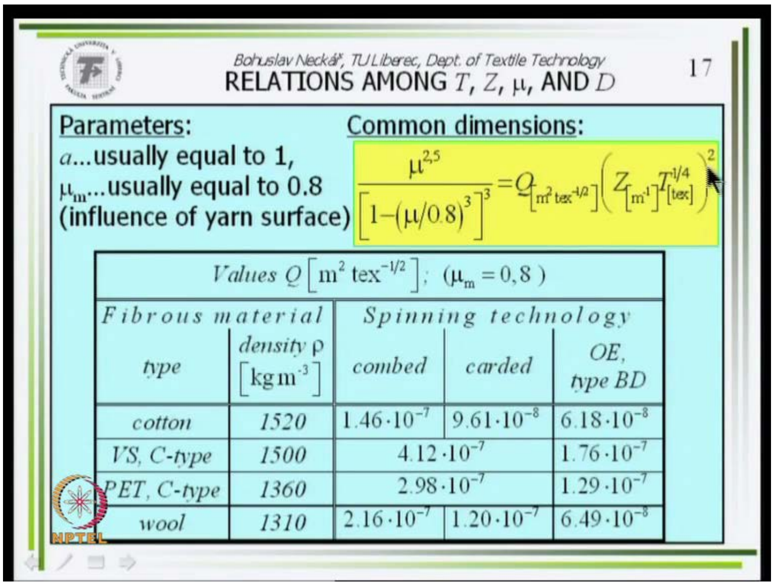

Using this equation (Refer Slide Time: 06:36) in common physical dimensions, we can iterate in such form. Now, here are the dimensions, which I can recommend for practical application. What is the value Q? It is I said is a material parameter. Based on our experiences, we can say that for different fibers and spinning technologies, the following values we can recommend to you. In more details, it is based on special type of your fibrous material, your situation in your spinning mill and so on. But, generally, you can use this here. Often say the values for wool yarn are rough, because we had not too much experimental material.

You can see, for example, for cotton combed yarn, 1.46 times 10 power to minus 7 for carded 9.61 times 10 power to minus 8 and so on. Combed and carded yarn have another, because another structure, so that they have another values; for viscose yarn, for polyester yarns, I have only one. It is not produced as a combed; it means it is here in the middle in this table, because also, the blends are used; it can be blend if it is carded as far as combed yarn. Therefore, it is in the middle here for open end, type BD; we have then this. This equation – because now, to derive this, my speech, I want to comment this equation; I know my students comment this equation; the meaning that application of this equation or this equation and its application is a little difficult. Therefore, they started to call it a horrible Neckar's equation. In check language, it is a better; and therefore, if also there is some shortening for this horrible Neckar's equation. It is horrible Neckar's equation number 1; later, it will be horrible equation number 2 also. So is the life in students' society.

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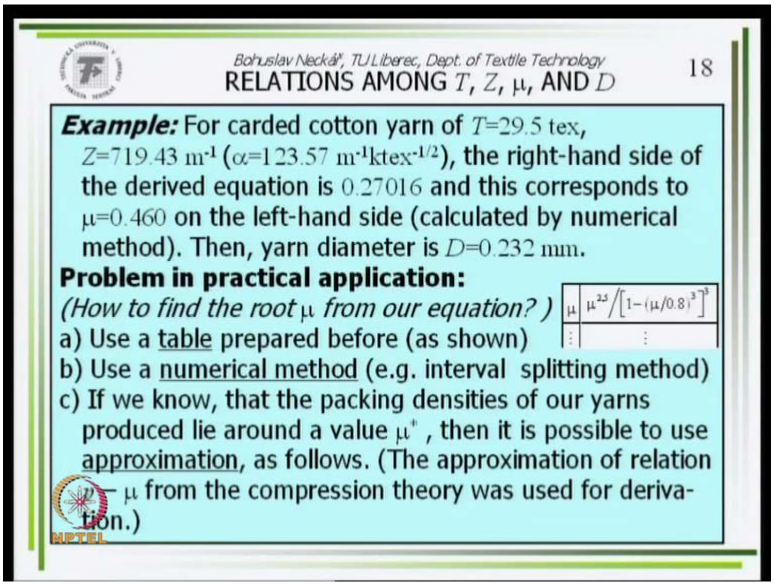

An example is here shown; when we know yarn twist, (Refer Slide Time: 09:57) yarn count and give a value of Q; for the yarn, we usually on the place of mu m, this maximum value – its limit value of packing density – we use… It is our practical experiences; its value 0.8; no 1; then, 0.8. Why? Because in each yarn also, very hard twisted yarn; on the vicinity round surface of the yarn, the packing density is smaller. Then, the mean value of packing density also in hard twisted yarn is a little smaller; it is not too near to value one. Therefore, based on our experiences, we can recommend to use on the place mu m – 0.1. And, for a, also based on our experiences, value – 1, so that we obtain then this – this expression, which we use practically. When we know yarn count, when we know yarn twist, no problem to calculate using such equation the packing density; no problem; we can calculate right hand side of our equation.

And now, we need to solve the question – which mu on the left-hand side corresponds to our right-hand side value. We have more possibilities – how to apply it in practice? You can prepare tables of left hand side of our equation. So, mu and left-hand side value (Refer Slide Time: 12:01) – like this here; prepare such table. When we know value of right-hand side, it must be equal to left-hand side; then, if we want this table, can say which of mu is corresponding to our equation. Second version is use a numerical method. For example, interval splitting method, but you need to know some basic tools from numerical mathematical to be able to (( )) and program it. Maybe you are, but lots of people are not especially in textile industry. Therefore, also, the first version for practical application is possible. When we know, the packing densities of our yarns produced lie around a value – some value mu star; what I mean? When you produce carded cotton yarns in your spinning mill, then you know that packing densities in all of your yarn will be maybe from 0.4 to 0.5. So, you can say, it will be no too far from value maybe 0.45 or 47.

Your yarn will not have packing density 0.2, for example. It is a question of… Maybe no far the products rowing, for example, so that we can say my yarn are nearer packing density and need to some value mu star, which I will choose based on my experiences. And then, it is possible to use an approximation. The approximation function to our original function – (Refer Slide Time: 14:11) this is our original function.

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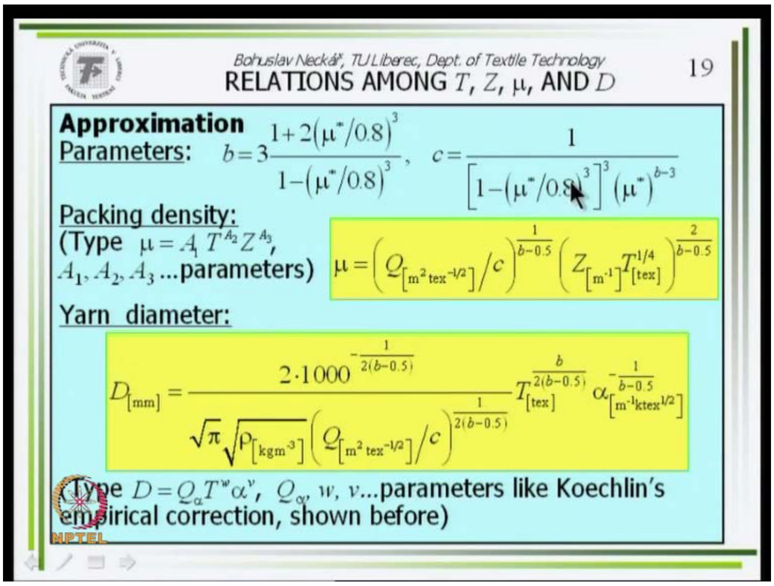

The approximation, which is valid around our packing density mu star, we obtain using following receipt. We calculate the value b; then, value c; then, when we have this here, we calculate packing density mu using such expression. It is very easy. Now, it is constant times twist times yarn count power to 1 by 4 whole power to 2 by b minus 0.5. And, yarn diameter is square root of 4 times 2 by pi mu rho; from D, we obtain this expression; practically, very easy. When we realize that it is constant times T power to something times alpha power to something, that the type, which was derived earlier as an empirical expression. Now, it is shown that it is an approximation – region of approximation of theoretical equation.

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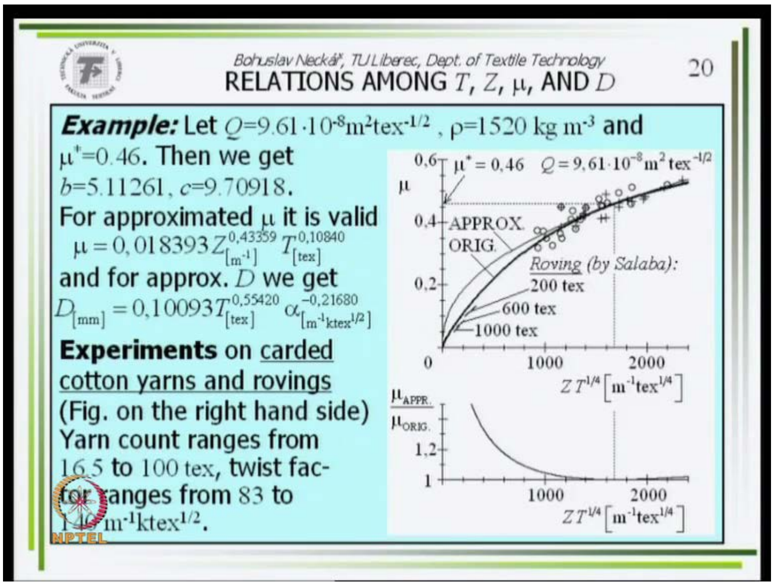

Here is an example, which for the approximation for the carded yarns, is often possible to use this equation for calculation of packing density. Then, diameter is evident. How is the relation between experimental result and our model. The first graph show the relation on the (( )) is twist times yarn count power to 1 by 4 – quarter. And, the ordinate is packing density mu. Using Q of this value, we obtained the original curve as this thick line; this is the thick line; varied our approximation function is this thin line. And, we measured lot of carded cotton yarns; diameter of these carded cotton yarns; and, we obtained from this measurement packing densities, which characterize the points here on this graph. So, you can see that the thick line follows the tendency of experimental values very well. But, because the yarns are only roughly for this value – 1000 or something under 1000; from this value, no for very small packing densities from… I do not know; this is 35 for extremely small twist in the yarn, because the yarns are not in whole region of whole area in this graph. The approximation curve, which is precise only in our point mu star – mu equals mu star is enough good for whole interval of yarns, for example.

But, it is not good for following experiment. Here the difference is very high my colleague earlier colleague in research institute mister Zalaba measured also the diameters of followings. His empirical equations of rowing diameter; the result of it are shown in these two short curves here. You can see that it is very far from our approximation, but very near to our theoretical curve. So, our theoretically derived curve is valid in acceptable comparison – this experiment in whole region from rowing to twisted yarns. The approximation equation – when you use the characteristical value mu star (( )) this. For yarn, is not possible to use for rowing and opposite. Write this relation of packing density; here is approximation by original – this ratio. You can see that in this region, it is very small difference.

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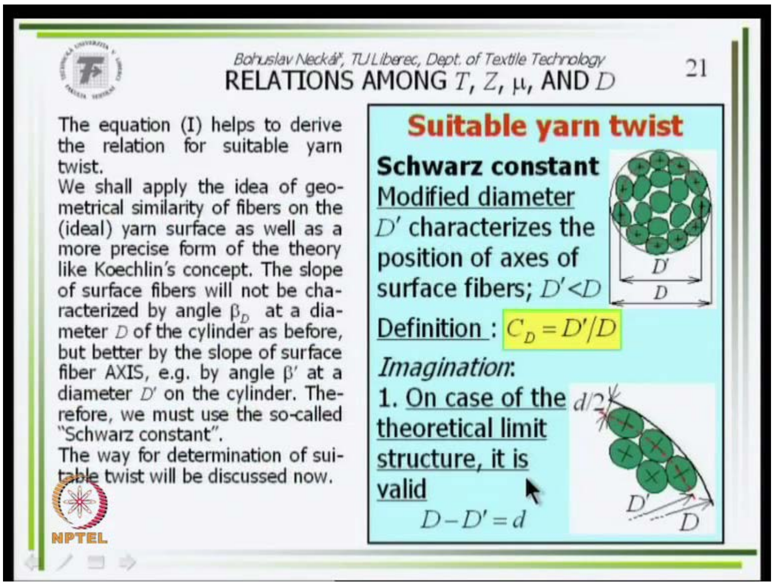

We derived an equation for calculation for the packing density inside of the yarn. The equation in which the influence of compression or generalized to one week's; one week's equation was used. The same equation is possible to rearrange and to obtain the second equation, which can help us to find the best; not the best, but, the good value of twist of the yarn twist – suitable yarn twist.

Let us start now with the rearranging of our equation to the second form, which is good for such work. Schematically, this is a yarn cross section; diameter of our idealized yarn is D. But, the axis of peripheral fibers are lying on the little smaller diameter. In the moment I call it D dash, we will find the ratio D dash by D. This ratio found mister

Schwarz at first. Therefore, it is known in later age as the Schwarz's constant. Let us imagine first step of our ideal; let us imagine – the yarn is limit packing density; then, the fibers are mutually (( )) in contact. The part of this structure seems like this here – D minus D dash. The diameter D is this; (Refer Slide Time: 22:05) diameter D dash is this here; D minus D dash in this case is equal to 1 half of fiber diameter, so that this is 1 half on the other side too.

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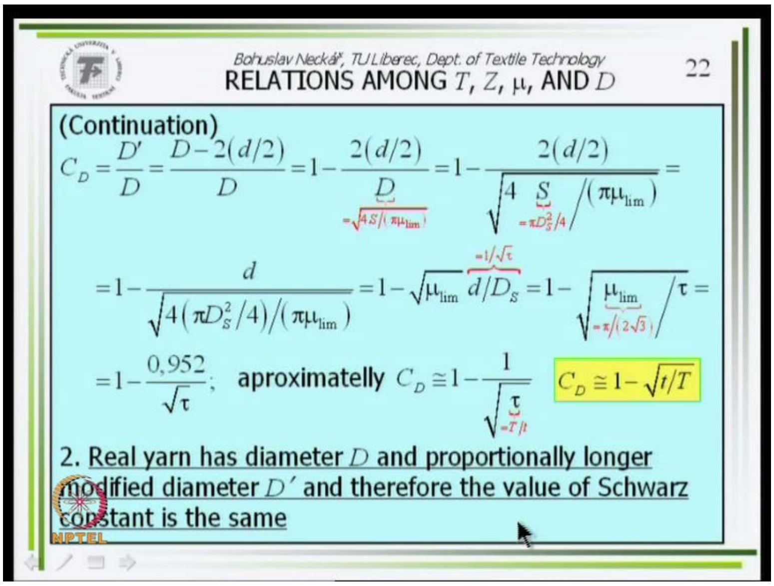

So, both together, D minus D dash is D. C D is D dash by D. So, it is this here; then, it is this here; using on the place of D, 4 times S by pi mu; mu limit with (( )) Now, we speak about the hypothetical yarn having limit value of packing density. So, then, after rearranging, on the place of S, pi D S square by 4 is rearranged. So, it is this here – d by D S – you can see from lecture 1; it is 1 by square root of tau. So, we obtain this here. And finally, because mu lima, we know it is something over 0.9; we obtained 1 minus 0.952 by square root of tau. And, because it is a little rough theory, we can (( )) too high. I do not say approximately that C D is 1 minus 1 by square root of tau.

When we use tau as T by small t, C D is 1 minus square root of small t by capital T. It was derived for yarn having limit packing density. And, when we go back to our real yarn, packing density is more. We can say that all in our dimensions can be elongated in the same percentage, so that also, in our real yarn roughly, this ratio can be same, so that this quantity C D – (Refer Slide Time: 24:22) we will use as an expression for our Schwarz's constant.

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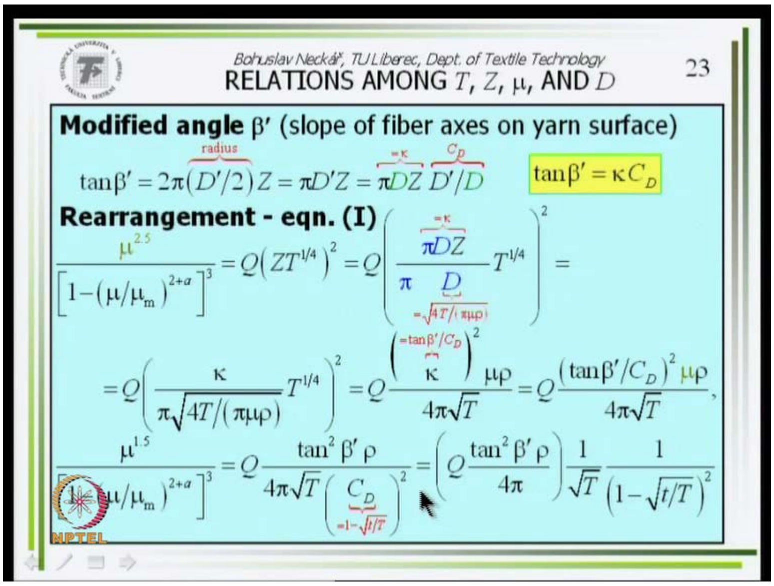

On the diameter D dash, (Refer Slide Time: 24:35) where a lying axis of our peripheral fibers – the axis of peripheral fibers have a little other angle than beta D – our earlier beta D. This angle is beta dash; and what its value tangents beta dash, is 2 pi D dash by 2 is radius; and, Z D dash after rearranging this, so that it is kappa times Schwarz's constant. Now, let us rearrange our earlier equation 1 – this equation (Refer Slide Time: 25:24). Let us rearrange. This is our earlier equation – Z times T power to a quarter; T power to a quarter is here; Z – it is pi D Z by pi by D; it is multiplied and divided by pi D; D – we know is square root of 4 T by pi mu rho. So, we obtained this expression (Refer Slide Time: 26:09). After graphically arranging this expression is black symbols – kappa from this expression is tangents beta dash by C D. So, after I obtained this here on the right hand side, left-hand side is this here (Refer Slide Time: 26:35). Using C D based on our earlier derivation, we obtained this expression. And, Q times tangent square beta dash times rho may be S by 4 pi – means this quantity.

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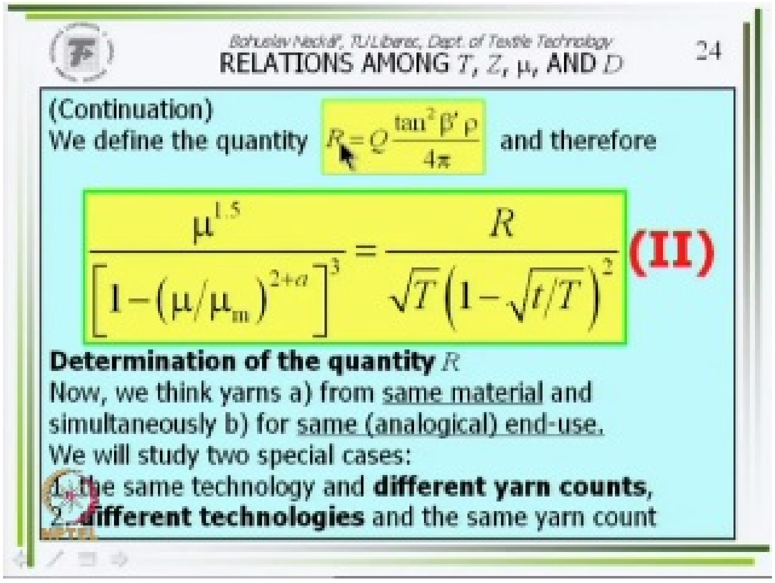

I will call under the symbol R, because here was (Refer Slide Time: 27:14) on right-hand side mu and this mu power to 2.5 for some left-hand side. Therefore, it is another exponent; it is 1.5 only. So, we obtained our equation in this form, (Refer Slide Time: 27:33) where R is here. In this moment, it is no more than only a rearrange form of our earlier equation; nothing new; only mathematically rearranged; the new will come. We can study how is the quantity R. Let us think, the yarns from the same material and simultaneously for same or analogical end-use. We will study two special cases. Case 1 – the same technology and different yarn counts; example – carded cotton technology, but one time 20 tex; one time 40 tex; one time 60.5 tex and so on. The second case, which we will study, is different technologies and the same yarn count; I said same material and similar use. For example, cotton material, cotton yarn – 20 tex; carded version, combed version, open end yarn version – different technologies.

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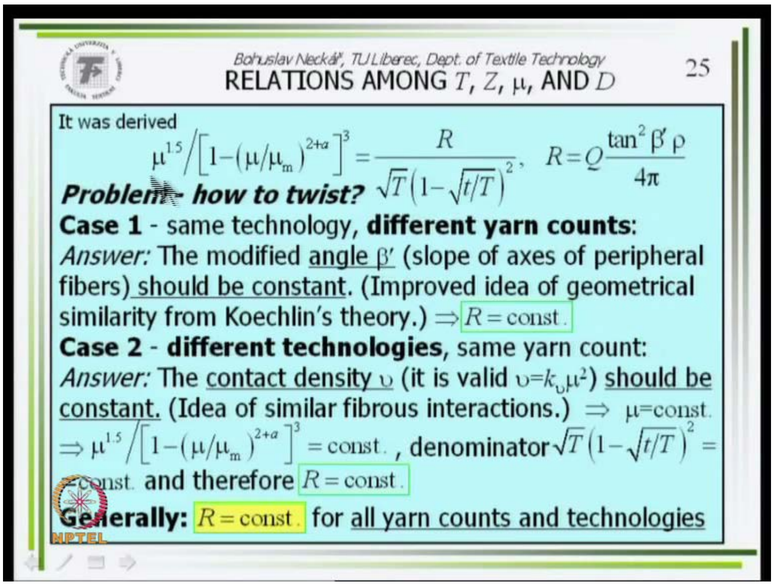

We said this is our equation – often equation. Case 1 – if the case, same technology and different yarn counts; Koechlin said that for different yarn counts is good when the yarn have a geometrical proportions, when we accept the geometrical similarity. Therefore, corresponding angles shall be same. This Koechlin's idea – 200 years old is very good also in these days; it is very good idea, but we must think now about the beta D angle on the surface of the yarn. Nevertheless, on the angles, which have the axis of peripheral fibers have angle beta dash, so that this assumption which may be very good have… We must interpret angle beta dash for the whole of this Yarn must be constant, because beta dash is constant rho; say material for pi; total constant, Q for given material is constant. So, R is constant. Resulting recommendation for such yarn from same technology and different yarn counts – R shall be constant; for suitable twisting, R must be constant.

Let us have the second case – different technologies and same yarn count. The contact density – density of contacts – number of contact per volume unit should be constant, because of the mutual influence fiber to fiber and so on. We said we use this yarn for similar analogical use and so on, because number of contacts shall be same, because number of contacts is parameter times mu square; we know it from earlier lectures. Then also, packing density shall be same. So, let us twist carded, combed as well as open-end yarn; same yarn count, so that it will have same packing density. Then, left-hand side of our equation is constant – same packing density mu. Denominator of right-hand side is we compare yarns having same yarn count; it is from same material. So, denominator is constant too. And, because this equation is valid, also R must be constant evidently. Understandable? Left-hand side – we say is good for suitable twist to have packing density constant; how will have left-hand side of our equation. Therefore, all this left-hand side is constant. Denominator, because all yarn counts are same, is constant too and because this is equivalency. Therefore, R must be constant also. So, R is constant. In both cases, which are quite different we obtained the same result; R should be constant.

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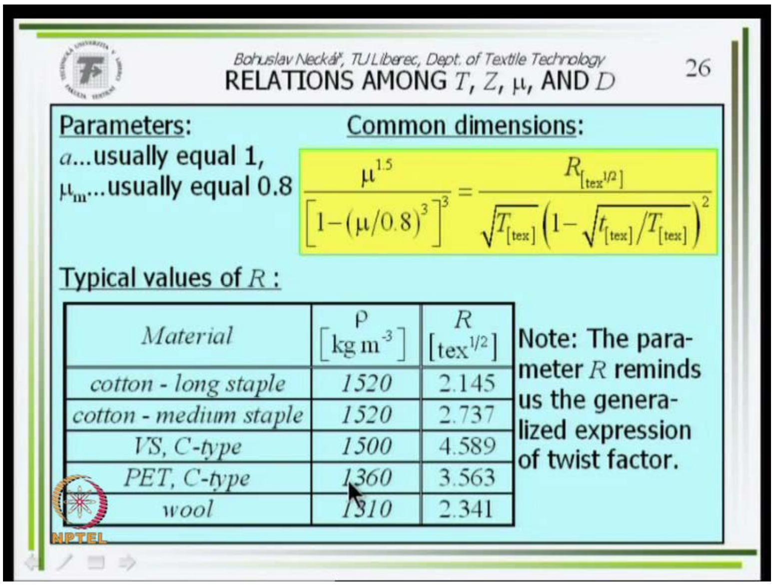

So, let us generalize this knowledge and say that for given material independently to yarn count and independently of type of technology, I speak about the technologies using twist for another group – fiber to fiber together and obtained some linear product; spinning technology used twist; no bonding silver or something. For each material and use of our yarn, the quantity R must be constant. What is good value? You can see in this table; for example, for cotton fiber - long staple – 2.1 tex power to 1 half; medium staple – 2.7 and so on.

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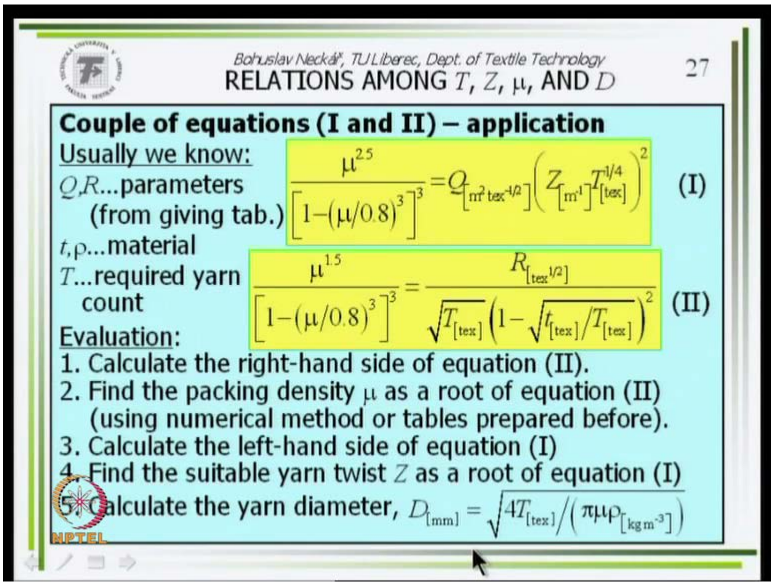

So, we have couple of equations. This is the first and this is the second. This couple called my students horrible Neckar's equations. In Czech language, [FT] In a shortening, it is [FT] S n R and to one time, one (( )), one student will come to my assistant and ask she, please she for some consultation. And, my assistant say yes and what the theme of which was… where is your problem. And, she said I want to have the consultation about [FT] Horrible (( )) So, it is very known in my university; (( )) is very known. Professor Ishteyak can comment it more. He knows it very well; yes. How is possibility how to apply these couple of equations? Usually, when Q and R… Maybe when you do not have more precise values, you can use some values from my tables. So, you know Q and R. Then, we know t and rho – fiber fineness and specific mass; that the mass density of all these materials.

And, we also know the required yarn count T. How to evaluate, how to calculate the future quantity of our yarn? That from point 1 to calculate right-hand side of equation 2; is it possible? Yes, is out problems. Point second – find the packing density mu as a root of equation 2 are worth find a mu, because the left-hand side was given the same number as our earlier calculated right-hand side using numerical method tables (( )) and so on. Then, we know mu. Point 3 – calculate the left-hand side on the equation 1. We know mu; we can calculate left-hand side value. And, point 4 – find the suitable yarn twist Z as a root of equation 1. We know all right-hand side, Q yarn fineness. So, Z is trivial to explicitly evaluate it; is not it? Now, Z square is what? This by this (Refer Slide Time:

38:18) and by Q and T power to 1 quarter times; the square is 1 one half; is not it? So, it is trivial to obtain Z from this equation. So, we obtained Z as a suitable twist of the yarn. And, calculate yarn diameter using packing density, which we derived from the equation from lesson 1. So, it is the way how to practically use this couple of expressions.

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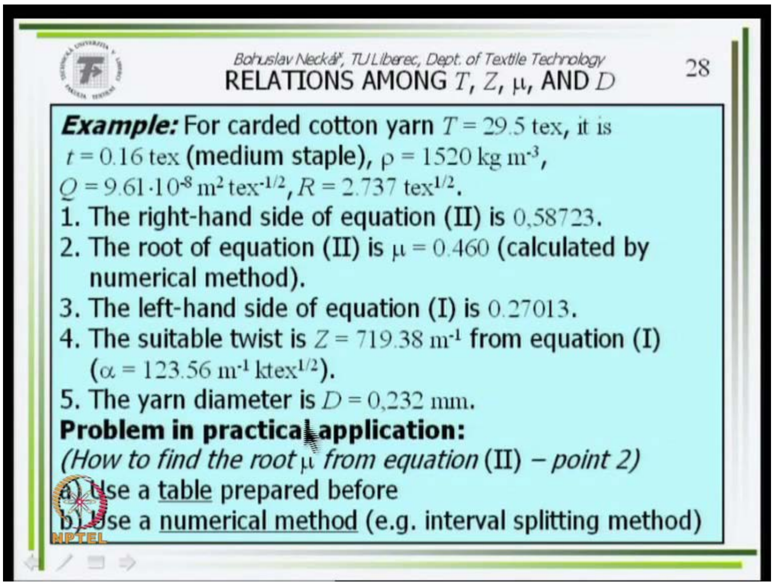

An example is here. Numerical example you can home study and verify that; I calculate it. The problem, which can you have, is the problem in practical application in evaluation. In point 2, find the packing density mu as a root of equation 2; find the mu on left-hand side, because we valid an equivalency, is known right-hand side. You can use tables; you can use numerical method.

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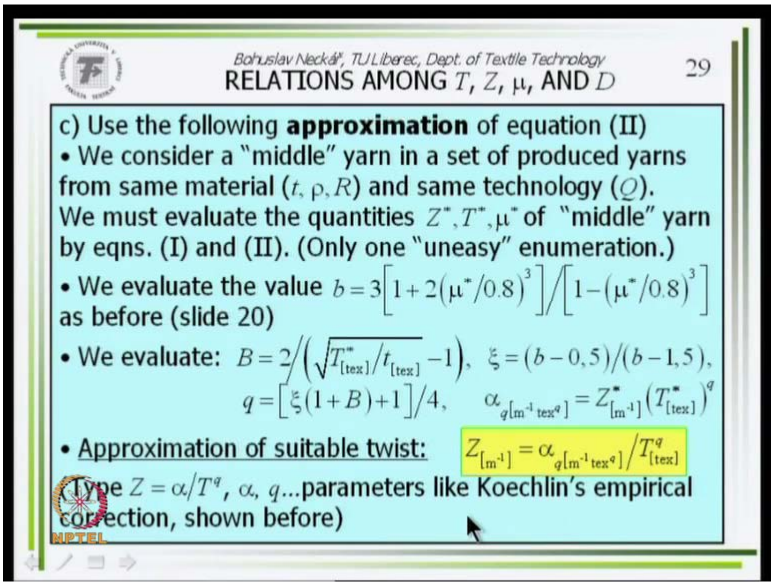

And, you can use also approximation. Similarly, we spoke about the approximation of our first equation, but it is possible to approximate also our second equation. The result is here. It is presented as a result. Both approximations are derived using following way. We said in some point, the first section – it was mu star; in some point, mu is the up most approximation. The approximation equation must give the same value than the original. And, the second assumption for approximation is or the second necessity is the first derivative of approximation function. And, the first derivative of original function in our point must be same. Based on these two equations: equivalency of value and equivalency of derivatives, we obtained the final equations, which are present; I am presenting now here. So, we can calculate the following.

We must evaluate the quantities of Z star, T star and mu star of middle yarn from equation 1 and 2. One yarn – let us say one Yarn I will calculate based on original equations include… I do not know tables or numerical method and so on; only one yarn. One typical (( )) yarn, which is in the middle of area of my activity in my spinning mill; this yarn is average or middle yarn; values have stars here (Refer Slide Time: 41:35). This is the result, which I presented. Now, we evaluate the value b, because we know mu star is possible; then, capital b also possible; Z x I also possible; all these three are helping quantities in our way. Then, q – it is very important; and, alpha q according to this equation. Having these quantities, we can formulate the approximation equation that the yarn twist is alpha q times T power to q.

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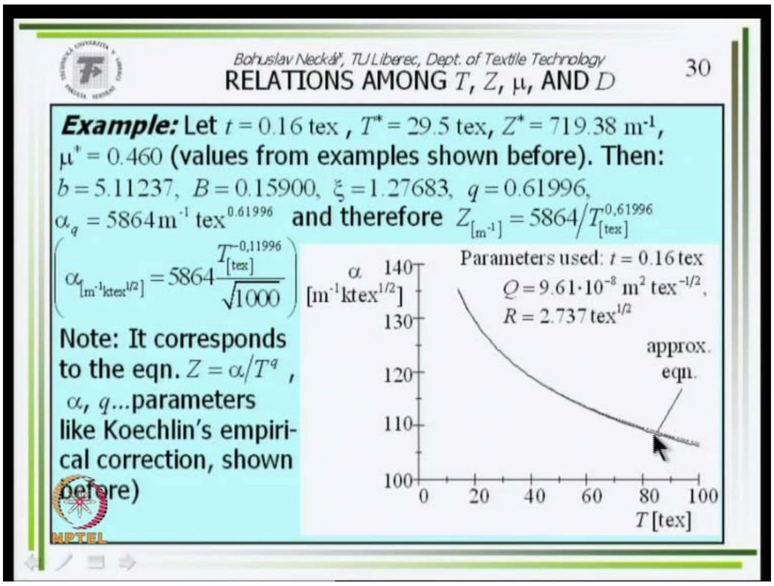

An example – numerical example shows that the correspondence between… this is numerical example and this is the graphical interpretation of two curves. One is alpha – recommended alpha as a function of yarn count; two lines – one is original theoretical equation; the second is approximation. Using that in large interval, both versions are no identical, but practically same; very good for application.

That is all for today. I presented you some model of packing density; I presented you a couple of horrible Neckar's equation. And, in short, also the way – how to apply it in practice in practical calculation? I did not explain one moment; I spoke about yarn diameter, but the question is what is it yarn diameter, because yarn diameter – where is the end radius for yarn diameter? Where is the maximum radius in the yarn? And, where started arial – this sphere of hairiness sphere? In reality, it has no strong borders; diameter is every time a little… the question of our convention. And, we need to solve it together; the modeling of external part of the yarn body, which is sphere of hairiness.

Our next lecture – we will study the models of hairiness. And, in connection with this model – we will find; we will explain in more detail the question about the yarn diameter too, but it will be in my next lecture.

Thank you very much for your attention.