

**Orientation of Fibers**  
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**Model No. # 01**

**Lecture No.# 19**

**Yarn Strength as a Stochastic Process**

As an introduction to this lecture, I want to ask you - do you know, why we do not use metal plates for our clothes? Of course, because they have not pores, because they are too hard and they have not pores; is it not? Pores are very specific or the gaps between among fiber is a very specific behavior of fibrous assembly which we use very intensively in our clothes; nevertheless, also for lot of technical applications, for example, by filtration and so on.

Today's lecture will be oriented to the possibility on how to model the pores and gaps among fibers.

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**PORES AMONG FIBERS** 1

**GENERAL DESCRIPTION**

*t*... fiber fineness, *s*... fiber cross-sectional area  
*ρ*... material fiber density, *d*... equivalent fiber diameter,  
*p*... fiber circumference, *q*... fiber shape factor,  
*a*... specific fiber surface area, *L*... total length of fibers,  
*A*... total surface area of fibers, *V*... total volume of fib.,  
*V<sub>c</sub>*... total volume of fiber assembly, *μ*... packing density

It was derived (lecture 1): :

1.  $t = s\rho$ ,  $s = \pi d^2/4$ ,  $d = \sqrt{4s/\pi} = \sqrt{4t/(\pi\rho)}$
2.  $q = p/(\pi d) - 1 \geq 0$ ,  $p = \pi d(1 + q)$
3.  $A = pL = \pi d(1 + q)L$ ,  $a = 4(1 + q)/(\rho d)$
4.  $A = V/V_c$ , where  $V = Ls = L\pi d^2/4$

Let us start with small repetition of our earlier relations which we derived in lecture 1 about times, definitions, relations and so on. We use symbols *t* for fiber fineness, *s* - fiber cross-sectional area, *ρ* - a material fiber density, *d* is equivalent fiber diameter, *p* is fiber

circumference,  $q$  - fiber shape factor,  $a$  is specific fiber surface area, capital  $L$  will be total lengths of fibers in our fibrous assembly,  $A$  is total surface area of fibers,  $V$  is total volume of fibers,  $V_c$  - total volume of fiber assembly and  $\mu$  is packing density.

We derived for fibers that fiber fineness  $t$  is cross section  $s$  times  $\rho$  so that all these relations are valid.  $q$  - fiber shape factor is defined as the perimeter of fiber cross section by  $\pi d$ , the shortest perimeter which is possible for the same area minus 1 so that fiber perimeter is given by this expression (Refer Slide Time: 03:07).

Total surface of fibers is fiber perimeter - perimeter of fiber cross section times total lengths of fibers. Using equation derived, their value of this equations and packing density was defined. Then, ratio - fiber volume by total volume where fiber volume is fiber cross section or  $\pi d^2$  by 4  $L$  times, of course, the total lengths.

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Furthermore let us define the surface area per unit volume of fiber:  $\gamma = \frac{A}{V} = \frac{\pi d(1+q)L}{L\pi d^2/4}$ ,  $\gamma = A/V = 4(1+q)/d = a\rho$

**Pores and their characteristics**

Volume of free space among fibers:  $V_p = V_c - V$   
**Porosity** (relative characteristic of this space):  
 $\psi = V_p/V_c = (V_c - V)/V_c = 1 - V/V_c$ ,  $\psi = V_p/V_c = 1 - \mu$

Porosity characterizes volume of free space among fibers, but not the size of gaps among fibers.

Therefore, we divide these spaces by **fi-ctive (imaginary) borders** (\) to produce some suitable bodies in the form of **small tubes** or **capillaries**, called **pores**.

Let us introduce more; one new quantity surface area per unit volume area, surface area of fibers in our fibrous assembly. We said it is this  $\pi d$  times  $1 + q$  times  $L$  and a volume of fibers is  $\pi d^2$  by 4 cross section times  $L$ . From this,  $\gamma$  which is  $A$  by  $V$  is given by such expression (Refer Slide Time: 04:14 to 04:26) and in comparison to  $a$ , what was  $a$ ?  $a$  is specific fiber surface area which is here. You can see that the surface per unit volume is specific area, specific fiber surface area times  $\rho$ .

Well, pores - relatively easy is to obtain a quantity which we call porosity. What is porosity? A model fiber has some gaps, some space of air; volume of this space is  $V_p$ ; yes,  $V_p$  which is total volume minus volume of fibers because what is not the fiber volume is the volume of air. I think air who **who** is giving inside for example **(())** usually had beside fibrous air is free air so that the porosity is the ratio volume of pore of air among fibers times total volume is this one, then this one (Refer Slide Time: 05:43) because this packing density porosity is  $1 - \mu$ . When the packing density is 40 percentage, then porosity 60 percentage. But more precisely, if porosity 0.6, then packing density 0.4 and so on.


It is easy and easy to obtain. Nevertheless, let us imagine two situations. One box, big box from compact polyester and on the center is one big hole. Can you imagine such box? Packing density inside this box can be, I do not know, may be 0.5. Let us imagine second box full of compressed polyester fibers. Packing density is also 0.5. Nevertheless, between these two boxes, high difference with behavior, the behavior of these two boxes is quite odd. Why? Because in the first case, the air is in one big hole; in the second case, the air is in a very long path of very small tubes beside fibers. Therefore, we need also to calculate some dimension of this **air space on of such airspace** air space, if it is big or very thin, so that we must create something which we will call pores.

Starting idea is following. Let us imagine that you have some red pencil. Can you imagine that you have some red pencil? Yes, you can. Make between fibers, some red lines as it is shown here in **the scheme of schematic** scheme of some section of fibrous assembly. Make this your red pen. Such lines, how you want - very subjectively; very subjectively, how you want?

This imagination is for this moment. Later, I take your red pencil back here. **Then say, but now in the moment because understand it**; please imagine it so. Using your red pencil, you divide the **the** region of air to some tubes having very different shape. Such tubes, such tubes, now, so created, we can call pores. **yeah** Thereal, **thethe** your red lines, we will call as a fictive or imaginary borders because they are not in reality; it is only your fiction; it is only used through your pencil.

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Each created pore (e.g. yellow):

- is in contact with fibers (black - **real borders**) and also with other pores (red - **fictive borders**).
- looks like an **"air fiber"**. (Therefore, all equations derived for real fibers are valid for these "air fibers", too. Subscript "p" like 'pore' will be used for "air fibers".)

Pore sectional area:  $s_p = \pi d_p^2 / 4$ ,  $d_p = \sqrt{4s_p / \pi}$

where  $d_p$ ... **equivalent pore diameter**

Perimeter of pore  $p_p$  - is defined as the length of **REAL (black) BORDERS ONLY!** (In fact, fictive borders do not exist.) Therefore  $p_p$  may be shorter than the perimeter of a circle having the same area;  $0 \leq p_p \leq \pi d_p^2 / 4$

So, each created pore is in contact with these fibers, black borders, real borders, and also we have the pores in our scheme. Using your red pencil, there are fictive borders and this tube, this air tube so created looks like an air fiber. **yeah** Therefore, practically, all equations which we derived for fiber are valid also for our air fiber, air tube.

These equations which are valid for air tubes, the quantities related to our air tube, we will have subscript p like pore because this **this** air fiber, this air tube we call pore. For example, here is one pore; black is real borders, red is fictive borders, area, cross sectional area of this pore is  $S_p$  which is  $\pi d_p^2 / 4$  where  $d_p$  is this. Here  $d_p$  is equivalent pore diameter. You know this equation from fibers. It was equivalent fiber diameter. Now, it is equivalent pore diameter.

One difference between our air fibers and three fibers exist in the theorem of parameter of pore. You know, opposite to fibers the parameter of pore to the parameter of pore, we calculate the real borders only so that the lengths of parameter of pore, our air fiber is **is** this length this black lengths plus this black length plus this black length (Refer Slide Time: 12:06). But now, this red parts therefore, the parameter lengths of the parameter lengths in pore is shorter than the parameter from how it this term defined in traditional geometry because the red parts are not our lengths related to **to** parameter, because it does not exist; it is only your subjective red pencil yes.

Therefore, the lengths of parameter can be higher, but also smaller than the  $\pi d_p^2$  by  $4d_p^2$  in a yarn in a fiber. It is not possible is it not? We discussed it. All other equations are valid.

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**Pore shape factor:**  $q_p = p_p / (\pi d_p^2) - 1, q_p \geq -1$   
 (Considering the definition of  $p_p$ , the pore shape factor could be negative.) It is also valid  $p_p = \pi d_p^2 (1 + q_p)$ .

**Assumption (simplification):** All pore shapes in a fiber assembly are the same. Then all equations are valid for each pore.

**Total pore volume:**  $V_p = s_p L_p, V_p = (\pi d_p^2 / 4) L_p$   
 where  $L_p$ ... **total length of pores** in a fiber assembly

**Total pore surface area:**  $A_p = p_p L_p, A_p = \pi d_p (1 + q_p) L_p$

**Surface area per unit volume of pore:**  $\gamma_p = \frac{4(1 + q_p)}{d_p}$

$\frac{A_p}{V_p} = \frac{[\pi d_p (1 + q_p) L_p]}{[(\pi d_p^2 / 4) L_p]}$

$q_p$  per shape factor - same equation is for fibers only subscript  $p$ . Nevertheless  $q_p$  shape factor of pore need not be higher than 0; can be also smaller than 0 because parameter is without red fictive borders and it must be higher equal minus 1. Volume, total pore volume is  $s_p$  times  $l_p$ . So, that is where  $l_p$  is total length of pores, some of all **some of all** paths **which you** through which you can go when you will be so smaller than microlevel.

Total pore surface area that is parameter times length is the total pore surface area, surface area per unit volume of pore is  $\gamma_p$  is  $A_p$  by  $V_p$ . Are we not using equation derived on the place of  $A_p$  and there, obviously,  $V_p$  which is here in a red to show after rearranging, we obtain  $\gamma_p$  and such for...


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### Effect of pore border choice

All air gaps are divided into  $n$  same pores (example:  $n = 3$ ). Each pore has the same parameters  $s_p, d_p, p_p, q_p, \gamma_p$  as pore 1.



If we **remove** the fictive borders among  $n$  pores (among pores 1,2,3) we get a **new big pore** having parameters denoted by  $'$ . It is valid:

Pore sectional area and perimeter:  $s_p' = ns_p, p_p' = np_p$

Equivalent pore diameter:  $d_p' = \sqrt{4s_p'/\pi} = \sqrt{n} \sqrt{4s_p/\pi}, d_p' = \sqrt{n} d_p$

Pore shape factor:  $1 + q_p' = \sqrt{n}(1 + q_p)$

Total length of (big) pores:  $L_p' = L_p/n$

This part we can jump. You can write it when you want and it is not too necessary.

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Total pore volume:  $V_p' = s_p' L_p' = s_p L_p, V_p' = V_p$

Total pore surface area:  $A_p' = p_p' L_p' = p_p L_p, A_p' = A_p$

Surface area per unit volume of pore:  $\gamma_p' = A_p'/V_p' = A_p/V_p, \gamma_p' = \gamma_p$

**Values  $V_p', A_p'$  and  $\gamma_p'$  are independent of the choice of fictive borders!**

### Conventional pore

The inverse value of surface area per unit volume of pore  $1/\gamma_p' = d_p/4(1 + q_p)$  has a length dimension. So, we introduce a variable  $4/\gamma_p'$ , according to which the pore size will be evaluated. This variable will be called

**conventional pore diameter**  $d_p^* = 4/\gamma_p' = d_p/(1 + q_p)$

So, that we jump **jumpin** to samethis formulation of conventional pore.

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**Note:** In contrary to  $d_{pr}$ , the conventional pore diameter  $d_p^*$  is independent of the choice of fictive borders, i.e. independent of the shape factor  $q_p$  of (real) pore!  
(We denoted parameters of conventional pore by \*)

Because  $V_p = (\pi d_p^2 / 4) L_p$ , similarly we use  $V_p^* = (\pi d_p^{*2} / 4) L_p^*$  for conventional pore. But  $V_p^* = V_p$  (independent of the choice of fictive borders). Then it is valid for **total length of conventional pores**:

$$V_p^* = V_p \cdot (\pi d_p^{*2} / 4) L_p^* = (\pi d_p^2 / 4) L_p \cdot \left( \frac{d_p(1+q_p)}{d_p^*} \right)^2 L_p^* = d_p^2 L_p \cdot L_p^* = L_p (1+q_p)^2$$

Because generally  $A_p = \pi d_p (1+q_p) L_p$ , analogically we use  $A_p^* = \pi d_p^* (1+q_p^*) L_p^*$  for conventional pore. But  $A_p^* = A_p$ .

$$A_p^* = A_p \cdot \pi \frac{d_p^* (1+q_p^*)}{d_p (1+q_p)} L_p^* = \pi d_p (1+q_p) L_p$$

I will define conventional pore more easier for you.

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$$d_p(1+q_p)(1+q_p^*)L_p = d_p(1+q_p)L_p, \quad 1+q_p^* = 1$$

**Shape factor of conventional pore:**  $q_p^* = 0$   
(Conventional pore can be considered as air cylinder!)

**Sectional area of conventional pore:**

$$s_p^* = \pi \left( \frac{d_p(1+q_p)}{d_p^*} \right)^2 / 4 = \left( \frac{\pi d_p^2}{4} \right) / (1+q_p)^2, \quad s_p^* = s_p / (1+q_p)^2$$

**Perimeter of conventional pore:**

$$p_p^* = \pi \frac{d_p(1+q_p)}{d_p^*} (1+q_p^*) = \pi d_p / (1+q_p) = \pi d_p (1+q_p) / (1+q_p)^2, \quad p_p^* = p_p / (1+q_p)^2$$

**Note:** All **parameters of conventional pore are independent of the choice of fictive borders**, which is of great importance in practice. (Other defined pore parameters depend on the choice of the fictive borders.)

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**Relationship between fibers and pores**

The total fiber surface area  $A$  (■) is generally higher than total pore surface area  $A_p$ . Namely the contact areas (■) are parts of fiber surfaces, but not parts of pore surfaces. But if the contact areas are very small, then it is possible roughly to use the following *assumption* (simplification):  $A_p = A$


Using equations derived before we find **surface area per unit volume of pore:**

$$\gamma_p = \frac{A_p}{V_p} = \frac{A}{V_p} = \frac{A}{V_c} \left( \frac{V_c}{V_p} \right)$$

$$\gamma_p = \frac{4(1+q)}{d} \mu / (1-\mu)$$

$$\gamma_p = \gamma \mu / (1-\mu)$$

$$\gamma_p = \frac{4(1+q)}{d} \frac{\mu}{(1-\mu)}$$



So, we derived equations which are valid for pores for our air fibers; we can say air tubes. These equations, these equations give together different quantities, different quantities  $s_p$ ,  $d_p$ ,  $V_p$ ,  $A_p$ ,  $q_p$ ,  $\gamma_p$  which are valid for pores. Nevertheless, we usually need, we do not know these characteristics. We know the characteristic of fibers, we know fiber diameter because we know fiber fineness, we know fiber surface and so on and so on.

How is the relation between pores and fibers? Can we obtain something; can we say something about the pores when we know enough quantities related to fibers? How is it?

Let us imagine that you are very small micro and you are working inside of our fiber structure through the pore tubes. Because down the go, down to your face, you your hand will be on the walls around, what is it? What are the walls around you? It is a surface of fibers; is it not? But this is also surface of pores, so that we can say that the surface of fibers is same than the surface of pores.

This idea is not fully clear. Why? Let us imagine the situation which is in our picture. By contact of 2 fibers, some contact area with red which is here related to fiber surface, but it is not; this contact area is not surface, pore surface. It is not part of pore surface; is it not? So that the pore surface is a little smaller than the surface of fibers, but if this contact places are not too large and limited to point, not too compressed fibrous material and so on. We can say that our starting idea can be as a assumption used. So, we were used that surface of fibers, total surface of fibers is equal to total surface of pores  $A_p$  is equal



A. Then  $\gamma_p$  was  $A_p$  by  $V_p$  because  $A_p$  is equal  $A$ , we can write this black here (Refer Slide Time: 18:46), we can multiply and divide by blue. It is blue; it is blue  $\frac{1}{\mu} V$  and by green  $V_c$ , so that we obtain this expression. But this is gamma; this is mu; this is 1 by psi; so, 1 by 1 minus mu. So, that you can write that gamma p is gamma times mu by 1 minus mu and gamma p also is when we use on the place of gamma. This expression also derived earlier; gamma p is this expression (Refer Slide Time: 19:21).

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Further  $\gamma_p = \frac{1}{\mu} \frac{A_p}{V_p} = \frac{1}{\mu} \frac{4(1+q_p)/d_p}{4(1+q)/d} = \frac{1}{\mu} \frac{(1+q_p)}{(1+q)} \frac{d}{d_p}$

$d_p/(1+q_p) = [d/(1+q)] [(1-\mu)/\mu]$

**Equivalent pore diameter:**  $d_p = \frac{1+q_p}{1+q} \frac{1-\mu}{\mu} d$

Especially for **conventional pore diameter**

$d_p^* = \frac{1}{1+q} \frac{1-\mu}{\mu} d$

It is also possible to use the following rearrangement:

$A_p = \pi d_p^2 (1+q_p) L_p$ ,  $\pi d^2 (1+q) L = \pi \left[ \frac{1+q_p}{1+q} \frac{1-\mu}{\mu} d \right]^2 (1+q_p) L_p$

**Total length of pores:**  $L_p = \frac{(1+q)^2}{(1+q_p)^2} \frac{\mu}{1-\mu} L$

Especially for **total length of conventional pores**  $L_p^* = (1+q)^2 \frac{\mu}{1-\mu} L$

Further using the  $\gamma_p$ , earlier derived equation for  $\gamma_2$  we obtain this equation. Here is 4; here is 4. It is going out and after rearranging we obtain such expression for  $d_p$ , equivalent pore diameter. It is nice.  $q$  is our cross as a shape factor of fibers;  $\mu$  which is a packing density how many how is the density of fibers in our structure;  $d$  is fiber equivalent, fiber diameter. Only one quantity on the right hand side bring us difficulties;  $q_p$ .

What is it  $q_p$ ?  $q_p$  is a shape factor of pore, but shape factor of pore related to two influences. The first is how is the structure of our fibrous assembly, if another fibrous assembly evidently in other shape? Second - how was your subjective idea to create fictive borders? It is your red pen because these 2 influences we do not know how is the value  $q_p$ . We do not know the value of  $q_p$ .

This moment is an idea how to very roughly say something to the dimension of these gaps and fibers independently to then that we do not know the quantity  $q_p$  and the people say, let us use some convention.

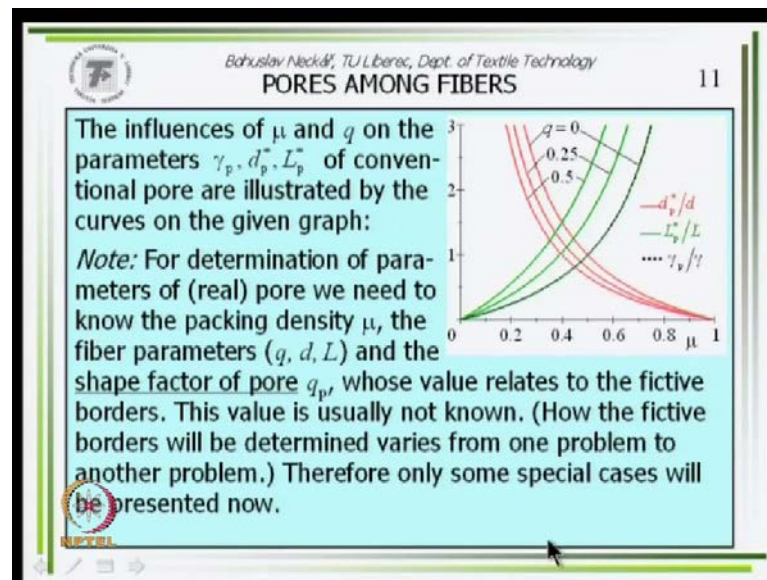
Let us say that  $q_p$  is equal 0 point; you know that it is not may be, but because we do not know what is it, let us use for a rough evaluation; let us use convention, convention only that  $q_p$  is equal 0. If  $q_p$  is equal 0, I obtain this equation (Refer Slide Time: 21:54); is it not? And it give me new program; such diameter to calculate; such diameter I call as a conventional pore diameter.

Conventional pore diameter you can calculate without knowledge of your, in the moment I say, knowledge of your a structure of your material, of your fibrous assembly and you need not have the knowledge about your earlier work with red pen. It is a convention pore.

How is the total length of pores, area of pores  $A_p$ , surface area of pores we said we assumed that it is equal to surface area of fibers  $A$  and for  $A$  we know that such equation and because  $A_p$  was  $\pi$  times  $d_p$  times this one (Refer Slide Time: 23:00). So, we can use it. On the place of  $d_p$ , we use this here and after rearranging we obtain we obtain  $l_p$  total length of pores as  $1 + q$  by  $1 + q^2$  by  $1 + q^3$  times  $\mu$  by  $1 - \mu$  times  $L$ .  $L$  is total fiber lengths possible to calculate it **it is from from** knowledge of our fibrous material, packing density  $q$ , but  $q_p$  also same problem and same idea when I know nothing. Then I will use some convention. You know the term convention. Only convention that  $q_p$  is equal 0; then I obtain for fiber pore total pore length such equation because this is the one -  $q_p$  equal 0; this is the length, total length of conventional pores in our fibrous material.

So, we define based on convention the quantities related to the conventional pore every times we are able to calculate conventional properties of conventional pore. The second question is, if in these are the textiles lecture, it corresponds what are bad to real dimension or lengths of pores? We will see it. Because to illustrate the convention in this moment, I do not know  $q_p$ , **So, that in I,** but I can say something about conventional pores. How is a conventional equivalent diameter of conventional pore diameter of conventional pore is shown here is based on our earlier equation these equation direct curves.

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So, these are packing density for 0 to 1 on the ordinate of quantities of functions used here.  $q$  equal 0. What it means? Cylindrical shape of a fiber cross section or fiber cylindrical fibers; then, the curve. The curve is now a  $d_p$ ; then  $d_p$  start by  $d$  by yarn by fiber diameter, this ratio (Refer Slide Time: 25:46) convention of pore diameter by fiber diameter; this ratio is possible to show **to tototo show** here. The function the course of this function is following. It is with higher packing density. So, smaller is diameter of conventional diameter of pore and theoretically if packing density is 1, then the volume of air is 0; in each pore must the volume of pores 0 certain diameter of pore is equal 0; it is well.

When we use another type of fiber shape, these different values of  $q$  here are shown. 0.25 and 0.5 shape of the fiber cross section; then the curves are little there, but not too much. So, is the **the** relation of conventional pore diameter in a relation to packing density. How is this total length? Here are the ratio total lengths of conventional pores by total lengths of fibers  $L$ , the green curves.

When  $q$  equal 0, the curve is increasing; what it means?

When we compress our structure, then the symbolically intuitively saying big hole is derived through fibers which are going inside to the set of small **small** parts within fibers, among fibers, so that the lengths is increasing. The length is really increasing. Your higher is a packing density. So, higher is increasing is increasing function. Yes. By the way,

also it is  $q_p$  which is the dotted black curve. Identity of it is green curve by  $q$  equal 0; we can see it too. So, it is the tendency with a conventional pore.

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**Pores with a constant shape factor**  
(Variant I)

*Assumption:* Pore shape factor  $q_p$  is independent of the packing density  $\mu$ . Then the value of pore shape factor is given by the equation  $1 + q_p = k \dots \text{const}$

**Equivalent pore diameter:**  $d_p = \frac{1+q_p}{1+q} \frac{1-\mu}{\mu} d$ ,  $d_p = \frac{k}{1+q} \frac{1-\mu}{\mu} d$

**Total length of pores:**  $L_p = \frac{(1+q)^2}{(1+q_p)^2} \frac{\mu}{1-\mu} L$ ,  $L_p = \frac{(1+q)^2}{k^2} \frac{\mu}{1-\mu} L$

*Note:*  
The conventional pore (diameter  $d_p^* = [1/(1+q)] [(1-\mu)/\mu] d$ ) is a special case of pore with constant shape factor, where  $q_p = q_p^* = 0$ . (Cylindrical shape of this pore.)

We want to be a little more precise and therefore, we will use now another idea than the easiest idea related to conventional pore that  $q_p$  is equal 0. The version 1 the variant 1 based on the assumption pore shape factor  $q_p$  is independent of the packing density  $\mu$ .

It means when I pressure a material, the dimensions of pores the pores will be smaller and smaller, but the shape, the character of shape shall be same. It is some board of idea how it will be if this is valid? Then  $q_p$  is permanently same value;  $1 + q_p$  is also for each packing density same value, so that I can and I call  $1 + q_p = 1 + q = k$ .  $q_p$  generally can be function of packing density. Now, based on my assumption I say it is constant, so that  $1 + q_p$  is constant too.

So, equivalent pore diameter using our starting equation is now this here (Refer Slide Time: 29:52) and total length of pores because  $1 + q_p = k$  is this here. You can say that conventional pore is a special case of  $q_p$  of this variant 1 when the  $k$  is equal 1. Well, this is version one.

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**Pores with a constant total length**  
(Variant II)

*Assumption:* Total pore length  $L_p$  is independent of packing density  $\mu$ . Because  $L_p = \left[ \frac{(1+q)^2}{(1+q_p)^2} \right] \left[ \frac{\mu}{(1-\mu)} \right] L$

it is valid  $L_p = \frac{(1+q)^2}{(1+q_p)^2} \frac{\mu}{1-\mu} L$ ,  $1+q_p = \sqrt{\frac{L(1+q)^2}{L_p} \frac{\mu}{1-\mu}}$

For pore shape factor it is valid  $1+q_p = k \sqrt{\frac{\mu}{1-\mu}}$ ,  $k \dots \text{const.}$

(Pore shape factor depends on the packing density now.)

**Total length of pores:**  $L_p = \frac{(1+q)^2}{(1+q_p)^2} \frac{\mu}{1-\mu} L = \frac{(1+q)^2}{k^2} \frac{\mu}{1-\mu} L$ ,  $L_p = \frac{(1+q)^2}{k^2} L$

Let us study another prior assumption. Is here somebody who has smoked sometimes? Somebody, sometimes; may be nevertheless you all know what is in the filter in cigarettes. In the filter of cigarettes, we can imagine that the air is going beside the fibers and therefore, the effect of filtration. This filter we can compress more or less nevertheless the total lengths of pores, the pores will be bigger or thinner, nevertheless the total lengths of pores may be same. Then from this is going to be called the second idea prior idea.

Let us imagine the total length of pores is independent of packing density; total lengths, now, shape of pore the total lengths of pores. So, we derived earlier such general equation we said the problem is our  $q_p$ .

Now, we must say this equation (Refer Slide Time: 31:36 to 31:48), this  $L_p$  is constant. This same equation, after rearranging, we can see this equation rearranged to this formula. It is only rearranging the same equation. Now, lengths of fibers is constant  $q$ ; fiber shape factor is constant  $q_p$ . Now, sorry  $q_p$  is not here,  $L_p$ ;  $L_p$  is total lengths of pores. Based on our assumption, it is constant. This square root is constant. So, is that  $1 + q_p$  is equal to some constant  $k$  times square root of  $\mu$  by  $1 - \mu$ .

Using  $1 + q_p$  in this form (Refer Slide Time: 32:36) to our equations, we obtain for  $L_p$  is only checking if our rearranging is well; we obtained this. Yes, it is right because right hand side is constant; it was our assumption and height is diameter.

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**Equivalent pore diameter:**

$$d_p = \left[ \frac{\sqrt{1-\mu}}{(1+q_p)/(1+q)} \right] \frac{1-\mu}{\mu} d = \frac{k}{1+q} \sqrt{\frac{\mu}{1-\mu}} \frac{1-\mu}{\mu} d, \quad d_p = \frac{k}{1+q} \sqrt{\frac{1-\mu}{\mu}} d$$

**Generalized pores**  
(Variant III)

We derived  $d_p = \frac{k}{1+q} \left( \frac{1-\mu}{\mu} \right)^1 d$  for var. (I) and  $d_p = \frac{k}{1+q} \left( \frac{1-\mu}{\mu} \right)^{0.5} d$  for var. (II); both are some special "limit" variants. But a right pore (i.e. right in relation to the physical problem studied) need not to follow these variants. Therefore we empirically generalize the equation for **equivalent pore diameter:**

$$d_p = \frac{k}{1+q} \left( \frac{1-\mu}{\mu} \right)^a d, \quad k, a \dots \text{const.}$$

For diameter we had this general equation. This equation we can, but, 1 plus q p is this here.

After rearranging this square root of mu by 1 minus mu and here we have 1 minus mu by mu, altogether rearranging and we obtained this here (Refer Slide Time: 33:23). This is the equation which is valid by our second version. It means if the lengths of pores are independent to packing density. We derived two different equations especially for pore diameter. This is the important quantity based on two quite different assumptions. Nevertheless, both equations are little similar. See inversion invariant, first in first variant we obtain from d p pore diameter; this equation 1 minus mu by mu I can write power to 1; it is possible.

The second is here (Refer Slide Time: it is this power to 0.5. It is similar. It is same structure, mathematical structure. Differences are only in the value of exponent or 1 or 0.5. Nobody know what may be the far variant are 1 is not in our situation is our fibrous assembly and the version 2 also is not well, but because this similarity may be that we can empirically generalize this knowledge and say the hypothesis that diameter, equivalent pore diameter d p is possibly obtained according to this equation, where beside the parameter k, also parameter a is used.

A is sometimes 1, sometimes 0.5 and may be sometimes another value. This kappa k and a are two parameters. I will discuss these parameters, how to obtain for variant 1, 2 and 3 later.

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Generally  $d_p = \left[ \frac{(1+q_p)}{(1+q)} \right] \left[ \frac{(1-\mu)}{\mu} \right] d$  and then now  
 $d_p = \left[ \frac{(1+q_p)}{(1+q)} \right] \left[ \frac{(1-\mu)}{\mu} \right] d = \left[ \frac{k}{(1+q)} \right] \left[ \frac{(1-\mu)}{\mu} \right]^a d$ .  $1+q_p = k \left[ \frac{(1-\mu)}{\mu} \right]^{a-1}$ .

**Pore shape factor:**  $q_p = k \left( \frac{1-\mu}{\mu} \right)^{a-1} - 1$

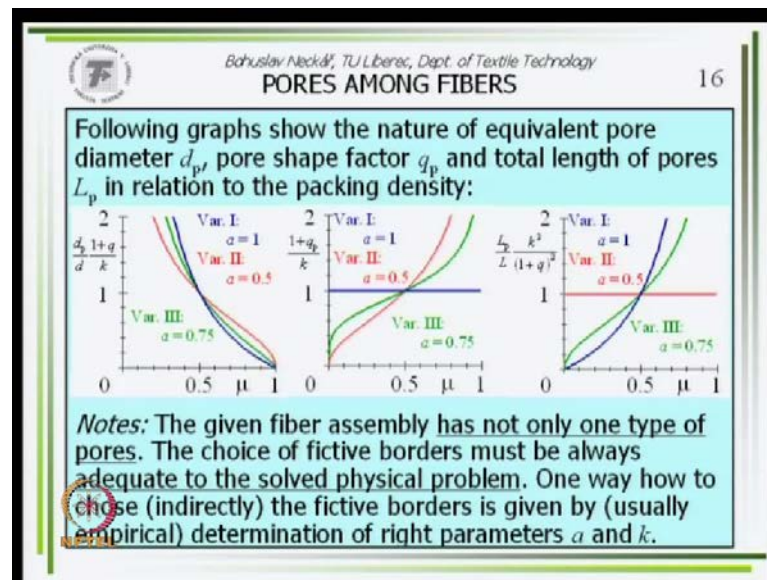
Generally  $L_p = \left[ \frac{(1+q)}{(1+q_p)} \right]^2 \left[ \frac{\mu}{(1-\mu)} \right] L$  and then now  
 $L_p = \left[ \frac{(1+q)^2}{\left( \frac{1+q_p}{1+q} \right)^2} \right] \left[ \frac{\mu}{(1-\mu)} \right] L = \left[ \frac{(1+q)^2}{k^2} \right] \left[ \frac{\mu}{(1-\mu)} \right]^{2a-1} \left[ \frac{\mu}{(1-\mu)} \right] L$ .

**Total length of pores:**  $L_p = \frac{(1+q)^2}{k^2} \left( \frac{\mu}{1-\mu} \right)^{2a-1} L$

*Note:* The value of parameter  $a$  should lie in the interval  $(0, 1)$ , but generally it need not to lie.

We can derive always another quantities because  $d_p$  now general  $d_p$  is this here and now we use for  $d_p$ . This expression we must write that this is equal to this is general equation. This is now our postulate equation and from this equivalent, we obtain  $1 + q_p$  is this here (Refer Slide Time: 36:10 to 36:22) pore shape factor, so that the pore shape factor is this higher here. Lengths of fiber are for total lengths of pores. General formula was this. Using using  $q_p$  from this this expression, we obtain for  $L_p$  this expression and after rearranging this, here it is only mathematical rearranging; nothing more; very easy. It is rearranging which you used on your high school before university. So, we have formula for total length of pores too.

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To illustrate it graphically, I have here 3 graphs: the first all are on the abscissa of all is packing density, so that it is from 0 to 1 packing density  $\mu$ ; the first graph on this ordinate scale is  $d_p$  by  $d$  times  $1 + q$  by  $k$ ;  $1 + q$  is constant is parameter;  $q$  is fiber shape factor;  $k$  is a suitable constant for this or that situation; this, our material is constant 2;  $d$  is equivalent pore diameter parameter value 2. So, all this quantity is proportional to  $d_p$  is proportional to pore diameter; is it not?

You can see how it is. Invariant  $1 - \alpha$  is equal 1. You obtained a blue curve. The version 2, the variant 2, where  $\alpha$  is 0.5, bring the red curve also decreasing curve, but a little other shape and general variant 3. Use it; for example,  $\alpha$  equal 0.75, where it is here is the green curve. Here, 0.75 something in between; then you obtain this green curve; acceptable decreasing curves.

How it is with length? No, shape factor? Here is a quantity  $1 + q_p$  by  $k$ . So, it is a linear function of  $q_p$ , shape factor of fiber pore. In the version 1, this blue we say the shape of pore is constant. So, the pore shape factor must be constant. Therefore, the blue curve is independent to packing density. Version 2 said lengths of pores is constant; in this version, the shape factor is increasing its packing density.

The shape of pores in higher compressed material is more far from idea of ring is it not? More something; so, you can imagine well. And in the version 3, using 0.75 on the place of  $\alpha$ , it is also increasing function; a little other shape. Now, how it is is the total lengths



of pores? It is shown here (Refer Slide Time: 40:18). Also you can see that this quantity which is possible to evaluate by using our equations, immediately is proportional to total lengths of pore  $L_p$  are related to 2 fibers. You can see the version 1, lengths of pores is increasing. Its packing density in version 2 the lengths of pore is constant; it is our cigarette filter idea. So, that red curve must be constant and by version 3, the green is a little larger shape, but also increasing function.

To this moment, we are living in the idea that we have red pencil and we create some borders, fictive borders; we create pores. Is it not? In our structure, but I am sorry, I must **your red pencil take back because we so is that we have not we haven't red pencil** We are not able to create pores. Who will do it when we are not? It will create physical process which we studied.


For each physical process for which is relevant the influence of pore dimensions, each physical process, this process had its own definition of pores, its own red pen and red fictive borders. For example, by wicking its capillarity, this physical process on the same structure, same **same** textile structure define the pores are there, so that the good dimension of pores are there. Then, for example, filtration process is other process.

This process use another other pores, other pore diameter than the wicking process for example, on the same fibrous structure textile fibrous assembly, have a not only one type of pores; it is not. It will have only one connector of space among fibers. No more. Adequate pore diameter is sometimes higher, sometimes smaller based on the process which used this pores. Can you imagine this situation? Therefore, how to calculate **how to calculate** our parameters  $k$  in version 3?  $k$  and  $a$ , we must calculate the suitable parameters in relation to the process which we solved.

When we solve, I do not know capillarity effect. Then, we must do our equations which we use for capillarity, give our general equation for pore, may be according version 3 from the 3, version 3 is the best. And then we on the whole mathematical model we have 2 parameters which characterize good pore dimension and this must be derived based on the study of this capillarity effect in laboratory. Using for example, I do not know statistical regression,  $y$   $r$  and so on. But, it is every time. So, when our colleagues in a mechanical engineering apply their equations for deformation of some metal objects, they derive lot of such equations by the Young's modulus. They must find in laboratory is it

not?iterated to situation and to material.The same is by us.Sorry, we havenot optour pencil, our red pencil,have this or that; his own red pencil have this or that physical process.

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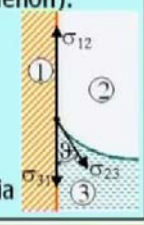
**SOME POSSIBLE APPLICATIONS**

Let us know: fiber diameter  $d$ , fiber shape factor  $q$ , total fiber length  $L$  and packing density of the fiber assembly  $\mu$ . Using equations derived earlier we can estimate: porosity  $\psi$ , pore surface area per unit volume  $\gamma_p$ , conventional pore diameter  $d_p^*$  and total length of conventional pores  $L_p^*$ .

**Wicking of textiles** (capillarity phenomenon).

... ① immersed wall (fiber surface),  
 ... ② air,  ... ③ fluid (water).

Surface tensions (at the surrounding place of contact – pore circumference  $p_p$ ):  
 $\sigma_{12}$  ...wall-air,  $\sigma_{23}$  ...air-fluid,  $\sigma_{31}$  ...fluid-wall  
**Tension equilibrium:**  
 $\sigma_{31} = \sigma_{23} \cos \vartheta$ ,  $\vartheta$ ...constant for given media



Some notes to possible applications, I want to say in a next lecture, so that in the moment, I thank you for your attention.