


Orientation of Fibers
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Bundle Theory of Yarn Unevenness
Lecture # 18

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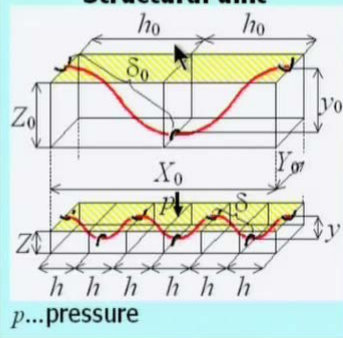
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COMPRESSION OF FIBROUS ASSEMBLIES

The distance between the adjacent contact places is considered as before. Considering these all assumptions, it is possible to interpret the compressed fibrous assembly as a "building" made from "bricks" - i.e., structural units - as shown. The initial vertical dimension of this unit is Z_0 , the initial deflection of fiber is y_0 (which can, but need not to be equal to Z_0), the initial packing density is μ_0 , the initial length of fiber segment is δ_0 , etc. By applying pressure p , a lot of new contacts are realized on the same fiber length; therefore, new values of those parameters are arisen and shown without the subscript 0. The geometry and deformation of the assembly will be derived now.

4. distance between adjacent contact places is $\delta = \bar{\delta} = k_c / \mu$

Structural unit



$p \dots$ pressure

Let us continue our theme about the compression of fibrous assemblies. In the end of our last lecture, we described some structural unit, some brick, which represents the whole fibrous structure which we compress. We said that in starting position this brick represents this picture of dimensions X_0, Y_0, Z_0 . The supports and the force realized using another fibers in contact at the contact places and the length between neighbor contact, agents contact, places is, let us imagine that it is a mean length, which we derived by the ratio $k \delta$, some parameter, by packing density.

After pressure of such brick, lots of new contacts are coming, because we prefer to idealize the shape of the fiber as a regular shape. We obtain something like this here; number of contacts is higher now, the height is shorter from Z_0 to the length Z , the distance between force and support is now not h_0 , but on the small one h . The number

of these segments is higher, this pressure is p and the length is delta. The deflection is from value Y 0 to y.

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Initial position: Total volume $V_{c,0} = X_0 Y_0 Z_0$, fiber volume... V'
Packing density $\mu_0 = V/V_{c,0} = V/(X_0 Y_0 Z_0)$

Final position: Total volume $V_c = X_0 Y_0 Z$, fiber volume... V' (same)
Packing density $\mu = V/V_c = V/(X_0 Y_0 Z)$

Relation: $V = \mu_0 X_0 Y_0 Z_0 = \mu X_0 Y_0 Z$, $\mu_0 Z_0 = \mu Z$, $Z = Z_0 \mu_0 / \mu$

Fiber length (const.): $(X_0 / h_0) \delta_0 = (X_0 / h) \delta$, $h/h_0 = \delta/\delta_0$

Length of the bending curve from mechanics:
 $\frac{\delta}{\delta_0} = \frac{h f(y/h)}{h_0 f(y_0/h_0)} = \frac{\delta f(y/h)}{\delta_0 f(y_0/h_0)}$, $1 = \frac{f(y/h)}{f(y_0/h_0)}$, $f\left(\frac{y}{h}\right) = f\left(\frac{y_0}{h_0}\right)$
 $\Rightarrow y/h = y_0/h_0$, $y/y_0 = h/h_0$

Together: $y/y_0 = h/h_0 = \delta/\delta_0 = (k_s/\mu)/(k_s/\mu_0) = \mu_0/\mu$

$y = y_0 \mu_0 / \mu$ and $h = h_0 \mu_0 / \mu$


From this picture, we can write lot of relations. For example, starting volume or total volume of our structural unit of our brick is evidently X_0 , Y_0 and Z_0 . Inside of this, the volume of our red fiber is Y . Sorry, the fiber volume is V . And we assume that this fiber volume is constant; is not changed, because the compression represents the smaller volume of air, not the change of volume of fibers. Packing density is μ_0 which is V by $V_{c,0}$, $V_{c,0}$ is starting volume and it is V by $X_0 Y_0 Z_0$.

In final position, this total volume is same X_0 , same Y_0 , but the height is now Z . So, V_c is $X_0 Y_0 Z$. The fiber volume is same, packing density is V by V_c , so that it is this here and because same fiber volume, we can write that this expression must be equal to this expression, so that $\mu_0 Z_0$ is equal to μZ or Z is Z_0 time's μ_0 by μ . Fiber length, which is constant is X_0 by h_0 times δ_0 . What is it the X_0 by h_0 ? What is it X_0 by h_0 ? Please, do not say me that it is two. Yes, in the picture it is two at the moment, but generally it is number of these segments or fiber segments between neighbor contacts. The number of this segments, it is this here times δ_0 , this is the length of fiber, number of such segments times lengths by one.

Number of such segments, this and this length by one, δ_0 (Refer Slide Time: 05:23). And the same is valid till after pressure. After pressure it is X_0 by h time δ and this

is from the same logic, so that from this equation h and both must be of the same fiber lengths; fiber lengths is not changed, so that h by h_0 is δ by δ_0 . The length of the bending curve from mechanics and what is δ by δ_0 ? This δ is this lengths of this portion fiber, length of this portion.

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COMPRESSION OF FIBROUS ASSEMBLIES

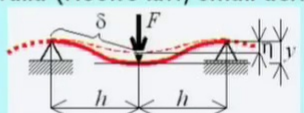
According to van Wyk, an idealized fiber is strained only by bending deformation. He imagined that each fiber is a regular loaded and infinitely long beam, with the same forces F are always applied in the middle between neighboring supports, having distance $2h$.

This situation is analyzed well in mechanics. Using a lot of assumptions, mainly Hook's law and small deformations, and the so-called "three-moment (Clapeyron) law", we obtain the equations shown on the right-hand side.

Note: The parameter k_F and the function f are Young modulus and second moment of area of beam cross-section, respectively.

Assumptions of van Wyk:

1. Bending def. of fibers only
2. Fiber - regularly loaded beam
3. Eqns. from mechanics are valid (Hook's law, small def.)



Force F – deflection y rel.

$F = k_F \delta / h^3$ k_F ...parameter

Length δ of bending curve

$\delta = h f(y/h)$ f ...increasing f .

But based on this result derived by our colleagues from mechanical department this δ is h times some increasing function like of quantity Y by h . So, we use it as δ is h times f of our quantity y by h by δ_0 , which is h_0 times f , this increasing function, same increasing function, but in other point or in other quantity, which is y_0 by h_0 . Nevertheless, h by h_0 is δ by δ_0 , so that h by h_0 is δ by δ_0 . So, that it must valid is that this ratio f of y by h by f of y_0 by h_0 is equal to one in our model.

So, that this function must be equal this function and because it is monotone increasing function, it must also validate y by h is equal to y_0 by h_0 . So, this is here and these relations together are shown here. Nevertheless, δ the distance between two neighbor contacts, was derived as a $k \delta$ by corresponding packing density, so that $k \delta$ by μ for δ by $k \delta$ by μ_0 for δ_0 , so it is μ_0 by μ . This relations we will use later into forms like y is y_0 times μ_0 by μ and h is h_0 times μ_0 by μ .

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Volume of the fiber segment δ : $V_\delta = \delta \pi d^2 / 4$

Number of segments in the compressed structural unit:

$$m_\delta = \frac{V}{V_\delta} = \frac{X_0 Y_0 Z_0 \mu_0}{\delta \pi d^2 / 4} = \frac{4 X_0 Y_0 Z_0 \mu_0}{\pi d^2 \delta} = \frac{4 X_0 Y_0 Z_0 \mu_0}{\pi d^2} \frac{1}{\delta} = \frac{4 X_0 Y_0 Z_0 \mu_0}{\pi d^2} \frac{\mu}{k_\delta}$$

Deformation energy (using equation from mechanics)

$$E_\delta = \frac{1}{2} \int_{\eta=0}^{\eta=y} dE_{2\delta} = \frac{1}{2} \int_0^y F d\eta = \frac{1}{2} \int_0^y k_F \frac{\eta}{h^3} d\eta = \frac{k_F}{2h^3} \left(\frac{\eta^2}{2} \right)_0^y = k_F y^2 / (4h^3)$$

$$E = E_\delta m_\delta = \frac{k_F y^2}{4h^3} \frac{4 X_0 Y_0 Z_0 \mu_0 \mu}{\pi d^2 k_\delta} = k_F y^2 \frac{1}{h^3} \frac{X_0 Y_0 Z_0 \mu_0 \mu}{\pi d^2 k_\delta} =$$

$$= k_F \left(y_0 \frac{\mu_0}{\mu} \right)^2 \left(\frac{\mu}{h_0 \mu_0} \right)^3 \frac{X_0 Y_0 Z_0 \mu_0 \mu}{\pi d^2 k_\delta} = k_F \frac{y_0^2 \mu_0^2}{\mu^2} \frac{\mu^3}{h_0^3 \mu_0^3} \frac{X_0 Y_0 Z_0 \mu_0 \mu}{\pi d^2 k_\delta}$$

$k_F y_0^2 X_0 Y_0 Z_0 / (h_0^3 \pi d^2 k_\delta)] \mu^2$...energy at final position (μ)

Volume of the fiber segment lengths delta Volume of fiber segment lengths delta, one of this segment for example, this segment or this segment or this segment are the same in our idealized structure. This volume V delta is a lengths delta time's cross section pi d square by four, we say d as the diameter of fiber, the equivalent diameter of fiber. Well, number of segments in the compressed structural unit; how many further segments are here? One, two, three, four, five, six; please do not say six; it is another example in my picture. Generally, how many such segments further segments are in our compressed structural unit?

This number is m delta; it is volume of fiber V by volume per one segment. It is well, is it not? Using equations derived, we obtained this or this or this (Refer Slide Time: 09:37). Using on the place of delta k delta by mu, we obtained is here, on rearranging and nothing more. We are ready to start with a formulation of mechanical relations; we will so call energy method. What it means?

When I have sorry I have some rubber for writing with this pencil, when I pressed this one I do some work. Is it so from physics? Physical work is work done. So, I gave some energy, I must give out some energy. Where is my earlier energy now? It is inside of this rubber and in which form? At most, so called deformation energy, in reality, it is small difficult, because some part of my work is transferred to thermal energy, for example or so on, but the dominant part is the deformation energy.

So, I can say, in idealized, I will then later generalize it. In idealized, the model I can say that my work must be equal to deformation energy inside of this rubber, when I neglect the others like the energy dissipation or thermal energy and so on. This way, we will use it for derivation of mechanical properties. Deformation energy, (Refer Slide Time: 11:43) we want to find that deformation energy in one of this segment, because we notice here, it is good to write it as based on deformation energy the double segment.

So, deformation energy in one segment is $E \Delta$ is one half of the deformation energy, which is calculated from here to here, between two supports, because this is same as this here. So, energy here and energy here, together is energy here and we can calculate the energy here at one half of this, it is energy related to the lengths Δ , deformation energy. So, it is one half of the energy $d e \text{ two } \Delta$, but now back to our picture? How it is this energy? The force started with value 0, it did not jump to final deflection y . The force was higher and higher and higher, deflection was also higher and higher and higher. Final position is that we have maximum value of force and maximum value of deflection y .


Generally, on this way from starting position to the final position, the deflection will be some deflection η ; some general deflection, and it was starting from 0 on this η was higher and higher to the final position y . Well, how is the energy, which is now in the beam or fiber; however you want. This is the actual force F ; we must when, I am in some general moment, the beam is in some general moment, then some deflection y , and then using the force F , I can increase little my deflection. So, the deflection increased to incremental value of $d \eta$, which is here.

So, force time $d \eta$ is elemental increase of energy of deformation energy in our beam. But, then the force must be a little higher, because the second $d \eta$ and so on and so on. So, that we can write it, so F times $d \eta$ where F is changed to general position η . Nevertheless, for F , for this force, our colleagues from mechanical engineering derived such equations. This is the equation from our colleagues. Where it is? It is here and only on the quantity of deflection I call it now as η .

So, it is this here (Refer Slide Time: 15:34). After, we must integrate this from starting position to the final deflection y , so that we obtain this here, finally here. This is the energy per one fiber portion length Δ . How is the total energy? Now, the total energy

is energy per one segment times the number of segment. So, the E delta times m delta; using this expression, you attend this after rearranging; after rearranging we obtain it here. Here, is y square and here is h power two three (Refer Slide Time: 16:17). Nevertheless, you know what this y is, for y we have this expression and for h we have this one. We use it and then it is here; after small rearranging we obtain that the energy is something bracket some global parameter times of mu square. It is energy at the final position on the structure of packing density mu.

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COMPRESSION OF FIBROUS ASSEMBLIES

The initial value of energy is given analogically. Then, the increment of energy is the difference between the initial and the final values.

We can also formulate the work done independent of the specific configuration inside our structural unit. The given table characterizes our used symbols. (See also the figure on the slide 15.)

$$E_0 = \left[k_F Y_0^2 X_0 Y_0 Z_0 / (h_0^3 \pi d^2 k_\delta) \right] \mu_0^2$$

...initial energy (μ_0)

Increment of energy

$$\Delta E = E - E_0 = \frac{k_F Y_0^2 X_0 Y_0 Z_0}{h_0^3 \pi d^2 k_\delta} (\mu^2 - \mu_0^2)$$

Work done

SYMBOLS	init.	general	final
pack. dens.	μ_0	μ^*	μ
vert. coord.	Z_0	$Z^* = Z_0 \mu_0 / \mu^*$	$Z = Z_0 \mu_0 / \mu$
trajectory	0	$\lambda^* = Z_0 - Z^*$	$\lambda = Z_0 - Z$
pressure	0	$p^* = p^*(\mu^*)$	$p = p^*(\mu)$
		$p^* = p^*(Z^*)$	$p = p^*(Z)$
		$p^* = p^*(\lambda^*)$	$p = p^*(\lambda)$

Thus on the starting position, the starting value of energy, which was in our fiber bundle in our structural unit is E 0 and it must valid the same equation with packing density equal to mu 0 and that is this one here. This is the initial energy, so that the increment of energy is delta E, which is this ratio times of mu square minus mu 0 square. Now, we need to formulate the work done.

When I use or when I work, it is a process. I have some starting situation then I give to some work and I finished on the final position in a deformed object. So, therefore, I must **have three tier I** use three types of quantities; starting value, general value on the way and the final value. For example, for packing density, for initial there are symbols, which we were use as initial value is mu 0, final value is mu, but through our activity, the general value on the way from mu 0 to mu is mu star. Vertical coordinate was Z, starting

was Z_0 , final was Z and on the way was Z^* . We derived that the Z is Z_0 times μ_0 by μ and similarly, it is valid also to this equation (Refer Slide Time: 18:38).

You know on the place of μ and Z , I use the star, **in subscript sorry** superscript star, which are the quantities on the way from starting position to end position. Now, trajectory; starting value of trajectory is 0, final value of trajectory is $\lambda = Z_0 \mu_0 / \mu$; this is trajectory. On the way it is $Z_0 \mu_0 / \mu^*$ because it is on the way. Final pressure; here starting pressure is 0. On the final pressure is p , as a function of packing density or of Z , of Z coordinate or of trajectory, λ ; however way you want. On the way the pressure is p^* . So, pressure p^* is starting from 0 and is higher, higher and higher and the final quantity is p^* equal p . The function p^* can be formulated as a function of packing density or as a function of vertical coordinate or as a function of trajectory. Others symbols, well are understandable. I hope.

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$$A = \int_{\lambda^*=0}^{\lambda^*=\lambda} dA = \int_0^\lambda p^*(\lambda^*) X_0 Y_0 d\lambda^* = X_0 Y_0 \int_0^\lambda p^*(\lambda^*) d\lambda^* =$$

Subst.: $\lambda^* = Z_0 - Z^*$, $d\lambda^*/dZ^* = -1$, $d\lambda^* = -dZ^*$

$$= -X_0 Y_0 \int_{Z_0}^Z p^*(Z^*) dZ^* = -X_0 Y_0 \int_{\mu_0}^\mu p^*(\mu^*) [-Z_0 (\mu_0/\mu^2)] d\mu^*$$

Subst.: $Z^* = Z_0 \mu_0 / \mu^*$, $dZ^* = -Z_0 (\mu_0/\mu^2) d\mu^*$

Work done: $A = X_0 Y_0 Z_0 \mu_0 \int_{\mu_0}^\mu [p^*(\mu^*)/\mu^2] d\mu^*$

Relation pressure – packing density
 Mechanics - conservative system: $A = \Delta E$
Assumption: The exerted work is only proportional to the increment of energy, where constant of proportionality $C > 1$
 (more general system) $A = C \Delta E$

How is the work done now? It is same idea. The elemental incremental increase of our work, what is it? It is the force times the differential quantity of the lengths; force times differential quantities of the length. How is the force? It is the general force on the way; pressure for example, p^* as a function of λ times of area, which is $X_0 Y_0$, was in our picture. Is it not? So, this is force, this is force, this is force times $d\lambda^*$, because some general characteristics of our way to which we press is this one. Now, we can apply two times apply some substitution inside of this integral, where is this here and

the second is this here, and finally we obtain work done is as $X_0 Y_0 Z_0$ times μ_0 times of integral, because we need sum this incremental works together. The integral is for μ_0 to μ , from this expression. Now, here we have the pressure as a function of packing density. It is this function, which we want to obtain; the relation between the pressure and packing density.

So, it is a formula for work done. Now, how it is the relation between pressure and packing density? In the mechanics, we know the term of conservative system. Conservative system is a theoretical assumption in which all our work done is transformed to the deformation energy. For example, rubber by the way is near to this ideal assumption; no other material in this world is a perfect conservative system. If it is a conservative system, then the work done must be equal to increase of energy.

In the textile, in our fibers assembly, this assumption is not too good, but we need not think so hard on simplification. We can assume that the work done is proportional to increase of deformation energy. So, that this C must not be equal to 1; C is equal to 1 for conservative system only. This C can be higher than one, so that our work done is higher than increase of energy, because part of our work is dissipated as a thermal energy and I do not know what.

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$$X_0 Y_0 Z_0 \mu_0 \int_{\mu_0}^{\mu} \frac{p^*(\mu^*)}{\mu^{*2}} d\mu^* = C \frac{k_F y_0^2 X_0 Y_0 Z_0}{h_0^3 \pi d^2 k_\delta} (\mu^2 - \mu_0^2)$$

$$\int_{\mu_0}^{\mu} \frac{p^*(\mu^*)}{\mu^{*2}} d\mu^* = \frac{C k_F y_0^2}{h_0^3 \pi d^2 k_\delta \mu_0} (\mu^2 - \mu_0^2)$$

After differentiation with respect to μ (it is $\frac{d \left[\int_k^y f(x) dx \right]}{dy} = f(y)$)

$$\frac{d \left[\int_{\mu_0}^{\mu} \frac{p^*(\mu^*)}{\mu^{*2}} d\mu^* \right]}{d\mu} = \frac{d \left[\frac{C k_F y_0^2}{h_0^3 \pi d^2 k_\delta \mu_0} (\mu^2 - \mu_0^2) \right]}{d\mu}$$

$$\frac{p^*(\mu)}{\mu^2} = \frac{C k_F y_0^2}{h_0^3 \pi d^2 k_\delta \mu_0} 2\mu, \quad \frac{p}{\mu^2} = \frac{2C k_F y_0^2}{h_0^3 \pi d^2 k_\delta \mu_0} \mu, \quad p = \frac{2C k_F y_0^2}{h_0^3 \pi d^2 k_\delta \mu_0} \mu^3$$

We consider $k_p = 2C k_F y_0^2 / (h_0^3 \pi d^2 k_\delta \mu_0)$ and so $p = k_p \mu^3$

So, let us use this equation and on the place of A and delta E, we use our expression derived earlier. So, we obtained A is equal C times delta E and this is here; after small

rearranging we obtained this well. It is nice, but not too nice, because our function pressure as a function of μ is inside of this integral. What to do then? Evidently, make some derivative, because it is inside the integral. So, make some derivative. Let us make the derivative of left hand side as well as right hand side. The derivative of this Z by μ ; derivative of right hand side by μ is very trivial. μ is only here, therefore, this derivative in our problem. The question is what the derivative of the left hand side by μ is. Here, where is μ ? No, μ^* ; μ^* is integrating variable and μ is the final variable.

μ is the upper limit of this integral. The question is how the derivative of integral where the variable is the upper limit is. You know, I hope you know from the mathematical analysis that this derivative (Refer Slide Time: 25:45), this is some expression known from mathematic. This derivative is this function where on the place of the integrating quantity x , is now the upper limit y . It is a formula from mathematic. After our lecture we can derive it mutually, it is very trivial. Because, this is valid we can also solve the derivative of left hand side. So, on the right hand side the derivative is this times two times μ , evidently this here. On the left hand side, the derivative is this function, but on the place μ^* , we must write μ . This is this here.

Nevertheless, the function p , pressure, general pressure by last value, the highest value of packing density or final value of packing density is our pressure p . We call it as a pressure p without star; the final pressure is p , so it is p , so that I obtain this equation here, then this equation here or this equation here. When this ratio (Refer Slide Time: 27:13), this ratio, which is not function of μ , it is only the function of material parameters; only quantities we subscript 0 and other parameters constants. If this quantity is called as a parameter k_p , then the result of Van WYK theory is very easy. Pressure is proportional to μ cube, μ to the power to three. This is the result of Van WYK theory, a direct relation between pressure and packing density.

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As p is not accurately equal to 0 when $\mu = \mu_0$, van Wyk suggested the empirical correction as shown. The derived equation holds good for smaller values of packing density (bale of fibers, etc.). But for higher values of μ , e.g., for packing density round 0.5, which is typical for yarns, this relation is not enough precise and for extremely high pressure, it brings logically nonsense expression. Therefore, it was necessary to correct and generalize this theory for higher values of packing density.

Note: Correction to initial position
 $p = k_p \mu^3 \rightarrow p = k_p (\mu^3 - \mu_0^3)$

Experiences: Good for small value of packing density

Problems of this theory

1. By $p > k_p$ is $\mu > 1$ (!?)
2. Not good for higher values of μ (approx. $\mu > 0.2$ or 0.3)

Van W Y K studied how is his experiment was and was very happy. Van W Y K worked in South Africa. In South Africa, there used wool fiber, the **merino** wool at most, you know it and they press it to bales and it was very good relation to his experimental experiences. So, eureka; this is perfect. From that time, lots of people use Van W Y K formula. Nevertheless, this formula is not well enough for all packing densities. You can see, first when you apply this formula to the modeling of yarn, the yarn internal structure, yarn packing density and so on, your result was not well and something is not good.

Second, let us see how is this equation, p is proportional to μ power to three is going over. So, when our high value of p represents, here k_p is some real constant, so when you use some **finite length** finite value of pressure, you obtain μ higher than one. This curve is showing here, so that here from this value of pressure, it is pressure and packing density. For higher values of pressure, the packing density is higher than one, which is logical non sense; packing density must not be higher than one, it is evident.

Why in a system, in model of Van W Y K, which is good for not only for wool fibers, when lot of others checked this equation and they said yes. This equation is good for packing densities to may be 0.2 to 0.3. When your packing density is 0.3 and smaller and you can say that this equation works very well. But why it is not good for higher values of packing density, which we use.

For example, in the yarn, in the yarn, we have packing density at most between 0.4 and 0.5, of 40 to 50 percent. Then, how it is possible? Because to solve this problem, to this moment, it was original, it was pure Van W Y K theoretical concept, only the **derivative is a** derivation of this, I used a little other way to obtain it. But as a theoretical model and the result is Van W Y K model. Because, this problem is such model, we derive some generalization of Van W Y K concept. It is based on following idea. The contact between two fibers is usually not only a point contact. It takes its some small area, contact area, between two fibers.

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We assume, the contact place is not a point only, but is an area, round which the fibrous volume is non-compressible. This is illustrated by the red colored space, which resembles a hard "stone" or "granule" inside the fibrous assembly.

Van Wyk's theory assumes purely the contact points (support - beam). Therefore, its result could be accepted, but only for compressible (deformable) part of volume. This compressible volume is the difference between the total volume of the fibrous assembly and summation of volumes of all "stones" - non-compressible volumes around the contact area (abbreviation NV).


GENERALISATION OF C.M. VAN WYK'S THEORY

Idea: Round the contact place (**area**, but not point • only), no more compressible material is present, but the so called **non-compressible volume (NV)** or "granule" (red) does exist.

Assumption: Van Wyk's eqn. is valid, but for deformable part of volume only.

And around this, the volume around this contact area, in this contact area, functions as a stone inside of it. Let us imagine it. This part of my finger, I can move with these two, but here it is like a stone, it is some small stone or some granule. So, this is some non compressible volume. Is it not? **So, that we said we said to us that.**

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COMPRESSION OF FIBROUS ASSEMBLIES

The used symbols are shown in the table on the right-hand side. We can write the original van Wyk's equation using total volume of fibrous assembly, as shown. Now, in place of total volume we put the deformable volume only and that's how we get the useful generalized equation.

SYMBOLS	one NV	all NV
Total vol.	$W_{c,g}$	W_c
Fiber vol.	W_g	W
No. of NV	1	n
Packing density	$\mu_g = \frac{W_g}{W_{c,g}} = \frac{W}{W_c}$	

Van Wyk: $p = k_p \mu^3 = k_p \frac{V^3}{V_c^3}$

Generalized: $p = k_p \frac{V^3}{(V_c - W_c)^3}$
(deformable vol.)



It is a non compressible volume and Van Wyk theory is valid. Van Wyk theory is well it is valid, but not for whole volume of fibrous assembly. But in each moment only for deformable volume, which is total volume minus the volume of such stones, which are non compressible. Can you imagine it? I think the principle is. This non compressible volume, in short is N V here. All granules, I call it home granula and granules; it is possible to call it here to. It gives the following idea. Earlier, Van W Y K's original Van Wyk's equation is k p times of mu power to three; mu is fiber volume by total volume. So, both times it is V. This equation is the relation of two quantities; p and total volume, volumes of fibers is same.

Now, we say no to total volume, we must in this place, give deformable volume, which is total volume minus volume of all our stones, our granules. Now, symbols; for one non compressible volume are here, for all non compressible volumes for one granule is here, for all granules here. Total volume has subscript c, so total volume pair one granule is **V c g**, total volumes of all granules is W of course is W c. Fiber volume per inside of one granule is W g and fiber volume of whole granules is W.

Number of granules of non compressible volumes is one per one granule, and generally n in our material. Packing density inside of our granule is very high and is every time near to one, but some gaps must be there and therefore, we must also have the quantity packing density inside of our granule. It is mu g; it is all ratio of W g by W c g or W by

W_c ; both are possible. And this is our idea. The problem is now to formulate how is the non compressible volume of all granules, the volume W_c .

This W_g volume, fiber volume inside of our one stone, one granule, one compressible volume, generally say it is not constant. Because, due to it subjective with your fingers. By light contact, you can feel that this volume is small and is non compressible volume is small. Also the volumes of fibers, the whole granule is small, so that the fiber volume inside of our fiber granule will be small. From other right, when we have some contact, which is very intensive, then the contact area is large, our granule is big and fiber material inside of our granule, the volume of this is high.

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The fiber volume inside one NV is changed with packing density of the fibrous assembly. Contact place in lightly compressed material is probably small and therefore, the fiber volume W_g is small too. Similarly in heavily compressed material the fiber volume W_g is high. This relation is not yet exactly known and so we will use an empirical relation as shown. Because total volume of NV is the product of number of NV (equal to number of contacts) and whole volume of one NV, the total volume of all NV can be expressed as shown.

The most theoretically compressed fibrous assembly is called the limiting state

Assumption (empirical):
 $W_g = f(\mu) = K\mu^a$
 where $K > 0, a > 0$...parameters
 $\left. \begin{array}{l} a > 1 \\ a < 1 \end{array} \right\}$

No. of NV: $n = \nu V_c = k_0 \mu^2 V_c$
Total volume of NV
 $W_c = n W_{cg} = \overbrace{(k_0 \mu^2 V_c)}^{-n} \overbrace{(W_g / \mu_g)}^{-W_{cg}} = k_0 \mu^2 V_c \frac{K \mu^a}{\mu_g} = K \frac{k_0 \mu^{2+a} V_c}{\mu_g}$

Limiting state = the most theoretically compressed fibrous assembly

So, volume of fibers inside one granule relate to the packing density of the whole structure. When we have higher packing density then the volume is higher. So, it is a function, increasing function of packing density. It must be so. We have another theoretical model for this moment. We only know that it is some increasing function. Therefore, in this moment, we will use some empirical function and we chose the function K times μ power to a ; K and a are two suitable parameters. Why, is because it is evident. If a , is higher, then it is convex increasing function. If a , is smaller, then it is concave and if a , is equal one, it is straight line.

So, these formulas can characterize each existing relation, roughly. Nevertheless, this formula is empirical. It is written here, it is empirical. Using this, the total volume of non

compressible volume of granule, which is of number of granules time total volume of per one granule. Using this derived the number of granules, it is same then the number of contacts in our fiber assembly and number of contacts is the density of contacts, which is this time volume. Density of contact is per one volume unit, so times volume. So, is number of granules and this is for W_c g. After rearranging and using this equation we obtain that W_c . Total volume of all granules is given by such equations. The parameter K is possible to derive using the idea of limit state, which is under this term, I think the most compressed fiber assembly.

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For limiting state these are valid

1. Total volume is minimum, $V_c = V_{c, \min}$
2. Packing density is maximum, $\mu = \mu_m = V/V_{c, \min}$
3. NV fill total volume of fibrous assembly, $W_c = V_{c, \min}$
 (In other case, the compressible volume $V_{c, \min} - W_c > 0$)
4. Therefore, the packing density of NV $\mu_g = \mu_m$

Using these relations we get for limiting state

$$V_{c, \min} = K \frac{k_u \mu_m^{2+a} V_{c, \min}}{\mu_g}, \quad 1 = K k_u \mu_m^{1+a}, \quad K = \frac{1}{k_u \mu_m^{1+a}}$$

and using this K

$$W_c = K \frac{k_u \mu_m^{2+a} V_c}{\mu_g} = \frac{1}{k_u \mu_m^{1+a}} \frac{k_u \mu_m^{2+a} V_c}{\mu_g}, \quad W_c = V_c \frac{\mu_m^{2+a}}{\mu_m^{1+a} \mu_g}$$

How it is in the limit state? The total volume let us imagine the limit say the maximum of maximum compressed fiber assembly, which is possible. No more is possible. Total volume of such structure must be of minimum. Packing density is maximum, the maximum value of packing density, I will call μ_m . it must be fiber volume by minimum of total volume and it is definition of packing density. Our granule fills total volume of fibrous assembly. Why? Let us imagine it is not. Then among of our granules there must be some compressible volume. If there exists some compressible volume then it is possible for this material to be more compressed. If it is possible for the material to be more compressed, then it is not the limit case; limit case is that it is not possible to compress more our material.

So, in this limit situation it is so that our fibrous assembly is like as a wall, from brick to brick; one brick, second brick, third brick; nothing among this, only stone beside stone. Therefore, W_c is equal to $V_c \mu_m$, and the packing density of this structure is $\mu_g = \mu_m$, because it is from the bricks from our granules only; it is the wall from our bricks. So, that our earlier equation, general equation is here, we can write now in this black form. The red, say what was earlier here; earlier on left hand side was W_c . Now, in the place of W_c , in this place of this is now $V_c \mu_m$. Similarly, we substitute all here.

From this we obtain this equation, so that K is $1/k \mu_m$ times m power to one plus a . we can explain the parameter K as a function of, without the parameter, as a function of maximum packing density. Using this expression here, in our general formula, which is this one, we obtain this here. This is our earlier formula. In the place of K , we use this here. W_c is V_c times of this and now we can have two packing densities here; the maximum value of packing density and packing density inside the granule. We can well imagine that both are same that inside of granule is every times a maximum packing density, only a small necessity gaps beside fibers.

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Assumption: Also in general fibrous assembly (not limit state), the packing density of NV is maximum; $\mu_g = \mu_m$

Then $W_c = V_c \frac{\mu^{2+a}}{\mu_m^{1+a} \mu_g} = V_c \frac{\mu^{2+a}}{\mu_m^{1+a} \mu_m} \quad W_c = V_c \left(\frac{\mu}{\mu_m} \right)^{2+a}$

Using last equation in the generalized formula of p we get:

$$p = k_p \frac{V^3}{(V_c - W_c)^3} = k_p \frac{V^3}{[V_c - V_c (\mu/\mu_m)^{2+a}]^3} = k_p \frac{(V/V_c)^3}{[1 - (\mu/\mu_m)^{2+a}]^3}$$

Pressure $p = k_p \frac{\mu^3}{[1 - (\mu/\mu_m)^{2+a}]^3}$ Usually, $a \approx 1$ and $\mu_m \approx 1$

Then using this assumption we obtain the W_c is V_c times μ by μ_m power to two plus a , and this is used on the place of our generalized idea; total volume minus volume of all non compressible parts. After small rearranging, using that the

packing densities which is fiber volume by total volume, we obtain this here and finally, we obtain this equation. Based on our experiences, μ_m is usually very near to value one; maximum packing density, and the parameter a , which is here is based on our laboratory experience is near to one. It means this relation is straight.

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Characteristics of the derived functions

- small value of μ - the characteristics are identical
- higher value of μ - the characteristics are different

Problem: invers. funct. $\mu = f(p)$ is not in analytical form.

Approximation round $\mu = \mu^*$:

$$b = 3 \frac{1 + (1+a)(\mu^*/\mu_m)^{2+a}}{1 - (\mu^*/\mu_m)^{2+a}}$$

$$P_{approx} = k_p c \mu^{b-3} \left[1 - (\mu^*/\mu_m)^{2+a} \right]^{-3} (\mu^*)^{b-3}$$

van Wyk ... $p = k_p \mu^3$

$p = k_p \frac{\mu^3}{[1 - (\mu/\mu_m)^{2+a}]^3}$

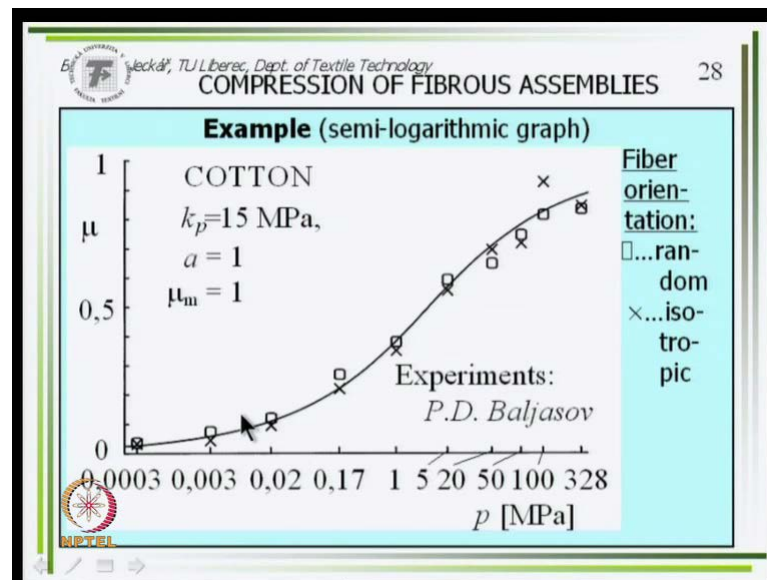
original $\mu_m=1$ $a=1$

1: $\mu^* = 0,02$ ($\cong \mu_b$)
2: $\mu^* = 0,4$
3: $\mu^* = 0,6$

P_{approx}/k_p P/k_p P_{approx}/k_p

And then we obtain such curve; such diagram. Pressure, packing density, this is our mu generalize function and this is original Van WyK curve. In the region of small packing density, we obtain roughly same graph, same shape. Then by higher values, our mu curve limited to maximum packing density, some think which is very near to one.

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It is possible to do some approximation because we formally need it. You can read it later in home. I want to comment a relation between our generalized equation and experiments, which was doing by Baljasov, Professor Baljasov, was earlier known name in Moscow and Moscow textile institute. He studied experimentally this relation of pressure and packing density in very large interval of pressure from 0.0003 mega pascals to 328 mega pascals. He used for smaller values our braking machines, for higher pressure he used some special machines from mechanical department.


His experimental results are shown at the point in this diagram. It is semi logarithmic graph, so that on the abscissa the logarithmic scale of pressure, on the ordinate is the linear scale of packing density. When we use the original Van Wyk, then from here the curve is increasing so, over the value one. Nevertheless, our curve corresponding to this experiment needs curve, which parameter k_p is 15, a is equal one, μ_m equal one is this. You can see that this comparison is experimental results, we compare it also. This is experiment by Baljasov, but we have lot of our own experiment from our lab and same character of result.

So, you can see that may be till packing densities of 0.8 is our type of curve, is in good comparison to experimental data. Some differences you can see by very high value of pressure and packing density near to one, this here (Refer Slide Time: 48:38). Why it is because the pressure is so high that quite other physical processes are there. Like the

destructions of fibers, it is a pressure, which creates from fibers powder. It is deformation destruction of fiber material and so on.

Therefore it is so high values, but there are unreal to obtain in textile processes. Yesterday, we discussed Professor Isthiaque, some research problems and we both know that the value around 0.8, packing density around 0.8, is the maximum, which we can obtain using all textile processes. Higher value, it is the production of fiber balls, it is not textile technology. So, you can see that the result is very good, acceptable. We will use it in one of our later lecture about a yarn structure. We will use this final equation, this one here, (Refer Slide Time: 50:15) also as an input to the model of internal yarn structure.

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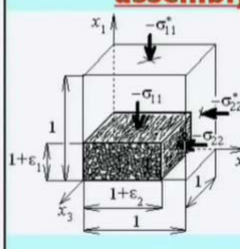
Till now, the one-dimensional deformation is studied. More general and complicated problem arises in case of two-dimensional deformation of fibrous assembly.

Let us assume, that the normal stresses act perpendicular to the dominant fiber direction and in the plane of stresses, the fibrous material is so-called "transverse isotropic".

To solve this problem we must use two types of stresses – Lagrange's fictive stresses, related to the initial areas (with superscript *) and Cauchy's real stresses, related to the final areas (without superscript).

If both stresses are equal, we will speak about homogenous stress.

Two-dimensional deformation of fibrous assembly



Stress:


Lagrange (fictive)
 $\sigma_{11}^*, \sigma_{22}^*$

Cauchy (real)
 σ_{11}, σ_{22}

Homogenous stress
special case, where $\sigma_{11} = \sigma_{22} = \sigma$

To this one, now only some short notes. It is possible also to generalize this result to two dimensional deformations, which is schematically shown here. The deformation of fiber assembly in two dimension or directions, 2D case.

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It is possible to show, that the value of the first and third additive components are very small, because the initial value of packing density is very small. Therefore, we can write the equation of homogenous pressure in the easier form, as shown on the right-hand side.


Note: Let us notice that the homogenous pressure is equal to b -times of the pressure corresponding to one-dimensional deformation.

The 1. and the 3. additive components are very small, because $\mu_0 \ll 1$. Therefore

$$p = -\sigma = k_p \left[\overbrace{q \xi(\sqrt{\mu \mu_0})}^{\text{small}} \sqrt{\mu / \mu_0} + \overbrace{+ b \xi(\mu) - (q \sqrt{\mu / \mu_0} + b) \xi(\mu_0)}^{\text{small}} \right] = \cong k_p b \xi(\mu)$$

and

$$p = k_p b \frac{\mu^3}{\left[1 - (\mu / \mu_m)^{2+a} \right]^3}$$



And then and we as (()) we obtain such equation, by some small approximation we obtains such equation. For pressure; for this pressure, homogenous stress corresponds also to the compression as shown here. For homogenous stress, we obtain roughly this equation, which is identical; all we need some other parameter before this relation here. So, therefore, it is possible for this type of equation to use also in yarn, by modeling of the yarn, which we will see in our later in another lecture.

Well it is all for today and thank for your attention. .