**Orientation of Fibers**

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**Lecture No. # 03**

**Compression of Fibrous Assemblies**

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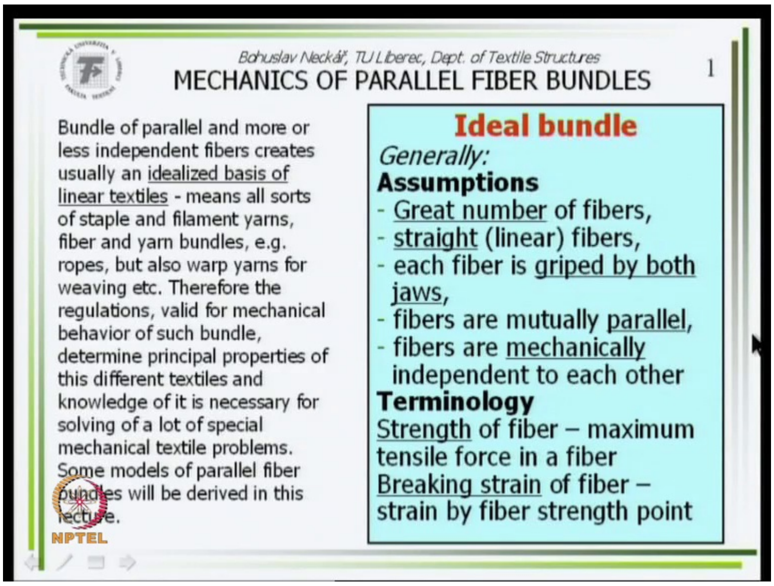


Let us start today's lecture, which isof mechanics of Parallel fiber bundles.You know that the fiber bundles are very important type of textile's textures.Bundles are basis of alllinear textiles, but also fordifferent other types.We will speak today, about the ideal bundle with parallel fibers.To solve some model in this direction iseither easy or very difficult.We planto show you one model, which is relatively very easy and this isknown as the Hamburger's model.Then,we will introduce also some probabilistic model which is a littlemore complicated.

So, let us start our first idea,how to modelfiber bundle mechanics.Let us use a generalassumption which are they. We assume that our bundleis created from greatnumber of fibers.Each fiber is straight, is linear.Each fiber is gripped in by both jaws.Fibers are mutually parallel,so parallel fiber bundle.Fibers are mechanically independent to each other.It means something like friction amount.Fiber’s is not used in our model or this model of Mr.Hamburgerterminologically.We were speaking about the strength of fiber andfrom strength of fiberwe understand their maximum tensile force in a fiber andbreaking strain of fiber which is strain by fiber strength pointok.

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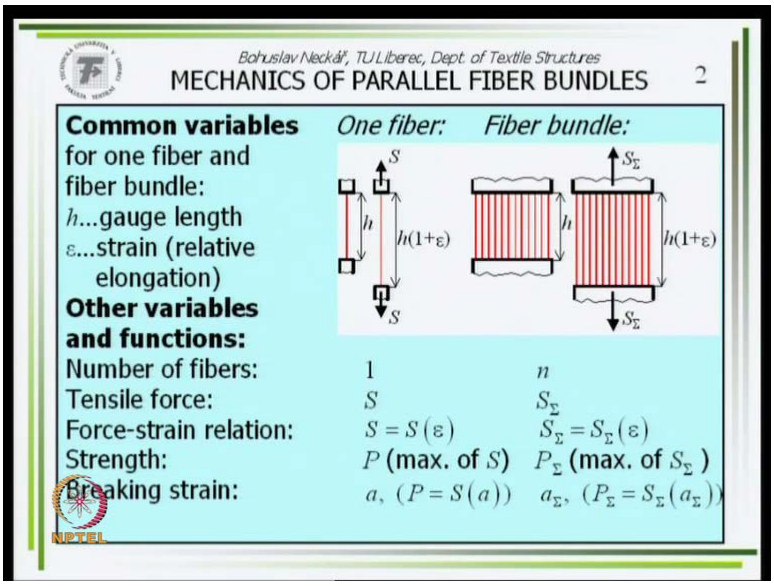


In this case, we will speak about the variance, but at first, some terms, some symbols.Let us imagineoneeasiest bundle having only one fiber,so thatone fiberbetween two jaws of breaking machine.The gauge length, we call h and strain orrelative elongation, we call epsilon. Then,we will speak about number of fibers in bundlein this case, which is here, number of fibers.Red fiber isone, thentensile force.Tensile force is s.We will speak aboutforce strain relation of fiber. So, the forces is the function of epsilon.Isn’t it,some function?Next term is strength.What is strength?Strength is the maximum value of force s.Isn’t it?

Finally, we will speak about the breaking strain and breaking strain quote a. It is a special value of epsilon ofstrain of fiber in the point in which the force is equal to strength P. So, the P is the function S in the point a. This is a case with 1 fiber.The second isfiber assembly having more fibers. So, then the number of fibers isin fiber bundles schematically here.Lot of red fibers is here.Number of fibers is n.We call it n.

Tensile force is s sigma, capital sigma as memo technique symbol subscript for summation all forces together. So, force is s sigma. This force is function ofepsilon,the bundle offorces.Therefore, a sigma must bea sigma epsilonfunction. Strength of bundle maximum force of bundle is P sigma which is maximumofa sigma.Breaking strain is called a sigma.Breaking strains of bundle,of course a sigma,so that if Psigma is equal to the function S sigma in the point a sigma.It is evident.

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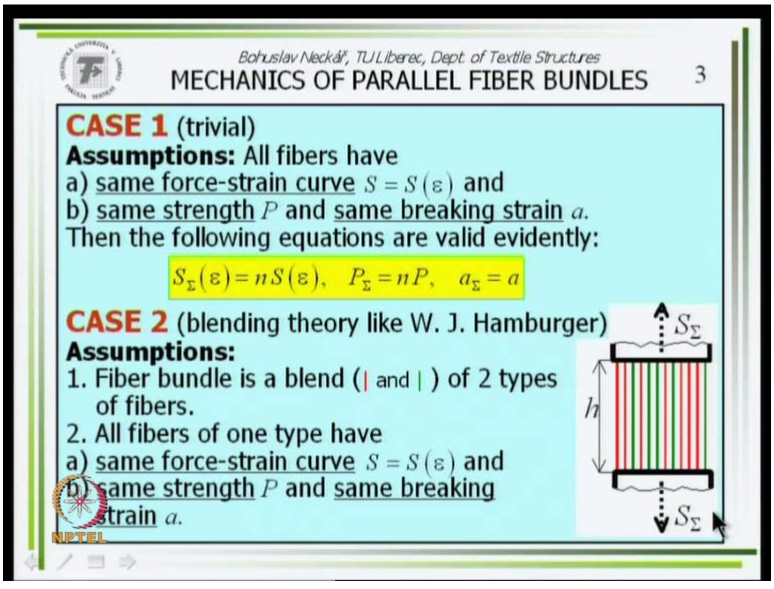


We will speak about three cases.In this theory, case 1 is trivial.Case 2 is very easy and case 3 is not too easy for you.

Case 1 the trivial case.Let us assume that all fibers have same force strain curve, the same strength P andsame breaking straina, at each fiber is same as each other fiber. All fibers are same.How is then it is evident?It is really trivial case.It is evident that the force in fiber bundle, what is the force in fiber bundle. Now, it isforce per 1fiber time's number offibers. So, that n times s epsilon.Strength,what is the strength?Howtheloading curve of such bundle is longer inone moment ping and all fibers are destroyedinone moment?

So, that the strength is evidently strength of bundle is evidently n times strength of fiber.Of course, strain thatthe breaking strain, the breaking strain of bundleis same than the breaking strain of each fiber.It is as this case, trivial.

Case 2 is solved in1949 year byHamburger and it is known asHamburger's linear theory.Let us imagine a bundlefrom 2types of fibers.Here on outer scheme,the fibers arered and green yeah.Bundle from 2 types of fibers.All fibers of 1type have same force strain curve, same strength P and same breaking strain a.Let us imagine for example, in realitythe bundle from viscose fibersand polyester fibers.

All viscose fibers,youmean thatall viscose fibers have same properties.Also,howpolyester fibers have same properties, but between viscose fiber and polyester fiber arevery high differentproperties,have very fine, verysignificant differences.

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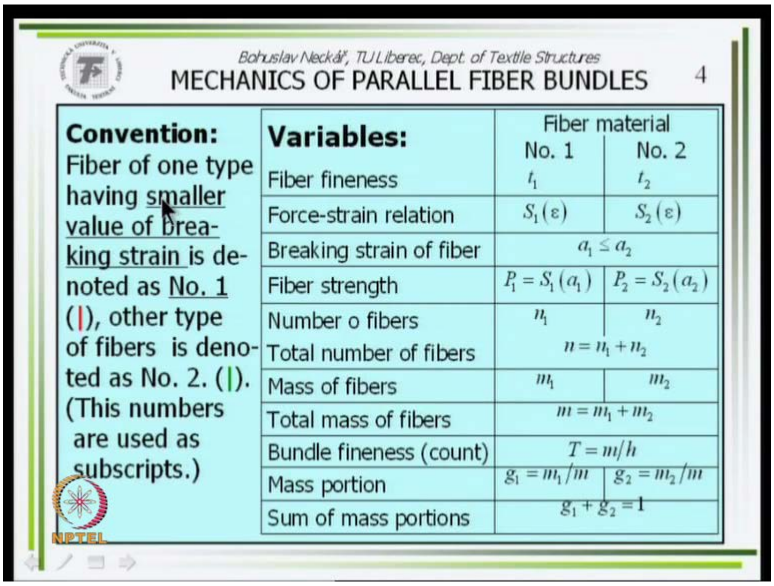


Well,let us formulateone convention.Now, fiber of 1 type having smaller value of breaking strain is denoted as number 1.In the brand viscosepolyester fiber,evidently viscose fiberhaving small value of breaking strain,isn't it? Therefore, viscose fiber will be fiber number 1 in our schemes.Let usassume that they arered fibers.The second fibers, green fibers will have a number 2.We will use subscripts 1 and 2for first and second type of fibers material.Symbolsfor the finenesst 1 ort 2, there are symbols for material number 1.There are symbols for material number 2.

So, t 1 andt 2.For strain relation isparallel fiber is S1 epsilon and are S2 epsilon.For material number 2, a breaking strain is a1 and a2 and based on our convention, a1 is smaller than a2yeah.Then, fiber strength is P1 or P2.Number of fibers in our bundleis n1 and n2.Total number of fibers in bundleis n which is n1 plus n2.

Mass of fibers in our bundle, all is related to our bundle among thecouple of jaws.Mass of fiber is m1 and m2.Total mass is m sum of both.Bundle fineness is bundle count is capital T which is total mass of our bundle by lengths of our bundle.Length of our bundle is h lengths. Mass portion,we have spoken of first lessons about the mass portions.Mass portion offirst material is m1 mass of first fibers by total mass.Similarly,g2 is m2 by m.Let us rememberthatg1 plus g2 must be equal 1.

You know in the industrywe used Parasynchuk values,so that wein our theoretical like g1, g2 can be 0.4, 0.6 something between 0and 1 inParasynchuk, sometimes40percentages,then60percentages here. So, in theoretical way if we speak about a dimensionless quantityfrom interval 0 to 1,this is g1 and g2.

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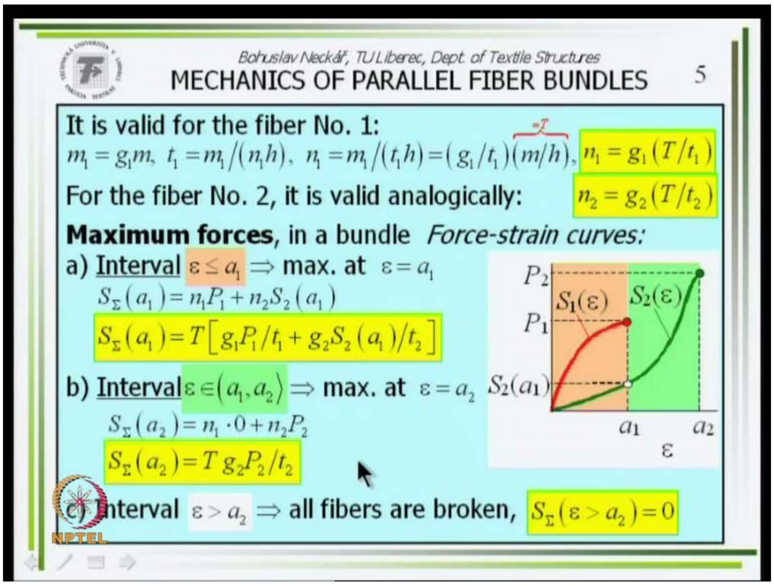


Well, how it is this number of fibers in our fiber bundle?You know that m1 mass of fibers from first material is g1times m.It is going cut from definition of g from mass portion.Isn’t it?Then, also it is t1 fineness is mass by lengths.How is the length of fibers in our bundle?Number of fiber times lengths of each 1n times h1h andtimes h.From the second equation we obtained 1 is and 1 byt1 h, but n1 from here, from this equation and 1 is g1 times m, g1 times m, but the ratio m by h, it was the fineness, the linear density of our bundle capital T.

Then, we can write n1 is g1 time capital T by t1 evidently.Similarly, we can derive n2, n2is g2 times T byt2.This equation we will use for number of fibers from first and second materials in our blendedbundle.Now, let us think aboutour scheme whichthisone here.On the other side,this is the scale of epsilon, strain fiber strain and though on the ordinate our forces.Schematically, let us imagine that the red curve is force strain relation of fiber number 1from first material red curve.The green curve is similar.Similarly,theforce strain relation of our fiber number 2 fromsecond material.

The first curve is increasing from 0 to some endpoint which represents the break of fiber.This end point has 2 coordinates.Epsilon is equal a1 breaking strain of red fiber and the force is p1. So, that it isstrength of our red fiber number 1.Similarly, green fiberis increased haveanotherforce strainrelation.End point has the coordinatea2 breaking strain of green fiber and p2 strength of green fiber.

On this, this scheme is well because a1 is more than a2.Our conventionis valid.On the red,sorry on the green function, we have 1white point is herewhich shall be important.What is it?Which of point is it?The green fiber in this moment have the strain epsilon equal a1 as a breaking strain of red fiber, but for green fiber, it is not breaking strain. It is onlysome general strain.In this moment,a green fiber has some force S2 because S2is whole this green function in the point epsilon equal to,a1 S2 a1.Let usnow divide this scheme to three parts.

First part is the part from 0to epsilon equal a1 breaking strain of red fiber.It is a littlearea in my picture.The secondinterval is from a1to a2.It is green color and the third is over a2.It is white.Do your study separately, the forces in these three intervals.

In the first interval from 0 to a1,in which point from this interval is the total force in bundle, the maximum force in which point?You see each fiber is epsilon take higher and higher forcered as well as green. So, the highest force must be in the point epsilon equal a1.Isn’t it?Is it logically clear?

Well. So,which of force bundle is when epsilon is equal a1, it is shown here.It is force Ssigma and the point epsilon equal,a1. So,a sigma a1,isn’t it?What is it?Logically, how many fibers, red fibers are in our bundle n1? Each fiber takesthe force which is its strength force p1. So, n times p1,it is the part from red fiber,same bundle.Which forcetake in the green fibers?Each fiberhasor takes the force S2 a1.Is itso?

So, total force is n times p1plus n2 times S2 a1yeah.After usingthis couple of equations here,we obtained this,this, this, this,this expression. So, we know that in interval from 0 to a1, the highest force in bundle which is in momentepsilon equal a1 is given by this formula,by this equation.

Now, let us solve the second interval for a1 to a2.How it is here?If epsilon is higher than a1, evidently all red fibers are broken.Only green fibers are working.Nevertheless, with increasing of epsilon, the force in each green fiber is increasing too and their maximum of force in each green fiber is in the point epsilon equal a2.Clear?

So, how is the total forcein this moment epsilon equal a2 in our bundle?How it is?Where are the forcesin our bundle?A number of red fiberstimes for 0, all are broken plus number of green fibers timesthe maximum possible force,it is strength of fiber.So, we can write n1 times 0 plus n2 times P2using n2.From this equation, we obtain s sigma a2 is t times, g2 P2byt2and for completeness, if epsilon is higher than a2, all fibers are broken.So, it is evident that the force in the bundle is equal 0.Clear?

Now, letus solve the problem. What is the strength of bundle?We said that strength is the maximum force.It must be one of our earliertwo forces.It can be thisone or thisone.May be this, may be this.In the moment nobody knows.

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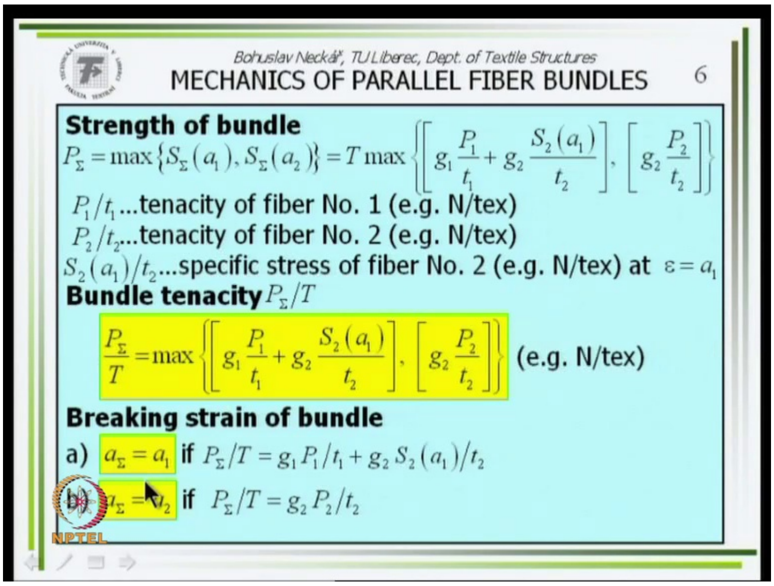

So, if we must write that strength of bundle is maximum from 2 values S sum a1 and S sum a2.Which wasright using expressionsderived?We can write thatstrength of bundle isT because we can give before brackets, before the operator of maximumT times, maximum of thesetwo expressions.Well,what we have here,it isthe first thing.Here is a ratio P1 by t1.What is P1?It is strength of fiber by fiber linear density by fiber fineness.It is tenacity.It is known as a tenacity of fiber,for exampleor somethingso on.

Similarly, what is it P2 by t2?It is a tenacity of fiber number 2, green fiber.When weget the t on the left hand side of our equation, we obtain ratio P sigma by T.What is this?It isevidently tenacity of our bundle.So,that we can write our equations in such form and we can say that thebundletenacity,it is given by such expression is maximum of these 2 values in which we havethe breakingtenacity of firstfiber, tenacity of second fiber and also ratio S2 a1 by t2 force in our earlier white point on the green curveby linear density by fineness and it is called as a specific stress.

You know, from earlier lecture that the quantities forlinear density is equal tostress by rho by specific mass and it is called generally in the theory of mechanics is the specific stress. So, ourtenacity is also something, is also specific stress, but in an end point of force strain curve. Now,let us solve the breaking strain of bundle.If the first member here is higher than the second, then from compacts, from logical contact is evident that the breaking strain of bundlewill be same as the breakingstrain of red fiber.It is a1 and similarly, if the second number ishigher than first,then it will be a2.

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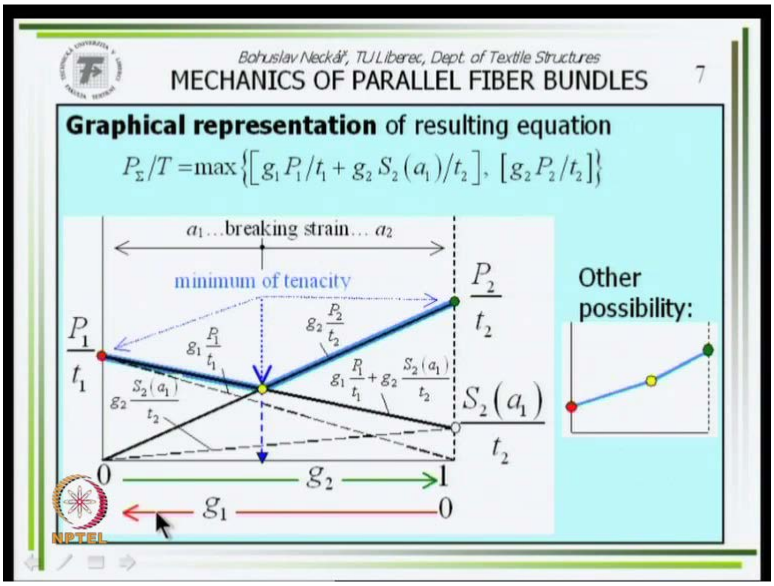


Now, let us solve the graphicallyinterpretation of our equation.We write it here.It is the same as this one.Well,we want to create some graphical interpretation of the equation. The g1 and g2 are the mass portions of first and second material.Let us give on their quantity g2 from left to right, from 0 to 10 percentage of fiber number 2 green fibers.Therefore, green arrow 2,1,100 percentagesyeah.

G1 must go from right hand sideto left,also from 0 to 1.Isn't it? Ido not know if g2 is 70percentages, 0.7for example. Here, then g2 must be at the percentage 0.3,thenis clear on or the right we willhavetenacities.

Let us study now this expression.The first member here,have 2 numbers.Let us study the first of this g1 time g1 times P1 by t1,P1 byt1.It is tenacity of fiber number 1.It is a given value here by P1 by t1 and this value is multiplied by g1,so that if g1 is equal 0 that this member is equal 0.If g1 is maximum, is equal 1.This member is equal P1 byt1 tenacity and linear relation,so thatthis member Linear Aincreased with g1from 0 to 1.

This one is this straight line which represents this member here.It starts from 0 and this g1 increasing from 0 to 1 is increasing to the valueP1 by t1.Clear? Similarly, the second member is here, if g2 is equal 0into 0, if g2 is1, then it is S2 a1 byt2 and it is linear function. So, that picture of this part of this member is increasing to this g2 from 0 to S2a1 byt2.It isincreasing from this point to this here to the value S2 a1 byt2, but our first member issum of both. Sum of these two lines, evidently this thick black line.

So, is thepicture of our first memberin relation to g1 g2proportions?How it is withsecond member?Second member is g2 times P to byt2 p2 times. T2 is tenacity of second fiber given value for a,this of that fiber which we use from the place of our green fibers. This member value of this member is increasing with g2 from 0 to P2 byt2.So, we canhavethe line from 0g20with g2 increased it to P2 by t2.

Now, what is the bundle tenacity P sigma by capital T?It ismaximum of thesetwoblack thick lines on our picture when we are in this region.What is higher, this thick line or this thick line? Higher value is evidently on this thick line.Yes. So, in this region, this line istenacity ofbundle tothis yellow point here. How it is on the righthand side?This region which this couple ofthick black line has the higher position, evidently this,so it is from this yellow point to the right end.This line represents the tenacity of bundle.Isn't it?Altogether,the tenacity of bundle is given by such blue curve sign which is a breakshape.Isn't it?

It is interesting.Why?See, let us imagine we start this 100 percentage of fibers number 1,then the tenacity is P1 by t1.Then, on the price of red fibers, we give some portion ofgreen fibers on the price of Ido not know viscose fibers.We give some fibers from polyester.Polyester is of higher value ofits stand.

For example, we use this g1 and g2.What do we obtain?We obtain value which is smaller than earlier.Starting bundle is not, it is not right when somebody is meaning that when on the price of one fiber, weuse some fibers which have higher strength.Then, the bundlethe blend together will increase in strength.You can see that it can be also a situation in which it is decreased thanthe tenacity is smaller. So, this iswhen we create in spinning mill.For example, the blanks because when we choose no good portions, mass portions of material, our final yarn is not ideal fiber bundle, but similarlycan have smaller tenacity than earlier result blending.

Thisblue curve, break curve is typical for blending, but not every time.It is possible also to obtain such picture in this case really direct point is the minimum point.Let us study, now how is the minimum bundle tenacity?It is the point in which mechanical properties thehighest is not.

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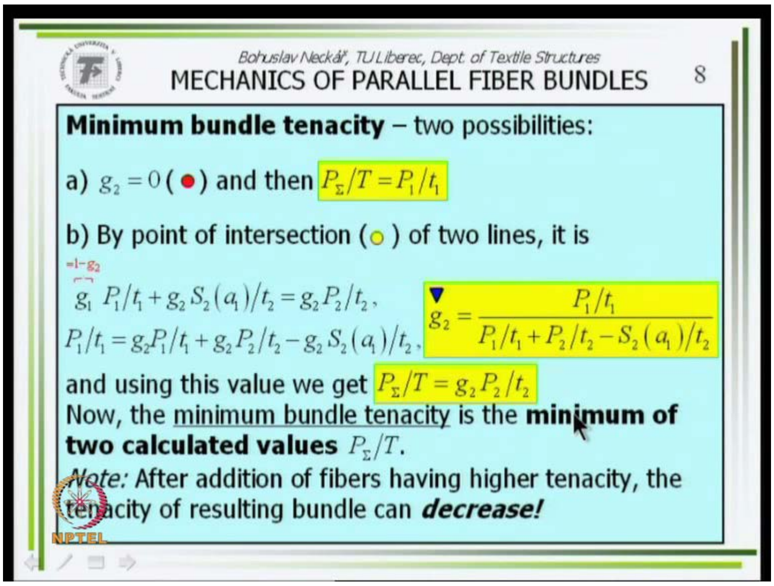


What is minimum of bundle tenacity?Minimum of bundle tenacity can be or if this structure is at 12 or in our red point, if the result is this 1 or in our yellow point, usually in our yellow point. So, let us calculatethisin this points the quantities which in the red point.It is very easy in the red point.Everytime it is P1 byt1 tenacity of first fiber's material in yellow point.What is this yellow point? It is section of two lines.One line,equation of line is given by this expression.Equation for the second is given by this expression.

In yellow point, our yellow point here both must be validbecause it is section oftwo lines. So, thatis valid that the first member g1 P1 by t1 plus g2 S2 a1 byt1 must be equal to g2P2 byt2.No.Well, of three arranging of this using g1 is 1 minus g2 because g1 plus g2 equal 1 and after rearranging, we obtain the mass portion for second material.Our green material in our lectureas shown in our equation here,it is to be rearranging.We know g2 in this positiong1 is 1 minus g2 evidently.

Usingthis value, we can calculate theminimum tenacityof bundle for which, for this line, but also from this line because yellow point is section of both. I recommend you to use this line because mechanically, it is easier you need not. So, long write by numerical calculation.Therefore, this tenacity is g1 times P2 by t2.It is the minimum tenacity.

No,precisely minimum tenacity isminimum fromtwo values or thisone from red points or thisone from our yellow points of tenacity of fiber bundle. I said after addition of fibers having higher tenacity, the tenacity of resulting bundle can decrease.Yes, this story can be applied for rough estimation of blended staple yarns too.Of course, staple yarn is not ideal parallel fiber bundle, but the preferential direction in yarn is longitudinal.So, as it is little similar to our ideal bundle.Therefore, our result can be roughly used also for evaluation of yarn tenacity of blended yarn. How it isapplied?

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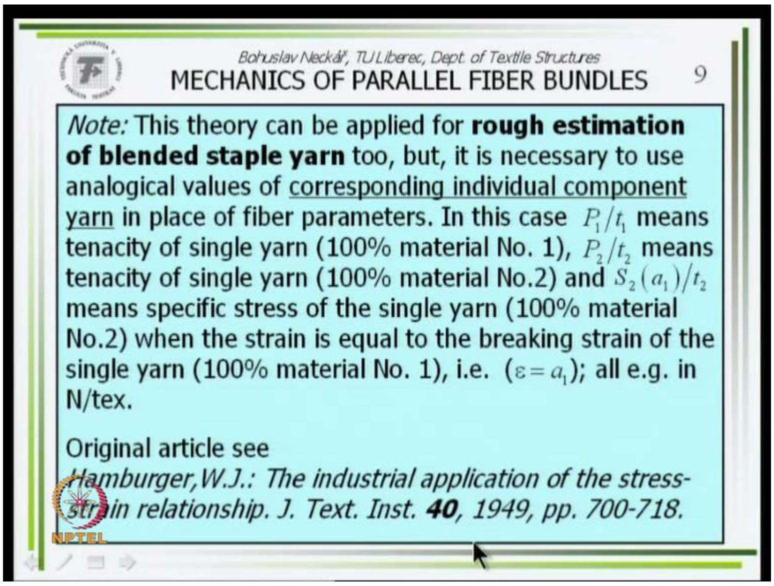


On the place of P1 byt1,earlier to this moment, it was tenacity of fiber number 1.We use the tenacity of single yarn from100 percentage of material number 1 on the place P2 by t2.Now, we use the mean tenacity of single yarn from second,only from second material.On the place of value S2 a1 by t2,our earlierwhite pointwhich means a specific stress of the single yarn, now from 100 percentage of material number 2.When the strain is equal to the breaking strain of the single yarnfrom 100 percentageofmaterial number 1,it is epsilon is a1 or for example, in N/tex.

So, similarly only on the position of earlier tenacities and breaking corrugation of fibers, we useanalogical quantities from yarns.We make on my universitysome experiments with blended yarns.Of course, we did not obtain so idealize break graph, but really such curve experimentally measured have such, usually it isso that our curve is going but it have half minimum, roughlynear to our yellow point, this expression is often used.For our work in industry is Hamburger'stheory, bringone importantresult desired possibility to calculate it numerically.

It is shown that when we prepare some blend, the tenacity of such blend may be yarn tenacity of such blendcanbe higher than earlier tenacity of100 percentage of yarn from 1 component.When you will prepare some blend in your brain must start some red light carefully that the strengthof your yarn will not beenough well for followingapplication.

Thisyou must prove it and check it and be sure that your idea of this or that blend is fully useful also from the point of your mechanical properties.Therefore, thistheoretical concept is veryuseful for industry.You can calculate when you have the starting values.You can calculate it and say itquantitatively, but in your brain,when you will be some technologies in industry must start by blending some,I saidred light in your brain be carefully re-strength isstrength means tenacity of yarn.Well, it is about the Hamburger's theory. (())is original work ofHamburger1949.

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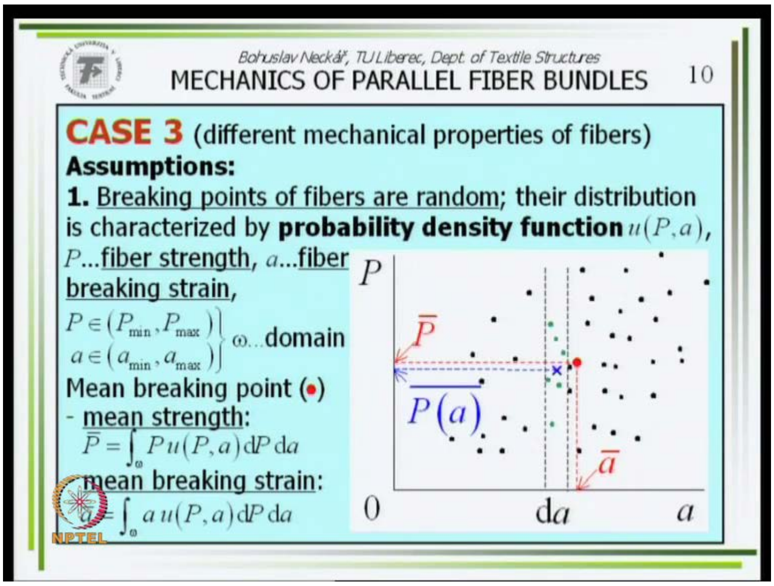


The fact,the case 3 which in short, we want to start now and in our other lecture, we will continue with this.It is notsoeasy.That is very trivial but useful Hamburger's theoretical model.Some intuitive introduction to case 3.We spoke in Hamburger's model abouttwo components.It was red and green fibers, yeah only blend from two components.

Similarly, it is possible to derive in corresponding equations for three components.Similar logical way for five components, for ten components, for thousand components, for million components theoretical,isn't it?Now, let us see bycotton fibermaterial,each fiber from its natural fiber.Each fiber has another value oftenacity and other value of breaking strain.Isn't it?Let us make from the fiber theoretically.Not practically.It is too difficult.Let us make a separate of fibers.The groups, the fiber is having same tenacity and same breaking strain.

You may have, may be thousand different groups.Then, make blend.It is our originalmaterial. So, the material having variabletenacity and variablebreaking strains of fibers is some sink like Hamburger's case, but with no tools andthen, thousandsvery much components intuitively.Is it intuitively clear, this idea?

So, this similar effect by Hamburger must be also in the case when we use fibers having the distribution oftenacity and distribution ofbreaking strain.Breaking points of fibers are random usual, breaking points I mean these couplesforce breaking strain at this case.In this graph on the(())is a breakingstrains of fibers, on the ordinate is force strength of fibers and each fiber of another end point by break. So, that altogether we obtain such set ofred point as the symbolic set of allcouple'sstrengthbreaking strain.Symbols which we will use, Pis fiber strength,a is fiber breaking strain. Let us imagine that P is from some interval from P min to P max and a is from some interval from a min to a max. No,because write itin shorterform thisdomain we will call under the symbol omega.

Omega, it means P from interval P min to P max,a from interval a min to a max. Thisdistribution,the distribution of all such points here, of all pointsstrength breaking elongation of fibers have somejoint probability density function of this couples. Thisprobability density function, joint probability density function we callUPA.It is probability density functionU of two parameter random variables.First random variable is P fiberstrength.Second random variable is a fiber breaking strain.

Well, I think this introduction to our third case in this lecture is finished.In following lecture, we will continue with the relation ofsolving of this problem. Well,thank you for your attention.