Orientation of Fibers
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Lecture No. # 02
Fibers and Yarns: Terms, Definitions and Relations

Let us continue our earlier theme about fiber orientation. We derived a set of equation probability and density and so on. Number of fibers for fiber structures for for planar planar types of of fiber structures, this distribution of directions is very important for lot of properties at most for mechanical properties of fibers assemblies.

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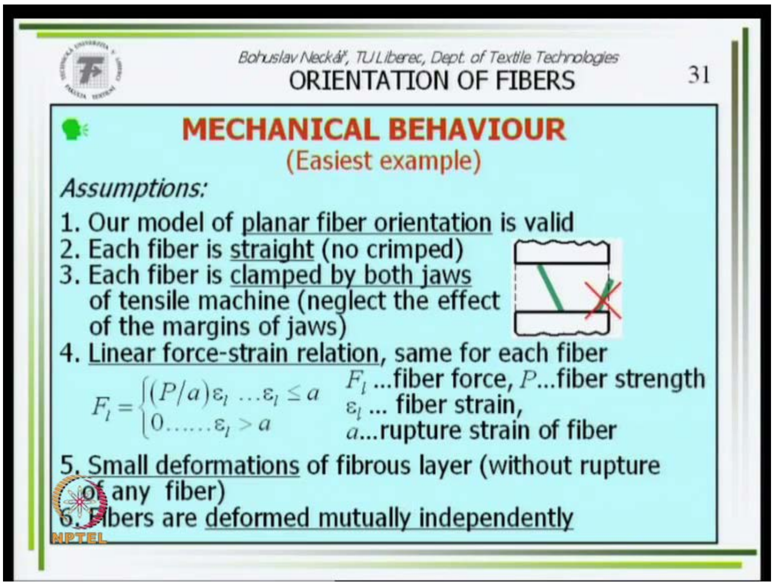

I want in this lecture to present one case, how is possible to apply our earlier knowledge to the mechanical behavior of fibers assembly, but I must say in our lecture, it will be an easiest case. So easy, that it is on the border of unreality, each real structure is much more complicated. So that, but I want to present it to you because it can show, how is the style of our work, when we when we when we must to derive some model for mechanical property often fiber assembly with with with some some distribution significant distribution of fiber direction because this methodological sense, I want to present you this easiest case. On the final sentences, I will in short comment what is possible to do more and go more to the real means more complicated structure.

So, that is easiest case of mechanical behavior of planar fiber assembly. Let us let us accept six assumptions, which which make our problem easy. First, our model is planar fiber orientation is the planar fiber, second each fiber is straight, it means no crimped also cotton have cotton fiber have some small crimp, but we assume that that the each fiber is straight then third each fiber is clamped by both jaws of tensile machine or breaking machine. And we neglect the effect of the margins of jaws.

You know, when we have something like non-woven some some warp or something so. In in couple of jaws by in breaking machine, we must cut this this structure, where the ends of jaws know. So, that some fibers are like this here, we do not want to calculate these fibers, you can imagine that the the jaws are very very very long and this age between this this this value between between between couple of jaws is very small fourth because I said easiest model, let us imagine linear force-strain relation same for each fiber. So, between the fiber force F l and the fiber elongation epsilon l is their relation strained by breaking strained times epsilon, this is the constant for fiber times epsilon l and 0 of course, after breaking the this next next assumption small deformations so small that that no one fiber will rupture it destroy it through our process. And the last of our assumption is that the fibers are deformed mutually independently, we do not calculate the friction fiber to fiber friction in in our structure and so on.

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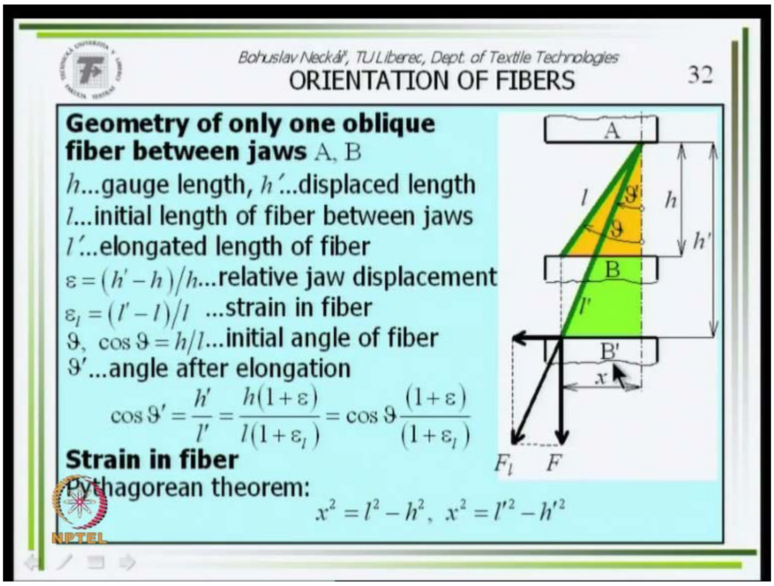

Well, first step let us do derive do derive the function of of pair one fiber one general fiber between jaws. The, we have some couple of of jaws to jaws in breaking machine (( )) is is clumped between these two, two between these two, two, two jaws. Lengths of this fiber is l, starting angle we we will need, it is enough to know non oriented angle theta is is is shown on the on the picture here. To vertical vertical axis is our earlier y axis.

Well, so, h is vision after first intuitively after elongation after after jaw displacement, the jaw B is changed the position to the new position B dash. Therefore, the fiber is fiber is now this here, plus one dash higher than length. So, dash epsilon, we call as a relative jaw displacement and it is from h lengths and h dash defined, the traditional here. Jaw displacement in opposite to them, the strain in fiber strain in fiber work is l length is. So, that it is l dash l minus l by l evident here. Starting angle theta for the starting angle theta is specific to write that it is cosine of this is h by l it is shown from the figure.

The final angle theta is the angle after elongation, which is similarly, from the picture here h dash by l dash and because h dash is h times 1 plus epsilon and l is 1 dash is l times 1 plus epsilon l fiber strain. Then we can write that the cosine is theta dash is cosine is theta starting angle times this ratio 1 plus epsilon by 1 plus epsilon l.

You know, the Pythagorean Theorem is not it? It is very known theorem in whole world. Therefore, we want to use it two times, first time from this triangle from this yellow triangle and what we obtain from Pythagorean Theorem x square is l square minus h square is not it? Well, from the second right green triangle, we obtain x square is l dash square minus h square because this length x is same for yellow triangle assess for for for white green triangle. Why? Because our jaws are on are from some metal deformable (( )) moving of the jaw B.

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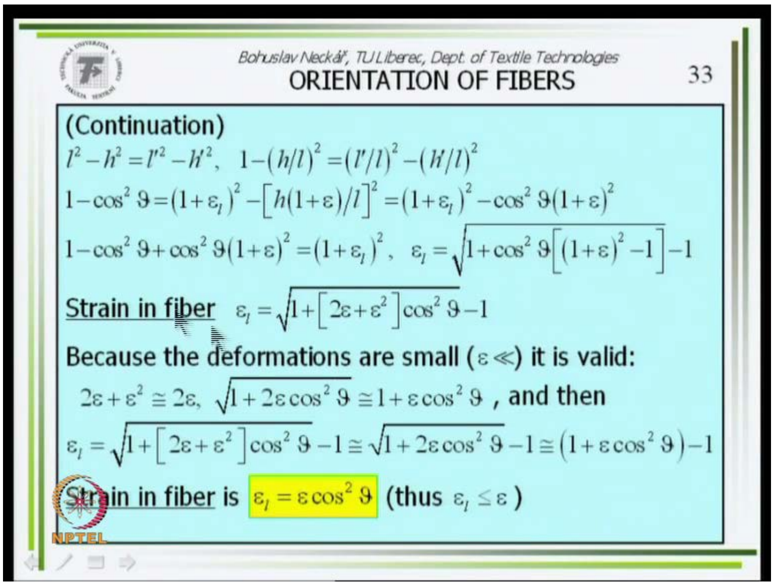

So, that it must be valid, this is equal to this expression is here, it is here. Now some small rearranging, which of we divide this equation by l. So, that we obtain this here, sorry l square of course, by l square we divide using our symbols our our equations now from here, we can write 1 minus cosine is square theta is 1 plus epsilon l square minus h times 1 plus epsilon by l square. So that, it is it is this one because h by l is cosine. So, I have this here from this equation, this is possible to write in this to this form trivially and epsilon one is this here or epsilon l we obtain epsilon or epsilon l is possible to explain using this expression. That is well, that is good. It is shown that the strain of fiber is not the same than jaw displacement epsilon.

And that is the function of jaw of angle theta, it based on the orientation of our fiber segment, our fiber. Well, we said our model will be the easiest as possible. Therefore, let us imagine small deformations, very small deformations then epsilon is very small and then the the approximate value, approximate equations are valid. You know that we can construct different approximate formulas using Taylor series is not it? It is known from mathematics. And using such, but exist also the some content of different often used approximated, approximation equations. The one say, 2 times epsilon plus epsilon square is roughly for very small epsilon 2 times epsilon because this square is extremely small then we can neglect it, square root of 1 plus 2 times epsilon cosine square theta if epsilon is small, if this part is very small is possible to write approximated s 1 plus 1 half of this.

The third is nothing this 2 is in the moment enough then our epsilon epsilon, l was this here using these approximation equations, we obtained that it is. This is this is approximately this? So, that epsilon one is epsilon times cosine square theta for small deformations.

In this easy equation, you can see you can see that the function of angle theta, when we have when we have fiber is higher angle theta then cosine is is smaller than 1, cosine square is much more smaller than than 1. So, that the epsilon l is smaller than epsilon, the highest value of strain of fiber is, when the fiber is parallel to jaw axis a higher this angle theta from the direction of jaw axis. So, smaller is fiber elongation.

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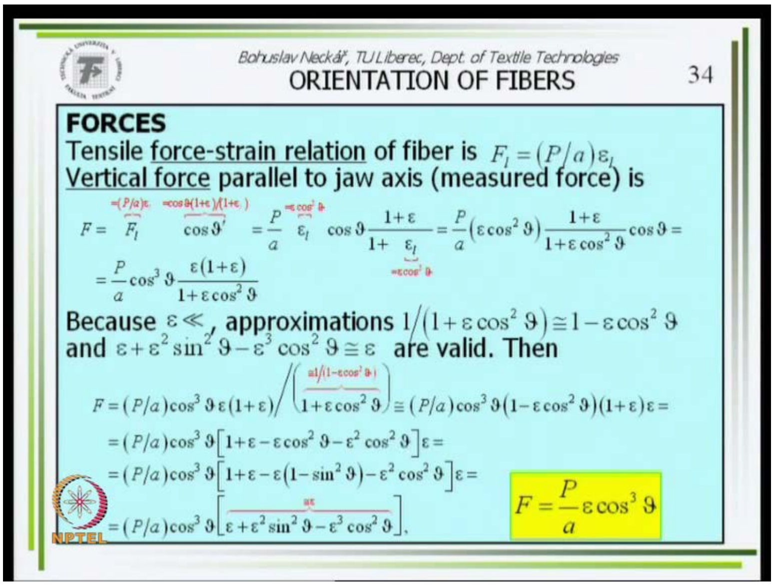

Is there is out of this equation? It was something about the fiber, about fiber strain, now about fiber force forces. We said that the force in fiber follows the linear function. So, that the force in fiber F l is proportioned to P by a fiber strain by fiber breaking strain times fiber strain across fiber strain epsilon l. How is the force, what I can say vertical force? This force F l have 2 components and we measure in our breaking machine, we measure this vertical force. It can be the spacious speech about this horizontal force, it helps together this this moment frictional moment in yarn, but it is no in yarn, in each structure specially in yarn is the plays interesting role.

But we spoke about our vertical force emulation to our picture. So that so that, how is the vertical force no to force F l times cosine of our angle theta dash, from our picture using our equations, we obtained this here. Using our we we assume this small deformation than epsilon l is given by such equation after rearranging, we obtained this equation. And because small deformation, we can also write that 1 by 1 plus something small is roughly 1 minus something small, over some approximation formula known formula.

And we can also write that epsilon plus epsilon square time sine square (( )) minus epsilon power, epsilon cube times cosine square and so on, is roughly equal to epsilon because epsilon square is small and epsilon power periphery is much more smaller is not it? So, we can write epsilon using this approximation, we can this function rearrange as follows, I think I need to command this rearranging this rearranging on the level of your high school. On the end we we obtain this this expression and because this is approximately epsilon, we obtained that the vertical force (( )) proportional to epsilon is imaginable and for cosine is of angle theta power to 3.

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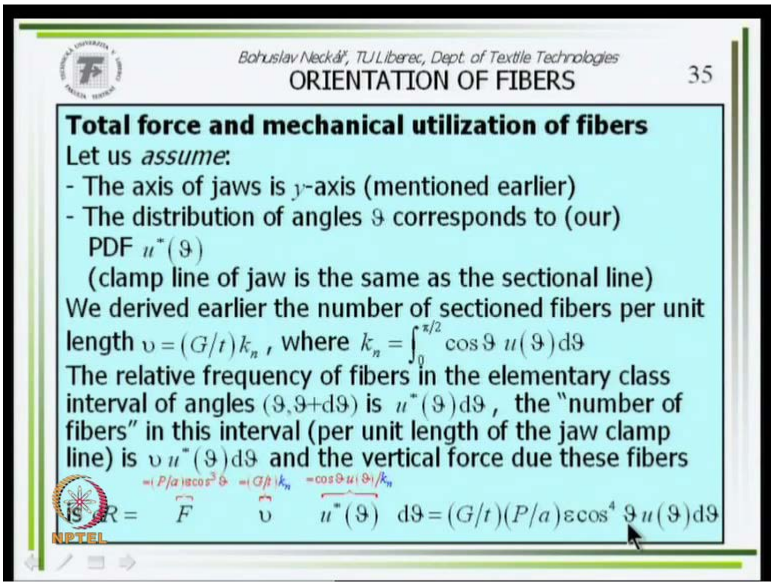

Now, it was one fiber, we derived a vertical force per one fiber. Now, how is the total force on the breaking machine by jaw displacement epsilon? I say the, we assume that the vertical axis, the axis of jaws is y our earlier, y axis the distribution of angles theta corresponds to our probability density function u star theta, which we derived in in our earlier lecture. Because clump line of jaw is the same as the section line evidently, it is not the same, but in the model it is the same.

We derived earlier, the number of sectioned fibers per unit length. It was g by t times k n, where k n was also this integral and what was G? G was mass weight of our our planar textile per mass unit, t is fiber fineness, you know it mostly in decitex or something so is not it? And what is k? And we discussed long time in last in the last lecture.

Now, we will speak about fibers, in short I say having the direction theta. What I mean, I mean that the in more precisely let us imagine, the group elemental group of fibers, which have angles from some value theta to theta plus d theta, some elemental angular class, class interval. So, that in this class interval, the relative frequency of fibers is u star theta times u star theta times differential quantity d theta. So, that the number of fibers in this interval per unit length of jaw is what? Total number of fibers per unit length of jaw, it was in our earlier lecture nu, the times probability density in earlier lecture we remember that this is evident and this is valid. And then the vertical force due these fibers is d R, which is force per one fiber times number of fibers times, number of all fibers times relative frequency of these 2 2 numbers together means number of fibers having angle theta. It is a number of all fibers per unit length of jaw times relative frequency of fibers having angle theta.

Using these expressions, we know the equation for each each of these quantities; we obtain d R in such. It is not well may be I have here, I have here one one one mistake, sorry nobody is perfect. It must be, no no no no no no no all back, all all back, all is well all is well, it is not new equation, it is the same equation, it is continued after rearranging.

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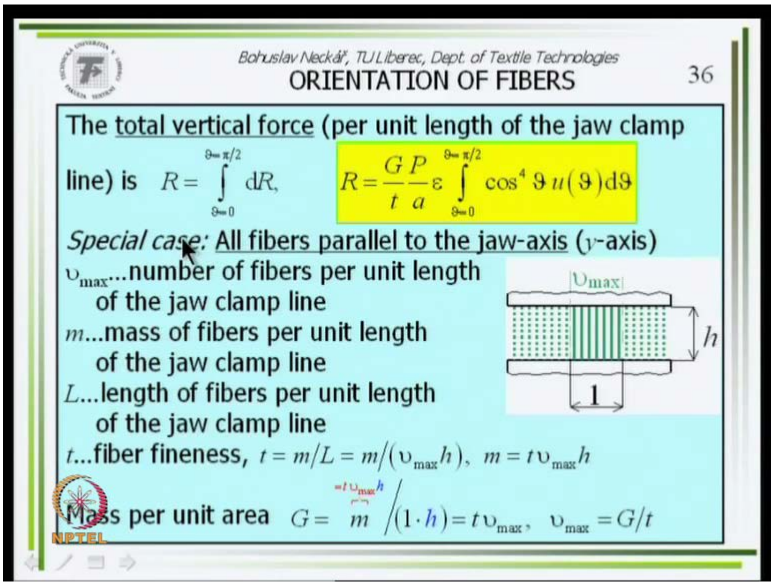

Well, and now how is the total force? Total force is not the force only from the fibers having our angle theta, but for all fibers. So, the force per unit lengths of jaw are must be an integral, must be an integral from d R, is not it? Over all angles theta, it is un-oriented and obtain for theta equal 0 to theta 90 degree, theta pi by 2. I must remember that in theoretical works every times, we we (( )) degrees, we can say degrees because it is better for our imagination, but we must work this radiance.

Well, using this, we obtain such equation as a resulting equation for a force, which we need because realize the displacement between jaws equal epsilon. So, it based not only to our epsilon, it is based also to distribution of orientation of fiber segments in our in our structure. It is evident that if G is higher, if mass of unit area mass per square meter; for example, is higher than we have more mass and the force will be higher and so on and so on.

It is not too important important. Therefore, let us calculate a special case and then something like it will became some utilization coefficient, which can say utilization of mechanic utilization fibers material. In opposite to earlier case, in which all fibers in between jaws have its own, each fiber have its own special angle theta. Let us imagine another structure in which all fibers be parallel to jaw axis, this structure is shown here, all green both at as far as came to the line are fibers here.

So, let us imagine the situation in which between our jaws, our fibers are parallel to jaw axis. Let us imagine that we take each fiber and we rotate each fiber to the position to be to be parallel to to to jaw axis A, then we obtain such structure. Same h length unit and jaw, the number of fibers in the lengths unit of jaw in this case is evidently maximum. Mass area is mass by mass area in this in this rectangle one times h is each G is mass here, of this fibers times area, one times h, but the mass, what is the mass? The mass is the mass is t times nu max times h, nu max is number of fibers here, in lengths unit of jaw, each have fiber have length h.

So that so that l times nu max, its total length of fibers in our rectangle, one times h total length and nu times and from the definition of from the definition of fineness, we can write that the mass is 3 times nu max times h, using it we obtain this here t times nu max. So, that number of fibers nu max is mass mass per R R unit by fiber by fiber fineness. Well, this is the number of such fibers.

How is the force, the strain of fiber generally at rest epsilon l is now equal epsilon because all fibers are parallel is equal to jaw displacement epsilon. So, that I can write that one fiber, it is very easy, one fiber on one fiber is the force f max, which is our linear equation for a force strain relation by fiber, but times epsilon epsilon l is no needed, it now because epsilon l is equal to epsilon.

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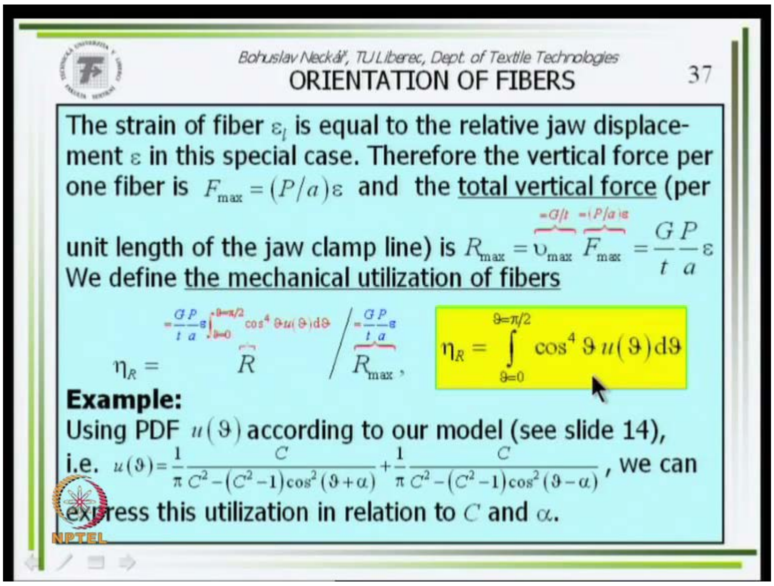

So, pair 1 fiber we obtain the first f max and then the total vertical force per unit length of the jaw clamp line, what is it? It is number of fibers times force per one fiber, using it we obtain this easy expression.

We have 2 forces, one is for our, let us imagine real structure having the orientation and second is for the structure from same fibers with same mass areal mass, but orient it parallel to to jaw axis. One is R, second is R max, we can construct the ratio R by R max. It can say us, how is how is the the mechanical utilization of fibers during the effect of fiber orientation, is not it? Here we have real include include affect of orientation and in denominator, it is without the affect of the orientation and using this you can see that this blue card before integral here is the same that R max. So, that we can write that the mechanical utilization in our easiest case, which we solved is given by integral from theta from 0 to pi by 2 from cosine is power to 4, where it is strong very hard effect of cosine times u theta d theta.

So, you know, you can see that this effect is utilization is can be very can be sometime very small, is going from 0 to 1, is not it? But sometimes it can be very small, when we use on the place of u theta, our u theta from our model (( )) imaginary flexible belt and so on. So, that you use this u theta according this this expression.

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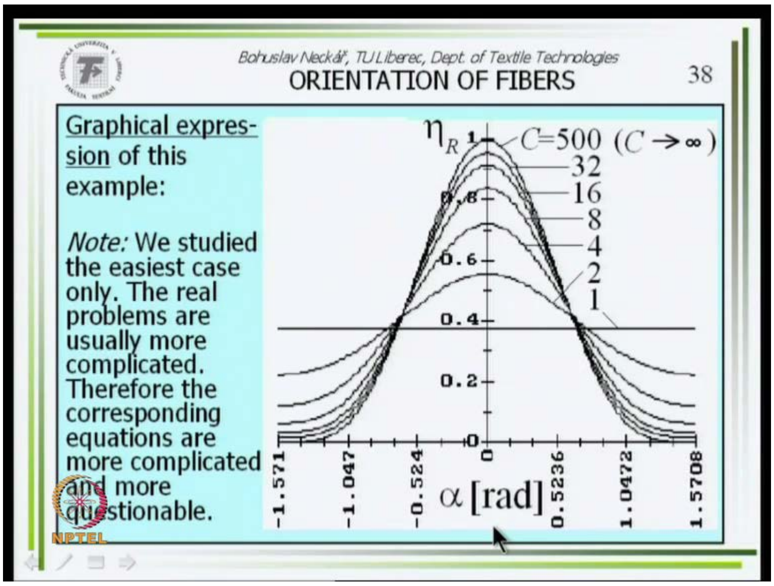

We can to to calculate it and you obtained based on C value following following curves. This is alpha angle from minus 90 degree to 0 to plus means, let us imagine something like web or from web some freeze or I do not know what? May be all those slides today we discussed is Professor Ishteyak about the possibility to apply such equation to and so on.

Let us imagine some structure and now, when it is planar structure, you can take your experimental part to breaking machine in different angles. You can in jaw with to longitudinal direction or right or left, it is small angle, higher angle, much more higher angle, clamp it in the. There is also (( )) different based on this angle alpha, why because the angle alpha is, it is here, the alpha is here here we integrate, we are integrating over theta, but angles steady here is a constant is a parameter of orientation of our web; for example, in relation to jaw axis. So, we obtain this curves based for different way of C.

For example, for evidently for C equal 1, what it means isotropic structure, you can rotate this structure how you want. The utilization will be permanently same, constant. When you use web then maybe you obtain C, C equal to roughly, you show that the typical web have C roughly equal to 2 then you your work is this function. The the the value here by the the utilization value is roughly 2 times higher than here. The utilization value value by longitudinal direction is roughly 2 times higher than by cross direction.

It is often used in in textile industry, the people do not have time and instruments for deeper study of similar problems, but they have breaking machines and they have the products which they produce. In a non-woven often used, the strength in longitudinal direction and strength in cross direction and ratio constructed from these 2 values. Longitudinal to to cross cross direction, it can show it is the very easy the practical the practical way, how to characterize the the to say intuitive and the degree of orientation, here the intensity of orientation which was used.

So, we can see on this on this picture, how it is by C equal to and so on and so on. So, fiber orientations have significant role or play significant role to mechanical properties of fibers assemblies, different fibers assemblies. Now to our easiest case and reality some last words. Normally, we often work with stepper fibers. So, that some of fibers are not clamp into in both in both jaws.

So, you need to formulate this problem mathematically are reduced the number of fibers in jaw only to fibers, which are clump by both jaws. Second on the end of jaws, on the end of jaws some fibers on the end of jaws here, on the other side too, some fibers are cut it. Therefore, they are not in both jaws because was section that your preparation of your experiment material, it is possible to calculate how how many fibers and so on and give it in our model 2, the second influence.

Third, you said fibers are straight, no fibers are not straight; usually, fibers have some crimping may be very important, may be small, but every time I am saying, it is. Then the fiber for strain curve is modified through this scrimping, through this scrimp also in parallel fiber bundle about which we will speak later. When the fibers have some distribution of such scrimp then the mechanical effects are significant. Doctor does help me prepare a special publication to to this problem. Well, this is the, it is I do not know third, may be third influence which we do not to use.

Linear force strain relation, it is it is only on easiest theoretical example, but in reality the force strain curve of fiber is fiber to fiber different. So, that you need to use similar way, but use inside of our equations non-linear non-linear curve of for strain relation related to specially your fibers. Well, all is this all is possible may be good may be not so good, but to formulate using some theoretical tools and make some modified model, which is much more which is much more complicated, but nearer to the reality.

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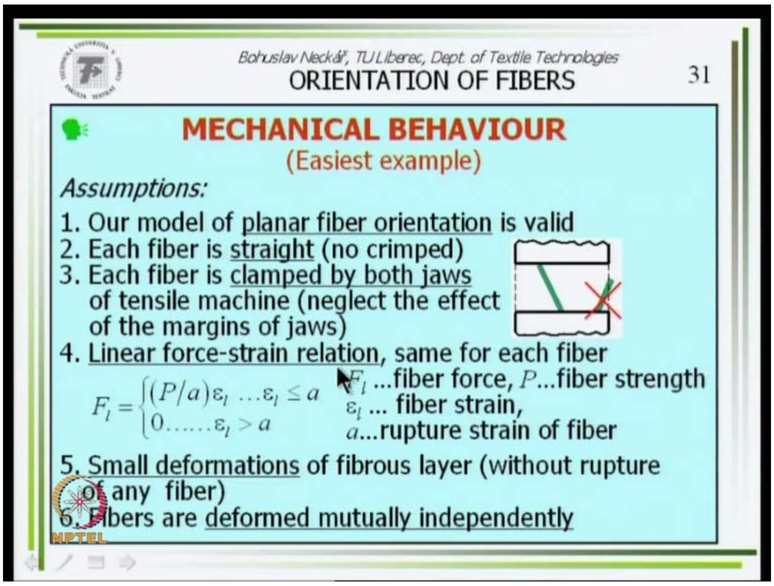

The last assumption, the fibers deformed mutually, it is it is very difficult to to to to give out because to this time, I personally, I do not know and neither good model, how to input to our model of friction phenomenon between fibers. I know only that the traditional equations like a coulomb equation forces friction forces proportional to normal force. Or the the the other equation is based on the coulomb idea means no other friction and so on are not enough well for textile structures, but what is well from point of view of friction, it is quite open question, which is waiting for you may be. Some of you will be researchers, you will be scientist in future and then you must solve the problems, which we all generation did it to solve.

Well, I think this is for this theme. All thank very much for your attention. Be happy and good bye.