Orientation of Fibers

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Module No.# 01

Lecture No. # 04

Compression of Fibrous Assemblies

Let us continue our fiber bundles of, idea of fiber bundles.On the end oflast lecture, wediscussed the scheme.Wesaid that afiberstrength P and fiber breaking strain a,are random quantitiesand exists some joint probabilitydensity function UPA. So, that P is from interval Pmin P max, as well as a is from some interval a min a max. This domain we call omega.

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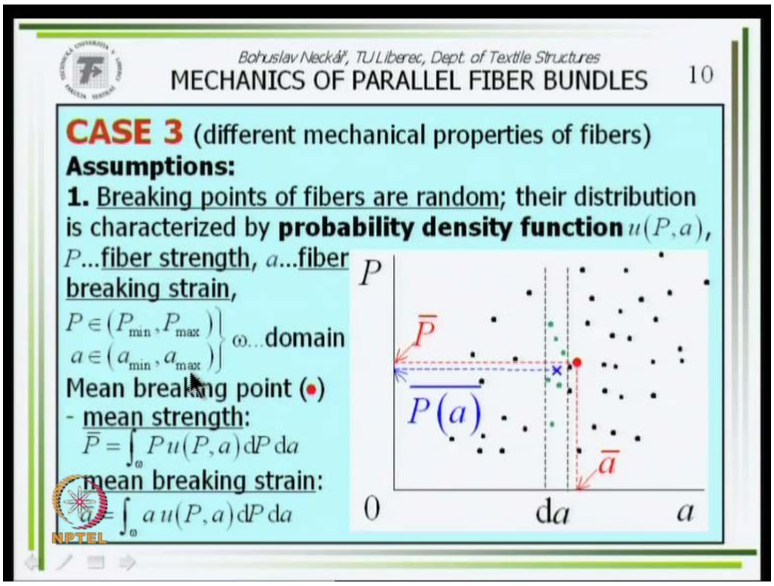

Evidently, the mean strength P bar is integral over our domain P times uPa,dPda as well as the mean breaking strain is given by this integral is ageneral definition of mean value.

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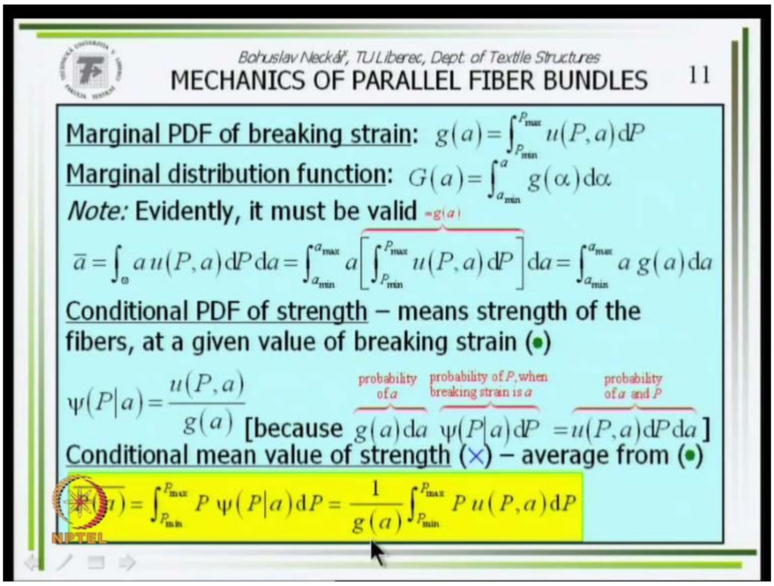

We willalso need a marginal probability density function of breaking strain gawhich is as you know the integral from uPadPover all Pvalues.Alsomarginal distribution function we can useas integral from small g.Small ga is a probability density, capital GA is function is a distribution function and this integral from this function g only because a isupper limit in this integral.Therefore, I changedthe integrating quantity to another symbol may be alpha.Yes,it is shown here that the mean value,it is only for our sureness that mean value a offiber breaking strain a bar which is quite a few definition given by this equation.After using of this here,we obtained this one which isand must be. So, we are right.No mistakes in our equation.

We will also use a conditional probability density function of strength means strength of the fibers at a given value of breaking strain.It is on this picture.What I mean?Let us imagine thatno all fibers, but also fibers having practically same value of breaking strain.I can say the breaking strain lying from some and to a plus da in an elemental interval, butstrength ofsuch fibers is different.These green points have some distribution, but only these green points.Yeahnot these points,only the green points which hasschematically have in my elemental thin strip.Thedistribution ofstrengths of such only this fibers,this subset,we call as a conditional PDF probability density function,conditionalprobability density functionand the symbol is psi P by a. P is a random variable,a is probability density function of random variable a P, but no, from all fibers, then only from fibers having given value of a is parameter ok.

This is theconditional probability density function.This function is very good known of probability and it is valid that this probability density function is uPaby ga.Joint probability density function by marginal probability density function.Why it is in short shown here?Because the relative frequency of ga da must time, it is shown here and it is written here.In each case, it is in eachteaching group for theory of probability.

We also willuse a condition on mean value of strength.What I mean out of them?Let us take all these green points in ourdifferentially teamstrip and let us make the mean value, but only from these green points mean value of strain.Sorry,nomean value of strength.Yeah mean strength valuebecause strain is same for each green points here,fibers from this green from this differential arrow.

So, this mean value from allgreen points is someblue value which is here.I willwrite it under the symbol Pa raise bar.It means mean value of strength from fibers having given value a, a’s parameter and this mean value, conditional mean value of strengthis as every times the definition of means. So, integral over P from P times probability density function of Pa is parameter, P is random variable atimes dP.Using thisratio,we obtain also Pabar in this form, in thisexpression.

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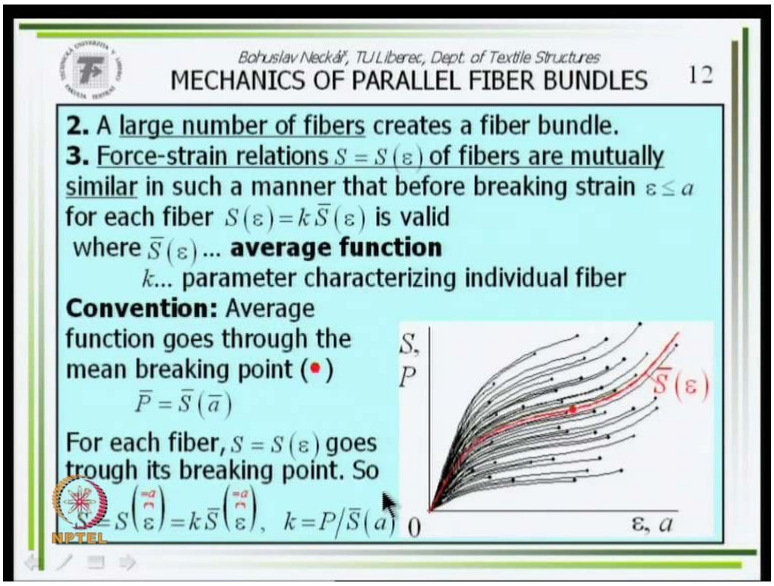

So, second we assume that a large number of fibers create a fiber bundle.No, two no ten no fifteen than thousand million or more.How you want very largenumber of fibers?Third assumption force strain relationsof our fibers S is a function of epsilon are mutually similar in such a manner that before breaking strain epsilon smaller than breaking strain a, for each fiber is valid that our function S epsilon for strain function, is proportional coefficient of proportionality key to someaverage function as bar epsilon.

So, as bar epsilon is an average function, K is parameter characterizing individual fiber.What I mean?Let us see my gender, the set for force strain.Force strain function of fiberisthis set of red of blackcurves from a quite few of shape similar,so that it exists.Some red function we call it as average function.Each order, each individual for strain function of fiber can be interpreted asK times as bar epsilon.This function,is it acceptable?Each black curve issome like magnification ofour redaverage function.Let us use a convention to our average function. So, that on this function is also mean breakpoint.It is the point, mean break point coordinates a bar P bar mean value, mean offiber breaking strain and mean of fiber in strength. So, let us construct our average function, so that this point is lying on this red curve.

Well, then it is valid.We said S is Sepsilon is k times, is bar epsilon.It is here, butifon theend point of fiber made before the break of fiber, the force is equal to strength s is equal to P.Epsilon is equal tobreaking strain a, but we said it is k times as bar epsilon.Now, k times as bar on the epsilon v right on theplace of epsilon we write a.Clear?

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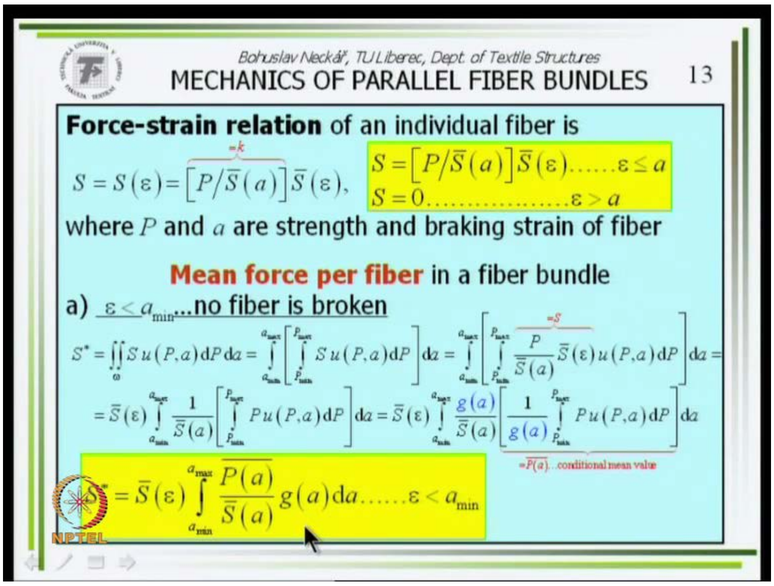

So, we have from this P is equalk times as bar a.We obtain k as a ratio P by as bar a.Finally, we can write before break of fibers.If epsilon small equal a, we can write an S.This is fairlyP by S bar a times S bar epsilon after break it is 0.Of course, what we need to know now for each fiber.We need to know couple of quantities P fiber breaking the strain.Sorry,the fiberstrength and a fiber breaking strain two scalars, no whole function forstraining all two scalarsP and a land for each for all fibers together.We need to know a1 function S bar,our average function, our earlier red functionok

Now, we want to construct mean force parallel fiber in a fiber bundle.Whenwe roll in some bundle fiber from such fibers in each fiber,it is an order force in themoment.Isn't it? So, that we want to calculate a mean force parallel fiber on our bundle.It will formulate into steps.

Infirst step,epsilon our strain of bundle ismore.Epsilon is morethan minimum value of breaking strain from set of our fibers inside a.What it means?No 1 fiber is broken.All fibers are functionable.Our epsilon is under the aim in or under the minimum of breaking strain of fibers. So, the mean force parallel fiber in bundle, we will call as S star.Generally, subscribe starourin this lecture for quantity similarity to bundle. So, it is mean force parallel fiber in bundle S star.What is it?As each mean, it is s force in general fiber times probability density function joint probability density function times to both differential quantities an integral over domain of couples Pa.

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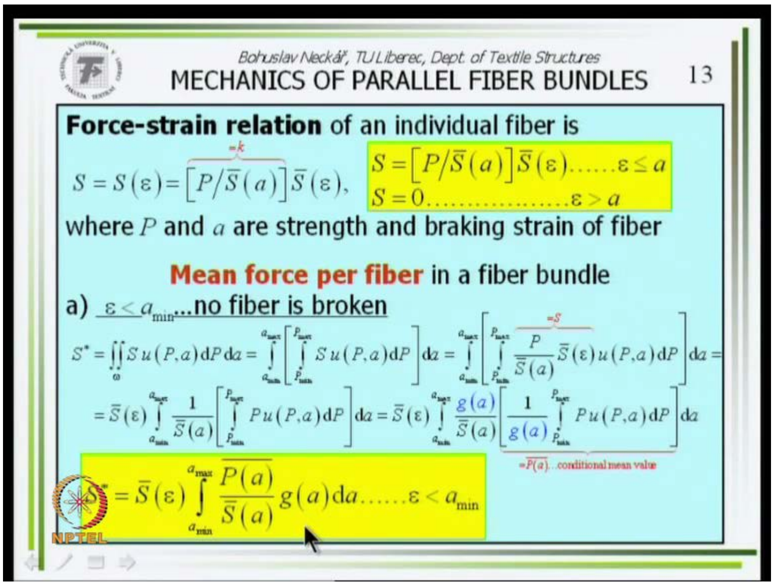

After rearranging, we obtained is, here it is evidenton the place ofPu Pa, sorry on the place of S, we can use this expression. So, we obtain this here because no fiber is broken, no fiber have S equal 0, nofibers have some force.This force we obtain this here, after rearranging this.Here, this equation in this equation, wemultiply and divide by marginal distributionmarginal probability densityfunction ga.Therefore, we can see, we multiply and divide by the same expression.These here in the brackets we know. So, is iton the anti?We can write as far is given by such equation.It has mean force Para fiber in ever in a fiber bundle line.Epsilon is smaller than a mean when all fibers are before brokenwell.

Now,the second part of this derivation.Let us imagine that epsilon of bundle strain of bundle is lying between two borders a mean and a max.What it means?Intuitively, how arestrains of bundle is? So, high that some, but no all fibers are broken and other fibersare not broken.It isbetween two interval from a mean to amax between where you a meanand where you a max.

Now, the mean force Para one fiber in fiber bundle. So, the fiber bundle,of course, you must start with the same expression.S star is integral of S times u Pa DPD a as in case a.Also, this equation is same as in a case at this.Here yeah is, but now the integral over a willrearrange as a sum of two integrals.It is definite integralfora mean to a max. So, that it must be also integral from a mean to our epsilon plus the same integral from epsilon to a max is not it yeahhere is.What isthe force s in our first integral?This oneupper limit is a mean lower limit.Lower limit is a mean upper limit is epsilon.

In this integral, all are smaller than epsilon,so that each fiber is brokenfor S.Weneed to use S equal 0 and because S equal 0, all this integral must be equal 0tie in the second integrala is epsilon is lower limit. So, each a is higher than epsilon fiber breaking strain is higherthan epsilon of our bundle. So, that on the place of S, we need to useearly equation is here. Sorry,other derivation in the same case a.On the end,we obtain this structure, these expressions.

How is the difference?This is the same than is here.Only lower limit here is a mean and lower limit here is epsilon.In order, the difference is only in thislowerlimit, lower board of our integral.For completeness of our ideas, if epsilon is higher than a max, then evidently all fibers are broken. So, is it s star mean force Para one fiber must be equal 0 is trivial and evident.

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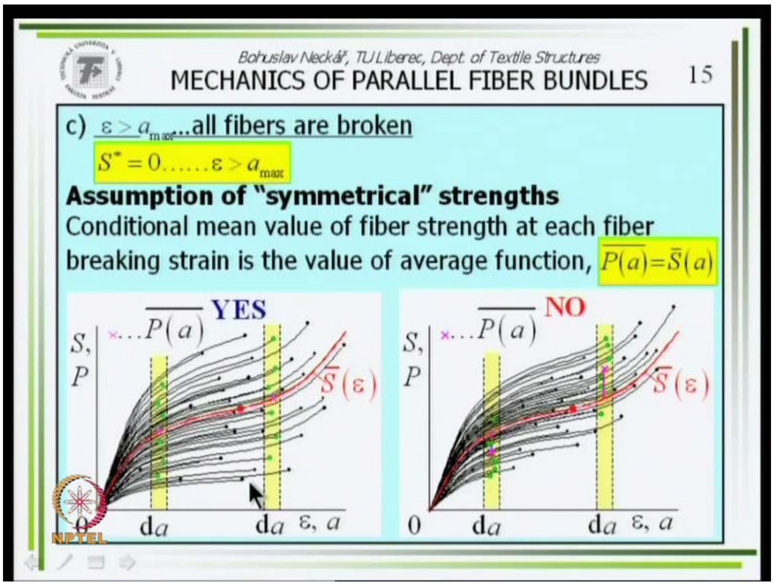

Sometimes,it is possible to use some assumptionwhichI call as a symmetrical strength assumption.It is valid on our left picture, but on our right picture,I will explain it.Let us see my gendersituation like on our left picture in this.Differentially, small strip exist some end points or some force strain curves or fiber.There are our green points here.

The conditional mean value from all of a strength from these green fibers only is lying onthe ouraverage functionhere.Also, in other strip differentialelemental strip the mean values of fibers which have itsstrengthwhich are in this elemental state.The mean conditional mean valueis lying on our average red curve.

In this case, this grass issymmetrical.Therefore, I call it symmetrical strength.Assumption of symmetrical strength, it neednot be valid.It can be, but it need not be one the second picture shown.That it is not valid here example the meanvalue of fromthisgreen pictures is here, but the value corresponding value of average function is here.It is not the same point here,the same here.It is from the other side.

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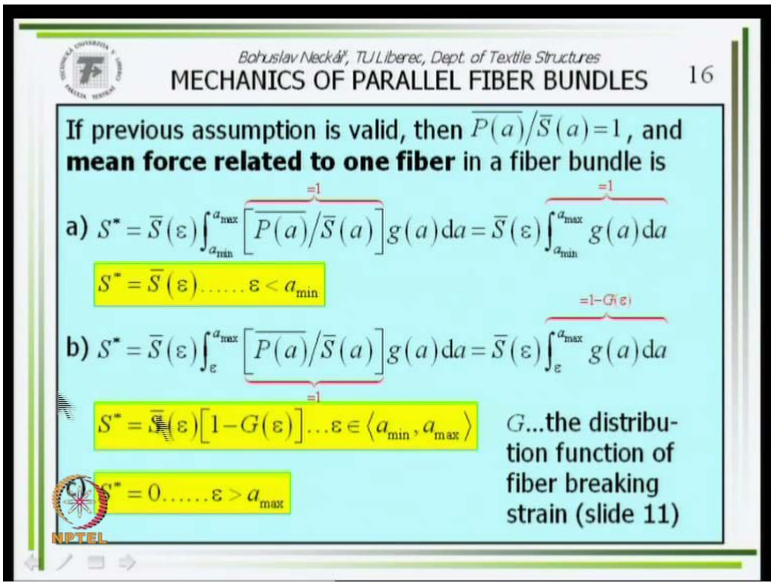

So, this assumption can that mean would be valid nevertheless, in the practice based in my experience. Often this assumption is roughly right.If it is valid, then we can say that the conditional mean value Pa bar is same.Then, the corresponding value on our average function is equal fto S bar a.

If yes, then forS star, this is our 3 cases a b c.The ratio Pa Pa bar by S bar a, which is in our earlier equation is now equal 1.It was here is this ratio, here is this ratio.Yeah is now equal 1.Therefore, we can write this one, but what is this one integral from probability density function ordomain. So, density is equal 1 and beforecoming epsilon equal a mean, we can write that theforce mean force parallel fiber corresponds to the average, our average function.

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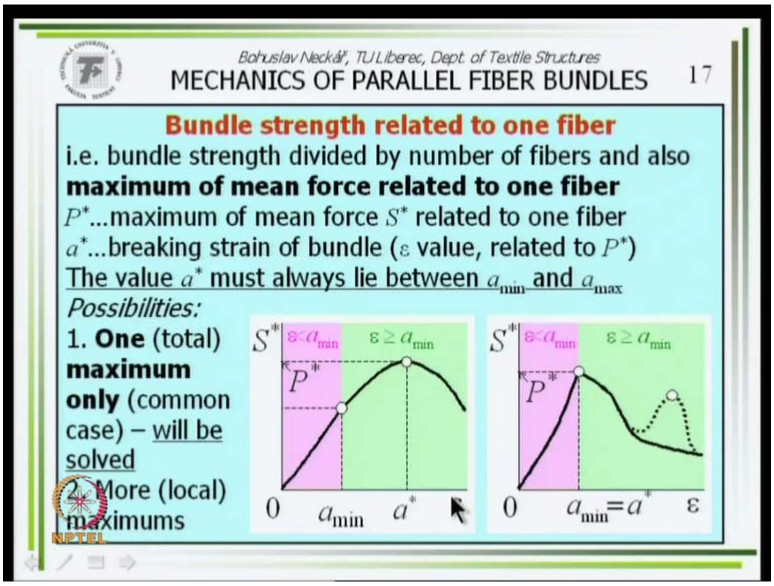

In the second case,it was the case epsilonand interval from a mean to a max.We derived this equation after hearingmeans this ratio equal 1.We obtained this and it is integral from epsilon to a max from ga integral from epsilon to maximum.What is it?It is evidently 1 minusdistribution function. The distribution function of margin ofmarginal distribution function of a, was called capital G here. So, that we have s star is s bar epsilon times one minus g epsilon and that is alleasier than in this case if the assumption of the symmetrical strength assumption is valid than the equation are more easier of course,.

If all the fibers are broken, then the force mean of parallel fiber is equal 0. So, we have the equations for bundlestrength parallel fiber in nostrength and the force parallel fiber in bundle in the relation to epsilon.We can calculate suchfunctions.S star is a function of epsilon.If epsilon is smaller than a mean,then it resultproblem.Whenepsilon is higher than a mean, then some fibers are broken and by increasing of epsilon more and more fibers is broken. So, its curve in I shape have one maximum and then is decreasing to 0.

Generally,it can be here more local maximums and also some broken in this border. So, it can be complicated.Theoretically, it is possible normal way by our type of bundles and fibers.This path has only one maximum,so that notice this maximum.It is the point ofmaximum force is strengthof bundle by number of fibersstrength where you pair one fiber well.

How to obtain?Let us imagine yourself.Only this case, no this case.It is too complicated.This case we will solve how to obtainthis point?How to obtain maximum of curve?The derivative must be equal 0.So, is it I need tothis maximum must be our interval from a mean to a max.Evidently, by a max all fibers are broken by a mean, noone fiber is broken. So, this maximum must be in our integral from a mean to a max.

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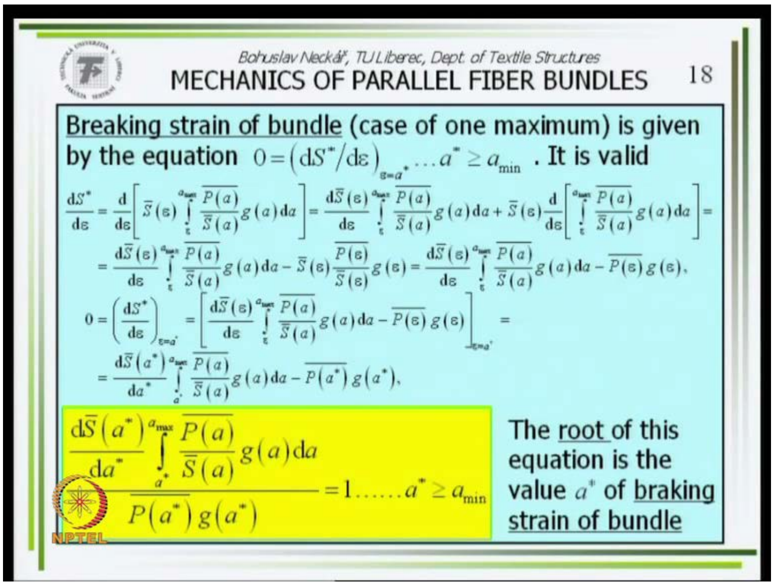

So, we need to derivederivative from this equation.Yeah, derivative from this equation and then,say this derivative must be equal 0.It is shown here.Nothing special.Here is derivative.I think I need not to commandmathematical steps.They are all here.You can quietly want to see the mathematical way.It is derivative nothing,nothing moreand after differentiation,even we have this resulting expression.For derivative,we say this derivative should be equal because equal 0,if epsilon is equal star in which point is the derivative equal 0.It is the maximum of force parallel fiber in bundle.

This moment corresponds to breaking strain of bundle breaking strain of bundle is a star, sothatin the point epsilon equal a star.This derivative must be equal 0.You know, this expression we use this and we made it and then, we obtain this expression 0 equal this as theequation rearranging.To rearrange this equation, is it good?Why?

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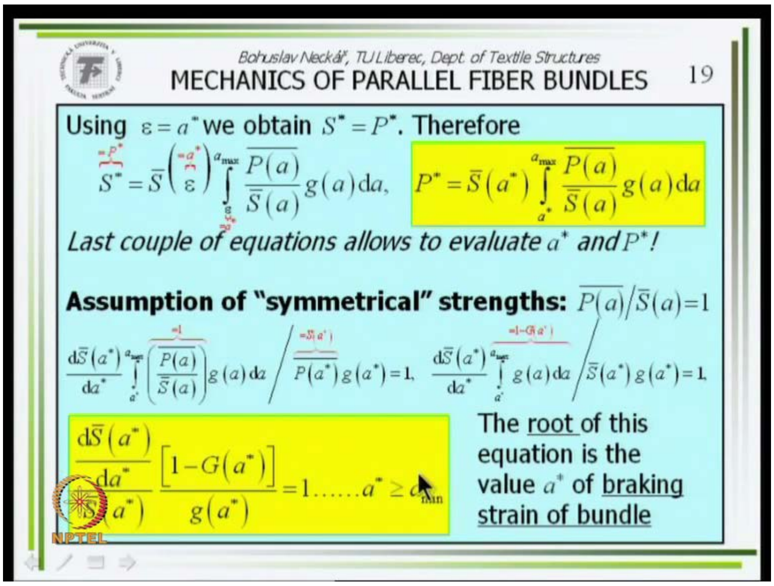

When we know our average function as star as bar a, when we knowthe function of conjugate mean value pa bar,when you know marginal probability density of breaking the strength of fibers, then only one quantities are known on our left hand side.It is a star breaking strain of bundle here.It is here in the derivative and here.Yeah, we can findwhich of a star.We must use on the right hand side because to obtain value one. So, this is the question offinding ofroot of this equation.

A star of this equation is the root.Yeah of this equation, you must do some numerical method for finding of root of equation, but not too difficult to use it.Usually, you need to do some numerical method. So, complicated equation is not possible to solve numerically.The root problems from this,it is clear principaling, sothat from these equations, we can obtaina star.We can obtain a breaking strain of bundle and when we have the breaking strain of bundle,then we use our general equation forforceparallelfibering bundle.On the place of epsilon, we give a star breaking strain,so that we obtain aP star which isP star whichforce breaking force pair one fiber by break moment of bundleyeah.

So, these last couple ofequations allow to evaluate a star. P star is there a case we obtainwhen we accept symmetrical strength as an assumption.If this one, then usingour earlier equation, this ratio is equal 1,so thatit is this here.Sorry yes and because this assumption Pa star bar must be S bar a starafter a small rearranging because this is one minus distribution function.We obtain this expression. Root of this expression is breaking strain of bundle and using this expression which is the exterior.

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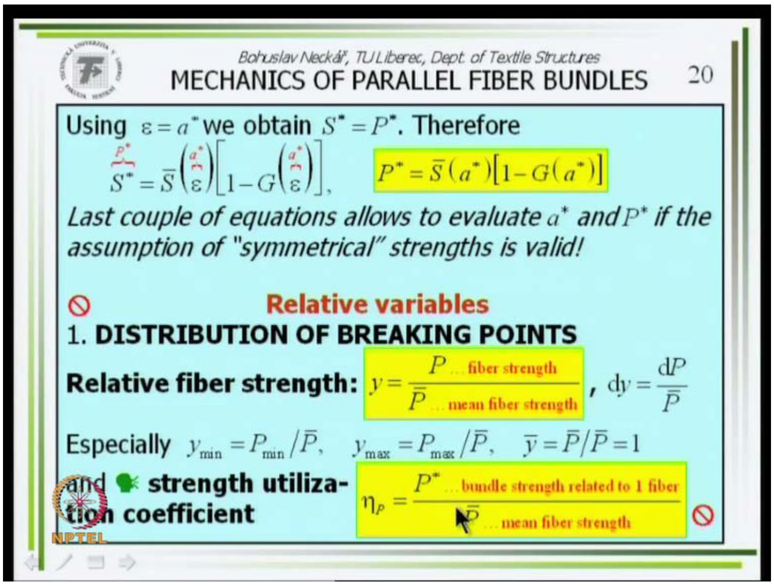

We obtain the force fiber by the moment of break of bundle. So, last couple of equations allows evaluating a bar a star and P star, ifthe assumption of symmetrical strength is valid. It is the way how to obtain, how to solvethe break of bundle?Next slides bring no logical moment.Next slides are only the mathematical rearranging.We introduce some relative variables and rearrange our equations to another form using this relative quantities andthe final equation is better for calculation.Therefore, it is here.Ido not want to comment it because if somebody will know you can read slowlymy slides and this rearranging upwith me result commentary.

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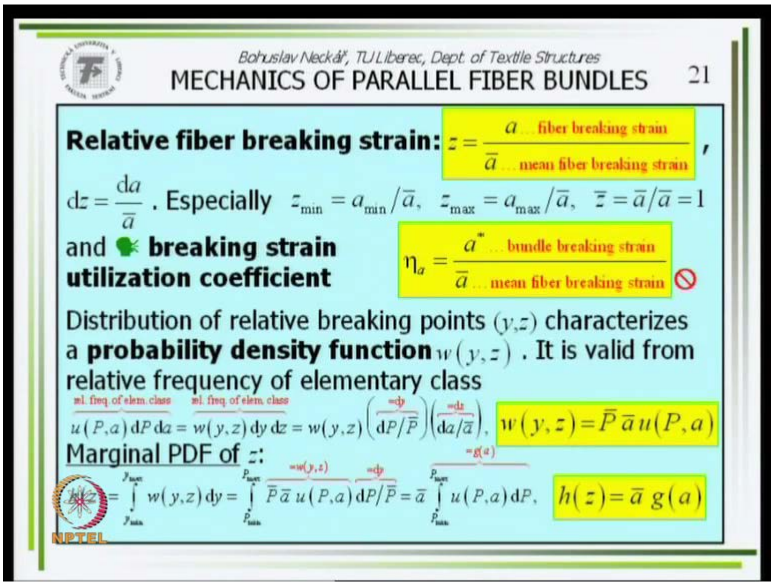

I would onlysay about two interesting quantities.It is the strength utilization.Coefficient eta p is also a1 of the relative quantities eta p which is a bundle of strength related to one fiber p star by p bar which is mean fiber strength.Yeah, by the way, when all fibers, our case number 1 our trivial case and all fibers are the sameproperties,thenthis ratio, this strength utilization coefficient is 1.Isn't it?This is the first, this the second which I want to say to you is the breaking strain utilization coefficientwhich is similarly etaa.Bundle breaking strain mean fiber breaking strain,bundle breaking strain by mean fiber breaking strain.

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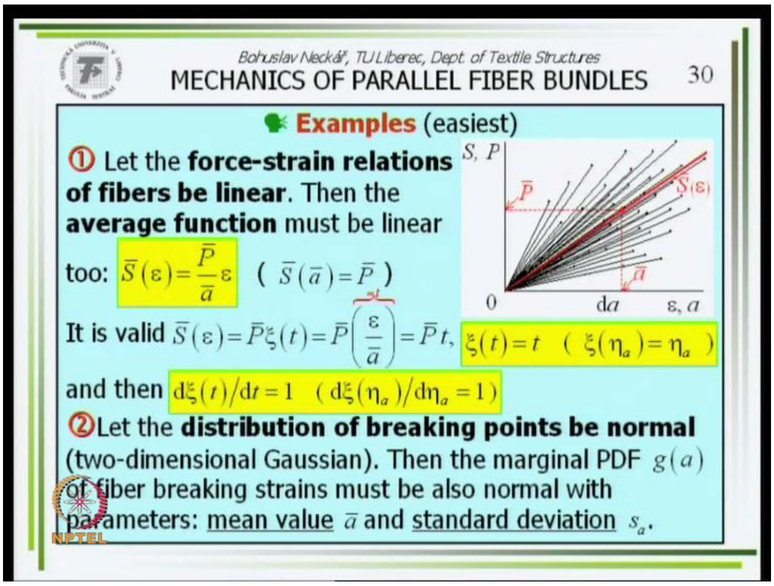

Here,is a lot of slides which rearrange our equations and say, we came to an example.A theoretical examplebecause I want toshow you only there isour result of our calculation and theoretical example which is here, based on really easiest.Why I want to present the result here because use the same way, you can also construct an order calculation.

We have lot of assumptions here.It means, assumption on another force strain relations of fibers be linear. So, this strainis force strain curves of each fiber are linear. So, is it also the mean?Mean function is the average function, is the linear.Here, second let the distribution of breaking point be normal means Gaussian two dimensionalprobability density functiondistribution of all and points of thisblack lines have the distribution alike two dimensionalGaussian distribution.

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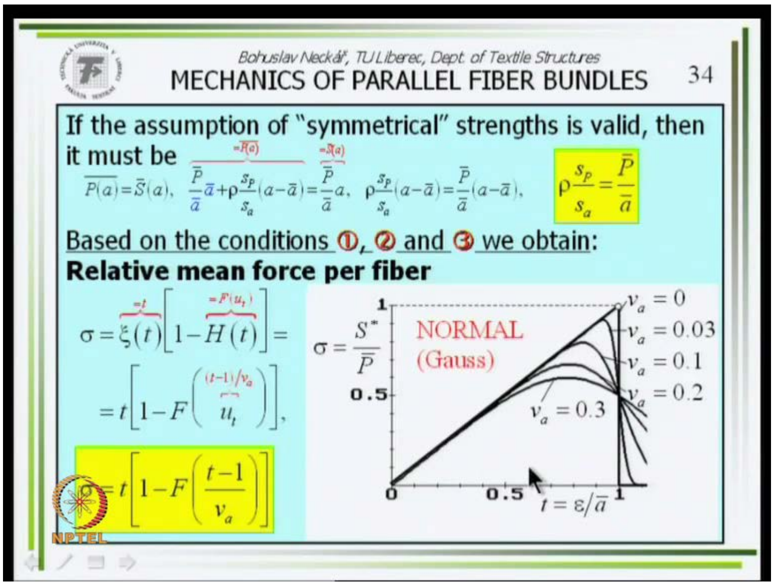

Therefore,let us accept the assumption of symmetrical strength.Let the assumption of symmetrical strength be valid and using these three assumptions, we obtain after application of our equations, this graph.By the way, this equation hereisshown. I will comment this equation later on.The epsilon by a bar,what is it?Strain by mean breaking strain parallel fiber.I call it in the moment t relative quantityand on the ordinate is S star force mean force per fiber bundle by mean value of fiberstrength.Itis when we calculate it.Result is that this function based only on the coefficient of the variation of fiberbreaking strain, no mean value,no standard deviation, but onlyco-efficient of variation.I use coefficient of variation as is usually is used in theoretical.For example, 15 percentages and 0.15, the measure is quantity yeah.

Iseehow it is?If coefficient variation is equal 0, then our case is reduced to our trivial case because linear force strainsfunction.Then,it is increasing to n point and pink all is this strain, but when you we have as we do not know on the second curve here, v a is 0.43 percentage of coefficient variation offiber breaking strain.It can be in some real natural fibersby cotton.We have some experience.Since, that it is perhaps round 0.2, but let us show this curve 0.3.This is related to the forceper one fiber is higher mean force is higher, but then is decreasing.Why?Because in the bundle more and more fibers is broken and the force which earlier takenthe broken fibers mustnow take on his body the fibers which are not broken. So, I think it is clear.

By the way, to obtain this function wasnot too difficult because it isphi its sigma and is capital f.It is distribution function of sum of standardized Gaussiandistribution absolutely known in each tablesor each computers software is the standard distribution,normal standard distribution,and standard normal distribution.What we obtain in ourexample forstrength and breaking strain utilization coefficients.

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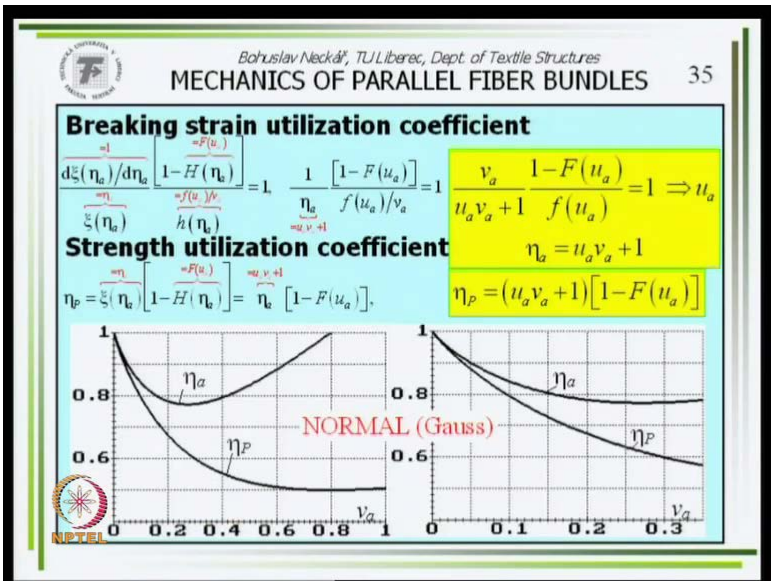

When we calculate it graphically,calculate it numerically,of course, we obtain following graphs.This function is a strength utilization coefficient.You can see that these both also are function of coefficient of variation of breakingstrain of fibers.This coefficient,this strength utilization coefficienteta P is permanently decreasing.It is very high interval from va coefficient variation from vasit is going from 0 to 1 is absolutely unreal value, but only to understand the general trend.

So, it is decreasing andutilization coefficient of strength of breaking strain is decreasing.Then, increasing for high values of coefficient variables, nevertheless in the real path which is roughly 0.3. So, that this part, it is shown in more details.Show on right hand, rightpicture.Both curves,Ican saydecreasingthe utilization is not small.You can see that for example, 0.2.Do not worry often used by cotton fibers this value is 0.7.

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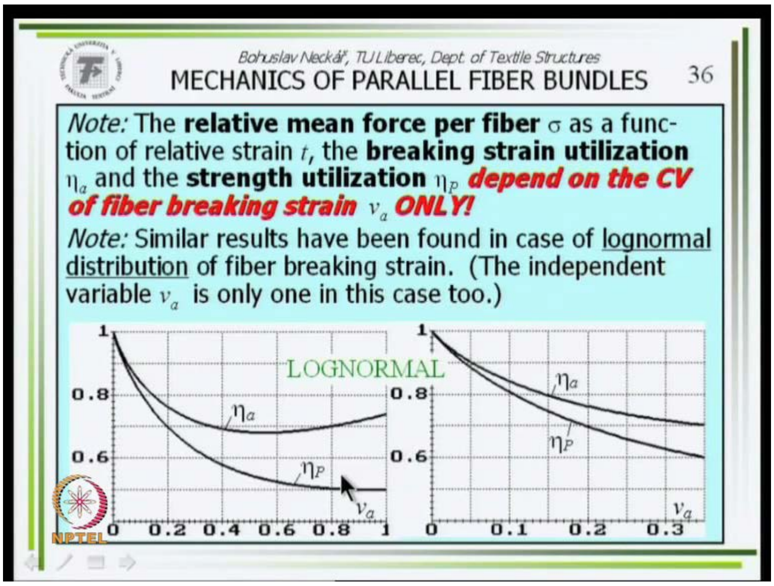

The bundle strength is only 70 percentages to this mechanismmean, meansofindividual fibers 30 percentage down only this affect.Well,you see both graphs.You see that such effect is very significant.Therefore, from this is going out to one practicalresult for you when we want to calculate numerically all this and we not have enough input variables.You see that if coefficient of variation of fiber breaking strain is high than the bundle,bundle strength is small.When you have some material having high value v a, we carefully re strength of your for example, yarn because this variability of fiber breakings strain can bring small tenacity of your yarn.It is intuitiveresultcheck.It is experimentally right now.

It is interesting also using no normally thandistribution, we obtain similar curvesalso using sometype of waybill distribution.You can also qualitativelycurve, but not too far from ourexample fromnormalthis result and our equations can apply also to the Hamburger's theory.Hamburger theory speaksaboutblend of two type fibers, but traditionally,what type of fibers?Each fiber has the same properties.Now, we can solve the problem.We have to take two fibers.For example, viscose and polyester, but the viscose fibers are not all same.They have somedistribution of properties and polyester fibers are all same.They have, theyown distribution.

Principleis also possible in this case.I want to show you that earlier, according tohamburger, you can obtain this thin line.It is the broad line which we construct in last lecture, but using this probabilistic model to Hamburger does generalize the Hamburger theoryusing this model,we obtain this line.The positionislarger.Here is an example.Here shown this inputandby an order type ofinput values, you can also on the place ofHamburger also obtainsuch,so that the curves are more nearer toour experimental experiences.So, I hope that the introduction because my speech was only on introduction to the probabilistic model of fiber bundle.Can show you the general way how to solve it?Also these slides are not theworst which you can meet inyour professional life.Dr. Ross and I prepared more general concept which respect not only the variability of strength andbreaking strain of fibers, but also of thevariability ofcrimp of fibers.

Let us see myfibers which haveshown same, strength same breaking oration when they are straight, but because the fibers have some difficultshape by elongation at first, practically these out force,we changed the shape to stretch a,and then startedforce. This affect can be from fiber to fiber different, also probabilistic model.