

Orientation of Fibers.
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Lecture No. # 16.

Relations among Yarn Count T, Twist Z, Packing Density, and Diameter D

Well, good afternoon, everybody. Last lecture we spoke about Pierces's model. This model introduced the general logic of the problem of strength distribution in different gauge lengths; Yarn Strength distribution (())

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YARN STRENGTH AS A STOCHASTIC PROCESS

How far Pierces's assumptions are valid on yarns?

Assumption No. 1) **Independent Strengths:**
Does not hold [J. L. Spencer-Smith, *J. Textile Inst.* **38**, P261-271, 1947]
He hypothesized that the fracture zone (actual breaking place) strengths are dependent, but this was not experimentally verified due to complications in measurement of the fracture zones in yarns.

Assumption No. 2) **Weakest-link principle:**
Not appropriate [J. L. Knox & J. C. Whitwell, *Text. Res. J.* **41**, 510-517, 1971]

Existing [D. F. Kapadia, *J. Textile Inst.* **25**, T355-370, 1934], [R. C. D. Kaushik et al., *Text. Res. J.* **59**, 97-100, 1989], [G. F. S. Hussain et al., *Text. Res. J.* **60**, 69-77, 1990]

Nevertheless, Mr. Pierces use some set of assumptions. One of his assumptions was on independent strengths. Based on the idea of Pierces, each short fiber portion, the strength of each short fiber portion is independent to each other; each is independent. During the experimental works, he was doing, the results were harder. The strengths of short segments are not independent. If one segment or one part length is 0, have relatively high value of strength, the probability that the neighbor will have also a little higher than the mean, than the average value of strength. The probability is higher, and then this is absolutely independent. You can imagine also intuitively that this relation is possible good to see,

because if this yarn is good oriented fiber then its neighbor also, the fibers will be good oriented and something so.

So, the independency, this is a problem which is not valid. Weakest-link principle, of which I spoke it in the last lecture; of course, we must work on which gauge length higher than longest fiber lengths. When we use the shorter, then the Weakest-link principle is absolutely out, because we measure something other. Nevertheless, when we use such, then different authors have different meaning. At most, they mean it that we can accept Weakest-link principle as a good theoretical assumption for our modeling.

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Assumption No. 3) **Normal distribution of strength of short Specimens:**

Not adequate [D. F. Kapadia, *J. Textile Inst.* **26**, T242-260, 1935]

Adequate [K. E. Perepelkin, *Fiber Chem.* **23**, 115-133, 1991], [N N Truevtsev et al., *J. Textile Inst.* **88**, 400-414, 1997]

NOW WE WILL PRESENT A NOTES TO OUR STOCHASTIC MODEL OF YARN STRENGTH.

The third is normal distribution of strengths of short specimens. In short specimens, the experiments results show that often this assumption is acceptable; not every time, but very often it is yes. So, what are the highest differences? It is the point one, which means the independent strengths, no they are not independent (Refer Slide Time: 04:37).

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Let us consider a long length of yarn, which is equally divided into small sections of length l_0 , as shown below. These sections are serially designated by the numbers $i = 1, 2, 3, \dots, k, \dots$. If strength of these successive sections is measured one after another, then a time series of strength values $S_1, S_2, S_3, \dots, S_k, \dots$ is obtained. S_i

Section No.	1	2	3	i	k
Strength	S_1	S_2	S_3	S_i	S_k
Distance from section No. 1	l_0	$2l_0$	$(i-1)l_0$	$(k-1)l_0$	

So, it was necessary to create some more general model in which among the short fiber portions, I mean the strength of the short fiber portions, had some dependency. Let us imagine a yarn symbolically divided into some portions lengths like l_0 . The distance from here is one times l_0 , two times l_0 , three times l_0 , till i minus 1 times of l_0 and so on. Each portion, each segment has its own strength as S_1, S_2, S_3 and so on. So, we obtain some series of quantities like S_1, S_2, S_3 , up to S_k and so on.

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Now consider a longer specimen of length l consisting of $k+1$ number of short specimens of length l_0 , such that $l = l_0(k+1)$. Now strength S^* of the longer specimen is the minimum of those strengths of shorter specimens. (This is the principle of the **weakest link theory**.) Mathematically,

$$S_i^* = \min \{ S_{i+j} \}_{j=0}^{j=k}$$

Thus, we can generate a huge number of strengths S^* for a particular value of k , i.e. for a particular gauge length l . Using these values, the PDF $g(S^*, k)$ and statistical characteristics (mean, variance, CV) are obtained. The same technique can be repeated for different values of $\{0, 1, 2, \dots\}$, i.e. for different gauge lengths $l = l_0, 2l_0, \dots$. Thus strengths at different gauge lengths are obtained.

What is the strength of some fiber lengths in which are the segments from with segments to k plus k ? Let us imagine some lengths, in which we have k plus 1 segments, from subscript j equal 0 to j equal k . How is the strength? It is the minimum from strengths, from all the strengths. Is it not? Automatically, say this is minimum of this strength set. Is it clear?

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YARN STRENGTH AS A STOCHASTIC PROCESS

An Example (7.4 tex combed cotton ring yarn): The stochastic measurement was realized as follows: the successive yarn sections each of 50 mm length was marked by 1,2,3,...,60; and strength of the sections marked by 1,3,...,59 was measured; the remaining sections were used for clamping. Thus strength of 30 sections was realized. This procedure was repeated 30 times at different places from the randomly selected 6 bobbins so as to obtain 900 strength values. Strength at other gauge lengths was also measured. The **descriptive strength parameters** are shown in the table. Evidently, strength is decreasing with increase in gauge length.

Statistical Parameters	Gauge length [mm]					
	50	100	200	350	500	700
σ_m [cN/tex]	21.81	20.22	20.44	19.61	18.74	18.11
V [%]	12.25	11.76	10.90	11.96	9.91	9.56

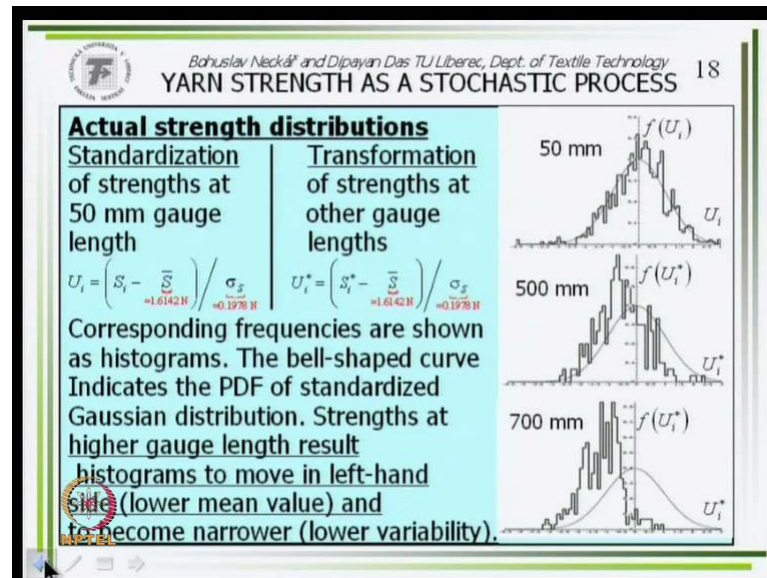
(effect of the weakest-link theory)

Our experimental work is as following. The table illustrates experimental values. We have some instrument which helps us to use five centimeter gauge lengths; l_0 equals 5 centimeter, and we measure strength, then second five centimeter was a technical length because summons is necessary, because in both jaws we must clamp the yarn segment. Is it not? So, in our system it was so that the jaw was here and here and we measured strength of S 2 (Refer Slide Time: 07:35). Then the jaw was here and I think here, and we measure the strength of S 4 and so on. So, one beside the other, we knew the distances between neighbor portions is length of five centimeter.

Then we evaluate on how the strength of longer lengths are, because we had this small. We obtain experimentally; experimentally we obtain this. This is immediately measured and it is from different gauge lengths. It is on breaking machine. For the same yarn and we obtain such values like the mean value and coefficient of variation. For five centimeter, it was 21 centi-newton per tex, and CV is 12 percent. By 77 centimeter, which was longest possible in our breaking machine, we obtain 18 centi-newton per tex and 9.5 percent as

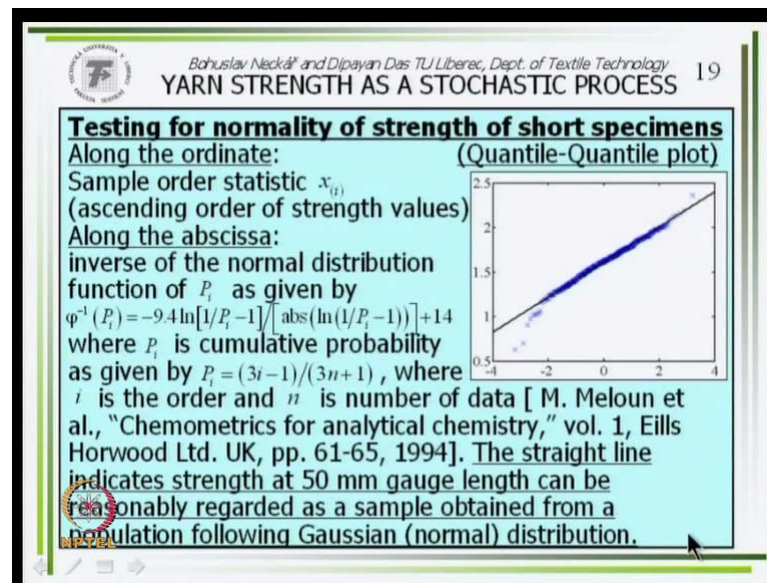
CV. You can see that really, in reality, the strength is decreasing as gauge length is increasing. This is the gauge length. The mean value of strength is decreasing and CV coefficient variation of this quantity is decreasing to.

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You can see how the distribution is by 50 centimeter, by 500 and by 700 centimeter. It was same yarn but different know how for measurement; One time it is 5 centimeter gauge length, then 500 mm and then 700 mm. Can you see this histogram, how is going on this left hand side? This histogram is recalculated to the standardize values. So, you can see this with linear transformations, which we did so that you can see it, is going to left, more and more to the left hand side in opposite to this; that is Gaussian standard probability density function.

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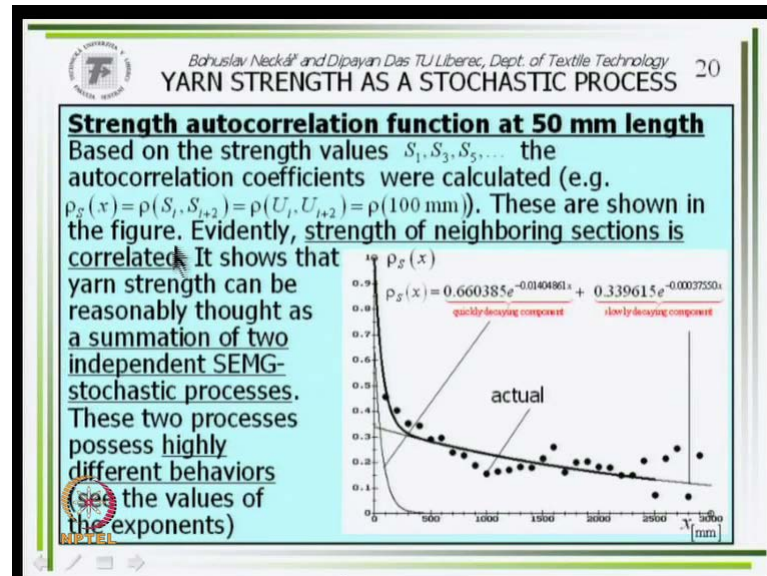


Gauss, the idea of Gaussian distribution in short lengths, in this case is five centimeters lengths are relatively good. As shown, in this example, it is the same yarn and Quantile-Quantile plot, may be you know what it is. It is some graphical system through which you can watch if this or that distribution is or not Gaussian. You can see that the dominant part of points are lying on straight line, which says us that this distribution is practically very good comparable with the Gaussian distribution. So, the idea of Gaussian distribution **can be...** But, how it is with dependency or independency? We use so-called auto correlation function. Do you know what the auto correlation function is? May be you know what is the coefficient of correlation, you know what is the coefficient of correlation, you know it, yeah. So, it is good.

Let us imagine our yarn. (Refer Slide Time: 11:46) We measure, let us imagine theoretically, we measure, we have the set of quantities S_1, S_2, S_3 . We can create some couples; S_1 and S_2 as second couple, S_2 and S_3 as third couple, S_3 plus S_4 and so on. Every time, this couple of quantities, represents two parts in which the distance is one times l_0 . Well from these couples, we can evaluate a coefficient of correlation. Is it possible? Yes, we evaluated and we say this is the coefficient of correlation for distance of this couple of strengths one times l_0 . Nevertheless, from the same set of experiments we can evaluate, we can create other couples; S_1 and S_3 , S_2 and S_4 , and so on. The two values in each couple have the distance on the yarn of two times l_0 . Is it not? Is it imaginable? We evaluate coefficient of correlation and we can say this is the coefficient

correlation for the distance $2l_0$. Similarly, from the distance three times l_0 , to all possible distances.

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Like this and we can create some graphs. On the abscissa, we can give the distance between our couple values like one times l_0 or two times l_0 or three times l_0 ; this is distance. On the ordinate is the value of coefficient correlation, and we obtained such points like this which are here. This is one real example. Can you imagine this way? I hope yes, it is possible. Evidently, the coefficient correlation at the distance 0 must be one, because when you have couples from one value and the same value, the second value and the same value, then the correlation of couples where both values are every time the same, then it is 1. Is it not? So, this point is also one, of point from this coefficient of correlations.

The function, which says how, is the change in the coefficient correlation in relation to the distance is the correlation function. We call it as a correlation function; nothing more. And because both these values are from the same yarn, when we speak about other yarn it can be quite out of problem. Now, from the same set here means in order from the same yarn. Not the first from the blue yarn and the second from the yellow yarn. Here, they are the same yarn. Therefore, we must call these functions as auto correlation function. It is correlation in the same yarn.

For this auto correlation function and the experimental trend of auto correlation function is given by this black points here is real experimental results; this is experimental results. Now, how to formulate the model; mathematical model, stochastic model of this auto correlation function? We use set of ideas; I want only here to present the name of these assumptions, which we used. We think that the series of our quantities, like those S_1 , S_2 and S_3 and so on, it is this series represents of some stochastic; represents of stochastic process.

In the theory of stochastic processes are known in following trends like stationarity; stationary process, stochastic process. I want to explain it only very vaguely or very intuitively. So, if somebody of you wants to know it in the mathematical precision then please forgive me. Stationary is the process when it is a random process, but it oscillates around some quantities, around some value. For example, processes, which are permanently in random and by general, the strength is increasing then it is not stationary.

So we assume it as the stationary process. The second is ergodic process; ergodic is intuitively is very difficult to say; very intuitively, I can say it is a normally imagined as what we understand, what we feel under the term stochastic process. Markovian process, it means that the influence of the following value in our series, the influence of following value have only the value which is immediately before. So, the strength of twelfth segment of the yarn is influenced through the strength of the eleventh, but not influenced through segment number three, for example. Only the neighbor can influence the following, the probability or the following value. Similar processes are named as Markovian processes.

And final is Gaussian; it means that we will use distribution according or normal distribution according to Gauss. Using this type of process, and using the set of different mathematical tools, we can derive (Refer Slide Time: 19:02) that the auto correlation function must be exponential curve; one exponentially decreasing curve. It is a mathematical result of process, which are Stationary, Ergodic, Markovian and Gaussian. Nevertheless, our experimental results do not correspond with exponentially decreasing function. Therefore, we say it is not well. In reality, it is a little harder. The situation or the real situation seems little harder.

We start to think about two independent; such two independent processes having another parameter and then we obtained very good result. It is shown in this example on this picture (Refer Slide Time: 20:04). One process has this character of auto correlation function. It is, in this moment, I do not know, is roughly 35 percent of this process plus 65 percent at the second process, which is going from here to here. The auto correlation function is slowly decreasing in one case and quickly decreasing from the other case. Sum of both together give the resulting thick curve, which could explain our strengths measured in our laboratory.

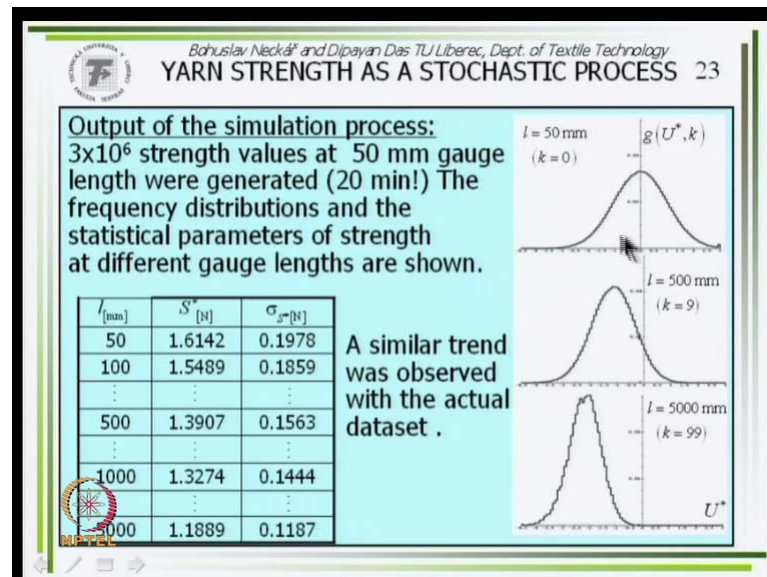
What it means from point of view of logic? The variability of our strengths in short ones is S_1 , S_2 , S_3 , which was in our first picture is a result of two independent influences. One influence decrease very slowly and second decrease very quickly in autocorrelation function or the correlation decreases slowly in the second case. How is the physical sense of these two influences? Nobody know to this time. Nevertheless, one of them, may be this more quickly downgoing part is a result of variability of a mass variability of the yarn. We compare it with the result which we obtained from (()) (Refer Slide Time: 21:54), we recalculate the primary values from (()) to auto correlation function; it is also difficult. Ok, jump these difficulties.

We obtained something which was similar to this curve (Refer Slide Time: 22:16), but was the sense of this influence; nobody knows. We can have lot of hypothesis, but none is verified. In each case, it exists of two independence influences, which create both together the resulting variability of strengths of our portions on the gauge lengths l_0 . Based on evaluation of such experimental curve, we obtained parameters through which it is possible to calculate all other equations and as finally, to calculate the probability density functions or standardized for different (()) (Refer Slide Time: 23:19). We use the simulation method because the analytical version is very difficult.

Therefore, we use the computer simulation. We simulate the strengths S_1 , S_2 , S_3 like this here, using computer. We give in to the computer, the corresponding equations and parameters and in computers; it is possible to produce the strengths S_1 , S_2 , S_3 with the same distributions S in original yarn, with this computer simulation. Then we can evaluate also the distributions of strengths for longer length easy from lengths 10 times of l_0 . Now, we produce by some our software, we simulate the first 10 values and minimum next link... Minimum of these 10 values is the strength of lengths, the first

lengths is ten times l_0 ; second 10 values generated from computer represent the second long lengths and minimum value, it is the strength of this one. So, using this way, using this simulation way, we can obtain, we can simulate the set of values of strength values for different lengths.

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And there are the probability density functions for such. For 50 millimeter it is here. For 500 millimeter it is here, from computer, and from 5000 millimeters, 5 meters, it is here. You can see similar effects as by Pierce's model. Now, when we have this set of values, which are obtained from computer using the simulation; using these set of simulated values, we can evaluate from this set, the same way as in laboratory, the mean value, standard deviation, coefficient correlation; all these standard statistical characteristics.

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Approximated empirical strength-length relation particular to this yarn

The descriptive statistical parameters obtained as an output of the simulation process were very well approximated by the following empirical relations

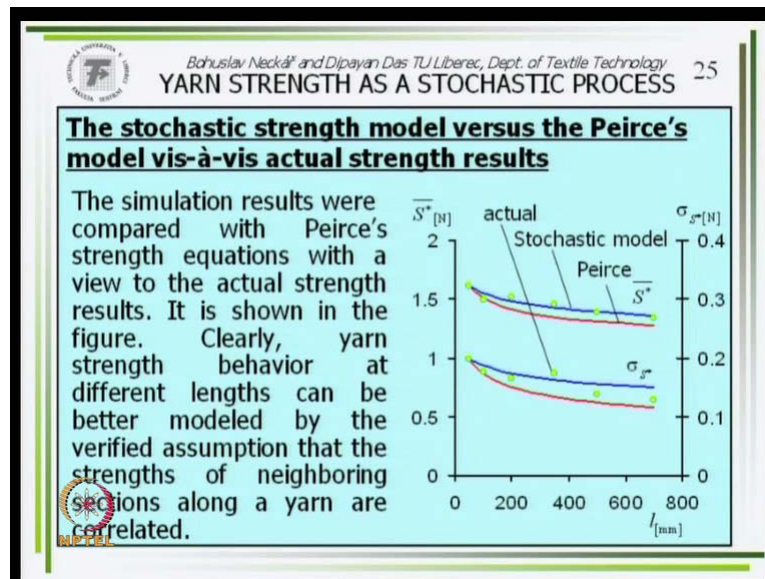
$$\bar{S}^* = \bar{S} + 5.33\sigma_S \left[\left(l/l_0 \right)^{-1/9.30} - 1 \right], \quad \text{and} \quad \sigma_{S^*} = \sigma_S \left(l/l_0 \right)^{-1/9.30}$$

Compared to Peirce's two unique values (4.2 and 1/5), we obtained the coefficient equal to 5.33 and the exponent equal to 9.30 in this particular yarn. Our experiences tells that these values are different in different yarns according to different degree of correlation of strength of successive sections along the yarns.

And when we had this, we prove to apply also similar equation then to Mr. Peirce and we obtained the following expressions. They are based on our experiences for yarn. They are good, we can use it. Nevertheless, the exponent here is now 1 by 5, then 1 by 9.3, and this coefficient is 5.33 and was 4.2. Peirce derived it for his model, individual **independent fiber, independent yarn portions**. Now, it is for independent, we call it SEMG-stationary ergodic, Markovian and Gaussian process or dependent, so that we obtain another parameter, which can approximate the resulting values like here.

What is interesting? Professor Rurek, very renowned name in the theory of yarn, he is very old man in Poland in the town Lodz, in University of Lodz. He derived, he measured this problem and he derived empirically. The value of this exponent is 1 by 7. (Refer Slide Time: 28:08) Based on our experience, we have 1 by 9. Now, it is written the reality have higher value, when he approximate the reality to this equation, and the reality need to use higher exponents than is the Peirce exponent which is 1 by 5 derived theoretically only. Some story, when Professor **Rurek** was 70, may be 75, I do not know. No, 70 maybe, it was an international conference in Lodz and I presented here some contribution that why Rurek value 1 by 7 is better than earlier Peirce number which is 1 by 5, because this story, say it theoretically.

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Now is the resulting; the examples of the results, points are same as earlier, points are experimental points. Mean value and standard deviation, in earlier Peirce model; it is the red curve here, this and this. Now, when we use our model, which accepts the mutually **dependence** of yarn segments, we obtain here the blue curve. You can see that there is blue curve is little better than the earlier red curve.

So, it is a short overview, nothing more. Short overview over this model of yarn strength is a Stationary, Ergodic Markovian Gaussian process. We call it SEMG process, which bring a little better result. Nevertheless, full derivation is relatively long and need to use **quite...** No easy methods for derivation. Therefore, I want to recommend you when such problem will be especially to somebody of you, I actually recommend you to call or invite Dr Dipayan Das from IIT D, and ask him, because he has 3 years of works on this theoretical, as well as these experimental concepts. He is in each detail very good informed. This is all for the program of yarn strength or for the problem of all textile questions about which was my set of lectures. My last words are my acknowledgements.

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LECTURES ON MODELLING OF FIBER ASSEMBLIES AND YARNS

The lectures delivered in this course represent some of my original research as well as didactic works. I would like to heartily thank to a lot of people for their support and help behind these works. Some of them were:


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- **Prof. Ishtiaque** from IIT Delhi, who inspired me for preparation of these lectures
- My earlier doctoral student and now **assistant Prof. D. Das** from IIT Delhi who was first reviewer and English corrector of all lectures

I want to thank. At first to my colleagues from our university namely from the department of Textile technology, who helped me in my research works very much. Then I want to thank to Professor Ishtiaque, your known professor from department of Textile on Indian institute of technology, Delhi, because he inspired me for preparation of this set of lectures. And third I wanted to thank to Dr Dipayan Das, to this assistant professor of the same department, who was first reviewer and English corrector of all of my lectures.

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But my greatest thank goes to my dear wife,

Mrs. Hana Neckářová

who was able to create a favorable family atmosphere and had extremely great understanding for my professional work during the 34 years. I am very sorry to say, she breathed her last on February in 2004th year.

(Our trip to the Jizera Mountains together with Mr. Dipayan Das, PhD.)

Well, my greatest thank goes to my dear wife, sorry, I am very sorry to say that she breathed her last on February 2004. She inspired the whole of my professional work and whole my life. Here, this woman is my wife, this person is known to you; it is Dr Dipayan Das. I am here today. You see my only my professor mask today. This is my nature face here and is for our trip to Jizera Mountain, which are the mountains by our town, where I am living and where is our university. This trip was when I was 60 years. Is it not? Then I was sixty years. So, this is my thanks. I thank also you, because you was very good auditorium for me. I wish you good luck, successful study and all good what you want.

Thank you.