Orientation of Fibers

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Bundle Theory of Yarn Unevenness

Lecture # 18

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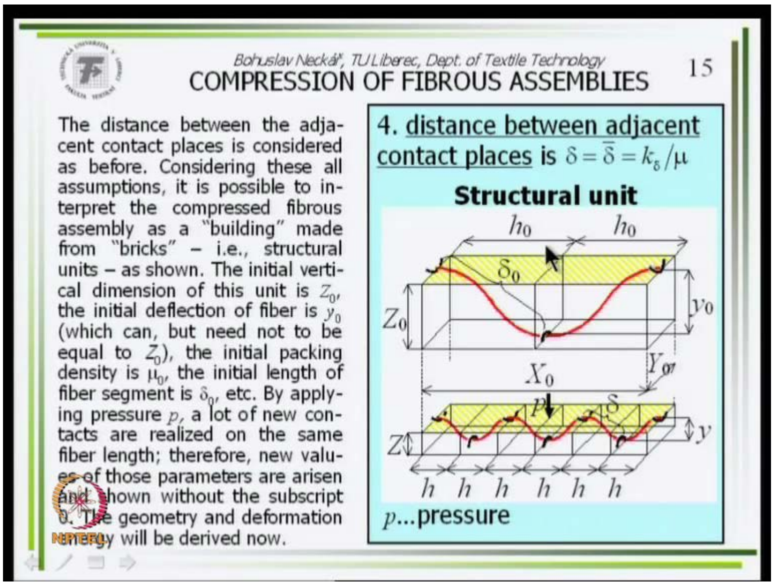

Let us continue our theme about the compression of fibrous assemblies. In the end of our last lecture, we described some structural unit, some brick, which represents the whole fibrous structure which we compress. We said that in starting position this brick represents this picture of dimensions X 0, Y 0 Z 0. The supports and the force realized using another fibers in contact at the contact places and the length between neighbor contact, agents contact, places is, let us imagine that it is a mean length, which we derived by the ratio k delta, some parameter, by packing density.

After pressure of such brick, lots of new contacts are coming, because we prefer to idealize the shape of the fiber as a regular shape. We obtain something like this here; number of contacts is higher now, the height is shorter from Z 0 to the length Z, the distance between force and support is now not h 0, but on the small one h. The number of these segments is higher, this pressure is p and the length is delta. The deflection is from value Y 0 to y.

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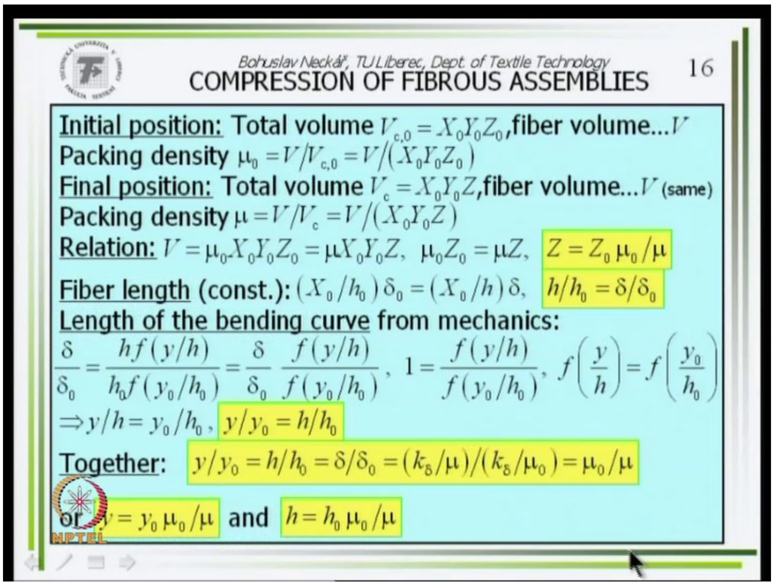

From this picture, we can write lot of relations. For example, starting volume or total volume of our structural unit of our brick is evidently X 0, Y 0 and Z 0. Inside of this, the volume of our red fiber is Y. Sorry, the fiber volume is V. And we assume that this fiber volume is constant; is not changed, because the compression represents the smaller volume of air, not the change of volume of fibers. Packing density is mu 0 which is V by V c 0, V c 0 is starting volume and it is V by X 0 Y 0 Z 0.

In final position, this total volume is same X 0, same Y 0, but the height is now Z. So, V c is X 0 Y 0 Z. The fiber volume is same, packing density is V by V c, so that it is this here and because same fiber volume, we can write that this expression must be equal to this expression, so that mu 0 Z 0 is equal to mu Z or Z is Z 0 time's mu 0 by mu. Fiber length, which is constant is X 0 by h 0 times delta 0 Y. What is it the X 0 by h 0? What is it X 0 by h 0? Please, do not say me that it is two. Yes, in the picture it is two at the moment, but generally it is number of these segments or fiber segments between neighbor contacts. The number of this segments, it is this here times delta 0, this is the length of fiber, number of such segments times lengths by one.

Number of such segments, this and this length by one, delta 0 (Refer Slide Time: 05:23). And the same is valid till after pressure. After pressure it is X 0 by h time delta and this is from the same logic, so that from this equation h and both must be of the same fiber lengths; fiber lengths is not changed, so that h by h 0 is delta by delta 0. The length of the bending curve from mechanics and what is delta by delta 0? This delta is this lengths of this portion fiber, length of this portion.

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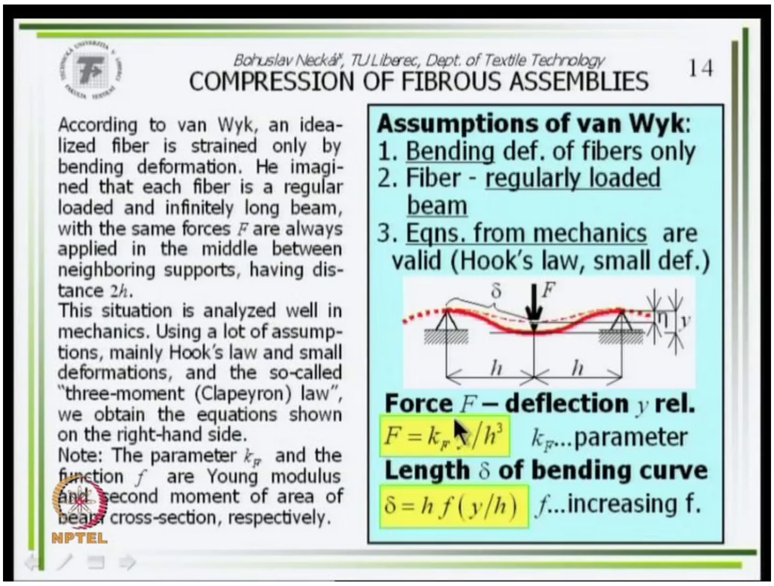

But based on this result derived by our colleagues from mechanical department this delta is h times some increasing function like of quantity Y by h. So, we use it as delta is h times f of our quantity y by h by delta 0, which is h 0 times f, this increasing function, same increasing function, but in other point or in other quantity, which is y 0 by h 0. Nevertheless, h by h 0 is delta by delta 0, so that h by h 0 is delta by delta 0. So, that it must valid is that this ratio f of y by h by f of y 0 by h 0 is equal to one in our model.

So, that this function must be equal this function and because it is monotone increasing function, it must also validate y by h is equal to y 0 by h 0. So, this is here and these relations together are shown here. Nevertheless, delta the distance between two neighbor contacts, was derived as a k delta by corresponding packing density, so that k delta by mu for delta by k delta by mu 0 for delta 0, so it is mu 0 by mu. This relations we will use later into forms like y is y 0 times mu 0 by mu and h is h 0 times mu 0 by mu.

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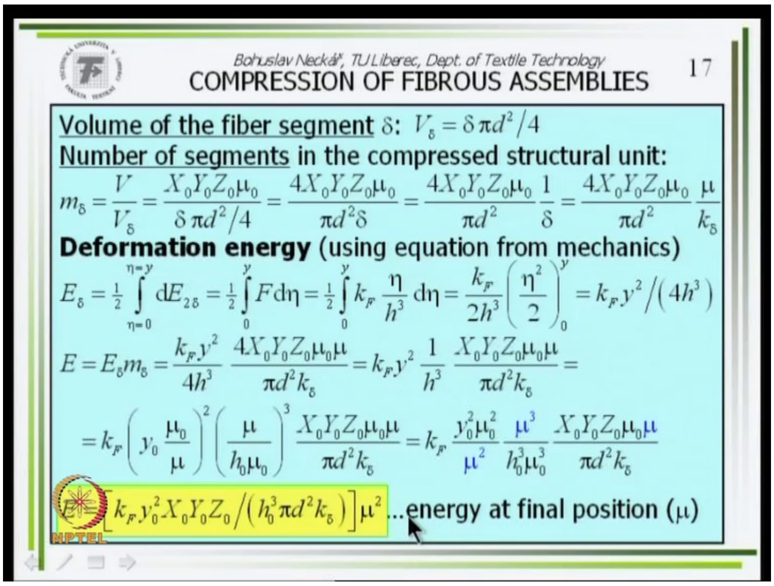

Volume of the fiber segment lengths delta Volume of fiber segment lengths delta, one of this segment for example, this segment or this segment or this segment are the same in our idealized structure. This volume V delta is a lengths delta time's cross section pi d square by four, we say d as the diameter of fiber, the equivalent diameter of fiber. Well, number of segments in the compressed structural unit; how many further segments are here? One, two, three, four, five, six; please do not say six; it is another example in my picture. Generally, how many such segments further segments are in our compressed structural unit?

This number is m delta; it is volume of fiber V by volume per one segment. It is well, is it not? Using equations derived, we obtained this or this or this (Refer Slide Time: 09:37). Using on the place of delta k delta by mu, we obtained is here, on rearranging and nothing more. We are ready to start with a formulation of mechanical relations; we will so call energy method. What it means?

When I have sorry I have some rubber for writing with this pencil, when I pressed this one I do some work. Is it so from physics? Physical work is work done. So, I gave some energy, I must give out some energy. Where is my earlier energy now? It is inside of this rubber and in which form? At most, so called deformation energy, in reality, it is small difficult, because some part of my work is transferred to thermal energy, for example or so on, but the dominant part is the deformation energy.

So, I can say, in idealized, I will then later generalize it. In idealized, the model I can say that my work must be equal to deformation energy inside of this rubber, when I neglect the others like the energy dissipation or thermal energy and so on. This way, we will use it for derivation of mechanical properties. Deformation energy, (Refer Slide Time: 11:43) we want to find that deformation energy in one of this segment, because we notice here, it is good to write it as based on deformation energy the double segment.

So, deformation energy in one segment is E delta is one half of the deformation energy, which is calculated from here to here, between two supports, because this is same as this here. So, energy here and energy here, together is energy here and we can calculate the energy here at one half of this, it is energy related to the lengths delta, deformation energy. So, it is one half of the energy d e two delta, but now back to our picture? How it is this energy? The force started with value 0, it did not jump to final deflection y. The force was higher and higher and higher, deflection was also higher and higher and higher. Final position is that we have maximum value of force and maximum value of deflection y.

Generally, on this way from starting position to the final position, the deflection will be some deflection eta; some general deflection, and it was starting from 0 on this eta was higher and higher to the final position y. Well, how is the energy, which is now in the beam or fiber; however you want. This is the actual force F; we must when, I am in some general moment, the beam is in some general moment, then some deflection y, and then using the force F, I can increase little my deflection. So, the deflection increased to incremental value of d eta, which is here.

So, force time d eta is elemental increase of energy of deformation energy in our beam. But, then the force must be a little higher, because the second d eta and so on and so on. So, that we can write it, so F times d eta where F is changed to general position eta. Nevertheless, for F, for this force, our colleagues from mechanical engineering derived such equations. This is the equation from our colleagues. Where it is? It is here and only on the quantity of deflection I call it now as eta.

So, it is this here (Refer Slide Time: 15:34). After, we must integrate this from starting position to the final deflection y, so that we obtain this here, finally here. This is the energy per one `fiber portion length delta. How is the total energy? Now, the total energy is energy per one segment times the number of segment. So, the E delta times m delta; using this expression, you attend this after rearranging; after rearranging we obtain it here. Here, is y square and here is h power two three (Refer Slide Time: 16:17). Nevertheless, you know what this y is, for y we have this expression and for h we have this one. We use it and then it is here; after small rearranging we obtain that the energy is something bracket some global parameter times of mu square. It is energy at the final position on the structure of packing density mu.

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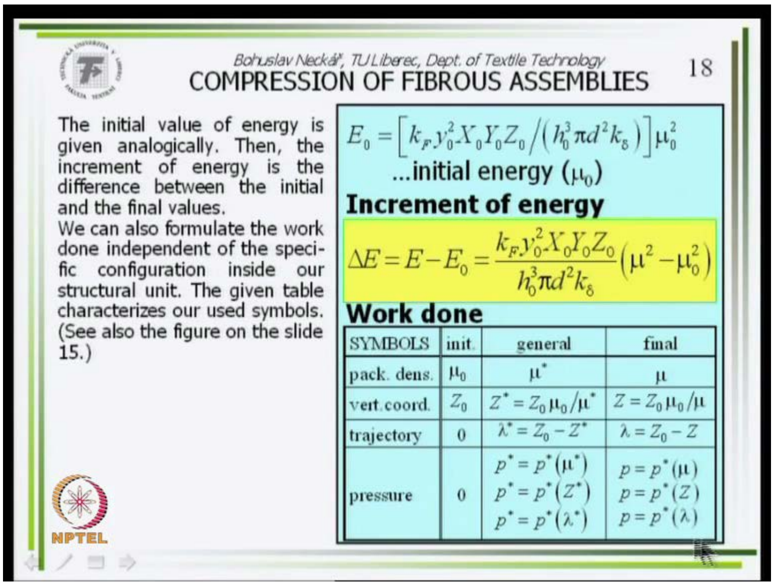

Thus on the starting position, the starting value of energy, which was in our fiber bundle in our structural unit is E 0 and it must valid the same equation with packing density equal to mu 0 and that is this one here. This is the initial energy, so that the increment of energy is delta E, which is this ratio times of mu square minus mu 0 square. Now, we need to formulate the work done.

When I use or when I work, it is a process. I have some starting situation then I give to some work and I finished on the final position in a deformed object. So, therefore, I must have three tier I use three types of quantities; starting value, general value on the way and the final value. For example, for packing density, for initial there are symbols, which we were use as initial value is mu 0, final value is mu, but through our activity, the general value on the way from mu 0 to mu is mu star. Vertical coordinate was Z, starting was Z 0, final was Z and on the way was Z star. We derived that the Z is Z 0 times mu 0 by mu and similarly, it is valid also to this equation (Refer Slide Time: 18:38).

You know on the place of mu and Z, I use the star, in subscript sorry superscript star, which are the quantities on the way from starting position to end position. Now, trajectory; staring value of trajectory is 0, final value of trajectory is lambda is Z 0 minus Z; this is trajectory. On the way it is Z 0 minus Z star because it is on the way. Final pressure; here starting pressure is 0. On the final pressure is p, as a function of packing density or of Z, of Z coordinate or of trajectory, lambda; however way you want. On the way the pressure is p star. So, pressure p star is starting from 0 and is higher, higher and higher and the final quantity is p star equal p. The function p star can be can be formulate as a function of packing density or as a function of vertical co ordinate or as a function of trajectory. Others symbols, well are understandable. I hope.

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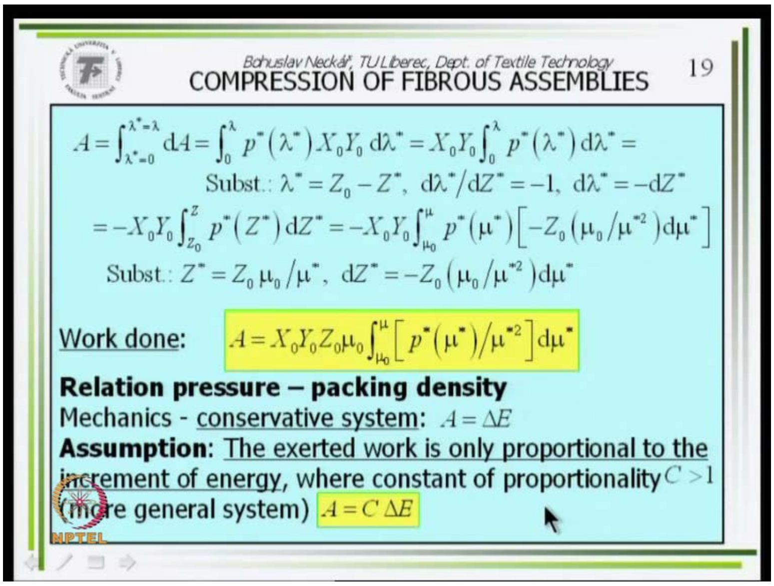

How is the work done now? It is same idea. The elemental incremental increase of our work, what is it? It is the force times the differential quantity of the lengths; force times differential quantities of the length. How is the force? It is the general force on the way; pressure for example, p star as a function of lambda times of area, which is X 0 Y 0, was in our picture. Is it not? So, this is force, this is force, this is force times d lambda star, because some general characteristics of our way to which we press is this one. Now, we can apply two times apply some substitution inside of this integral, where is this here and the second is this here, and finally we obtain work done is as X 0 Y 0 Z 0 times mu 0 times of integral, because we need sum this incremental works together. The integral is for mu 0 to mu, from this expression. Now, here we have the pressure as a function of packing density. It is this function, which we want to obtain; the relation between the pressure and packing density.

So, it is a formula for work done. Now, how it is the relation between pressure and packing density? In the mechanics, we know the term of conservative system. Conservative system is a theoretical assumption in which all our work done is transformed to the deformation energy. For example, rubber by the way is near to this ideal assumption; no other material in this world is a perfect conservative system. If it is a conservative system, then the work done must be equal to increase of energy.

In the textile, in our fibers assembly, this assumption is not too good, but we need not think so hard on simplification. We can assume that the work done is proportional to increase of deformation energy. So, that this C must not be equal to 1; C is equal to 1 for conservative system only. This C can be higher than one, so that our work done is higher than increase of energy, because part of our work is dissipated as a thermal energy and I do not know what.

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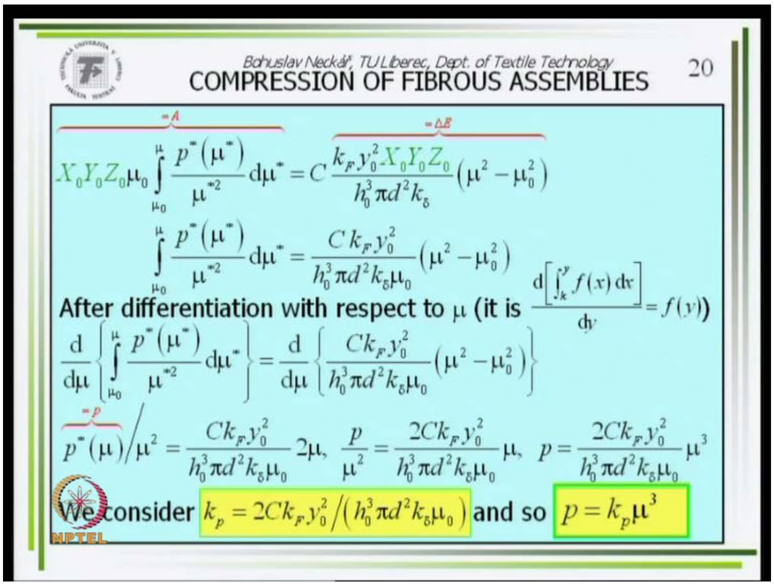

So, let us use this equation and on the place of A and delta E, we use our expression derived earlier. So, we obtained A is equal C times delta E and this is here; after small rearranging we obtained this well. It is nice, but not too nice, because our function pressure as a function of mu is inside of this integral. What to do then? Evidently, make some derivative, because it is inside the integral. So, make some derivative. Let us make the derivative of left hand side as well as right hand side. The derivative of this Z by mu; derivative of right hand side by mu is very trivial. Mu is only here, therefore, this derivative in our problem. The question is what the derivative of the left hand side by mu is. Here, where is mu? No, mu star; mu star is integrating variable and mu is the final variable.

Mu is the upper limit of this integral. The question is how the derivative of integral where the variable is the upper limit is. You know, I hope you know from the mathematical analysis that this derivative (Refer Slide Time: 25:45), this is some expression known from mathematic. This derivative is this function where on the place of the integrating quantity x, is now the upper limit y. It is a formula from mathematic. After our lecture we can derive it mutually, it is very trivial. Because, this is valid we can also solve the derivative of left hand side. So, on the right hand side the derivative is this times two times mu, evidently this here. On the left hand side, the derivative is this function, but on the place mu star, we must write mu. This is this here.

Nevertheless, the function p, pressure, general pressure by last value, the highest value of packing density or final value of packing density is our pressure p. We call it as a pressure p without star; the final pressure is p, so it is p, so that I obtain this equation here, then this equation here or this equation here. When this ratio (Refer Slide Time: 27:13), this ratio, which is not function of mu, it is only the function of material parameters; only quantities we subscript 0 and other parameters constants. If this quantity is called as a parameter k p, then the result of Van WYK theory is very easy. Pressure is proportional to mu cube, mu to the power to three. This is the result of Van WYK theory, a direct relation between pressure and packing density.

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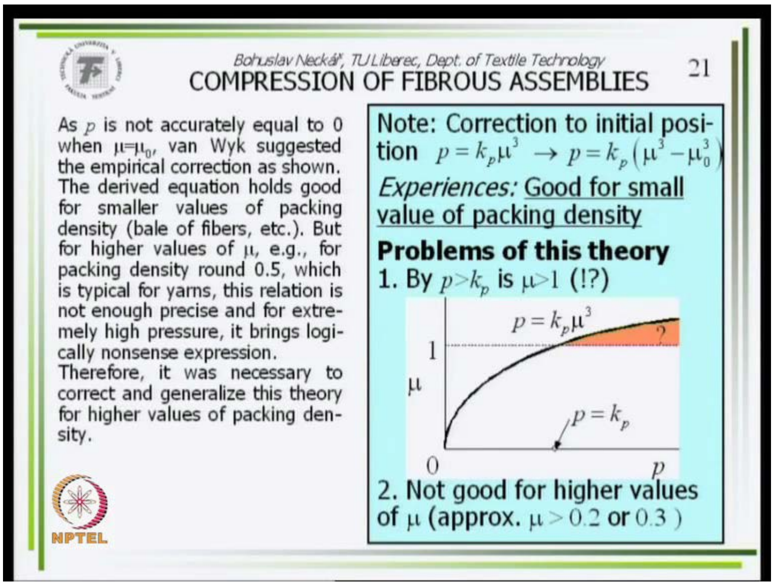

Van W Y K studied how is his experiment was and was very happy. Van W Y K worked in South Africa. In South Africa, there used wool fiber, the merino wool at most, you know it and they press it to bales and it was very good relation to his experimental experiences. So, eureka; this is perfect. From that time, lots of people use Van W Y K formula. Nevertheless, this formula is not well enough for all packing densities. You can see, first when you apply this formula to the modeling of yarn, the yarn internal structure, yarn packing density and so on, your result was not well and something is not good.

Second, let us see how is this equation, p is proportional to mu power to three is going over. So, when our high value of p represents, here k p is some real constant, so when you use some finite length finite value of pressure, you obtain mu higher than one. This curve is showing here, so that here from this value of pressure, it is pressure and packing density. For higher values of pressure, the packing density is higher than one, which is logical non sense; packing density must not be higher than one, it is evident.

Why in a system, in model of Van W Y K, which is good for not only for wool fibers, when lot of others checked this equation and they said yes. This equation is good for packing densities to may be 0.2 to 0.3. When your packing density is 0.3 and smaller and you can say that this equation works very well. But why it is not good for higher values of packing density, which we use.

For example, in the yarn, in the yarn, we have packing density at most between 0.4 and 0.5, of 40 to 50 percent. Then, how it is possible? Because to solve this problem, to this moment, it was original, it was pure Van W Y K theoretical concept, only the derivative is a derivation of this, I used a little other way to obtain it. But as a theoretical model and the result is Van W Y K model. Because, this problem is such model, we derive some generalization of Van W Y K concept. It is based on following idea. The contact between two fibers is usually not only a point contact. It takes its some small area, contact area, between two fibers.

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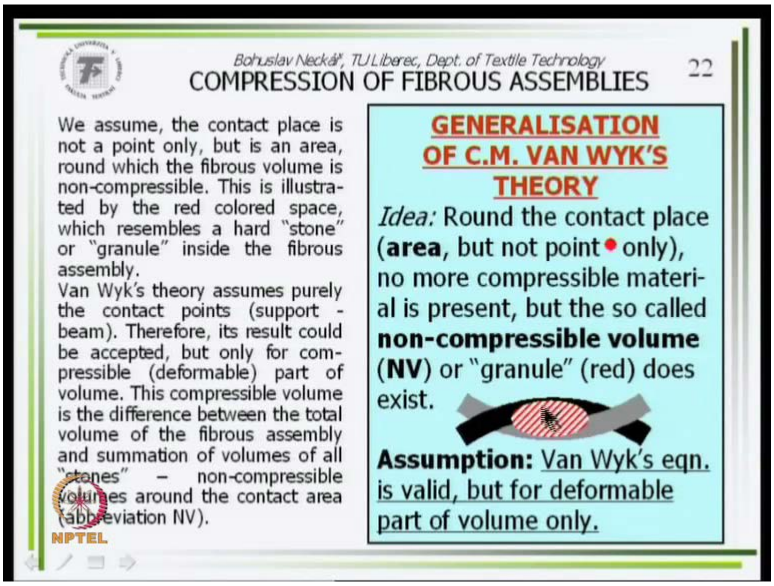

And around this, the volume around this contact area, in this contact area, functions as a stone inside of it. Let us imagine it. This part of my finger, I can move with these two, but here it is like a stone, it is some small stone or some granule. So, this is some non compressible volume. Is it not? So, that we said we said to us that.

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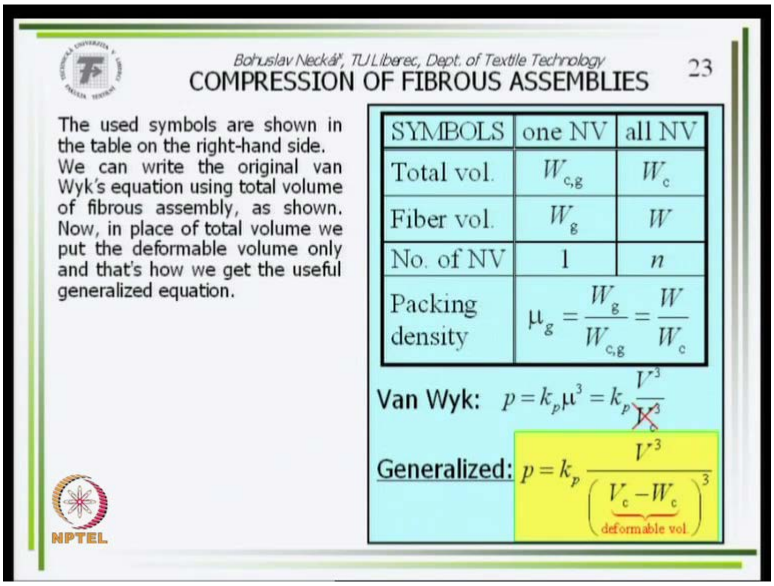

It is a non compressible volume and Van Wyk theory is valid. Van Wyk theory is well it is valid, but not for whole volume of fibrous assembly. But in each moment only for deformable volume, which is total volume minus the volume of such stones, which are non compressible. Can you imagine it? I think the principle is. This non compressible volume, in short is N V here. All granules, I call it home granula and granules; it is possible to call it here to. It gives the following idea. Earlier, Van W Y K's original Van Wyk's equation is k p times of mu power to three; mu is fiber volume by total volume. So, both times it is V. This equation is the relation of two quantities; p and total volume, volumes of fibers is same.

Now, we say no to total volume, we must in this place, give deformable volume, which is total volume minus volume of all our stones, our granules. Now, symbols; for one non compressible volume are here, for all non compressible volumes for one granule is here, for all granules here. Total volume has subscript c, so total volume pair one granule is V c g, total volumes of all granules is W of course is W c. Fiber volume per inside of one granule is W g and fiber volume of whole granules is W.

Number of granules of non compressible volumes is one per one granule, and generally n in our material. Packing density inside of our granule is very high and is every time near to one, but some gaps must be there and therefore, we must also have the quantity packing density inside of our granule. It is mu g; it is all ratio of W g by W c g or W by

W c; both are possible. And this is our idea. The problem is now to formulate how is the non compressible volume of all granules, the volume W c.

This W g volume, fiber volume inside of our one stone, one granule, one compressible volume, generally say it is not constant. Because, due to it subjective with your fingers. By light contact, you can feel that this volume is small and is non compressible volume is small. Also the volumes of fibers, the whole granule is small, so that the fiber volume inside of our fiber granule will be small. From other right, when we have some contact, which is very intensive, then the contact area is large, our granule is big and fiber material inside of our granule, the volume of this is high.

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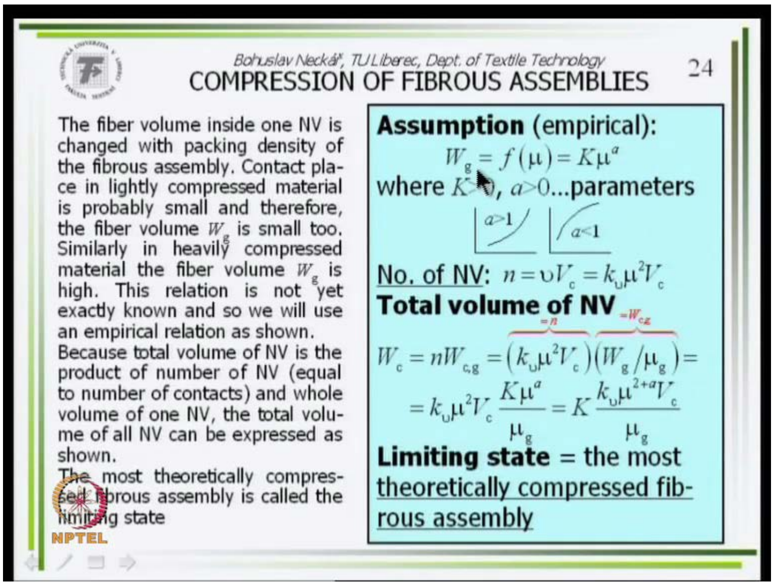

So, volume of fibers inside one granule relate to the packing density of the whole structure. When we have higher packing density then the volume is higher. So, it is a function, increasing function of packing density. It must be so. We have another theoretical model for this moment. We only know that it is some increasing function. Therefore, in this moment, we will use some empirical function and we chose the function K times mu power to a; K and a are two suitable parameters. Why, is because it is evident. If a, is higher, then it is convex increasing function. If a, is smaller, then it is concave and if a, is equal one, it is straight line.

So, these formulas can characterize each existing relation, roughly. Nevertheless, this formula is empirical. It is written here, it is empirical. Using this, the total volume of non compressible volume of granule, which is of number of granules time total volume of per one granule. Using this derived the number of granules, it is same then the number of contacts in our fiber assembly and number of contacts is the density of contacts, which is this time volume. Density of contact is per one volume unit, so times volume. So, is number of granules and this is for W c g. After rearranging and using this equation we obtain that W c. Total volume of all granules is given by such equations. The parameter K is possible to derive using the idea of limit state, which is under this term, I think the most compressed fiber assembly.

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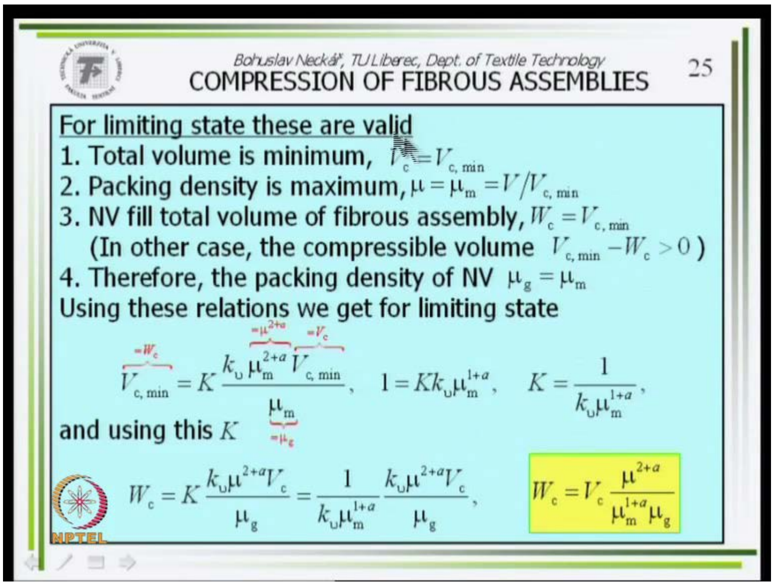

How it is in the limit state? The total volume let us imagine the limit say the maximum of maximum compressed fiber assembly, which is possible. No more is possible. Total volume of such structure must be of minimum. Packing density is maximum, the maximum value of packing density, I will call mu m. it must be fiber volume by minimum of total volume and it is definition of packing density. Our granule fills total volume of fibrous assembly. Why? Let us imagine it is not. Then among of our granules there must be some compressible volume. If there exists some compressible volume then it is possible for this material to be more compressed. If it is possible for the material to be more compressed, then it is not the limit case; limit case is that it is not possible to compress more our material.

So, in this limit situation it is so that our fibrous assembly is like as a wall, from brick to brick; one brick, second brick, third brick; nothing among this, only stone beside stone. Therefore, W c is equal to V c min, and the packing density of this structure is mu max, is mu m is equal to mu g, because it is from the bricks from our granules only; it is the wall from our bricks. So, that our earlier equation, general equation is here, we can write now in this black form. The red, say what was earlier here; earlier on left hand side was W c. Now, in the place of W sorry V c V W c, in this place of this is now V c min. Similarly, we substitute all here.

From this we obtain this equation, so that K is 1 by k mu times m power to one plus a. we can explain the parameter K as a function of, without the parameter, as a function of maximum packing density. Using this expression here, in our general formula, which is this one, we obtain this here. This is our earlier formula. In the place of K, we use this here. W c is V c times of this and now we can have two packing densities here; the maximum value of packing density and packing density inside the granule. We can well imagine that both are same that inside of granule is every times a maximum packing density, only a small necessity gaps beside fibers.

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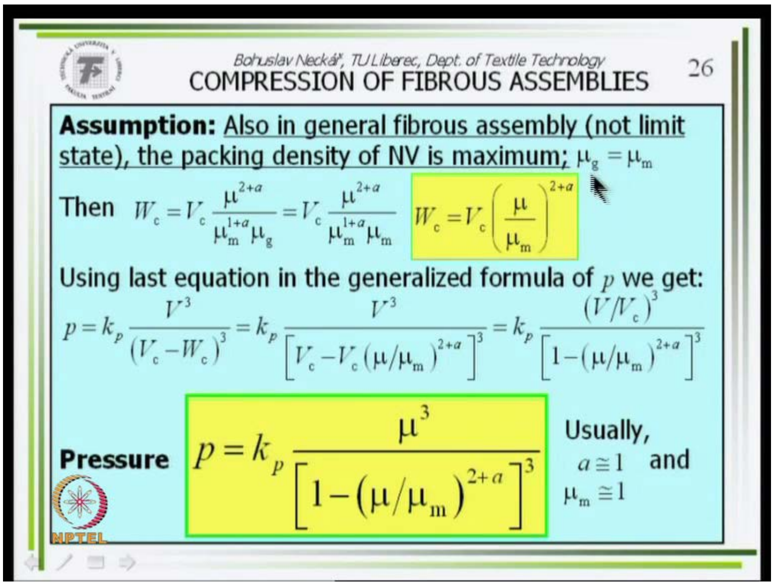

Then using this assumption we obtain the W the subscript c is V c times mu by mu m power to two plus a, and this is used on the place of our generalized idea; total volume minus volume of all non compressible parts. After small rearranging, using that the packing densities which is fiber volume by total volume, we obtain this here and finally, we obtain this equation. Based on our experiences, mu m is usually very near to value one; maximum packing density, and the parameter a, which is here is based on our laboratory experience is near to one. It means this relation is straight.

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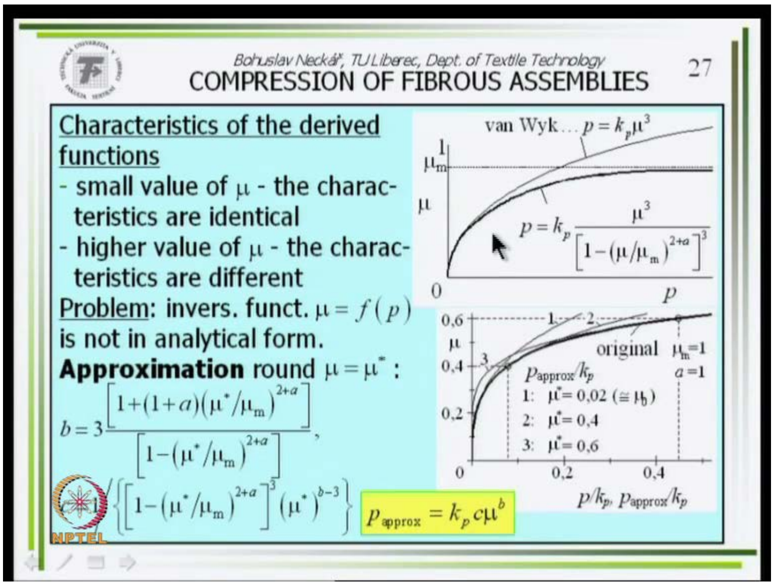

And then we obtain such curve; such diagram. Pressure, packing density, this is our mu generalize function and this is original Van WyK curve. In the region of small packing density, we obtain roughly same graph, same shape. Then by higher values, our mu curve limited to maximum packing density, some think which is very near to one.

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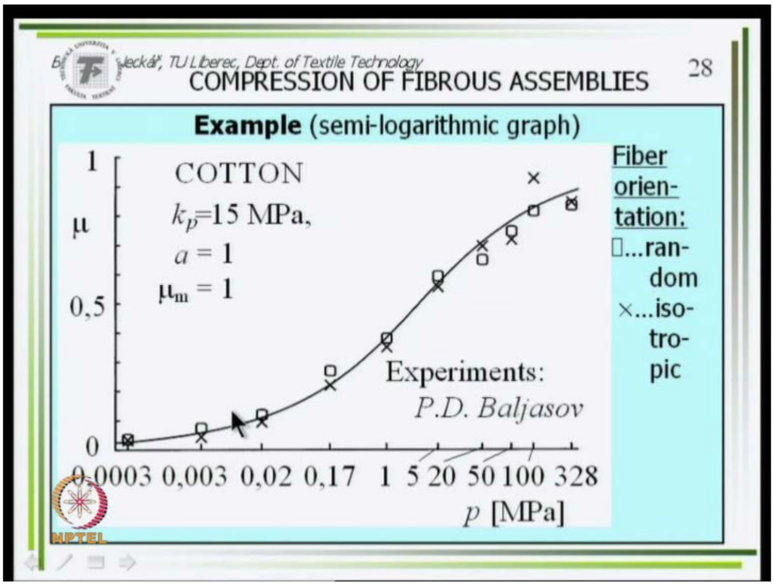

It is possible to do some approximation because we formally need it. You can read it later in home. I want to comment a relation between our generalized equation and experiments, which was doing by Baljasov, Professor Baljasov, was earlier known name in Moscow and Moscow textile institute. He studied experimentally this relation of pressure and packing density in very large interval of pressure from 0.0003 mega pascals to 328 mega pascals. He used for smaller values our braking machines, for higher pressure he used some special machines from mechanical department.

His experimental results are shown at the point in this diagram. It is semi logarithmic graph, so that on the abscissa the logarithmic scale of pressure, on the ordinate is the linear scale of packing density. When we use the original Van Wyk, then from here the curve is increasing so, over the value one. Nevertheless, our curve corresponding to this experiment needs curve, which parameter k p is 15, a is equal one, mu m equal one is this. You can see that this comparison is experimental results, we compare it also. This is experiment by Baljasov, but we have lot of our own experiment from our lab and same character of result.

So, you can see that may be till packing densities of 0.8 is our type of curve, is in good comparison to experimental data. Some differences you can see by very high value of pressure and packing density near to one, this here (Refer Slide Time: 48:38). Why it is because the pressure is so high that quite other physical processes are there. Like the destructions of fibers, it is a pressure, which creates from fibers powder. It is deformation destruction of fiber material and so on.

Therefore it is so high values, but there are unreal to obtain in textile processes. Yesterday, we discussed Professor Isthiaque, some research problems and we both know that the value around 0.8, packing density around 0.8, is the maximum, which we can obtain using all textile processes. Higher value, it is the production of fiber balls, it is not textile technology. So, you can see that the result is very good, acceptable. We will use it in one of our later lecture about a yarn structure. We will use this final equation, this one here, (Refer Slide Time: 50:15) also as an input to the model of internal yarn structure.

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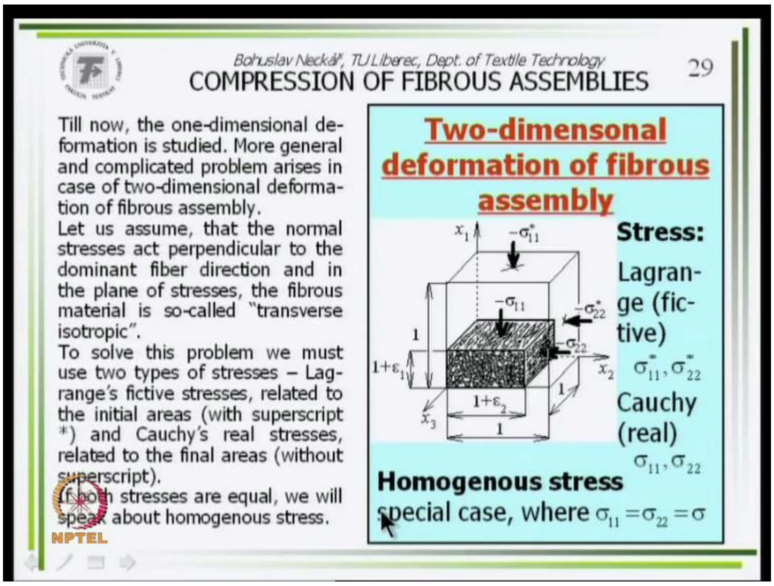

To this one, now only some short notes. It is possible also to generalize this result to two dimensional deformations, which is schematically shown here. The deformation of fiber assembly in two dimension or directions, 2D case.

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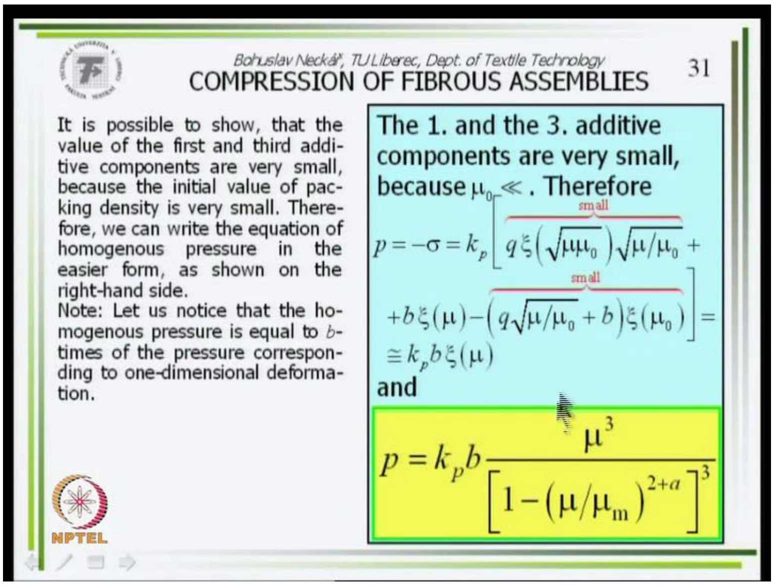

And then and we as (( )) we obtain such equation, by some small approximation we obtains such equation. For pressure; for this pressure, homogenous stress corresponds also to the compression as shown here. For homogenous stress, we obtain roughly this equation, which is identical; all we need some other parameter before this relation here. So, therefore, it is possible for this type of equation to use also in yarn, by modeling of the yarn, which we will see in our later in another lecture.

Well it is all for today and thank for your attention. .