Orientation of Fibers

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Lecture No. # 11

Modeling of Internal Yarn Geometry

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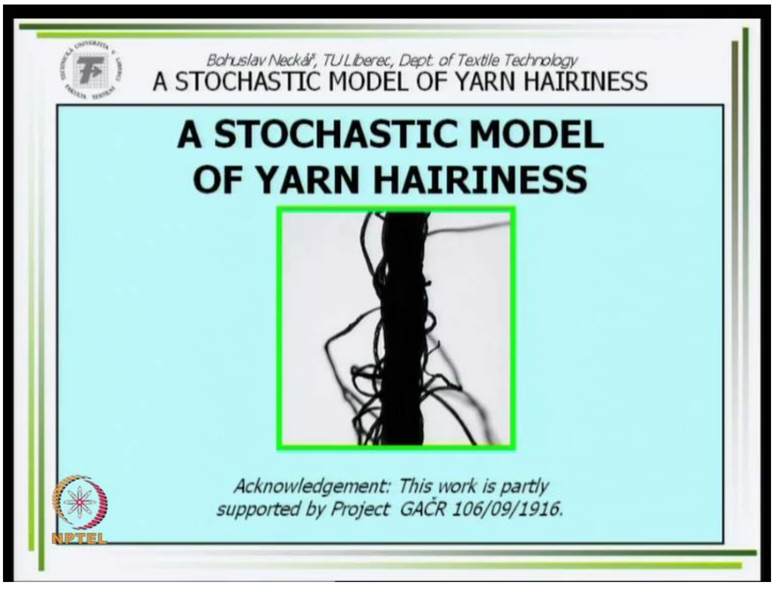

Today's theme is related to a model of yarn hairiness. You know, principally, that on the yarn surface is something, which we call as a hairiness sphere. It is this sphere in which our fibers have very probabilistic character of their shapes. Therefore, also the model of this sphere must be a stochastic model, it will be, may be little difficult for you, but I want to speak slowly and please concentrate your mind to logical operations, logical sense of the steps, which we will together to do.

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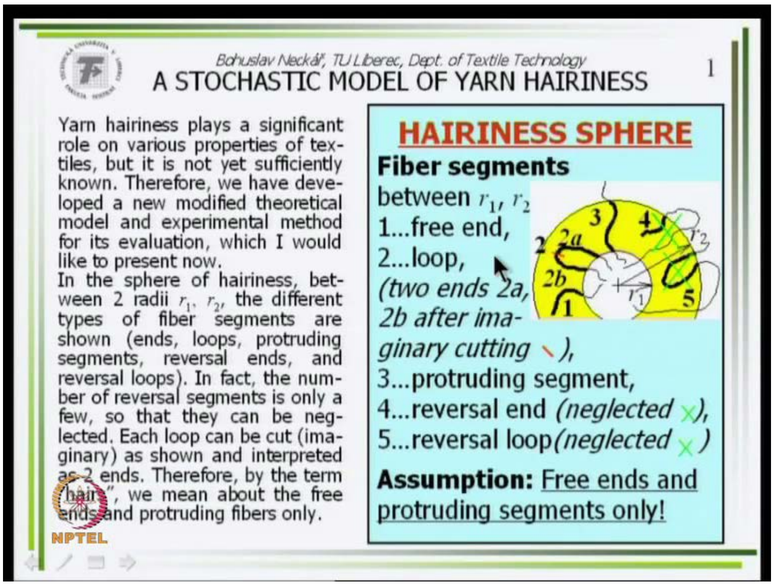

In hairiness sphere between some to radii r 1 and r 2 are different shapes of fibers, it is symbolically something in cross section. There are fibers, type 1 for example, as here, some free ends; then, fiber type 2 like some loop; also, the fibers type 3, some protruding segments, fiber segments, type 3; there are existing also fibers type 4 and 5 here, fibers type 4 are reversal ends and fibers type 5 are reversal loop.

We can assume, that the number of such, such shapes, reversal ends and reversal loops are not too high and therefore, we neglect this, I can say, atypical fiber shapes. So, we have only fiber shapes number 1, 2 and 3. The loop type 2, we can, hypothetically of course, no, really we have not knife for such fibers in the moment. Only hypothetically, we can divide based on such line, so that we obtain from 1 loop, 2 ends, 2 a and 2 b, yeah. So, then, we will solve the structure in hairiness sphere, where free, free ends and protruding segments are only.

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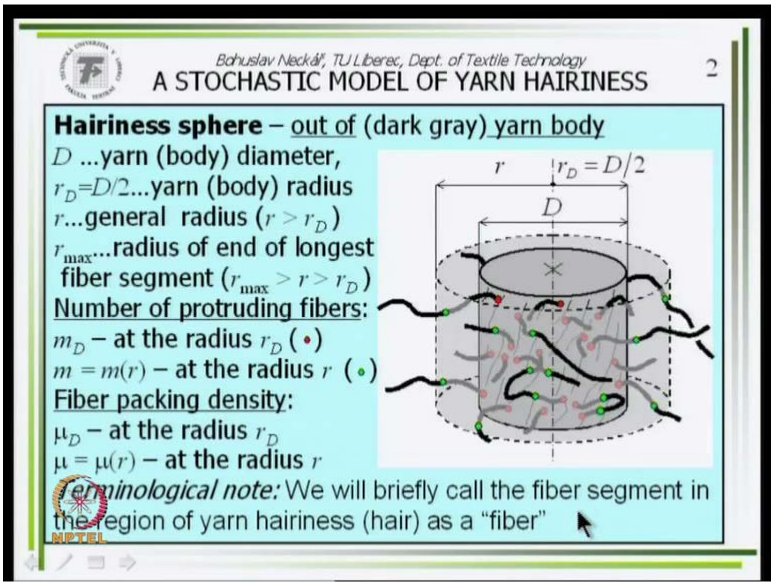

This is the scheme of some yarn and its hairiness sphere. Radius of yarn body is our known yarn diameter D, this here, yeah. We will often use also one half of this. So, yarn radius r D. All quantities, which are related to this radius r D, we will have subscript capital D on the yarn body surface on the diameter D, yeah. Well, r, some higher radius, radius r is a general radius, evidently high than r D, r D, this smallest radius for hairiness sphere and the fibers are going to the maximum of radius r max, which is radius of end of longest fiber segment, so that in the hairiness sphere in reality, the radius is going from r D through general r to the r max, yeah. A number of protruding fibers at the radius r D, it is surface on, of yarn body, compact part of yarn body, on this the number of protruding fibers is m D.

I can say number of our red points is m D, is it shown on the general radius r, number of protruding fibers is called s m and this quantity m based on radius r. Intuitively, we can say, that (( )) increasing of radius, the number of protruding fiber fibers are decreased, is not it, number of our, of our green points in our scheme, acceptable, understandable, well.

On the, in the differential layer on the radius r D on the yarn surface is some value of packing density. This packing density, we call mu D, is a packing density on this, in vicinity on this cylinder surface having radius r D, yeah. On general, in a general radius r, the corresponding fiber packing density is mu. You can imagine that this mu is smaller than mu D because some fibers are, have end between these radius r D and r.

So, packing density mu r is also some function of a radius, decreasing function of radius, is not it. Yes, terminologically, I will speak about the fibers, but in each case, I mean the segment of fibers, which is lying in hairiness sphere. So, under the fiber, in this, in this lecture, please understand, fiber hair in hairiness sphere, yeah, ok.

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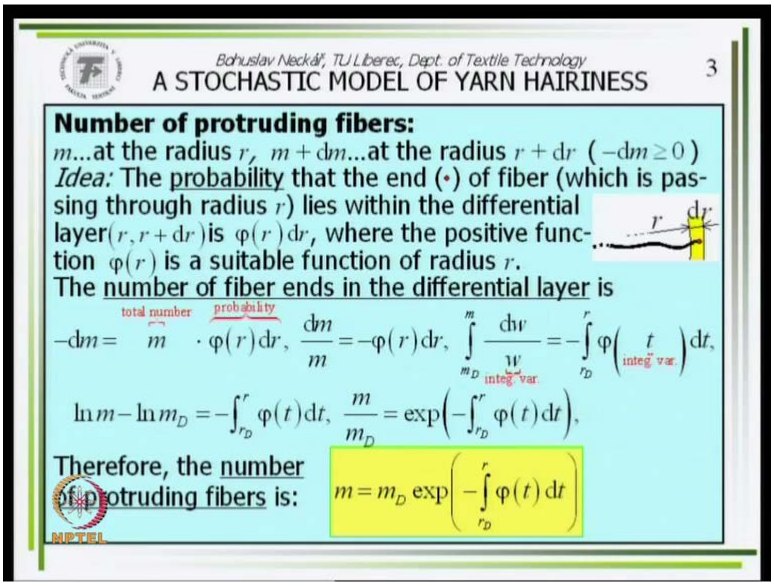

What about number of protruding fibers? It is m at the radius r, is not it? And generally, say m plus dm at the radius r plus dr because number is with radius decreasing, this dr dm must be in negative. So, minus dm is, elemental increasing is dm, this is negative. So, minus dm, it is higher than zero, is not it, well. The probability, that the end of fiber, which is passing through radius r lies within the differential layer between r and r plus dr, differential layer is known, known imaginationery, know it from, from helical model (( )), is not it and so on.

So, the probability that the end of fiber, which is passing through radius r, lies within the differential layer r. From r to r plus dr is some elemental quantity phi r dr, where the positive function phi r is a suitable function of radius r. What it means, it is shown on this, on this picture.

Let us imagine a fiber, which is protruding some general radius r, this black fiber, yeah. How is the probability, that it, its end will be immediately after its protruding, its protruding to, to, to radius r in a differential layer, from r to r plus dr. Schematically, high is the probability, that this end of this fiber is lying in this yellow, this, this red point, yeah end point is lying in this yellow differential layer. How is the probability is differentially small probability, is not it? So, it is something, which is, which related to dr and we can call this probability as phi r dr, where phi r is some real function and dr is differentially small quantity of radius.

So, probability, that the, this fiber have its end immediately after protruding the radius r is phi r dr. The number of fiber ends in the differential layer, in such differential layer is minus dm because to have the positive quantity, yeah, minus d m, what is it all? Total number of, total number of protruding fibers on radius r, we called m.

So, m times probability, no all fibers have the end in our differential layer, lot of fibers are having end in some higher radius, so total number m times probability phi r dr from this equation, it is possible to arrange to the such form and integrate it from, on left hand side from over m, from m D to m, from starting position on the radius r D to the general position on the radius r.

And the right hand side, from r D to r because do not have the same symbol for integrating variable and upper limit of our integrals, we use an other, other alphabets, but it is only integrating, name of integrating variable here. So, on the place of r, here I also use w because, sorry, on, on the place of m, I use w because m is now the upper limit of our integral and the same is here.

After integration of this both side we obtain then, logarithm m minus logarithm m D is minus integral from r D to r of the function phi t dt or m by m D is e power to, you know, that often, we often use graphically, if the exponent is too, too, too, too special, large and complicated, we do not write e an exponent, we usually write x and in brackets is this value of exponent. So, I hope, you, you all know this convention. So, m by m D is e power to minus this integral or the number of protruding fibers m on general radius r is m D, starting number of fibers, our earlier red points times e power to minus this minus such integral.

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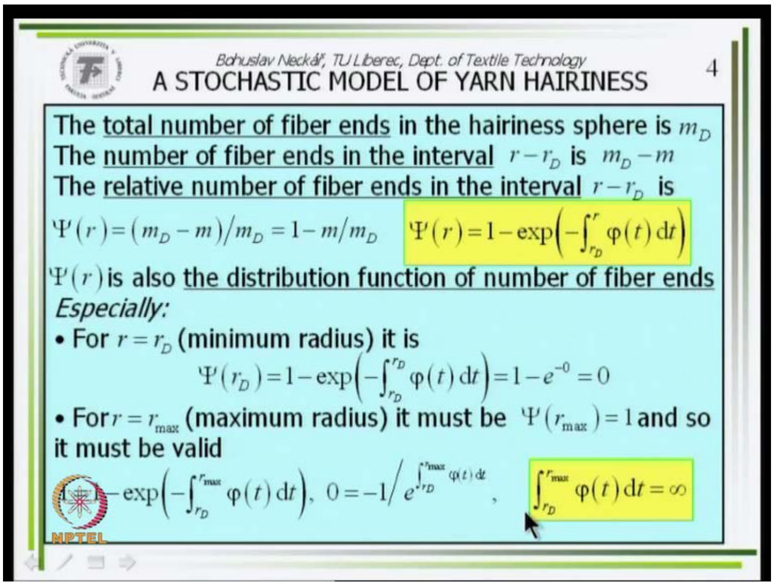

The total number of fiber ends in the hairiness sphere is m D. Now, evidently, each fibers is starting in yarn body surface must also end it in our space, yeah, so that total number of fiber ends in whole hairiness sphere is m D.

The number of fiber ends in the interval from r minus r D, it means from the minimum radius r D to our generally radius r, how many fiber ends is in this in this layer? From r D to r it must be m D minus m; m D is the maximum value on the radius r D; m is the number of total ring fibers on the radius r; the difference is, must be the, must be the fibers, which are finished earlier, then is coming the general radius r.

Well, let us call psi r as a ratio m D minus m number of fiber ends in interval from r D to r by total number, by total number of all fiber ends in hairiness sphere. So, this 1 minus m by m D using for this ratio earlier derived equation, we obtain psi r is 1 minus e power to minus this integral.

Well, when you think about the sense of this function capital psi, you must say, that psi is also the distribution function of number of fiber ends. Intuitively, on the radius r D, the value is 0, yeah, because from r D to r D is 0, fiber ends on r max in maximum possible radius, this value is 1 because all fibers had its ends before the maximum radius r max, yeah.

Well, so psi has the, the sense of the function psi is the distribution function of number of fiber ends. How this on the radius is r, formally when we write it, it is, if r is r D, the minimum radius, I must on the place of r write r D, so that I obtain this here, but integral from r D to r D is equal 0, evidently, yeah, so that it is this one minus 1 e 0.

Well, it corresponds to our intuitive theory. This function is equal 0 for r equal r D; for r equal r max, it must be equal 1. From point of view of logic, as far as formally we know, what is it the distribution function? It must be equal 1 and so it must be valid, that this one, one is now, it is 1; one is 1 minus e power to this integral, this integral, so is from r D to r max. So, that 1 is on the left hand side, 1 is on the right hand side, we can write 0 because 1 minus 1, yeah, is equal 0; 0 is minus 1 by e power to this integral. And what is going out, that this integral must limited to infinity because this ratio be 0, we will need it later.

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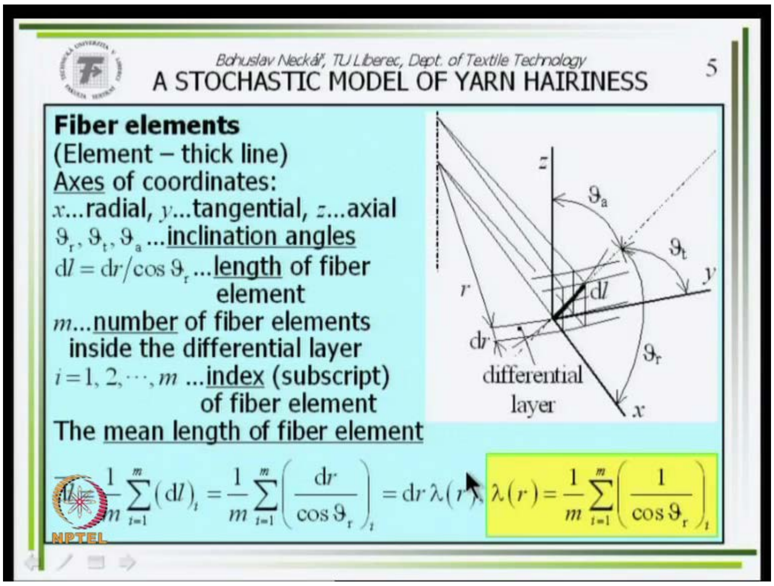

Well, (( )) we must prepare set of general equations, it will be our 1st part of our activity, then we will introduce is some assumptions and we were more precise our general equations. Now, we are in the 1st part of our activity, we formulate general equations. Let us imagine one element, elemental part, one element of fiber in hairiness sphere, which is on the radius r and this protruding some elemental layer thickness dr, yeah, imaginable, ok. This is some fiber element length dl, we can speak about the local Cartesian coordinates x, which is radial y, which is tangential and z, which is axial direction to our, in relation to our fiber element. Well, this fiber element have 3 angles to our local axis, to the axis s, to, sorry, to the axis x, it is the angle theta r; to the axis y, it is the angle theta t; tangents, theta direction and to the axis z to the axial direction, it is angle, tangent a.

Well, length of the fiber, of this fiber element is dr by cosine theta r as it is evident from this scheme. m was number of fiber elements inside the differential layer, inside this differential layer. These elements we can have some numbers 1, 2, 3, 4, generally subscript I, which is 1, 2 and so on to the last number m because m fiber elements is protruding our, our layer, it will be index subscript. How is the mean length of the fiber element?

Now, we must sum all fiber elements from 1st to the last, from 1 to m and divide by m as each, as each mean values. So, this is mean lengths of fiber element. We can also write it in this form, why not, because dl is this here and also is possible to write it formally in the, in the expression dr times lambda r, where lambda r is this expression, is the same. What is sense of lambda r? It is a mean value of all reciproc values of cosine theta from all fibers in our differential layer. Well, and it need not be same in each radius. Therefore, generally, this, this lambda is a function of radius, in another radius, such value is, have another, is another quantity.

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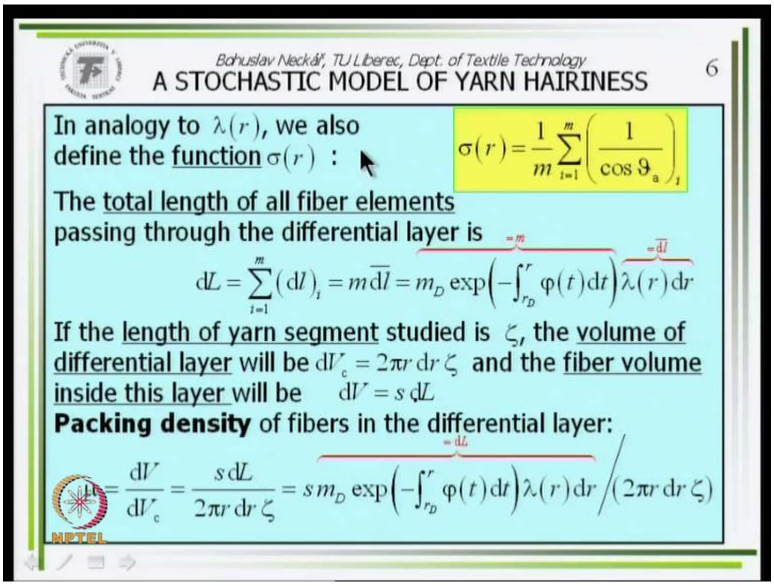

Well, let us, similar, similarly define also another function sigma r. It is similar than earlier, derived earlier, postulate equation, but here is this angle theta a. So, it is arithmetical mean from all reciproc values of angles theta a, over all fibers in our differential layer.

The total length of all fiber elements in our layer, what is it? d capital L, yeah, well it is sum of all all portions, which are elemental portions, which are lying in our differential layer, is it all imaginable. So, it is also number of such protruding elements times mean lengths per 1 element, is not it and this is based on earlier equation, and is this here and this is lambda r dr, yeah.

So, if the lengths of yarns segment is called zeta, it means, let us imagine, we analyze some yarn portion, yarn segment lengths zeta, lengths of our yarn is zeta, zeta. The volume of differential layer, what is the volume of differential layer? Our known differential annulus times lengths, differential annulus have the the area 2 pi r dr times 1, I know zeta lengths of yarn is zeta.

So, 2 pi r dr times lengths of the yarn zeta, this is, this is the, the volume of differential layer, whole volume, total volume of this differential layer. The fiber volume inside this layer is total lengths of fiber elements, which are protruding our differential layer d capital L, we derived it, yeah, times fiber cross-section s. So, dV is s times dL.

And using known definition, what is it, fiber packing density? It is every times fiber volume by total volume there. Now, it is fiber volume in our differential layer by total volume in our differential layer. So, we obtain, sL by 2 pi r dr zeta; on the place of dV use this equation and then, after rearranging we obtain this expression for packing density in hairiness sphere on general radius r, yeah.

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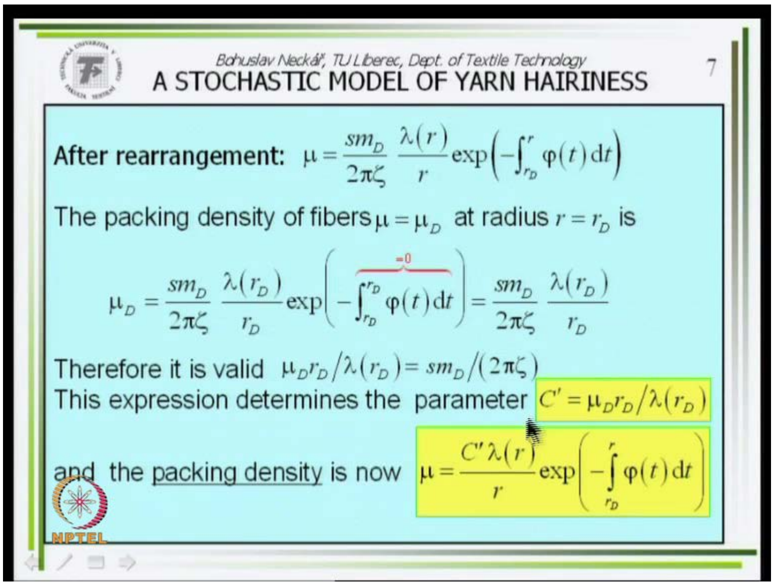

The packing density of fibers on the radius r equal r D is the starting packing density mu D, starting, I mean on the surface of yarn body. How is this packing density? Now, the, on the place of r here, here and here, we need to give quantity r D, it is here, this is 0, so that we obtain mu d is equal to this expression, or we can write from mu D is equal to this also, that mu D times r D by lambda r D is s times m D by 2 pi z, it is already arranged.

The ratio mu D times r D by lambda r D, mu D r D by lambda r D, let us call as some parameter C dash, yeah. It is the function of radius; it is some parameter of this hairiness sphere of this yarn. Then, the packing density can be expressed as shown, can be expressed through this equation, no more.

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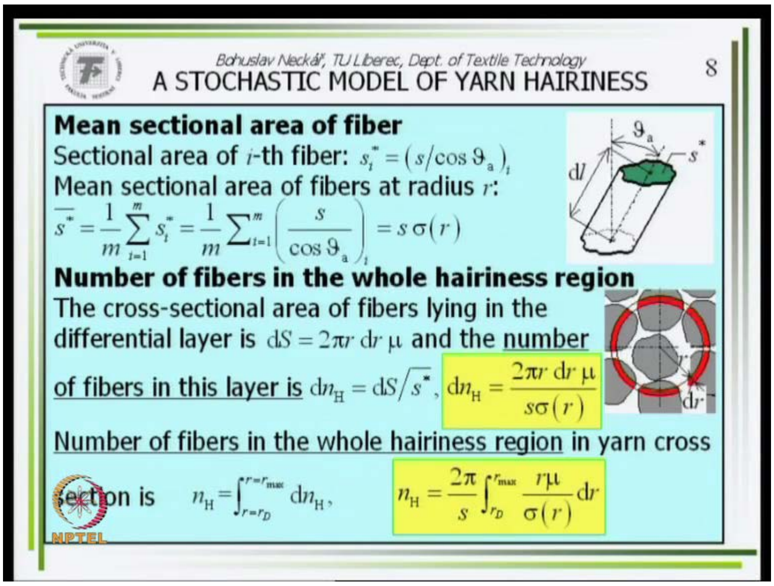

Well, what the mean section area of i-th fiber? i-th fiber, general fiber had some green section area s star, which is no first time in set of our lecture. We know it, which is cross-sectional area of fiber s by cosine of corresponding angle. In this case, it is an angle theta a, is it? You, yeah, this is the theta a, yeah, we are right.

So, it is cosine theta a, means sectional area of fibers at radius r is which, yeah, we must make the, we must make the, the, the, the mean value or average value, which is some of all sectional, this green sectional areas by number. How many other m, so that it is this? Here, we can write it because s star is s, so we obtain this here and because we earlier postulate the, the function sigma, sigma r we can write it also in the form s times sigma r.

Well, how is the fiber, number of fibers, in whole hairiness region? The cross-sectional area of fibers lying in the differential layer, we have here some differential layer in a cross-section, yeah. dS, what was dS? You know it from helical model, for example, total area 2 pi r dr times packing density times mu. So, is this, this is the red area in some general, general differential layer, yeah, this red area here.

Number of fibers in such layer is dS by s star bar mean value of mean section area of fiber on the radius r, is not it, yeah. Similar, similar, logic was in helical model when we derived number of fibers in the yarn based on idea of ideal helical model, similar logic. So, the number of fibers in this layer is d n H which is fiber area, this red area d s by a star bar, mean value per 1 fiber section.

So, d n is using equation derived earlier, d n H is given by such expression, i was only on the place of a star bar, this here and on the place of dS, where it is, dS, dS, dS is here, 2 pi r D r mu, yeah. How is the number of fibers in the whole hairiness region in yarn cross-section? In one differential layer, in cross-section, we see d n H fibers, but axial points of our fibers.

How is the total number of fibers in yarn cross-section in the sphere of hairiness? Now, it is some, it some, is not it, sum of all this numbers over all differential layers starting on the radius r D to the maximum possible radius, which is r max, it is clear. I think, I said you earlier, how, how was the history of symbol of integral.

So, from that time, you know, William Leibniz, very known mathematician knew, that the give together, sum together, infinity, number of infinity small parts is some special type of summation. So, therefore, (( )) used symbol s, capital S as a symbol of such type of summation and because was some mistakes, which alphabet S and this operator S, then he make this S longer, longer, longer, to today's integral, you know this history.

So, what is it the, how we must, to sum the number of fibers over all differential layers using integration, we must make integral from d n H, from r equal r d to r max, using equation derived earlier, this here, we obtain n H and such expression, yes.

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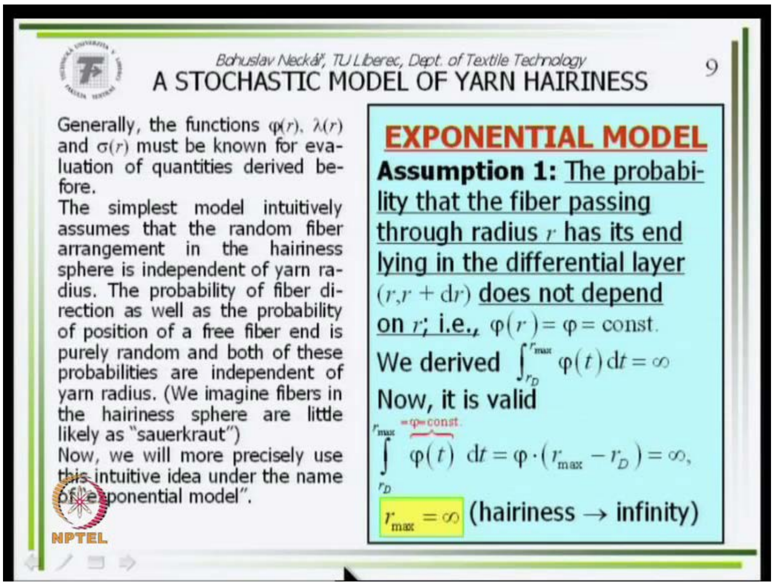

And we have, may be all necessary equations for our, for solving of our problem, but with the problem, there is some practical side of our model, in our model, we are 3 unknown functions.

One is the function phi r, which related to the probability of fiber end on radius r and then, the kappa of functions sigma r and, and the 2nd was, was it, one moment, sigma r and lambda r, yeah, lambda r, this is the function of radius. Then, sigma r, this is also the function of radius and then, phi r; these 3 functions we do not know.

So, let us accept some assumptions for simplification of whole our problem. Assumption 1, let us assume, that the probability, that the fiber passing through radius r has its end lying in the differential layer, for r to r plus dr does not depend on radius. In each radius, if the fiber is protruding this radius, the fiber have same chance to be finished independently to value of radius; do we imagine this assumption, yes.

So, then, the probability, our probability is independent radius, then the general, say function phi is based on this assumption constant, the function phi r is a constant, phi common constant for each radii, each radii we derived, that integral for r d to, from r d to r max phi t dt must be equal infinity or must limited to infinity, be more precise in formulation. Now, what is it? Integral of constant phi dt is phi times r max minus rd and it shall be equal infinity, must be equal infinity phi because it defines some probability and the fibers must finished phi, must be some real quantity, higher than 0.

Then, then, r max minus r D must be infinity r D is not, is given value, so that r max must be equal infinity. In such model when we accept this assumption, there hairiness sphere, we have smaller and smaller number of protruding fibers, but it is going to the infinity, the maximum radius is going to the infinity. It is not real, but no too difficult because the longest fibers are only a few and then, we will, in each case (( )) in long distance from yarn body. So, it is possible to accept.

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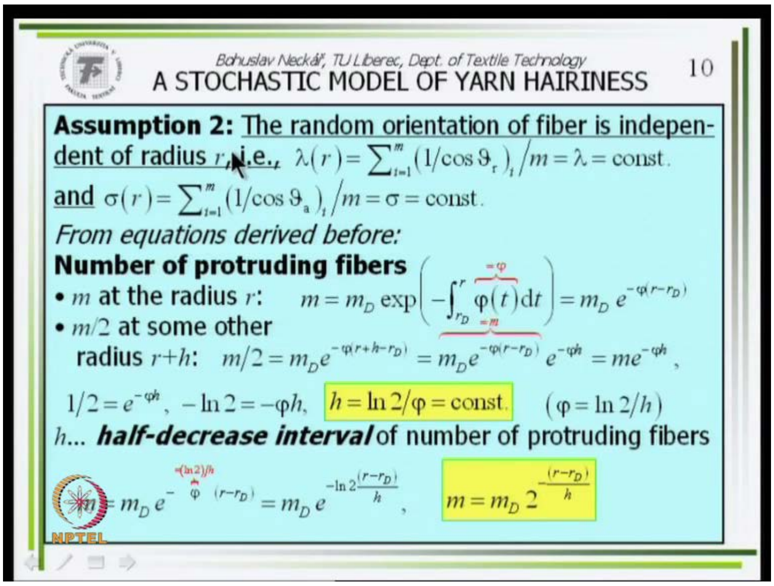

Assumption 2, the random orientation of fiber is independent of radius. Our fibers or fiber segments theoretically, infinitesimally small fiber portions in this or that radius, have very difficult directions, difficult to, to, to, to, to formulate some probability density function of orientation of fibers in a, in a sphere of hairiness. Each fiber have another direction, it exists some distribution of such directions.

And we here assume that the distribution of fiber directions is same in each radius. In small radius, I speak about the hairiness sphere only, in small radius, relatively small radius exist some distribution of directions on higher value of our radius; distribution of directions have the same character, is same, yeah. This is logical sense of our assumption 2.

If it is so, then the quantities lambda r, the functions lambda r and sigma r are not functions, they are constants, so that this is lambda, it is constant and this is also sigma, is constant, yeah. It is our 2nd assumption, is understandable, the logically and then, that the distribution of fiber orientation is independent to radii. Now, let us, based on such assumptions, of this couple of assumptions, let us rearrange our earlier general equations to new, new form, number of protruding fibers m, we derived, m is this expression, yeah, now phi is constant.

So, it is going before and after integrating, which is trivial. We obtain, that m is m d times e power to minus phi r minus r d. Well, well, on some general radius r and number of protruding fibers is m in an, in, in another higher radius, number of, of protruding fibers is only one-half of earlier quantity m, yeah. On radius r number of protruding fibers is m; in another radius, higher radius, number of protruding fibers is one-half of this earlier quantity m, yeah; this longer, higher radius is some radius r plus h. So, h is some distance in the radius.

So, on radius r is m, protruding fibers on the radius r plus h, some higher. I do not know in the moment, what is the age? Number of protruding fibers is m by 2, so it is valid, this equation is valid. And now, on the radius r plus h, the number of protruding fibers is m by 2 left hand side and it is based on this general equation. Here, it is m D times e, but on the place of r, I must give r plus h, here mu radius, higher value of radius.

Well, after rearranging, I obtain this here, but this part, it is earlier m. So, I have the equation m by 2 is m times e power to something, minus phi h. So, we obtain one-half is e power to minus phi h; after rearranging we obtain, that h is logarithm 2 by phi and because phi is constant, h is also constant, yeah, independent to radius, or we will use also, that the phi is logarithm 2 by h. So, it is the sense of h.

Every times, when I increase radius value plus h, radius plus h, I obtain a newer, new bigger cylinder, where number of protruding fibers is one-half. Therefore, we can call this quantity h as a half-decrease interval of fiber, of protruding fiber, of number of protruding fibers. Every times, when I jump from some radius r to the value r plus h, number of protruding fibers decreased to one-half.

Similar situation is in nuclear physics, you know, the half time or how it is in the English, yeah, half-decrease interval, using on the place of phi logarithm 2 by h, after small rearranging we obtain, that the number of protruding fibers is starting number m d times 2 power to r minus d by h, well.

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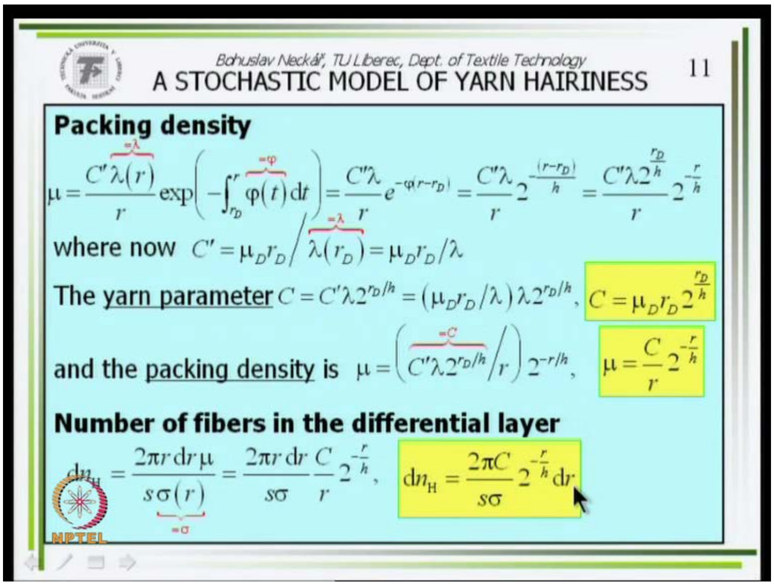

How it is now with packing density? For packing density, we derived in general part of our derivation this formula, this equation. Now, this is constant, this is constant 2, we can use it, rearrange, use on the place of phi, use this here and we obtain this expression, where C dash was this here. Now, because lambda is constant, it is mu D r D by lambda. We can also introduce, I can say final constant C, which is mu D r D 2 power to r d by h r D, so that it is also not function of radius.

And then, the packing density is mu, is, which is given by this formula because this is C we obtained. The formula mu is C by r 2 minus r by h. Of course, when you want to, to calculate the packing density on given radius r, you need to know 2 quantities, the characteristic h have decrease interval value and the characteristic constant C, yeah. Number of fibers in the differential layer, number of fibers in the differential layer d n H, it was this here, we derive it on an earlier slide. Sigma r is now constant sigma, so that it is this here and on the place of mu here, we use this expression, so that we obtain this here, and then, those d n H is equal to this expression.

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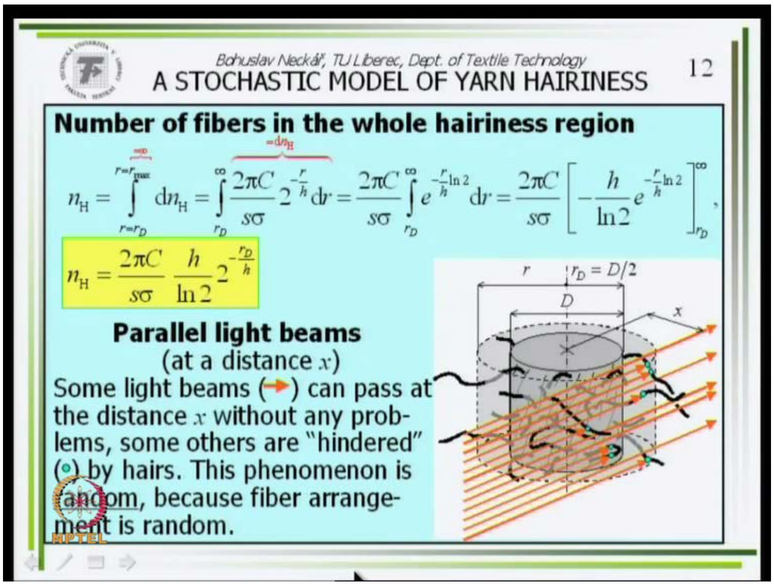

Well, total number of whole fibers in yarn cross-section, but in only in the sphere of hairiness, only in the sphere of hairiness is an H, which is integral from r equal r D 2, now infinity because r max is now infinity. From d n H, using it after small rearranging, we obtain such expression or and the finally, this expression. The derivation, which have jump are very trivial, it is only rearranging or as in this case, e power to something integrate, this is the toy for you.

We measure, we can measure, one method, how we can measure hairiness of the yarn is that we use some parallel fiber beams, some light and we make some projection of the yarn. What we see on the microscope? For example, in the central part we see, but we later, we show it more precisely, only intuitively in the central part, we see black, black points; then, we see some light windows among the black, black curves of fiber. The light windows are larger and larger, this radius and then, only light, is not it, is the typical picture of the yarn, one moment, one picture like this here, yeah.

From this, this is easy to obtain, it is easy to obtain this picture and evaluate this picture using for example, techniques on similar tools, which you know, from laboratory. Therefore, therefore, we want to derive, how is the possibility of light beams go beside the yarn, so that, let us imagine a set of light beams, orange light beams in our picture, which are going beside our yarn on the distance x. Can you imagine it, yeah, they are light beams. Some of them, some of them, this light beam, this light beam, this light beam, this, this can go beside the yarn without problems. Another, for example, this light beam is here, here, here, here, here, are hindered to, to fibers, to hairs, which are on the yarn surface, is not it. Symbolically, they are these light blue points in our picture, for the arrangement is random, of course.

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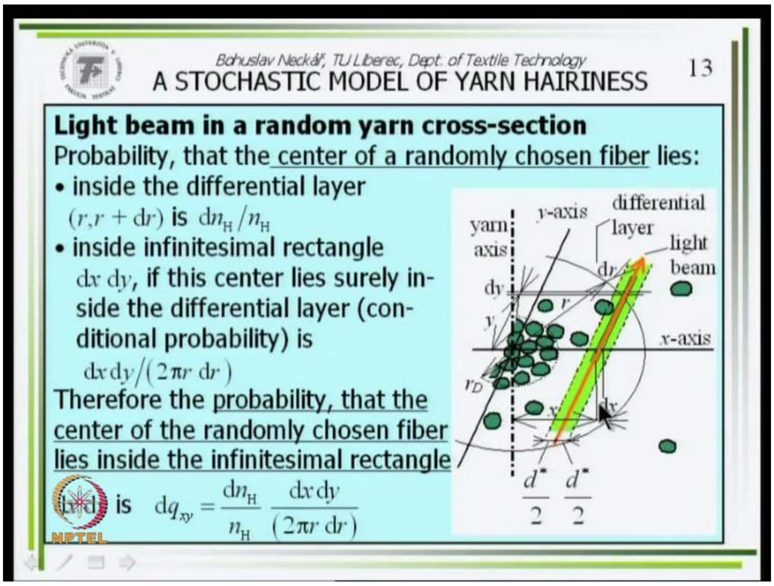

Yes, in this picture is one mistake, it shall be here in reality 2 rings because this is is my mistake, I proved yesterday to, to repair it, but because this picture is from older software and I have a newer software, it bring me no good result, results. It is typical, their relation between theory and practice. The companies, which produce software said, it is perfect compatible, but when we have a more sophisticated problem, you can see, that perfect is not compatible; excuse me, nobody is perfect.

And let us imagine, that we have here now only 1 ring, it is scheme of cross-section of yarn. We have not here only 1 ring, but 2 rings, with distance dr, elemental distance, so that this we have here some differential thing, annulus. Can you imagine it, I hope yes.

Well, how is the probability, that the center of our randomly chosen fiber, sometimes for (( )), that the fiber, I mean, the center of, of fiber, fiber section in this moment, that the center of randomly chosen fiber lies inside the differential layer r i-th rise, the dr in this differential annulus. How is the probability number of fibers in all protruding fibers, which are here, is dn H? All fibers, which are in hairiness sphere, which are lying in hairiness sphere are n H.

So, the probability, that randomly chosen fiber from, from hairiness sphere is lying in our differential annulus, must be dn H by n H, is it, well, yeah. Let us imagine, that inside of such elemental annulus is more smaller rectangle, dx times dy. This elemental rectangle, dx time times dy and let us formulate the probability, that the, the center, center of fiber, which surely is lying inside of our differential annulus, is lying also in our elementally small rectangle, the x, dx dy, yeah. Let us think only about the fibers, which are lying inside of our differential annulus, is sure in the moment.

And then, how is the probability, then the one chosen fiber, which is lying in our differential annulus, is lying, is especially in our rectangle dx dy? In our picture, this is, it is area of this rectangle dx dy by total area of differential annulus 2 pi r dr, sure, yes. Well, then, we can formulate the probability, that the center, am I say this is the, this is the conditional probability, the 2nd, is not it, so called conditional probability. The probability, that the center of the randomly chosen fiber lies inside the infinitesimal rectangle, this more is what is a probability, that the random chosen fiber is lying in our elemental annulus times the probability, that one, it is lying in differential annulus; it is lying also in our elemental rectangle.

Therefore, this, this probability, probability, that the center of the randomly chosen fiber lies inside the infinitesimal rectangle dx dy is dn H by n H. This here times this here times this, yeah, such probability we will call dq xy.

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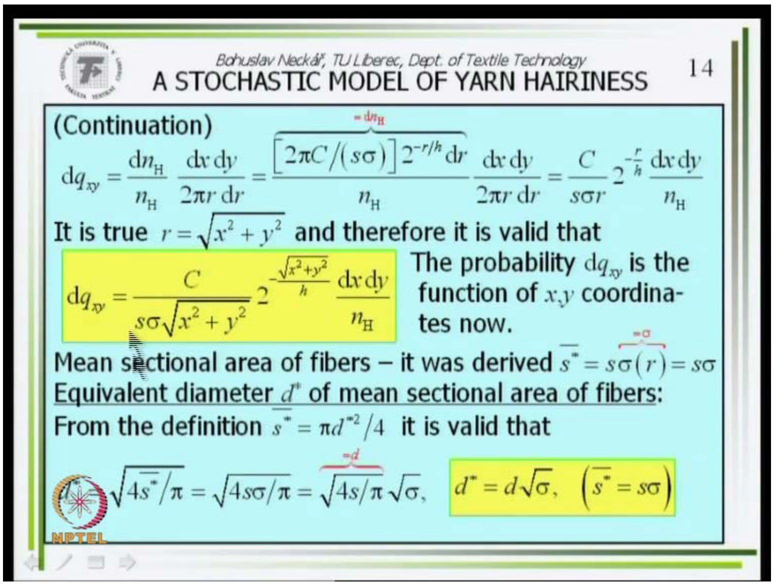

This repetition of our equation from last slide using known equation for dn H n H, we need not to explain its possible step by symbol n H.

But using the the equation for n H, we obtain this here on the place of n H. So, we obtain after rearranging this, this expression dq xy, this, this here. By the way, you know, from elemental geometry, from our pythagorean theorem, that radius is square root of x square plus y square, yeah no Pythagoras, all Mister Pythagoras.

Well, so that we can write on the place of, earlier r is here and here we can write square root of x y plus y x square plus y square, yeah. And so we obtain the probability, that random chosen fiber from hairiness area is lying in our elemental rectangle dx dy, this probability dq xy is given by such expression, yeah.

Well, I think time is running in this moment. We break our, our lecture and we will continue in the next lecture, we will finish this concept, this model concept and then, we will compare our results, these experimental experiences, yes. For this lecture it is all, thank you for your attention and see you in your next lecture.

Thank you.