Orientation of Fibers

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Lecture No. # 15

Relations Among Yarn Count t, Twist Z, Packing Density And Diameter D

Today's lecture is oriented to the yarn strength as a stochastic process. Itis known from practical experiences from textile laboratories and so on, that the mean value of yarn strength as well asyarn breaking strength andcoefficient of variation of yarn strength are changed in relation to gauge strengthwhich we use for making machines; why?It is a theme of our today's lecture.

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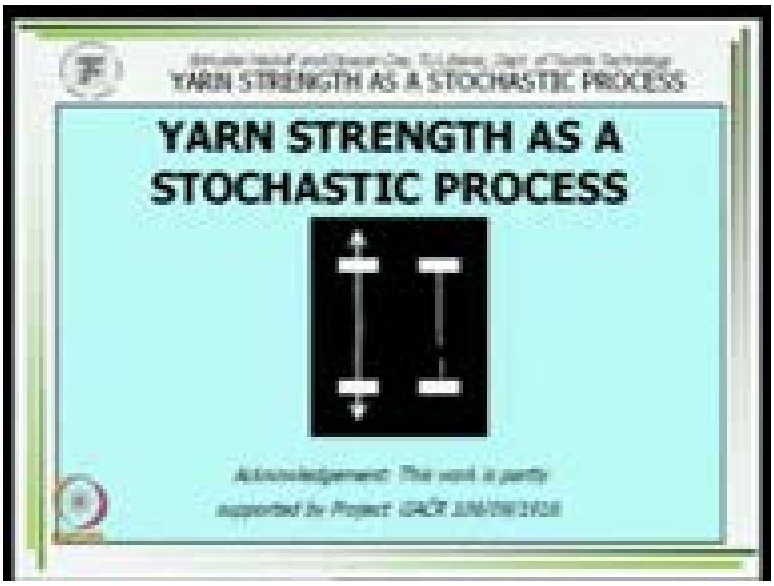

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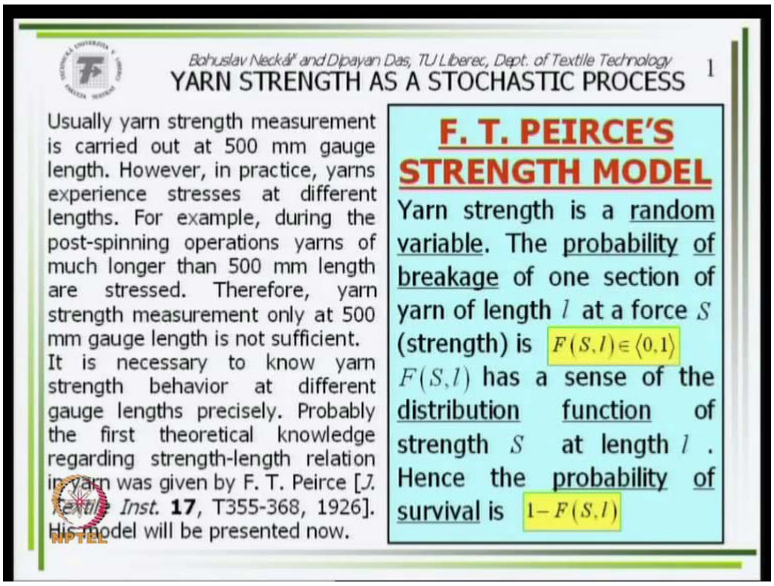

We will speak about a Peirce's model which is relatively old and the easiest model and the second part of this theme I wrote in short comment ourmodel which is much moredifficult and need to have understood the tools frommost special tools from theory of probability; especially from stochastic processes.

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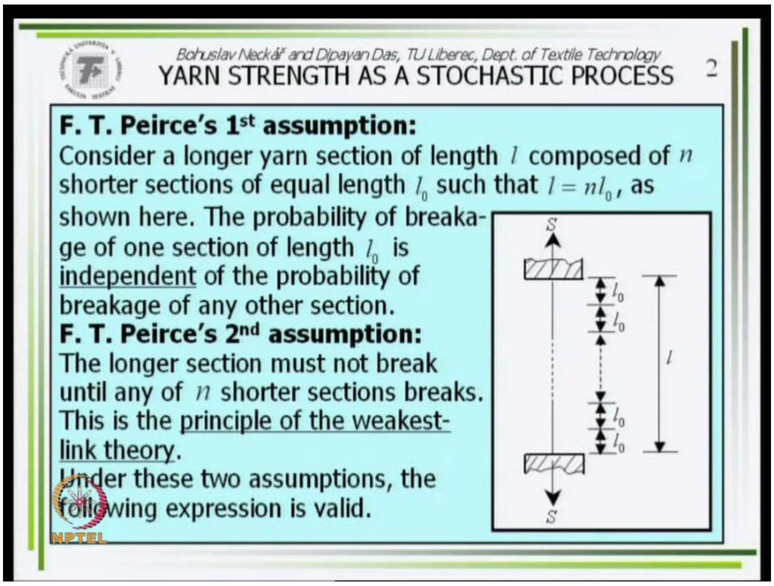

Well,let us startthe model according to Peirce. Youknow that yarn strength is a random variable; it is evident. Theprobability of breakage of one section of yarn of length l at a force s we can call under the symbol f s l; s is strength of yarn portion, yarn part, which is studied l is gauge length which we use ok. So, that s is random variable: l, it is some parameter which, we know in which lengths we have our yarn which is the gauge lengths by our experiment. So, that f s l has a sense as a distribution function of strength s, clear? Atthe lengths l and the probability ofsurvival is 1 minusf s l probability of break is f s l. So, that the probability of the portion that doesnot break is 1 minus.

Well,let us imagine some yarn lengths l like this line is here f n lengths l between couple of this lengths l lets imagine divide it to n small lengths is l 0; then evidently l 1 is n times 1 0 as is shown. Theprobability of breakage of l section of lengths 1 0 is independent ofthe probability of breakage of any other section.Thisassumption is very important and it create a basis of Peirce’s model. We assume that the short the strength of short parts of the yarn lengths 1 0 are mutually independent.

You can imagine abstractly when you cut it your yarnto set of short segments lengths 10. Then, you mix the segments together and then randomly you take one beside the other and glue it together you obtain new yarn. But, from point of view of break of strength your result will be the same because the strengthsare not mutuallydependent they are independent. Basedon this assumption, the model is created

The second assumption is with the longer section must not break until any of n shorter sections breaks. Thisis the principle of the weakest link theory. Weakest link theory started from the idea of chain. Let us imagine a long chain; create it from links; one link beside the other. So, how is the strength of this long chain? Thisis equal as the strength of this link which have the minimumlinks; which isminimum quality, mechanical quality like inside.

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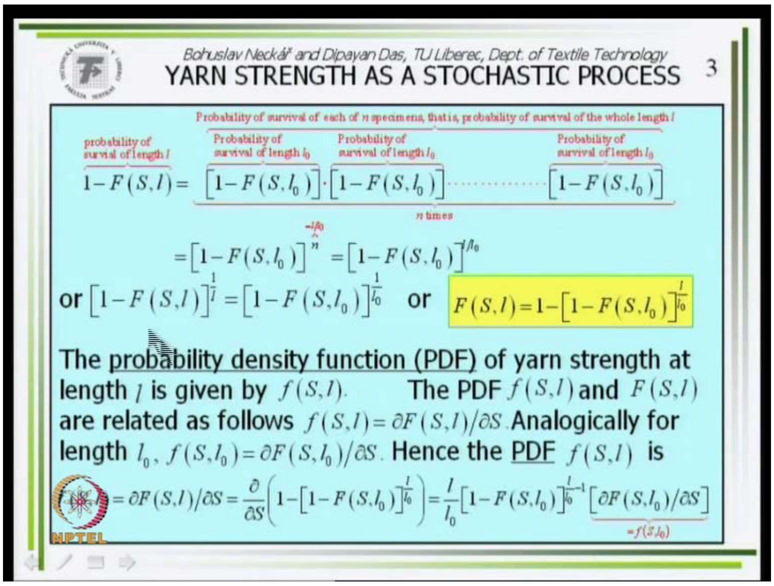

Well, so, this is the principle of the weakest link theory. Underthese two assumptions, the following expression is valid; it is shown here. What the sense of this equation F s l is distribution functionofstrengthon the yarnportion lengths – l 1, relatively long. Imaginethis are long what is 1 minus F s l is it a 1 minus F s l.

Now, let us see the probability that these long lengths will not be distrait will not be breakto our force sis it. So, is the distribution function and what is it lets imagine that this long lengths of the yarn is not break; it means that the strength of the first link must be higher than our force s. How is the probability of this? 1 minus F s l 0; thisis distribution function of the force F breaking force F by lengths of yarn 1 0 short lengths of the yarn l 0 is not it. So, first link the probability that will not break is 1 minus s F s l 0ok.

How it is with the second link? Same; the links are independent in our model now. So, the probability of the second link will notbe destroyed, is also 1 minus F s l 0 clear? Howis a probability that both will not be destroyed? Youmust multiply probabilitiesand so, we can go from first to the last link. Howmany links are there? 1 sorry n.

So, we must multiply this quantity and times (Refer Slide Time: 08:35) is from first link from second link third and then to the end. So, that it is 1 minus F s l 0 power to n, but n it was l by 1 0 is not it? So, it must be in the place of n we can write lby 1 0; we can obtain this equation as well as this equation F s l distribution function of strength of our yarn by lengths l it is 1 minus bracket is 1 minus F s l 0 distribution function of same force s byshort fiber segment l 0 powered to l by l 0

So, now we have summarized it how from distribution of the strength from short segments to obtain the distribution of strengths by longer portions of the yarn. Probabilitydensity function is principally clear? Question is now because probability density function it is a derivative of distribution function every times, is it not? So, we can say that the probability density function F s l, it is derivative of the function F s l by s. So, its partial because of the quantity is a parameter; it is notrandom quantity.

Well, analogically for length l 0 f s l 0 is the derivativefunction f s l 0 by l 0 bythe s. So, that the probability density function F s l is the you made the derivative length left as well as right hand side and we obtain F s l having this here, this here, then this (Refer Slide Time: 1108) by this wasp d f of short for short derivative for short analysis.

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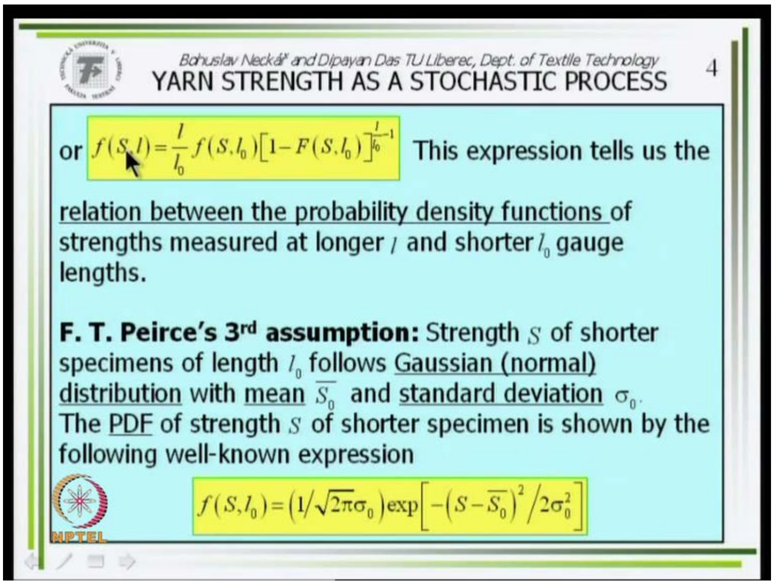

The probability density function of the distribution of yarn strengths using long gauge length l is given by this equation clear. So, it is good to know because in short lengths Ido not k now for example, for 5 centimetergauge lengths we can in laboratory do it; it is very difficult to say how it will be made. Someprediction how it will be for example, the distribution of yarn strengths using the yarn lengths Ido not know for example, 5 meteror 3 meter or 500 metervery long so.

So, constructed instrument we have not in our laboratory we have not breaking machines where its possible to have gauge lengths Ido not know,5 meteror 500 meter it is sometimes very important because (( )) for example, in modern weaving roomscan be 2 – 3 meters which are programs; now 50 centimeter its standardgauge lengths based on standards in laboratory.

So, we can recalculatethe probability density function or distributionfunction from short lengths right to short gauge lengths to the distribution on the long run the Peirce's. Assumedalso a third assumption he said that thein a short gauge lengths in our short gauge lengths 1 0 the distribution of strength of yarn strength is Gaussian normal Gaussian distribution and you know what is morefrequent in India or Gaussian distribution normal distribution, but I hope you know both these parts, yes. Then, the probability density function F S l must be given by known expression of probability densityfunction in a Gaussian distribution.

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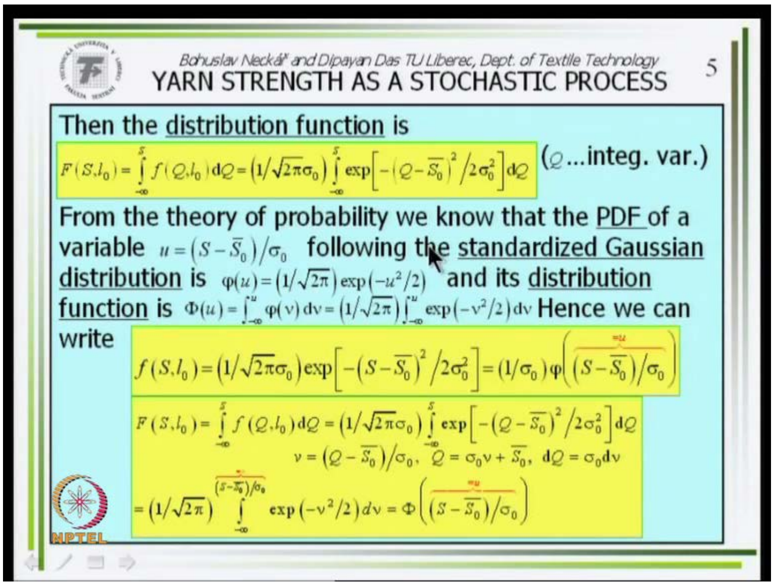

You can say in the moment that right hand side we have not 1 0;yes, explicitly we have not, but we have because on the 1 0 this is the 1 0 determent sigma 0 standard deviation and mean value in our short length 1 0 therefore, implicitly it is thereand the distribution function the distribution function F S l 0 is an integral of this. It is renamed1 and thequantity because integrating variable must be added in the border. So, that using integral from our probability density function we obtain this here (Refer Slide Time:

14:55).We know that this integral this innormal distribution,Gaussian distribution, this it is an integral have not the primitive function inanalytic form. That it is not possible to write it in an analytic form it is. So, it is called (( )) integral; under this theorem you can find it in some mathematical hand books and it takes it a lot of methods - how to numerically obtain the best approximation of this in general integral.

As you know, repetitive possible that we often write that so called standardized Gaussian distribution is not it; if our random quantity s f Gaussian distribution by the quantity s minus s bar 0, it means, mean value mean value of our yarn strength in a short fiber lengths by standard deviation that isstandard deviation it is a quantity u having standardized Gaussian distribution. Youknow from the theory of probability I think and thisquantity u haveprobability density function is phi u which is given by this expression. Also, very known from lighter age and this distribution function capital phi u it is integral from this to this here, the valuesexist some numerical method because probability integral (Refer Slide Time: 16:47). I said hence, we can write F S l 0 is this here or this here a very phi is aprobability density function of standardized Gaussiandistribution and for f capital F S l distribution function we obtain using integral of there is such substitution we obtain this formula, but this is distribution function of standardized normal distribution which we called here as a phi if whole function is very known is in each book.

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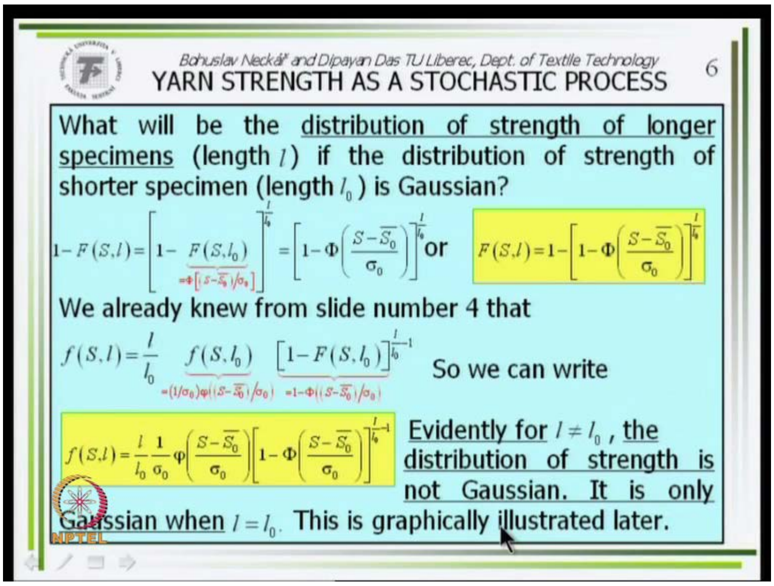

This is distribution function of standardized Gaussian distribution. Well, now how it is with our distribution of yarn strength on the long lengths. Wewill derive 1 minus F S l is 1 minus F S l 0power to l by l 0. Usingrelations derived on earlier slide, we obtain that it is 1 minus phi s minus s 0 bar by sigma 0 power to l by l 0 or F S l is 1 minus this here.

If it is right that the distribution of strength in short lengths isah givingaccording the Gaussian distribution then on the long gauge length l the distributionfunction is given by this expression. So, that we mustthe distribution function of standardized Gaussian distribution and we calculate it in the point s minus s 0 bar by sigma 0.

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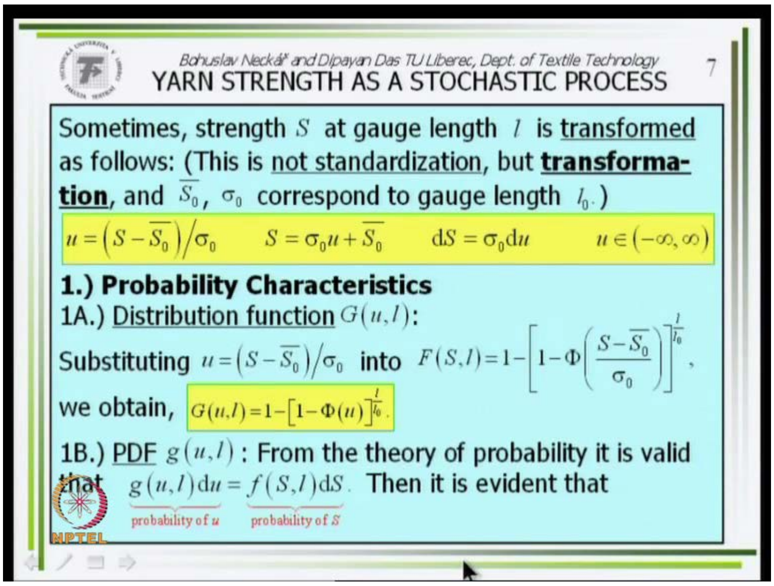

What we need to know?We need to know the mean value s bar 0 and standard deviation sigma 0 which have the short analysis for example, when we now we make some experiment there from this experiment we know thatmean value on 5 centimeter lets imagine that the short lengths is 5 centimeter. So, we will use the gauge length short gauge length 5 centimeter 1 0 and from alaboratory we obtain mean value ofstrength and standard deviationvery standard method for evaluation of set of data.

We have s as 0 bar and sigma 0 clear and now how is the value of distribution function and Ido not know half meter gauge lengthsconductively, we use half meter gauge lengths in yarnhalf meter gauge lengths l is 1 half meter how is this value. Howis this distribution function is easy you must calculate this functions phi standardized distribution function in all points orpoints numericalpoints s minus s 0 bar by sigma 0.

So, is it clear how to apply it and we obtain distribution function for long lines. Whenweshow through this way we when through this way we obtain the distribution function for one half meter gauge lengths we can compare it withexperimental resultstake it on one half meter standard gauge lengths. But, we can through this we also obtain the distribution of strengths which probablywe writefor the gaugelengths5 meter for example, and we have notchanced experimentallyobtained from laboratory. So, special breaking machine we have not, from this we need may be dance roomfor such breaking machine ok.

The distribution functionyou can see this over here that thisfunction this distribution function (Refer Slide Time: 22:18) and the gauge lengths l; another gauge lengths follows this expressions. So, that it is not Gaussian distribution; Gaussian is in this case only the distribution on the starting short lengths l is also 1 0.

We can alsoderive the distribution function which isderivative from one probability density function which is derivative from distribution function and so, we obtain suchusing this easy wayyes. Because, with understanding in the moment I do not know what it is your value as 0 bar and sigma 0 what is your value mean value of strength in short in short gauge lengths and what is your standard deviation.

Nevertheless we want to have some better picture how isalso graphically proved explain how are the relations how is the change thisof quantities of functions therefore, letsintroduce some linear transformation which is here u is s minus s 0 barsigma 0 when on the place of s breaking forces give the values from gauge lengths 1 0 then it is standardized random quantity clear, but when on the place of s you use random quantities from general from another gauge lengths gauge lengths l, then I cannot call it as a standardizing then it islinear transformation ok.

For example, I have values from one half meter breaking that the strengths from one half meter and now I construct the quantities u random quantities u every times the strength from the yarnby lengths one half meterminus mean value mean value ofmean strengths value by gauge lengths 5 centimeter by standard deviation by gauge lengths 5 centimeter by 1 0 clear it is a linear transformation.

It is possibleto rearrange our equation to anew a distribution function all distribution of function of s breaking force then distribution function of. So, defined linearly transformed quantity name u and I obtain the distribution function g u 1 it is evident this fromthis derivation very short and very easy, but this g u r is 1 minus 15 upower to 1 by 1; 1 by 1 0 and the probability density function we obtain.

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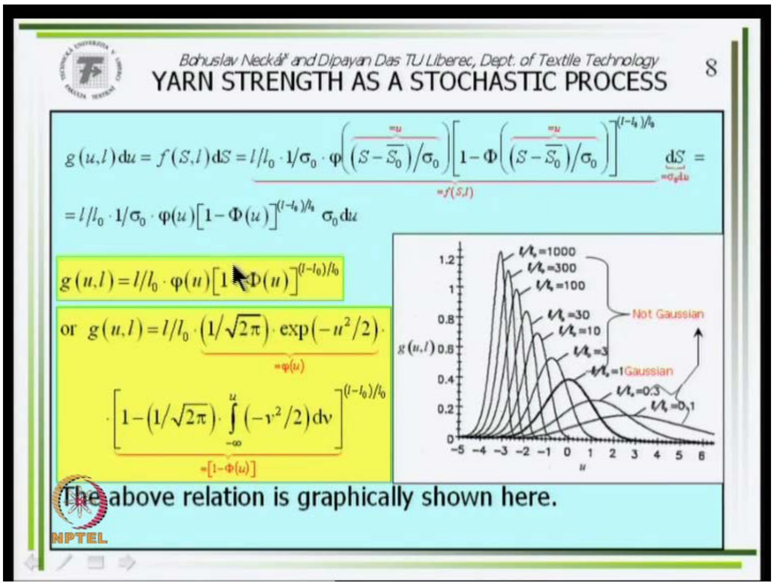

Similarly, its derivative from this here it is shown here after derivativeit was f this is under place F S l and this is from the s. So, that we obtain a probability density function n such from or especially because from the… because Gaussian distribution then capital phi is distribution function of standardized Gaussian distribution and we obtain we obtain these equation for practicalapplication graph and we can show how is the probability density function for different lengths l.

If the lengths l ison the graph, a quantity u, linear transformedbreakingforce strength of the yarn and on the ordinate is the probability p d f probability density function g u for different gauge length is l if l is equal 1 0. So, that our experiment is also on our minimum gauge lengths 1 0 then we must obtain because it was our assumption; we must obtain Gaussian distribution. Inthis case,it is standardized Gaussian distribution this stick cover here this stick cover here.

When wehave for example, length 50 centimeter and starting 1 0 was 5 centimeter then fifty centimeter is ten times longer gauge lengths. So, then one by one, 0 l by 1 0 where is the curve for by 1 0 is 10 it is this curve here, it is this curve, clear?It is another distribution it is another distribution the position of this hat ofprobably curve of the probability density function is going to the right hand side to a smaller values and a I can say thickness of this hat is a little smaller standard deviation is smaller as well as coefficient variation.

You can see that we can calculate it for different values 1 by 1 0 hereand it corresponds principally good to our laboratory experiences means when we use longer gauge length then the distributionhave the mean value is small and standard deviation is smaller too by the same products same yarn yes. So, you can see how it is you can also see thatthe curves this is right Gaussian distribution.

When we use very high value 1 by 1 0 it is also by our eyes to see thatthis curve is not Gaussian for example, this curve is not fully symmetric see it clear. So, its another distribution;its another type of distribution. Eachgauge lengthis have some other distribution only the lines gauge lengths lequal 1 0 have our starting Gaussian distribution.

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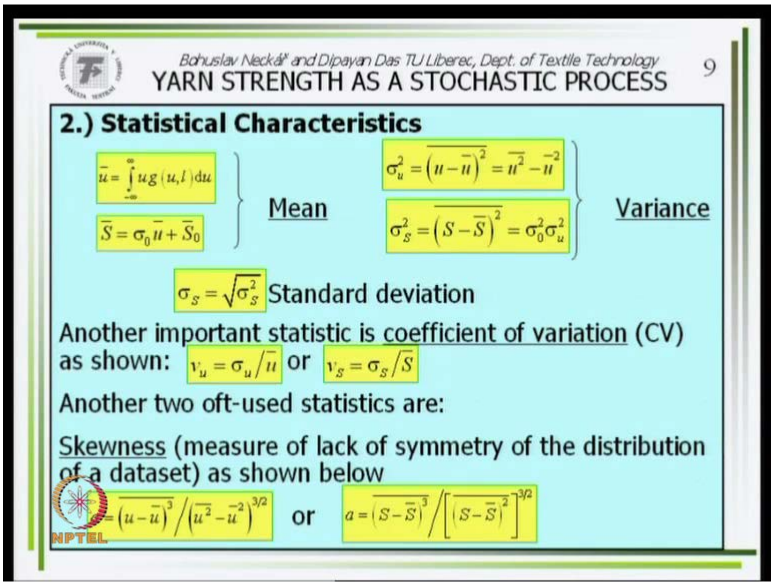

It is possible to evaluate also some statistical characteristics u bar which is even hereor ultimately s bar mean values which can bethen evaluate using this expression we can evaluate s square of standard deviation of this dispersion variants of uand using the this 1 also thevariants of s strength variants which is here then of course, standard deviation as a square root from dispersion variantvariants then we canevaluate.

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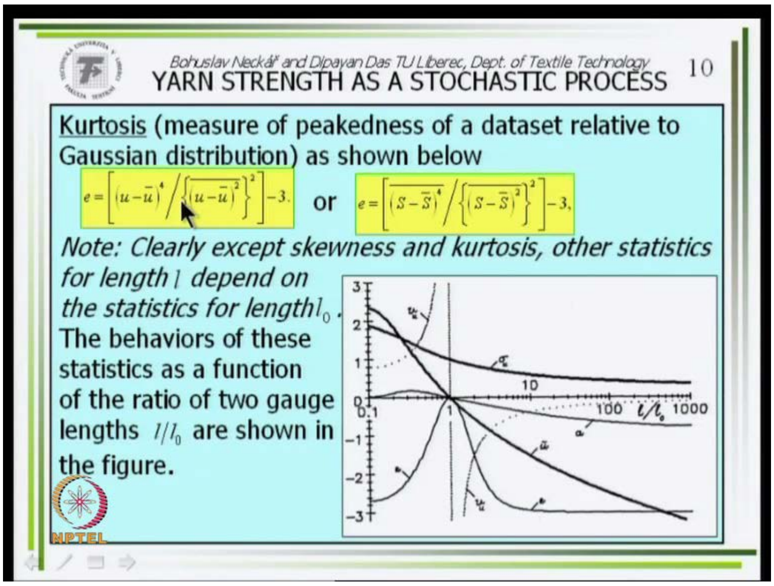

Anotherimportant statistic as for example, coefficient of variation v u as well as s we canobtain alsoskewers using this expression here or this one as well as Kurtosis curve forprobability density functionthis graph show different statisticalquantities as a function of gauge lengths important line is two thick curvesthe mean value oh sorry the mean value is this curve it is shown that the mean value of strength is increasing of the gauge lengths is ratio 1 by 1 0 is decreasing and the standard deviation is decreasing too there is also the path for shorter analysis then 1 0 thatis possible to go to another side, but for us is enough to understand how it from 1 0 to higher gauge lengths.

Wellthis all is nice, but it need to use a relatively know to nice numerical method by calculation today its principle possible nevertheless for people which are able to prepare somecomputer programmed some software for such evaluation Peirce's create it around nineteen thirty in that tine do not exist computers and I must say that based on my meaning mister Peirce's was very junior and very high educatedbecause this is was too complicated forto calculation thisfinal values by hand only.

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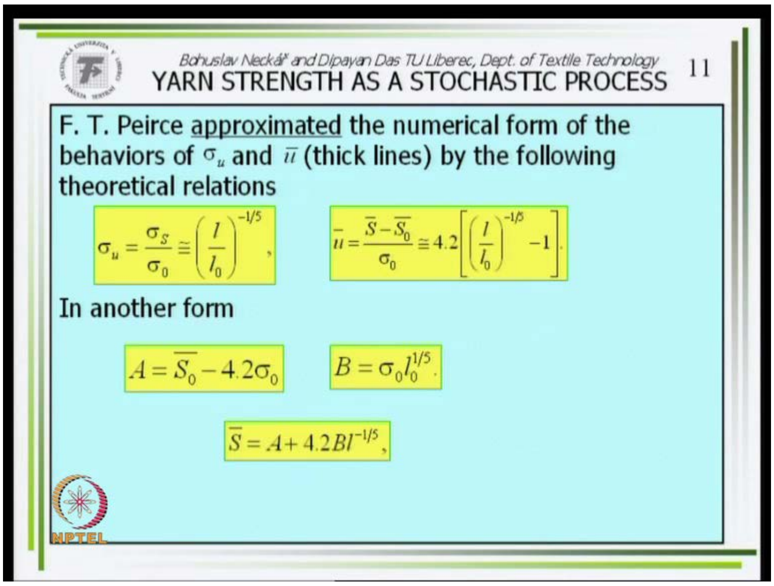

He proved to construct some approximation equations and when you studied thismaterial you can see how deep was education in theory of probability and properties ofprobability density functions and. So, on resultingequations which we which we obtain as a approximation of our way because.

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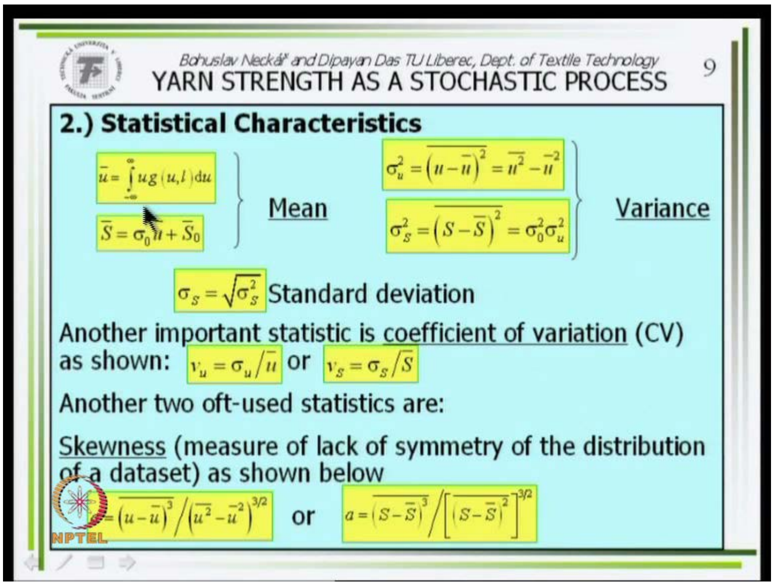

We need for we need for mean value this integral we need for standard deviation over yarnsthan this here. So, this very difficult integral is not analytical I said it is Gauss integral. So, it was very difficult, but he found something which is relatively good approximationin my book as well as in other books for example, cosigns structure of the yarn that polish bookis compact how is the difference between the approximation original results and I can say that not practically too important. So, we can go; we can very well use following equations.

He said that sigma e which is sigma s by sigma 0 sigma s is standard deviation of the strength by lengths l our lengths longer lengths l sigma 0 is standard deviation byof yarn strength by gauge length 1 0 and that is ratio is l by 1 0 times minus 1 by 5 the second equation is the u bar which is s star minus bar minus sbar 0 mean value of yarn strength and the cosigns 1 minus mean value of yarn strength by gauge lengths at 0 by sigma 0 standard deviation of yarn strength of short lengths 1 0therefore, 0 is four point two times 1 by 1 0 power to 1 minus by minus 1.

We can also this result of mister Peirce's rearrange a flowing from some parameter a is s 0 bar minus four point two times sigma 0 the other parameter b is c by 0 times 1 0 power to one by 5 and then s bar is a plus four time four by two b times one power two minus one by 5 how to practically use this equations for example, you are you have studied some weaving process lets imagine in our modern weaving loom weft yarn where the Ido not know free meter for example, long.

We need to have some information how is distribution ofyarn strength by gauge lengthsgauge lengths is this the lengths on which ison which we exponent our left yarn.

So, symbolically gauge length three meter its much possible is standardbreaking machine, but you can same yarn study in your laboratory and you can study it for example, using one half meter standard gauge lengths. So, you can thinkmy standard my short gauge lengths is one half meter 1 0 my long lengths 3 meters I observe the work in laboratory and I evaluate standard the values of yarn strength and what I obtain I obtain as 0 bar mean value of yarn strength by 1 half meter gauge length and standard deviation sigma 0 from our data know.

So, this two are known then can obtain a parameter a I can calculate the parameter a as well as the parameter b where on the place of 1 0 is now 0 point 5 meter our short lengths I able to calculate both and how is the mean how is the meanas strength by three meter length long gauge lengths you have these equations here.

And so on. So, that through this way is relatively easy possible to obtain themean value and standard deviation and.

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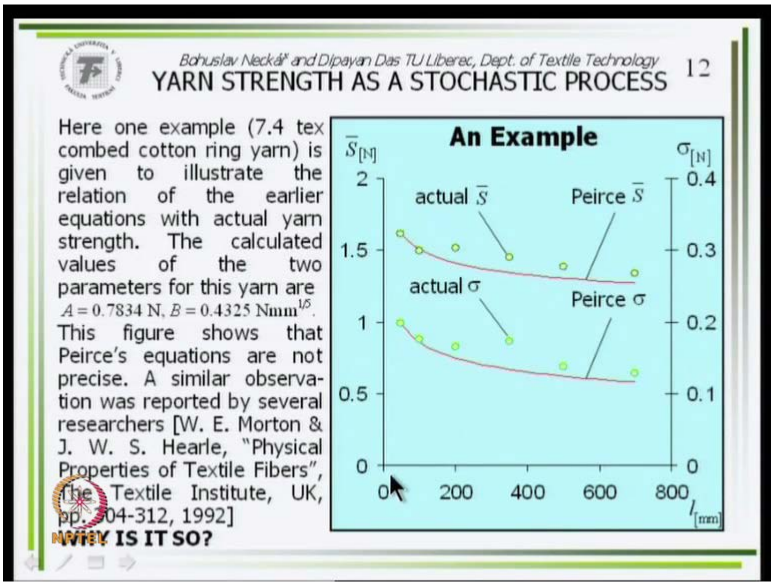

So, on to the gauge length we should do not measure based on the results which you measured this graph is one of graphs from work of our doctor dipayan das why it in the from here which he was university is not it.

The points here are experimental points it is an example from some yarn which of combed yarn seven point four from cotton it is the yellow point are experimental pointsbasedhow wehow we measured in ourmethod we used 5 centimeter 5 centimeter gauge lengths as a shortest 5 centimeter was for us was for us the shortest 1 0 gauge length why use 5 centimeter because the small is gauge length must be longer than each fiber.

In other case, you do not measurethe strength of the yarn then partly you measure strength of fibers when we use for example, 1 centimeter gauge lengths and lot of fibers you will have in both the jaws. So, it is not the strength of the yarn then the from fifty percent each strength of the fibers .

So, that we choose 5 centimeter as a short gauge lengths andah then we then we measure one beside other one portion 5 sentence second that and so on. So, we obtain lot of 5 centimeter lengths and. So, on and from this we evaluate also the strength on the different are the different are the lengthsthe experimental points from different gauge lengths. Yellow based on the shortest it means 5 centimeter gauge lengthswe derivethe parameters which is necessary and we derive the red curves which are here these red curves show the Peirce’s mean value in the relation tomean values measure experimental values and the second graph showthe standard deviation standard deviation which is measured the yellow points and the values constructed from the shortest gauge lengths.

Therefore, in this point 5 centimeter, it is 5 centimeter,15 millimeterthe experimental point as well as the red curve is in same point. Well, you can see principally this theoretical concept corresponds to the reality. Itcorresponds to the reality; nevertheless, some distances you can see the red curves are little under our experimentalis not itits typical for this equations why I want to say in our next lecture. So, in the moment thank you for your attention and the next lecture, we will continue it is notes to the possible modification of the Peirce’s concept; thank you for your attention.