Orientation of Fibers

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Module No. # 01

Lecture No. # 17

Bundle Theory Of Yarn Unevenness

We have today, professor Bohuslev Neckar.

Good morning.

Professor of technical university Liberec Czech Republic, it is indeed a pleasure to have a professor Neckar with us and we have an opportunity to get his wisdom, what he has put last 35 years.

Yes.

Of his work and what he is going to cover in this particular course, he is going to talk about his new ideas, new concept, logics to come out a particular aspect of a, this particular subject that is your theory of Textile Lecture and in this particular course that he will cover a good number of lectures, where that he is going to talk about helical model, integration migration model, compression model. And at the same time he is going to talk about the theories, but we have studied in the literature as like a one literary (()) and there that to with his whole idea that he has tried to drive with Cole's equations and a tried to work out what are these assumption, what are being considered earlier he has taking care of all these issues.

So, that these equations now or that information what we are doing, so we can say it is updated in today's contest and another aspect that, which can be noticed from his lecture what a modeling, that he is doing that excellently, that he tries to prove these concept, so that the logical concept which has, he has taking into consideration in his lecture can be experimentally seen, with this now I will I will request professor Neckar.

Thank you

To start

The Lecture

Thank you for your nice introduction.

Thank you

I want to say you hi everybody, well let come to our first theme, and let us come to the theme compression of fibrous assemblies.

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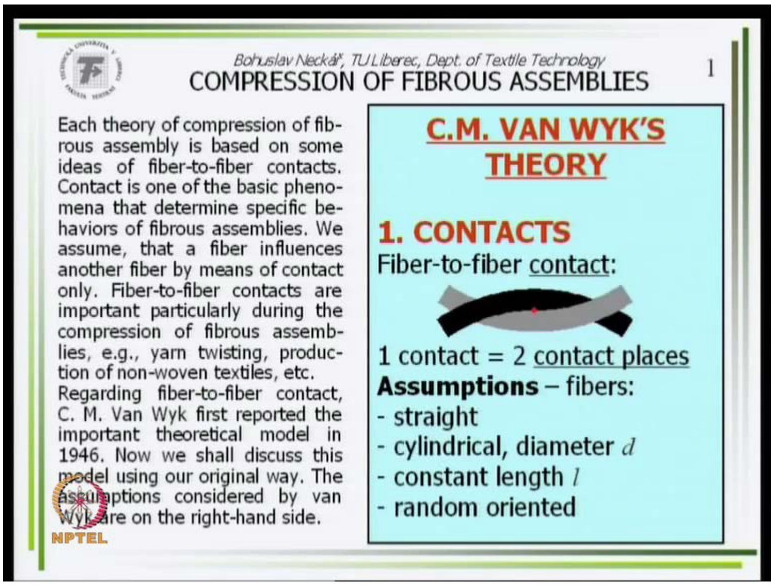

The compression is very very important process in textile technology, we compress fibrous material for example, by creation of a normally one textiles, we compress the fleece by bonding process, we compress the fibrous inside the yarn therefore, we twist and so and so also, that the compression is very interesting and very important process.

We can be sure only about the one in, only in a one moment it is that this compression regulation do not follow known Hooke's law. So, the linearity between the force and deformation effect, it was work of people earlier which studied this problem the known is name is C.M Van Wyk's theory, and about the Van Wyk theory I want speak in this first lecture, then I will mention also our own theoretic by some generalization of this theory, but it will be in the second lecture Van Wyk.

Van Wyk was some scientific person, it it is when I say Van Wyk it is like (()) some some Holland, Dutch Dutch painter, but he was not from Holland he was not painter, he was some scientist in South Africa and he studied the compression of fibers material to the bales, it was interesting to know how is the relation between the pressure and the volume of (()).

Theory of Van Wyk started with some assumption speech theory, he assumed that the fibers are straight are cylindrical having diameter d, have constant length l and a random oriented; what is it, the fiber contact you see on the scheme in our slide, it is this one the the contact of 2 fibers, 1 contact 1 contact represents, two contact points then you give some small drop of red color between these two fiber, then you will have two point two, two contact points, one contact is two contact points.

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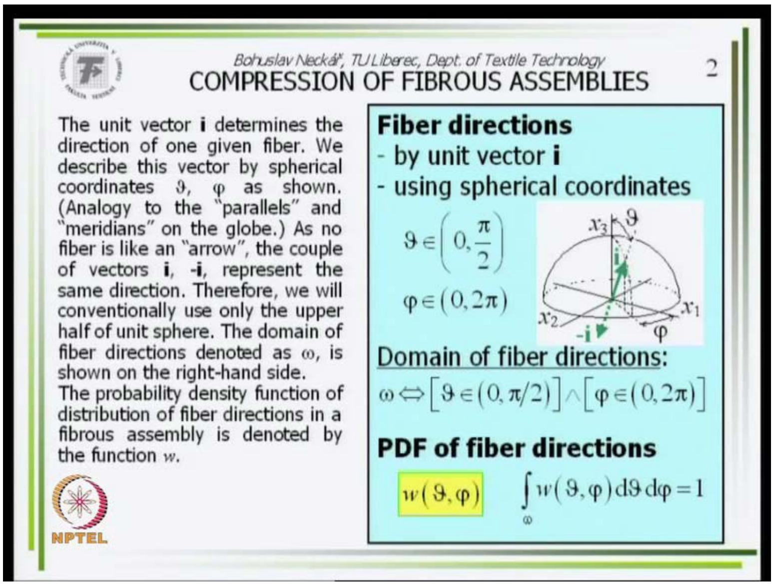

Fibers in the structure have have random distribution of the orientation, how to how to characterize this fiber orientation, it is necessary to characterize through some unit vector, such vector like vector i on our scheme of course, our fiber let us imagine that this is fiber, fiber having not is arrow, fiber have not connector of arrow. So, that this direction and this direction half of fiber only one direction, is not it (Refer Slide Time: 6:54).

Therefore, we use some convention that the, we will use the half square for for vectors which will characterize our our our orientation of our fiber, portion of fiber segment and it exists couple of two angles and angle theta to x 3 axis and angle phi to x y axis, which characterize direction of unit vector, and so direction of our fiber fiber portion, angle theta is going from 0 to pi by 2, because we only use half of our possible square, and phi is from 0 to pi, because no write too long form we will use symbol omega as a domain of fiber direction between omega is theta from 0 to pi by 2 and pi from 0 to 2 pi. Because, the orientation of fibers is random, we need to work with some joint probability density function of couple of quantities, couple of fibers, theta pi is PDF is called as W. So, that evidently integral over all directions from such a probability density function is equal 1.

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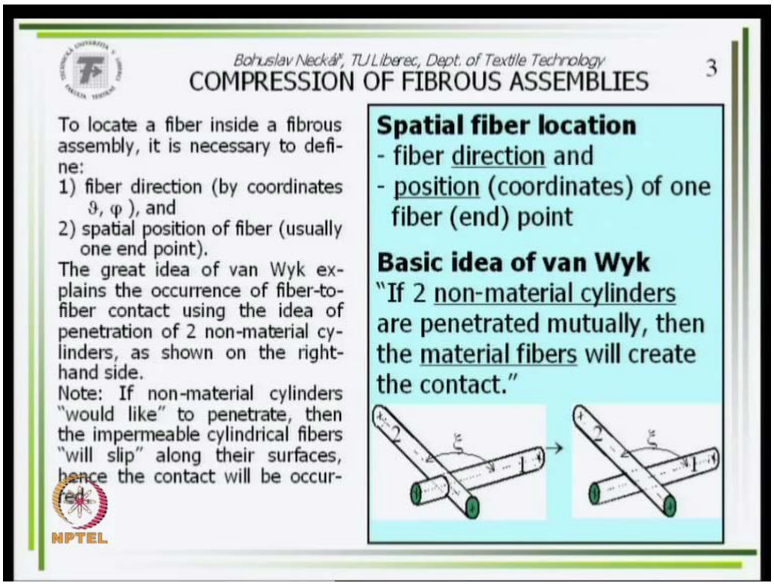

For spatial fiber location, we need to know two information, let us imagine this is some fiber, fiber portion; we need to know fiber direction may be this here, but the fiber segments can be here or here or here or here all these fiber segments have same, have same directions (Refer Slide Time: 09:10). So, that it is not enough, we need know also we need also fix one point of this fiber for example, at the end point, when I say the end point is here, then direction is (()) direction, then it determined together one original fiber segment.

(())

Each compression, each theory of compression need to solve two problem, the first is to solve contacts fiber to fiber contacts and the second is to solve the mechanical roles which are valid; the first one must be also about the fiber contact, van Wyk has (()) is a my meanings very genial, very genial idea he said we know from geometry, the situation which which is for example, in our first picture the penetration of couple of objects may be two, two cylinders, because our idealized fiber now, it is some cylindrical object. In geometry is possible penetrate cylinders you know, it from the descriptive geometry and similar courses and van Wyk is meaning, that if that two obstruct cylinders are mutually penetrate, then the couple of fibers it is not possible to penetrate (()) material or from then the couple of fibers may be in contact. So, on the place of this geometrical position he said in reality the final position is this here.

So, evidence for first question about the contact to the to the other question about the penetration of two cylinder, geometrical cylinders which represents geometrically our fibers, but I say that we speak about the random oriented fibers. So, how the probability that two fiber where we you have mutual contact, the idea is given in this picture.

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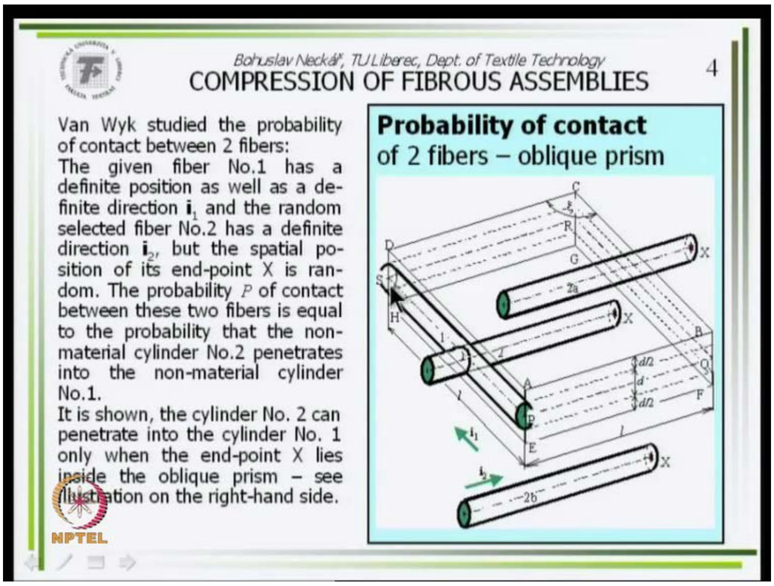

Let us imagine two fibers, fiber number 1, and fiber number 2, fiber number 1 is in our scheme this fiber idealize stripe cylindrical as we as we say in the, in this lecture yes, build this fiber number 1, let us imagine we know the direction of this fiber vector i 1 as well as the position in the space, so this fiber is fixed in space.

Let us imagine that we know also know direction of fiber number 2, you know direction, but we do not know the position of the fiber in our space, such fiber having the direction y 2, unit vector y 2 can be here, can be here, can be also here and therefore, I will speak about the fiber in this moment I mention the theoretical cylinder or the is possible (Refer Slide Time: 13:38). So, in this case in this case and group of cases such fiber number 2 will not penetrate fiber Number 1, never the less in this position fiber number 2 will penetrate the fiber number 1 is based on the position of end point x on the fiber 2.

It is evident first intuitively done, if this end point x is lying inside of this oblique prism then the fiber 2 will penetrate through fiber 1, when o will not penetrate, how to construct, now how to construct this such oblique prism it is write very easy we know the axis of the fiber 1, because fiber 1 is fixed in the space; we can each fiber have the same lengths l. So, from the point, which is point point point P, from the point P we reconstruct stripe lines length l in the direction of fiber 2.

We obtain the line P Q is not it? Similarly from the point S, we create single straight line and for we obtain S R and four points S R P Q we find some some line perpendicularly to this plane, perpendicularly to this plane reconstructs some line and on the distance D we we we have the point A. So, P A is lengths equal D, and similarly from the other side P E is also distance lengths D.

The same we make from point Q from point R and from point S and, so we obtain such such oblique prism, you can only to think that length is point x is lying inside of this oblique prism, then the then the contact sort of penetration exists if no, then this couple of fibers have not some contact. As the probability that fiber 2 which is randomly vacant in random position in our space with fiber number 2 will penetrate fiber number 1, it is a probability there is a point x is lying inside of this oblique prism.

And this probability must be given by a by the ratio of two volumes, volume of our oblique prism V 1 2 by total volume of whole fibrous assemblies, we see the volume of this oblique prism is given is 2 times d times l square times s sin as psi where psi is the angle of which is between given by the direction of both fibers, it is evident from elemental geometry never the less, let us remember that the angle of psi which is here must be the function of these two directions; it means first direction it is couple of information theta 1 phi 1, second direction it is theta 2 phi 2, so third, fourth quantities define our N psi theta valid psi 1 theta 2 psi 2 clear.

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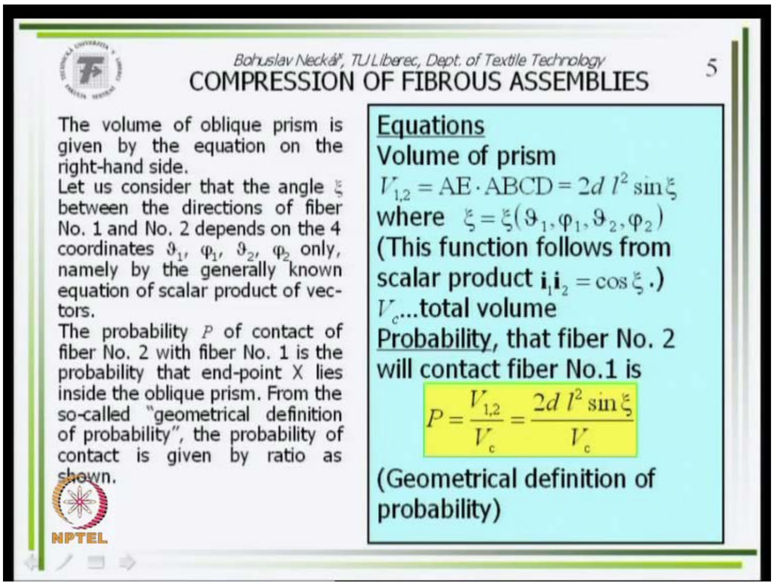

It is not unknown equation, it is not unknown relation, it is very known from basic of laws which are writing, which are a valid in geometry for example, we obtain cosines psi as a result of result of scalar product of this couple of unit vectors and scalar product is something very known.

So, when it is known function which give psi as a function of four four quantities theta 1 phi 1, theta 2 psi 2 well, this is the total volume and the probabilities V 1 2 by V c evidently. So, that the probability is given using this expression by such equation, well this is geometrical definition of probability yeah of course, we know the probability that fiber numbers 2 will be in contact with fiber number 1.

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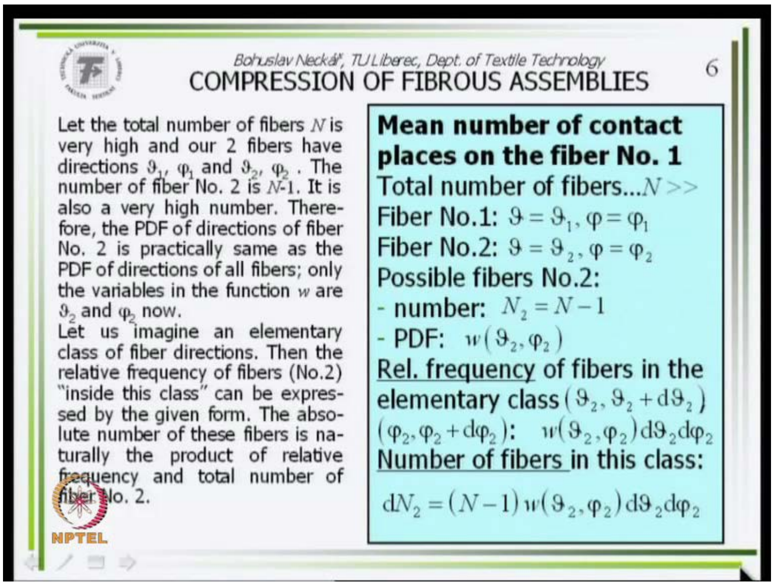

Now, let us study number of contact places on our fiber 1 from all fibers, fiber number 1 have theta 1 phi 1, fiber number 2 have directions given by by cup of theta theta 2 phi 2 1 fiber we select as a fiber number 1 from all, each other fiber can play the roles of fiber number 2, so that number of fibers which can play such role possible fibers number 2; another of such number is N 2 capital N 2 is total Number N minus 1, because 1 fiber we have, we select as a fiber number 1, probability density function of fibers number 2 is which of and PDF probability density function of all fibers direction is w function of general theta 5, let us imagine fishes in all oceans in the water, the lengths of the fishes have some probability density function, it is characterized by some probability density function.

Let us imagine that a fish 1 (()) and you give fish out from ocean, how change the probability density function of lengths of fishes in the oceans theoretically change, but its change will be so small, that we can say we do not change this probability, is not it? Similar it is here, when you think about a fiber assembly from very high very high of volume of fibers which usually is, so then we can say when we take 1 fibers we select 1 fiber to the position of fiber number 1, the probability density function is not changed.

Therefore also the fibers possible of fibers number 2 have the same function w and this probability density function as I call as a w theta, w theta theta 2 phi 2, now let us imagine some elemental class interval you know from laboratory what is a class interval you use it by evaluation of your experimental, you are experimental and voluntary and so on, let us imagine the idea that we have infinity small class interval.

Let us imagine that it is so small class, that the angle is going from some value theta to some value theta plus d theta, d theta is differential quantity which is higher than, which you can imagine something, which is higher than 0, but smaller that each real number differential quantity is infinity small. So, let us imagine that the the interval from theta 2 to theta 2 place d theta 2 elemental short interval and similarly phi is going from phi 2 to phi 2 d phi 2 such class, I call as a elemental class and from theory of probability is known that probability density function times this to differential quantities have some logical sense it represents logically something.

It is relative frequency, relative frequency in our element of class, relative frequency and our probability, because relative frequency and probability are too very similar similar times when we when we obtain such value after our experimental experiences, we prefer the time relative frequency when in the same quantity we use as a as a how to characterize possible future, can we speak about a probability well.

So, that the the relative frequency of fibers, are you in if for short say fibers having direction theta 2 pi 2 more precisely length in elemental of class from theta 2 d theta 2 d d theta 2 from pi 2 2 pi 2 plus d pi 2 yeah; in short I will say fibers having direction theta 2 pi 2. number of fibers in this elemental class is which of total number of fibers number 2 N minus 1 times relative frequency that the theta 2 pi 2 d theta 2 d pi 2.

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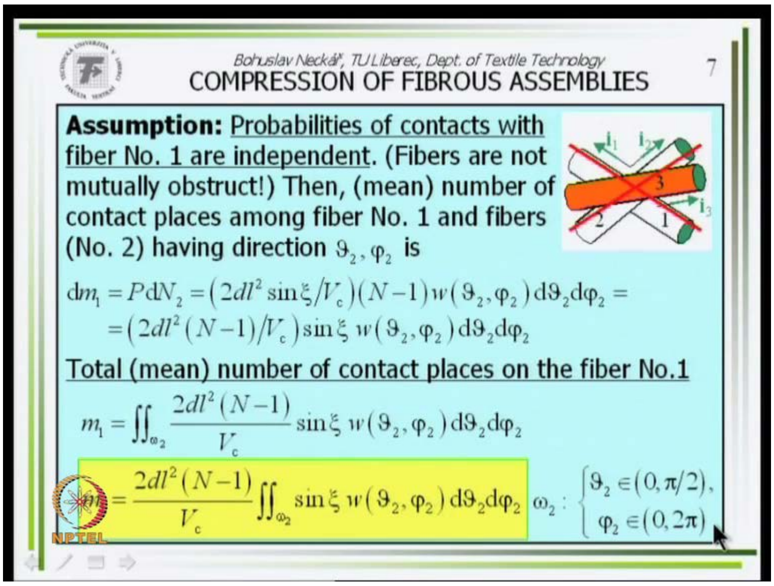

Well for following variation, let us assume that such configuration which is here, which are not about such configuration, which are nothing it means; if 2 fibers are in contact and such fiber would like be also in that orange fiber want Want or relativity is contact the fiber number 1 the situation is not think. So, that we can write, how is the, is this assumption is valid?

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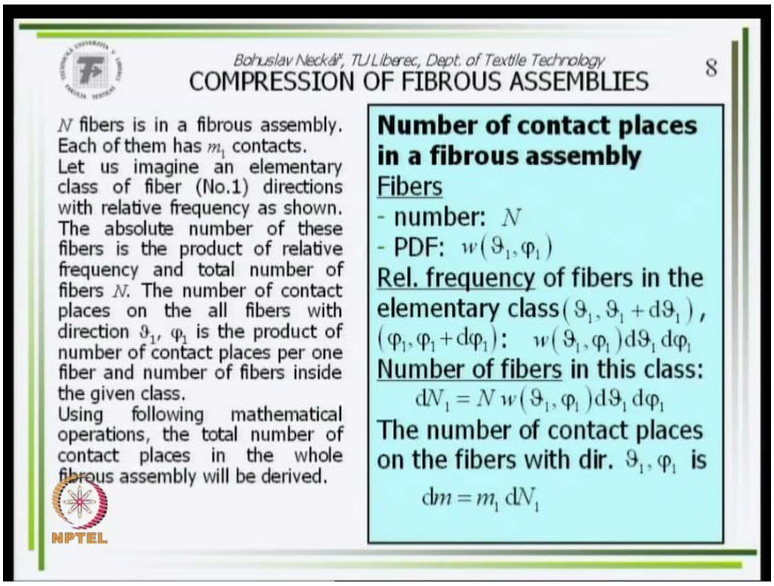

Then we can say how many contact places on the fiber number 1 is from the group of fibers having direction theta 2 pi 2, all fibers number of all fibers was P d N 2, but no all fiber and contact is our fiber number 1 only some of them it based on the probability evidently, the probability with the number of contact places is probability times total number is not it, yeah. So, using our equation for probability and our equation for number of fibers having direction theta 2 pi 2 in our elemental class, we obtain this expression after small rearranging this expression.

What it it is not too important for us, we want to know how is the total number of contact places on our fiber number 1, what we need to do? We need to sum this contact places over all fibers all fibers number 2 or other fibers because 2 things to solve total number of contact places of fiber number 1, what is it to sum it over all directions well it eliminates this special sum quote by the alphabet s, but because it was a little mixed it another alphabet, they used more and more longer alphabet s and today we call is very long on slim alphabet s, until the integral therefore, we use symbol of integral s, we must integrate it.

So, that the number of fiber contacts on the fiber number 1 is integral from this expression, it is here over all direction after rearranging we obtain N 1 and this and this equation, like this equation; we solved the question, how many contact places are on our fiber number 1, that we need to know how many contact places is lying on on all fibers in our fibrous assembly, how to do it, we will use very similar very similar why, logical why? Number of all fibers is N, PDF probability density function is W theta 1 pi 1, because each fiber can be you can choose each fiber as a fiber number 1, so it is W theta 1 pi 1.

Element of class interval of fibers number 1 is given by this two intervals and probability density function. So, is w so that the relative frequency is V function w theta theta 1 pi 1 times d theta 1 times d pi 1 well. Number of fibers in such class is d N 1 total number of all fibers times related frequency sure sure and the number of contact places on the fibers with this direction theta 1 pi 1, how is number of contact places on the sub group of fibers, but only this fibers which have the direction theta 1 pi 1 this direction no other. How it is, it is number of contact places per yarn fiber times number of fibers having this direction d N 1 it is it is d m.

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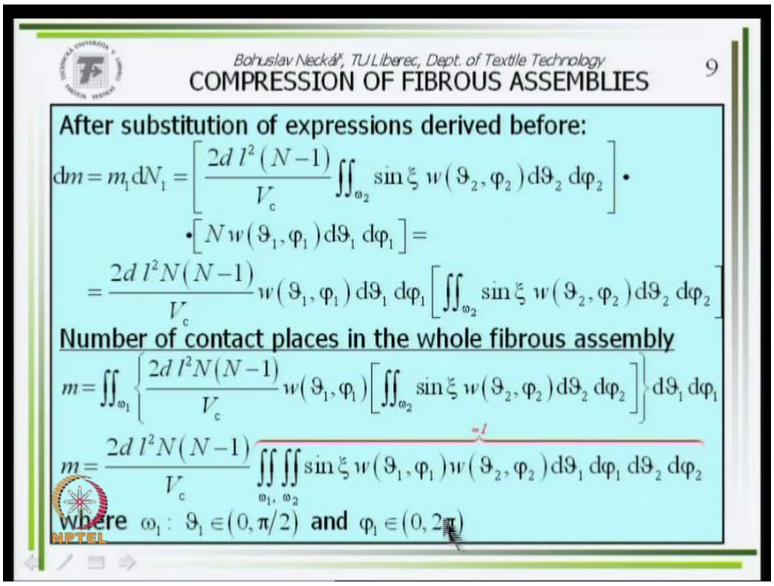

So, that m using expressions derived, earlier is this here or this here yes and finally, how is the number of contact places in the whole fibrous assembly, we must sum contact places over all fibers, so that it is integral over all fibers which comply the rho s our fiber number 1. So, over omega 1 we obtain this and then this horrible very very very treble yeah expression never the less then d y it is much to to difficult.

This is formula for number of contact places in all fibers in our fiber assembly, what is a what seem so horrible it is, this integral, but what is in N x I it is known function of theta 1 pi 1, theta 2 pi 2 we mentioned it earlier, then we have here only one function function w in the first position the the variables are theta 1 pi, 1 in the second in the same ah same function only the name of variables are are there theta 2 and pi 2. When somebody of you, this the mean the analytical formula for function w, I can to solve this integral and and the result say you high this value, this is a value, this is definite integral resulting resulting it is value some some some real value, something between 0 on 1 it is, so this value this integral which we, by the way we call this integral I is the function of a fiber orientation only it is enough to known probability, joint probability density function of couple theta pi, the function w no more need not more, Well we can say that the I, value I is some characteristic of fiber oriented in our fibrous assembly; it is some orientation characteristics you know what is statistical characteristic, like mean value standard variation, I do not know well correlative variation only this a little, a typical part of the some statistic characteristic of of distribution given by by this joint probability density function w well.

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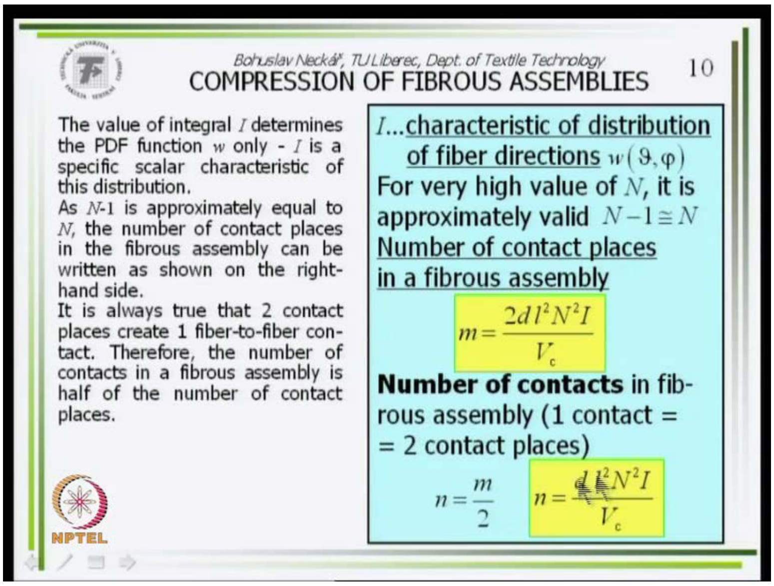

Therefore, this equation we can write symbolically in this easier showed, where I is some characteristic of suitable characteristic of fiber distribution of fiber direction of fiber orientation. It is number of contact place in all fibrous assembly, number of contact contacts, because every times two contact places create one contact is one half this, so number of contact and is given by such expression.

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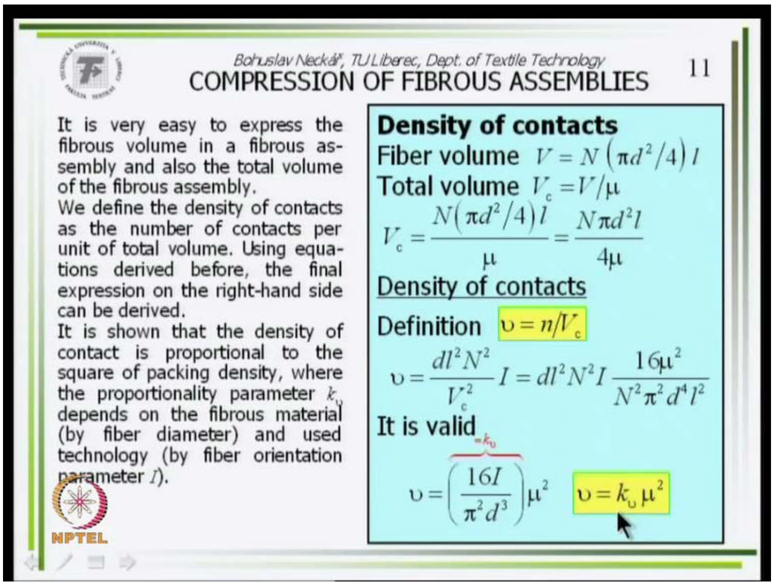

Now, let us so the density of contacts, density of contacts which fiber volume we obtain using this, why fiber volume is, which of pi d square by 4 fiber cross section times l volume per 1 fiber constant of fibers it is fiber volume s, we see is a total volume and you know from lesson from first lesson that this is fiber volume by packing density, fiber packing density.

So, that total volume using on the place of this equation is this or this density of contacts, it is as a number of contacts per volume n-eth of fiber assembly and by this we using such equations equations derived, we obtain after rearranging this here or this here it is only using rearranging. And what is this this is independent 2 packing density, its quantity, so it is some parameter related to fiber orientation as well as for the diameter, so some material some material characteristic, which we call K mu, and we can say that the density of contact number of contacts per volume unit is proportional to square root of packing density.

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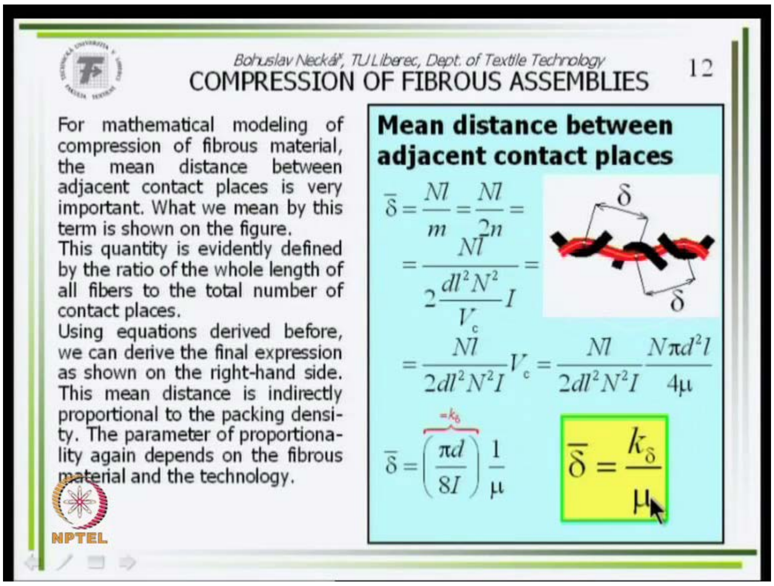

In mean distance between adjacent contact places, how it is this mean distance of contact places, it is it is easy let us imagine the red fiber which is in contacts with the black fibers, here this lengths this portion from contact to contact we call as a delta. So, mean value of delta is which of, when we, when you imaginary bond it together all fibers, let us imagine that I give to each contact some drop of a curve and then I take each fiber and bond it 1 fiber beside the others I I expand my fiber assembly very long. Fiber lengths of such fiber is N times l, and fibers per 1 fiber and on this very long fiber where the number of all contact places m.

So, that distance between adjacent contact places delta bar is N l by m N is 2 times N and using our equation after rearranging small rearranging, we obtain delta bar is this here, this is parameter in the dependent to packing density. So, we can call it as the K delta and we can say that the mean distance between adjacent contact places in a inverse proportional to packing density well.

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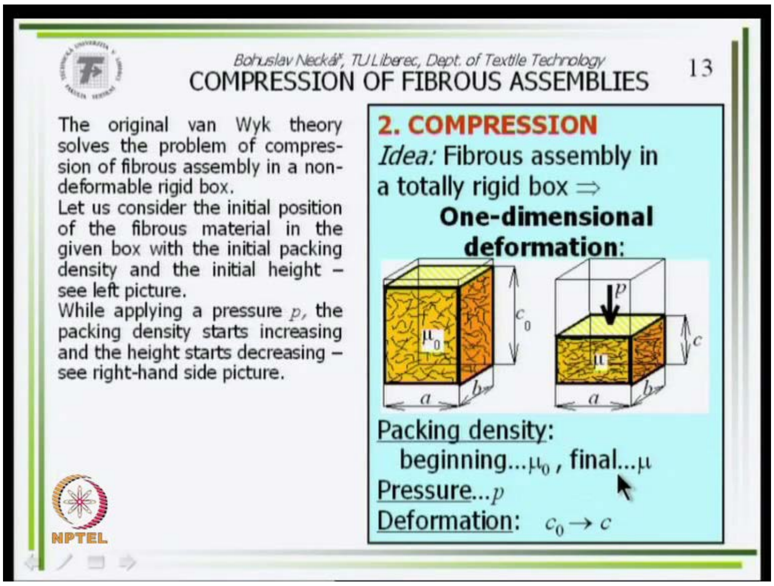

We solve the half first part of our problem, we solve the questions about the contacts why, because by pressure the first which we the of the the the the the pressure which we use is going to other fibers, the force is going to coming to the other to the other fibers through contact places between fibers no through are are is no well. So, contacts we know, now we start it is compression we have solved the easiest case, so one-dimensional deformation, one-dimensional deformation is shown schematically here let us imagine some starting starting material starting this equation of our fibers material in some known deformities box packing density mu 0, now using the pressure P our material is pressed and then their finals equation is shown here.

The fibrous material have smaller height now, c is 0 then c and the packing density is increase from mu 0 to mu based on the influence of pressure p. So, packing density beginning it is mu 0 finally, it is mu pressure is P and the deformation is from c 0 to c, van Wyk means that they are dominant the formation of the fibers by pressure is a bending deformation of course, generally all imaginable forms of deformation can be, but may be it is well that that dominant character of deformation is a bending deformation of fiber. Van Wyk also thought that, we can roughly of course, his assumption for simplification.

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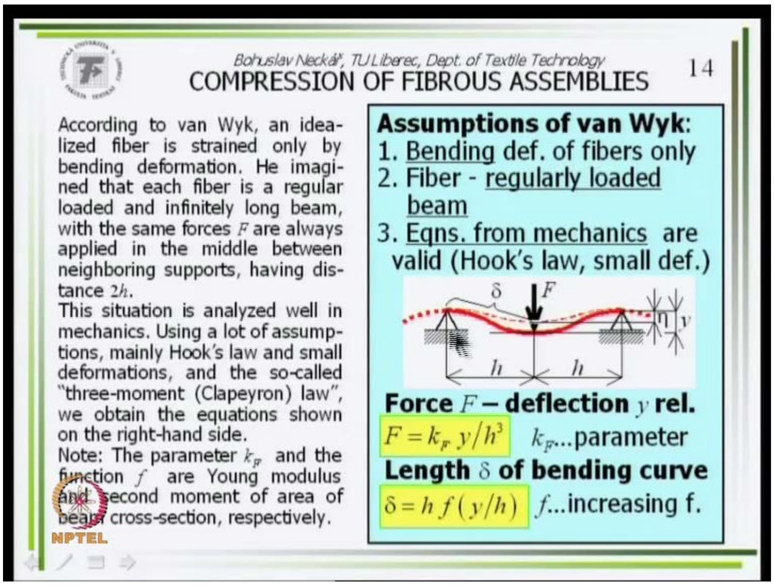

We can use the knowledge which now our colleagues from mechanical engineering, in a mechanical engineering is solved a some beam, some infinity want to beam, which is shown here where the force is is lying in the middle of of this fixed point here. This is the traditional traditional problem forum our colleague from mechanical engineering, they solve it using, so for using such clapper on, clapper theorem or three moment theorem, we will not to repeat this only you were right now the result from our colleague from mechanical engineering, they are assuring of such deformed beam, this following couple of equation the acting force f is proportional of to the ratio y, the ratio y is shown here it is a deflection of this this beam by this distance h power to 3 yeah.

So, this is the first result of our colleague, and the second is the the lengths of beam, this lengths this, this lengths is h times, some function, some increasing function one upon increasing function f of ratio y by h of course, when we read some of teaching book for students of mechanical engineering on the place of K f you will have some some more structural quantity, some some ratio in which is a shape of cross section of beam and hook's law and so, this e this yarn modules and so on and so on, but all together we can this function call only f and this parameter call as a K f.

This K equation we will use also, but we must accept that this equation was derived for very small deformation, and using hook's law, so it is for us only rough result, but because in the moment we have not something better it must be enough good for us yeah well.

The structure unit, structure of fiber assembly it is random and it is very very complicated, we must know it our picture of the structure little more easy and regularly, so random, so that let us imagine make our our structure.

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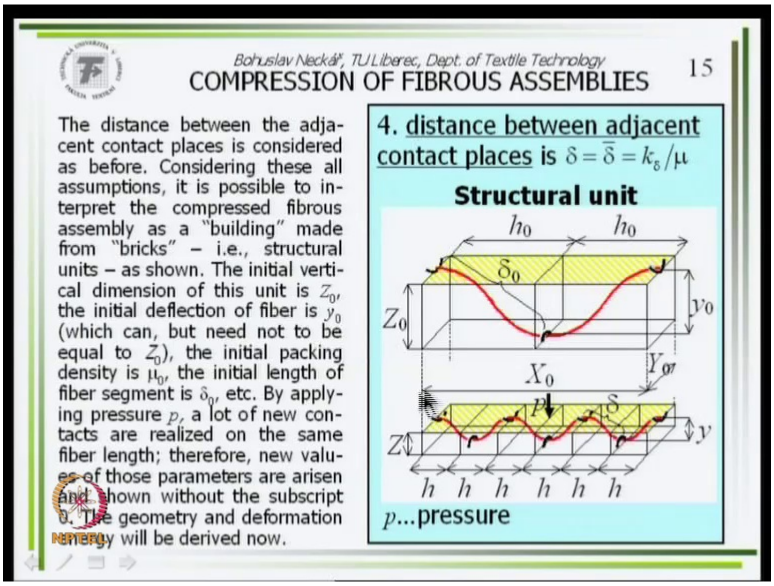

Let us imagine our structure as a more regular, let us imagine that our fiber have some shape like some beam and around our there are another fibers here, so this is single then this picture until our fiber exists some area, then X 0, y 0, Z 0 in starting position y is y is some area. So, that the packing density of this unit this structural unit it is mu 0, after compression let us imagine that from such structural bricks, whole structure is created you can repeat this bricks left hand side, right hand side before behind of this yeah and, so from this bricks to construct the whole fiber, whole fiber structure idealize, because regional on the random.

After pressure on this yellow on this yellow area using the pressure P here, the brick is deformed, because we solve the 1 dimension deformation, we imagine that our our box is non deformable box from some steel, then we obtain from this position a new final position of our structural unit of our of our brick and also number of contacts, because higher packing density will be higher well. How we solve this this question, we will explain in our second lecture, so I thank you for your attention, be happy.