Orientation of Fibers

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Lecture No. # 13

Relations among Yarn Count T, Twist Z, Packing Density, and Diameter D

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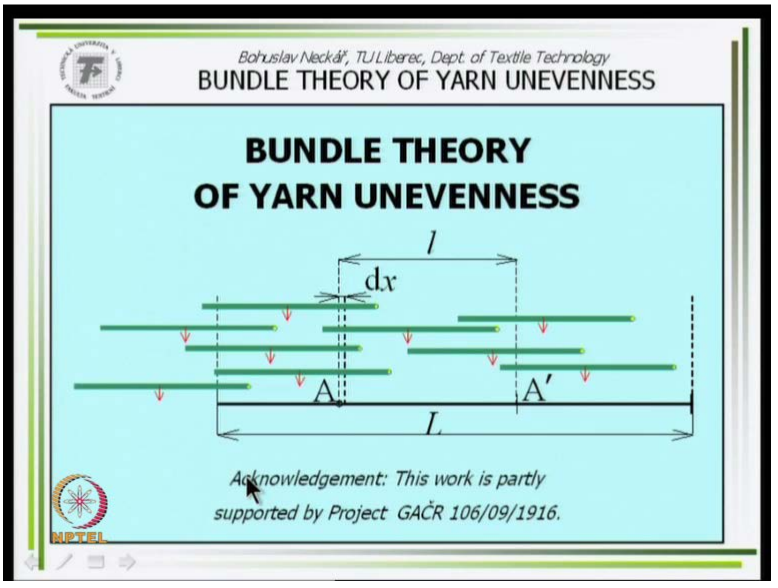

Let us start today's lecture. Today's theme related to yarn unevenness. It is evident, that we need to use lot of probabilistic and statistic tools for, analyzing, analysis of such problem.

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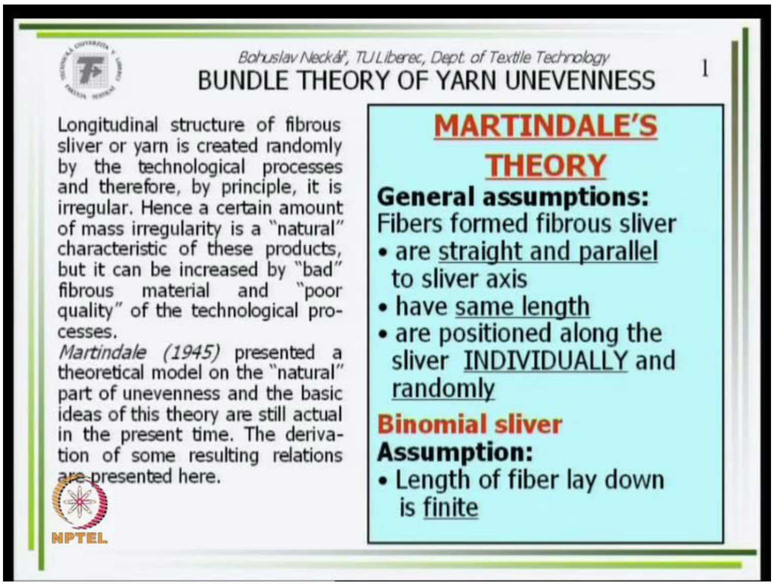

We will start about so called Martindale's theory, theory of Professor Martindale and then, I want to introduce some new model based on, on bundle character of fibrous material.

Let us start with an easiest version with bundle is a Martindale's theory. Our general assumptions are following: all fibers are straight and parallel, that is, it imagine it, so parallel to sliver axis; of course, all fibers have same length, we call it L; all fibers are positioned along the sliver individually, individually and randomly. When we accept the idea of so called Binomial sliver, we need to also say the assumption length of fiber of, fiber lay down is finite, yeah.

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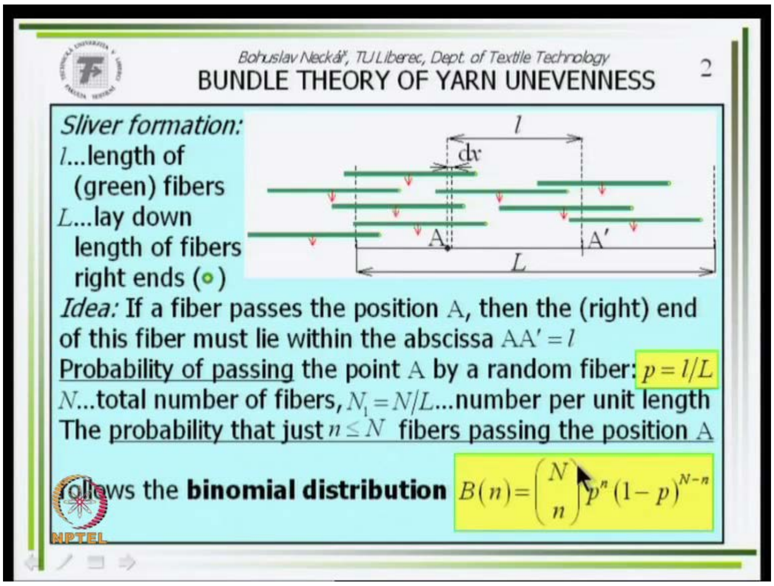

Is shown on our picture here, l, l it is a length of each green fiber; capital L from here to here to this, this lines, this straight line, it is lay down length of fibers right end; right end of each fiber is shown by this yellow point. Our idea or Martindale's idea is following: if a fiber passes the position A, then the right end of this fiber must lie within the abscissa, within the part from A to A dash, is it shown.

If the fiber like this here have right end, right, it is right end in the distance from A to A dash, then, then cover our point A for elemental interval here, when know for example, this fiber or I do not know, this fiber, then do not cover our point A. Probability of passing the point A by a random fiber, it is evident, it is small l, lengths of fiber by capital L, why? It is because possible is the, in that interval from to A to A dash and we can be sure, that our fiber, which we gave inside lengths capital L must lie in the lines capital, capital L, so that the probability here is small l by capital L.

N is total number of fibers, which we gave to our length's capital L. We call N 1 the, I can say, average number per unit length, it means 4 number of all fibers by our length L, number of fibers related to unit length. How is the probability, that just small n, which is smaller than all number of all fibers, capital N, so the probability, that just small n fibers passing the position A? Now, it means, small fibers passing the position A and Capital N minus small n fibers do not pass, sure, well.

It is evident, that the distribution of number of fibers covering the position A must follow the binomial distribution. It is evident from your 1st lectures about the theory of probability, binomial distribution.

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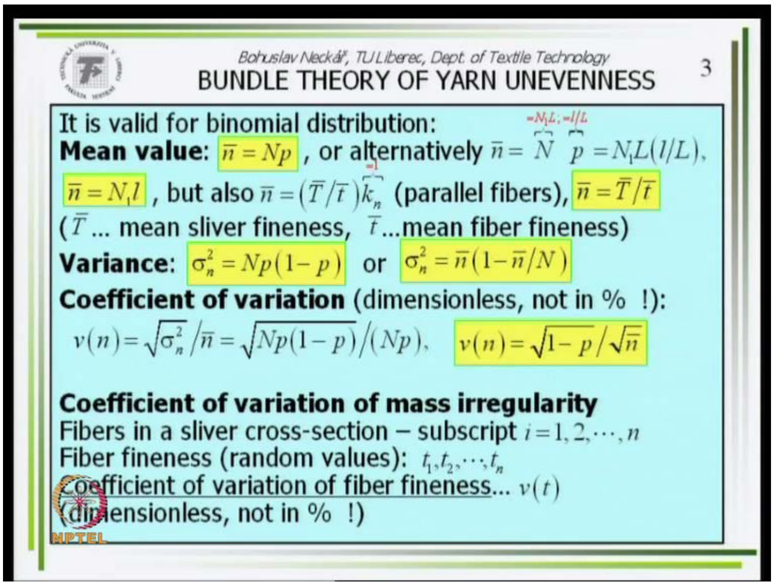

And you know about a binomial distribution, that it is valid, that they are, for example, the mean value of binomial distribution n bar is Capital N times p, number of all fiber times probability. It is derived in this order and book about the theory of probability or alternatively, we can write n bar is Capital N times p bar; N is N 1 times L and p, it is small l by capital L, so that we can write it also in this form. And your bar is n 1 capital L1, number of fibers average number 2 related to (( )) times fiber length, well.

But also, we know it from earlier lectures, number of fibers in cross-section is generally capital T to small t times k n count, or that the fineness of our bundle by a mean, by fineness of fiber times our known k n factor, because parallel fibers k n must be equal 1 and so, so that n bar is capital T by small t, both bar.

Variance, variance or dispersion of number n, number of fibers covering to the point A is based on our statistical and book n times p times 1 minus p. Or this variance is also n times p is n bar and probability we can write as n bar by n, as we show, well. Coefficient of variation, which we in theory every time use as a dimensionless quantity, so, no, for example, 15 percentage than 0.15 and is coefficient variation, it is, this is given by known ratio, square root of a variance by mean. Using our equations we obtained this, this expression for coefficient of variation, yes.

Fibers, this is coefficient of variation of what? Of number of fibers, number of fibers n v n, number of fibers. It characterize the variability of number of fibers, but usually, each fiber have a rather own fiber fineness. Therefore, we must study coefficient of variation of mass irregularity, you know, number of fibers irregularity. Fibers in a sliver cross-section are from the n, small n fibers. Let us subscript, for each fiber, i is 1, 2, 3 and so on to n; fiber fineness, fineness is now t 1, t 2, t 3 to t n in our section of sliver.

Coefficient of variation of fiber fineness, we will call v t, variation of t, no, n and then t fiber fineness, dimensionless circle of curves.

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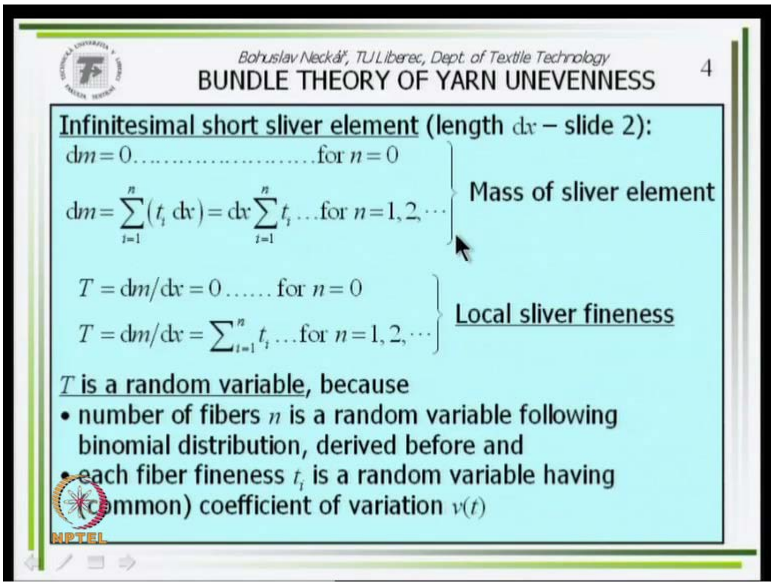

Let us now imagine our infinitesimally small distance dx in our scheme. If number of covering fibers is equal 0, then evidently, the mass in our small, differentially small length is 0. When no, then mass dm from all fiber element L, parts in our element L, distance dx is which one, mass per one, but general light fiber is, is t i times lengths, so the t i times dx, sure, yeah. And because dx is common for each fiber, for all fibers, so that it is dx times sum for i equal 1 to n of t i. If n is 1, 2 and so on to dm, how is the, the count? The fineness is, the local fineness is, local fineness is of sliver in our elemental portion dx.

How this is the mass by length as each fineness? It is 0, if n equal 0, if number of fibers is equal 0 and it is dm by dx is dm is this 1 by dx, so that it is the sum of di for n equal 1, 2 and so on to, yeah, t. You can see that the quantity t, the local sliver fineness is a random quantity from 2 points of view. First, number of fibers n is a random variable following in this moment of binomial distribution derived before, so that n here, from 1 to n is n here is a random quantity.

And 2nd, each fiber fineness is fiber fineness is, when we have more such differential lays, the first fibers in different random tau dx lengths have some distribution, t 1 have some distribution, t 2 have some distribution and so on, so t i is also a random variable. Nevertheless, because from the same material having the common coefficient of variation, v t, is it clear.

Therefore, from 2 point of view because n is random, each t i is random quantity of also the fineness of the sliver, local fineness of the sliver mass, the random quantity. We have not enough time to derive some probabilistic relations, therefore let me in short inform you about the possibility, what is possible to derive in theory of probability for one special type of random quantity.

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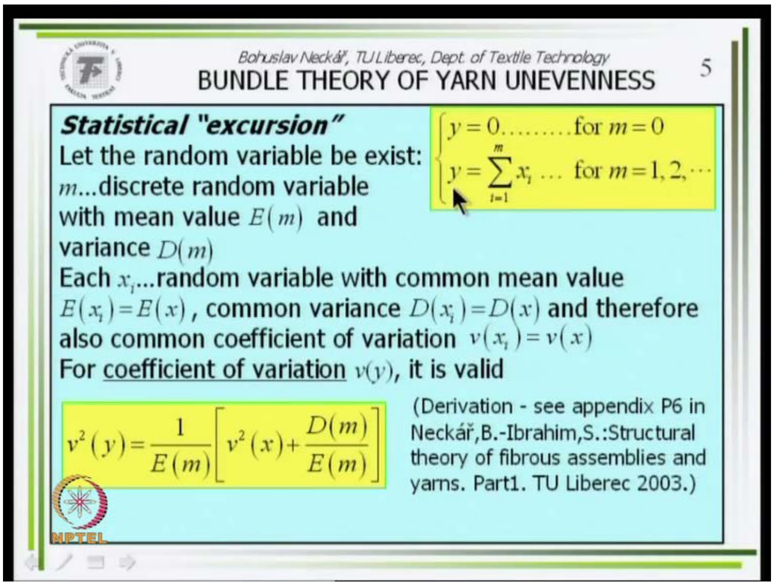

So, now, we are not in textile problems, now we jump to, to statistical, to some statistical hand book or statistical teaching book, yeah.

Let us imagine a random quantity y defined here, y is 0, if some quantity m is 0 and y is sum for 1, m I by, by m x y, from x y, if m is 1, 2, 3 and so on. Sum of this value, let the random variable be exist, m is discreet random variable, with means, with mean value of this E m, you know, that for mean we usually use some operator E. So, E m is the mean value of random quantity m and variance of m is D m excursion.

Each x is also random variable with common mean value; each E x is equal to E x only 1 quantity and common variance. Each D x i is equal to D x, only 1, 1 value and therefore, also common coefficient variation v x i is some v x, common for all fibers, 1st as well as, as 5th, as well as 13th and so on.

Well, when we derive the coefficient of variation of such random quantity using the (( )) from the theory of probability on mathematical statistic, finally we obtain this equation, it is, let us, brief me it when no, we find this. For example, our, our book about the structure of fibrous assemblies as an attachment, as an appendix in this book, it is derived. Professor Ishtiaq has this book in his library, well, what is it?

Square root of coefficient of variation of such random quantity y is 1 by mean of m times square root of coefficient of variation of x plus variance dispersion of m by mean value of m. So, this is the result from theory of probability; this expression we will use more times today.

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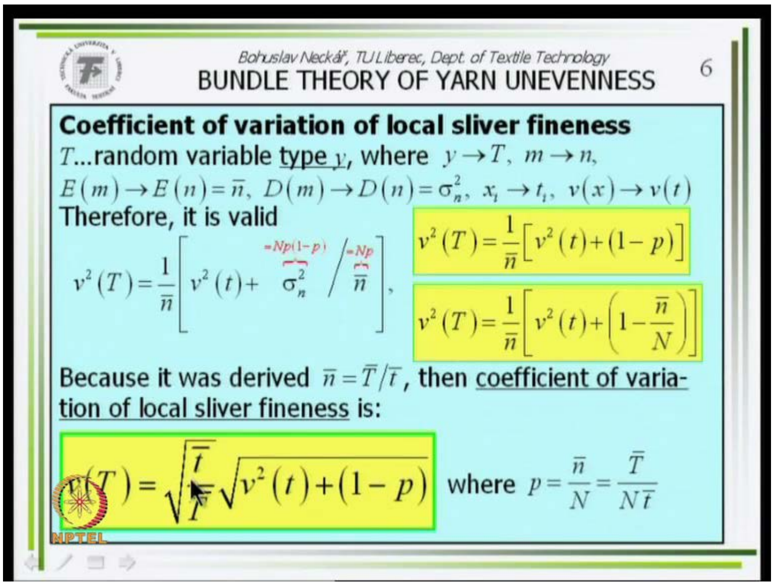

And now, back to our problem. The coefficient variation of local sliver fineness was this here, yeah, but this is, this is random variable type y. Therefore, we must use such general formula for variation of coefficient variation. Now, especially on the place of y is capital T; on the place of m is small n; and on the place of E m is evidently, E n; mean value is n bar, we call it n bar; on the place of D, D m is D n and we equal it sigma square n; and on the place of each x i we, we now use the t i; and of course, on the place of v x, in general formula we have now, specially, v t.

What we obtain applying our result from theory of probability? What is the equation; is this the equation? Yeah, because we derive sigma, sigma n or sigma n square is this from, binomial distribution and n bar, which is n times P, we obtain all this or this (( )) for square of coefficient of variation of local sliver fineness because it was derived. And n bar is capital T bar by t bar ratio, mean values of sliver fineness of fiber fineness.

The coefficient variation of local sliver fineness is v t, this equation, sorry, this equation and we obtain this here for probability, for P is given by this or this, how you want, yes. So, it is binomial distribution we need to know; we need to know the mean value of sliver fineness, of fiber fineness; we need to, to know the value of coefficient of variability, coefficient of not v for fiber fineness, coefficient of variation, of course. And we need also to know the parameter P, it was discussed earlier, it, write it to, to lengths, capital L, which we used in our starting idea, especially the parameter P is difficult to, to formulate. Nevertheless, usually we can imagine the process of creation of, of sliver as a process, which is actual on very long lengths, capital L.

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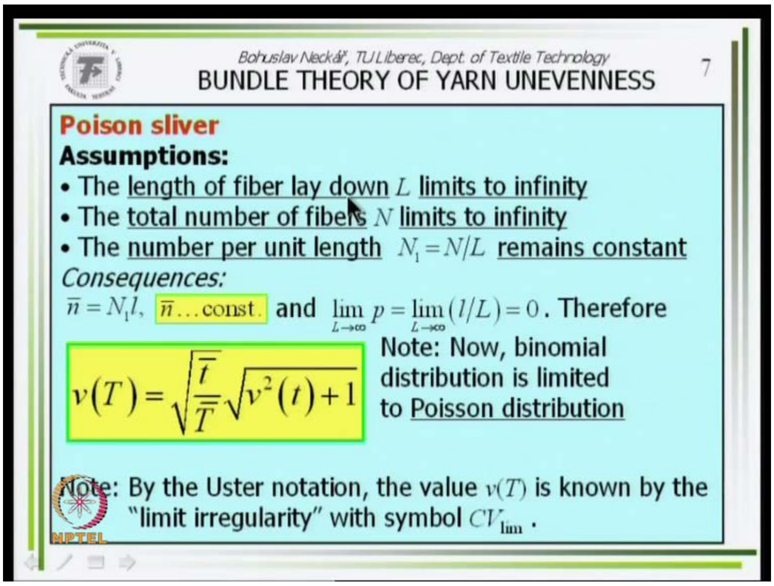

Therefore, let us imagine, that the lengths fiber lay, the fiber lay down lengths, capital L, limits to infinity. It is very, very long, it limits to infinity. The total number of fibers, capital N, it is also limited to infinity, yeah, it is higher and higher and higher. Nevertheless, the number per unit lengths N L, which was number of all fibers by lengths remains constant, is constant, clear.

So, as I, I elongate capital L 10 times and number of fibers, I increase also 10 times, so that N 1 stay be same, its mean number of fibers per lengths unit, yes, how is the consequences of this? One mean value of fibers in a sliver cross-section, small n bar we derive is N 1 times L, N 1 is stable, L 1 is stable, so that N is constant, no changed.

Nevertheless, the probability p, which was small l by capital L; when L is higher and higher and increased to infinity is a limit of this ratio and because L is limited to infinity, then this ratio is limited to 0, the probability p is limited to 0. Therefore, I can use this expression in the special moment in which the p is equal 0 and I obtain this, this equation.

This equation corresponds to the, this limiting, is the limiting from Binomial to Poisson distribution, so that we recall. Usually, we speak usually about the Poisson sliver and this, this equation correspond to the, I can say natural unevenness of Poisson silver, yes. The note by Uster notation, you know, what is Uster, instrument Uster, the Uster and so on.

By Uster notation, the value v T is known by the so called limit irregularity with symbol CV lim, in the materials of this company. What we need, we need to know the coefficient variation of fiber fineness?

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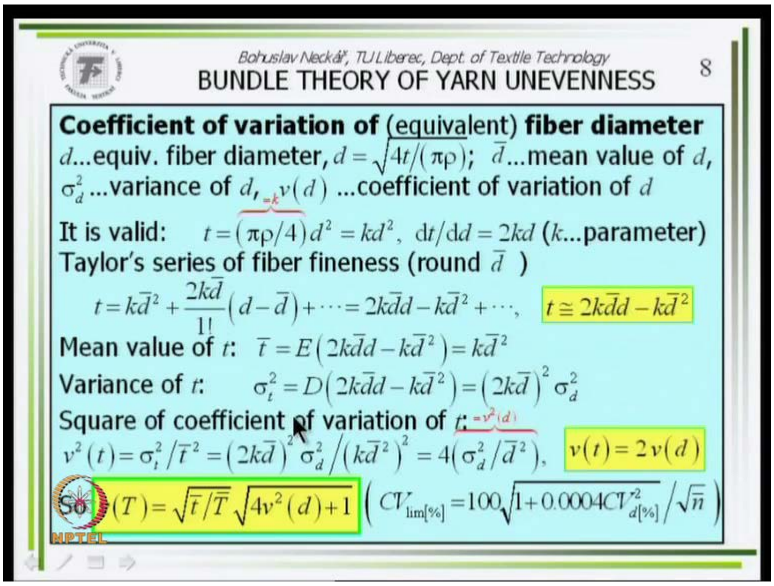

Nevertheless, usually, we have not it, it is difficult to, directly to obtain. Therefore, we can use some approximation and apply no coefficient variation of fineness, of fiber fineness, then fiber diameter, how?

Coefficient variation of fiber diameter is now our d, let us show, that d. Let us think, that d is equivalent fiber diameter. Fiber diameter we derive is 4 times t by pi rho d bar is mean value of d fiber diameter and sigma square; d is variance of fiber diameter; d and v d is coefficient of variation of fiber diameter d, it is valid. This is our one of our 1st equations, it is valid at t. Fiber fineness must be pi rho by 4 times d square because d square or some 6, you know it from lesson, you, so this relation is valid; this expression is valid; this is not function of d. So, let us for sure, make sure, our, our formal graphic form, let us call a sum parameter k, k times d square, make derivative of this equation, so that we obtain dt by d is 2 k d, sure, yeah.

And using Taylor's series of Fiber fineness is, we, we use round d bar. You know, what is the Taylor's theory? Yeah, I think, yeah, you heard it, minimum, ok.

We obtain some series, which is, t is k times d k bar square plus 2 k d bar by 1 factorial times d minus d bar plus and so on and so on. This is, this, here, let us say approximation, our approximation will be enough when we use first 2 numbers in the series. Therefore, let us use t is 2 k d bar d minus k d bar square. When you use it, then we can, from this equation, to obtain mean value of t, what is it? Mean value of this here, this is, this is absolute, yeah. It is evident, that this is, this is k times d d bar square because you obtain 2 times in here minus 1 times here, yeah.

Variance, it is d from this expression, the random quantity is this d is here and because constant do not play a role and the parameter 2 k d bar by variance, by variance of some constant times random quantity is the constant square times this random variance quantity. You know it from probability; theory of probability we obtained is here, so that square of coefficient variation of s, which, of coefficient variation of t was sigma t square by t bar, but sigma t square, what is sigma t square? Sigma t square, it is here by t bar square, it is this here from here.

And so after rearranging, we obtain v t is 2 times v d on the price of coefficient of variation of fiber fineness. We can roughly, because this is approximation, we can roughly write 2 times coefficient of variation of fiber diameter, yeah, well. So, our equation have now such form, it is the final equation for us in the moment and same equation you can read in the professional material of (( )) Uster, this is the length in such form because in percentage and so on. Therefore, this 0.00, so we know why it is this number 4. It is, because, because this way, this approximation of coefficient variation of fiber fineness, well. This is all for in the moment for sliver having parallel fibers.

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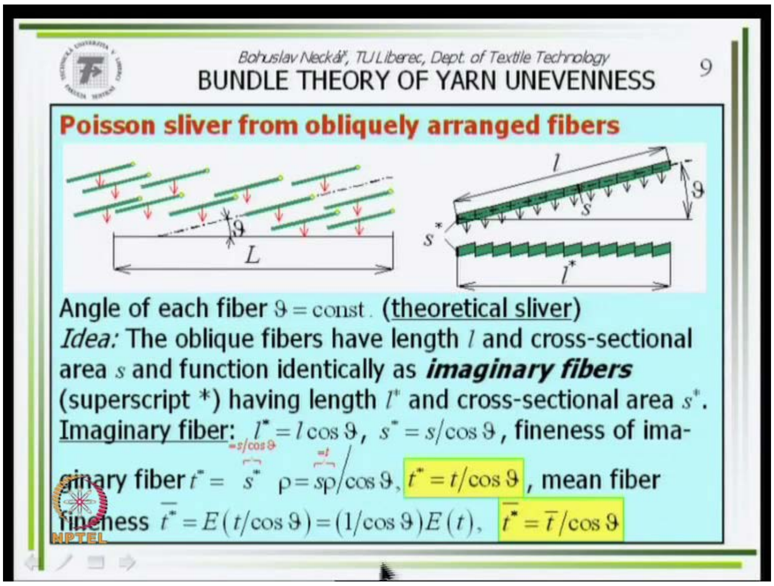

Let us solve the theoretical case, I do not imagine, how it is possible in technological process to realize, but the theoretical sliver, where each fiber like this, created like this here, based on this scheme, where each fiber has same angle theta, angle to longitudinal direction of sliver, yeah.

Each fiber is an average fiber having the same angle theta to longitudinal direction of, of sliver. The scheme, I think can show you, what is our theoretical imagination? 1 fiber seems like this here, yeah. Now, let us imagine that I cut this fiber to very short elements, vertically perpendicularly to longitudinal direction of sliver here, here, here, here, here and so on. All fiber I cut to lot of very small such fiber segments and now I, the fiber segments remove to this position here and I grew it together, can you imagine it? Yeah, then what I obtain? I obtain a fiber, a new fiber, a constructed fiber, hypothetic fiber, imaginary fiber having lengths l star, having another fiber fineness t star, another fiber cross-section s star, but this fiber is parallel, this fiber is parallel with longitudinal direction of yarn, sorry, of sliver, sliver direction, is not it.

Let us calculate this parameter of our imaginary fibers, reconstructed fiber and theta is constant, so that lengths from this, from this picture is evident, that the new lengths l, l star, it is l times cosines theta, trivial geometrical triangle. s star, we know it from earlier lectures, s star must be s star by cosines theta. Fineness of imaginary fiber t star, we know, that the fineness is cross-section times rho. Now, cross-section of our new imaginary function is s star, sorry, s star and for this s star, we know this. This expression shows, that s times rho by rho cosines theta and because this s times rho is t, starting value of, of fiber fineness, we obtain, that t star is t by cosines theta, is it well, clear. And we construct our average fiber to another fiber, which is parallel to, to fiber axis.

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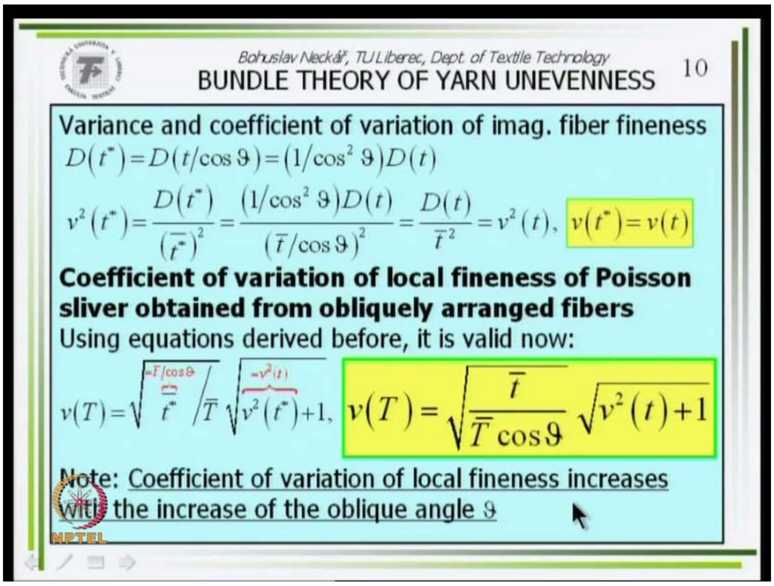

And we can, to use the same, the same quantity as earlier, yeah, as v square t bar, we can say, it is the same we derived using equations, which we derived in last slide. And we obtain finally, because by cosines, by cosines square by cosines square, so that we obtain, that v square t is same as v square t star. The coefficient variation is now changed after our hypothetic cutting and rearranging of our fibers, well.

Because same we can use, we can use also the same, the same equation. Our equation is, Poisson sliver was this break here, only new is, that I have here on the place of v t star and on the, this here to, well. But we know that the t star bar is t by t star by cosines, the same, this is equivalent with this. So, I obtained finally, this equation, is not it. You can see, that here the angle theta play some role, it is in the denominator of this ratio, so that coefficient variation of local fineness increases with the increase of the oblique angle theta. If our theta is starting, graph is increasing, then cosine is decreasing and 1 by cosines is increasing, so that v t is increasing. The shape, the end of theta plays a role to the final sliver irregularity, to the final variation coefficient of variation, yes.

Our earlier case was pure theoretical to have the sliver with constant angle theta, it is only theoretical idea. Nevertheless, in a real sliver, we have local directions, we can imagine, that our final sliver is result from the break of load of partial fiber slivers and each part of fiber sliver have same angle theta, clear. So, let us now imagine the idea of doubling of Poisson of slivers with oblique arranged fibers.

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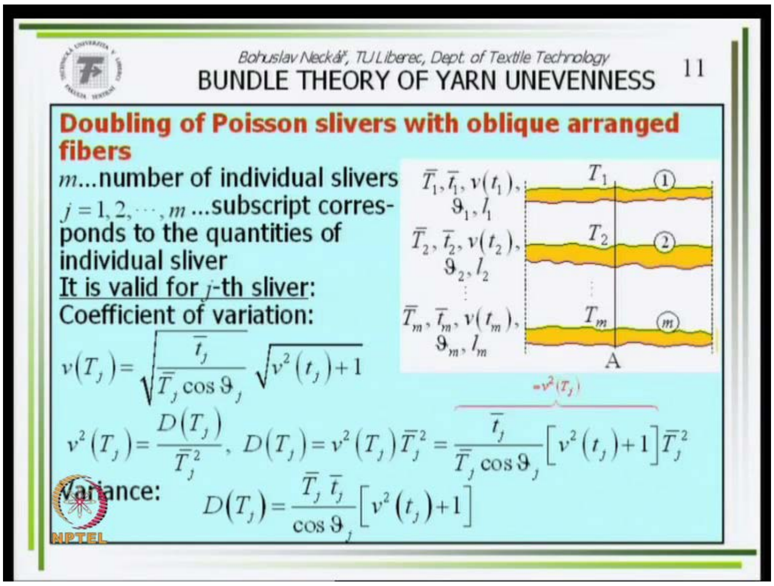

We have, we have m, m individual slivers subscript 1, 2, 3 and so on to m. Each, each sliver have, for example, the 1st sliver have, the mean sliver fineness is T 1 bar, mean fiber fineness is small t 1 bar, coefficient variation of local, of, sorry, coefficient variation of fiber fineness is v t 1 angle, angle oblique theta 1 and length, length of fibers is l 1 and so it is in all, all slivers. So, it is in all silvers and that, so that we can formulate the coefficient of variation.

Coefficient of variation of local sliver fineness is generally of jth sliver, j is from 1 to, to, to, to m, it is our known equation. Nevertheless, subscripts j here is, subscript j here, here, as well as here, because it is jth sliver. The square of D T j is this here, evidently. So, variance is this here and from this equation, variance is this here, and we know this v square T j here from earlier, v T j is here.

So, using it we obtain this expression for variance in a jth sliver, it is, well, so the variance is finally this here.

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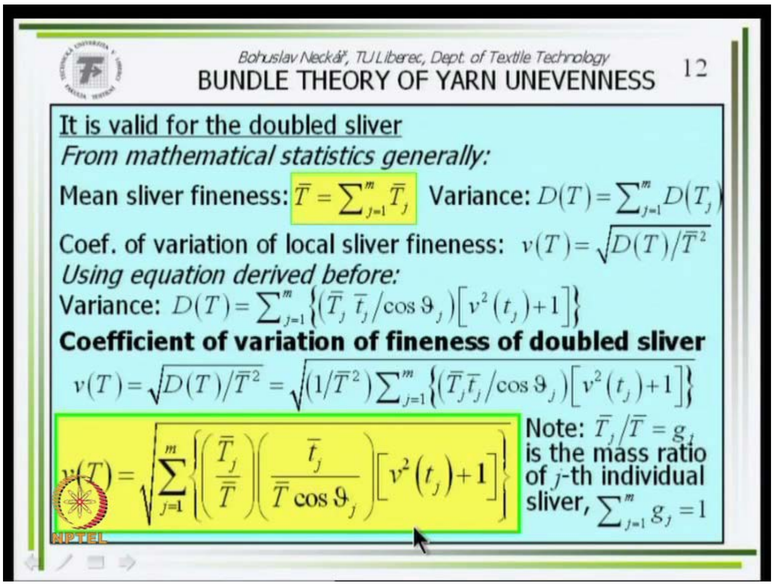

And you know, that it is valid for the doubled sliver, that the, the resulting mean value t bar is sum of partial mean values T j bar and variance is sum of variances. Using this, this expressions, especially for variance, we obtain, because v T is this here, this square root of dispersion by, by, by mean value square, we obtain d T. Using the equation derived in last slide, we obtained this here and for coefficient of variation f, fineness of doubled sliver, we obtain as each coefficient of variation. We use this expression here, so that this here and finally, we obtain this expression.

The angles theta j play some role, so that the orientation of fibers play some role for final coefficient of variation v T. If the distribution of this angle is relatively very, very significant, then also we can see, that the, the coefficient of variation of local sliver fineness is increasing because the denominator is of these quantities.

Let us remember, that the ratio of T j bar by T bar, it is g j, which was the mass ratio of jth individual sliver. Here, this, this ratio here, if it is a mass ratio of jth sliver in all silver, well. This is called general equation.

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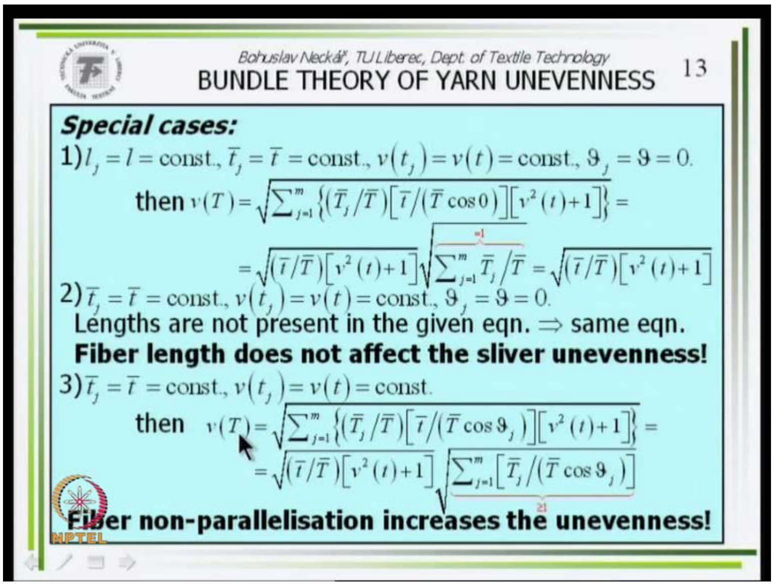

We can solve some, may be, free special cases using this. The easiest case is, in each partial sliver, the lengths is constantly same for each fiber sliver; mean fiber fineness is same; coefficient of variation of fiber fineness is same; the angle of, the oblique angle, the angle of fiber (( )) to longitudinal direction of fiber is same and equal 0. What is it?

This is Para Fiber bundle or area, you can, our this for checking of course, you can use our equation, which is here and, and based on this, this values, after small rearranging you obtain, that it is this here, our earlier starting equation, is not it, well. We have not some mistake in our some formal, some formal error, in our equation, is also a checking.

Second, fiber fine, mean fiber fineness is same, coefficient variation is, of fiber fineness is same for each sliver, partial silver, and is same for equal 0. Nevertheless, I say nothing, I said nothing about the fiber lengths, may be, the each partial fiber sliver have another fiber lengths. Nevertheless, the lengths is not in our equation, we can have the coefficient of variation is not changed, fiber lengths is not in our right hand side for coefficient variation of local sliver fineness. Lengths of fibers do not play role for our value v T, so that you obtain same results. We show, that lengths is not important for coefficient of variation, length does not affect the silver unevenness.

Third is, here fineness is constant for some special cases, fiber, mean fiber fineness is constant, coefficient variation is constant, but different about the angle side. I did say they are 0; each angle can be another, no, well, sorry. Then, I obtain using our starting equation, such equation from which is evident, that the, that the fiber non parallelization increases the unevenness.

The unevenness is best for parallel fiber bundle, parallel sliver. For non-parallel slivers, the unevenness is increasing because the cosines theta, if this is the sum of some expressions having cosines, cos, sine of theta j in denominator, from this it is, it is possible to see, it is not necessary, that every time speak about, speak about the calculation, calculate numerically. In your industrial practices also, possible to understand, why is this, this result and to know, that when you have too high distribution of fiber angles to, to significant distribution of orientation, fiber orientation, it can have the, the, it can influence to final value of unevenness, is not it, yeah, well.

To the end of this lecture I want to postulate the Hubert's index of irregularity. It is theoretical concept based on Binomial, Poisson's sliver and we obtain some equations from other side. We are able to measure sliver or yarn unevenness, yarn irregularity, using may be the (( )) instrument from (( )).

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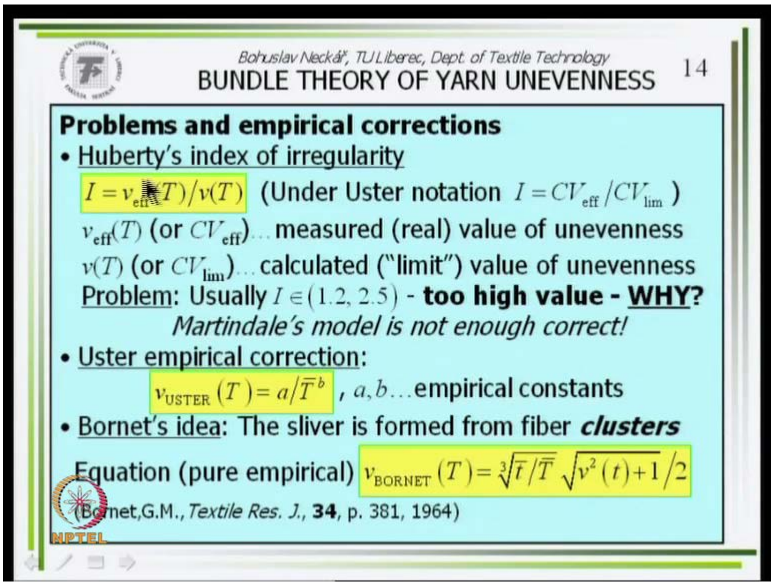

We obtain 2 values, 1 is practical value, measured value of unevenness, I call it as effective, and 2nd is the theoretical value, which we derived. We can create such ratio, need to measure to calculate it, it is known under the Hubert's index of irregularity, index of irregularity according to Hubert, it is name of author.

In Uster, the material from this company, they call it CV eff or CV lim. Yes, value of this index by some practical experiences is enlarged from, from may be 1.2 to 2.5, sometimes also it is very high value, that this difference between our theoretical result and the result of experiment is very high. It is not 5 percentage for example, 50 percentage, 80 percentage, 100 percentage. We calculate, I do not know what, some value, and the reality is 2 times higher. It means, it means, that something in the Martindale's theoretical concept, we have not some influence, something is force, I can say.

What is force? Some influence, which to, to our final experimental values of unevenness. We do not have in our model, we must make some corrections. Therefore, (( )) we make some corrections and about this, a new or modified model, I want to speak in our next lecture.

Yes, in this lecture it is all, thank you for your attention. Be happy and see you in our next lecture. Bye.