

Orientation of Fibers
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Lecture No. # 05
Pores Among Fibers

We start as set of information, set of lectures about the yarns **about the yarns**. Yarns are very important objects in textile and very old objects in textile practices. Around 25000 years, the people of society know to produce some yarns. So, it is very old and very **very** original type of fibers assembly. In my speech, I want first to introduce some general vision about the possibility, how to model the structure of the yarn, then we will speak about the special type of models, so called helical models of the yarn, then something about the alternative to helical **helical** model, which is called as a migration model of the yarn.

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The practically useful yarn properties are the result of fiber properties, the mutual fiber interactions inside the yarn, and the interactions between yarn and outer influences. The internal structure of yarn is very important especially for geometrical and mechanical properties of yarn.

We observe that the specific regulations of internal yarn geometry are relatively complicated due to the complex nature of deterministic and random influences. Lots of mathematical models on this topic were created during the last two centuries. Some of the well-known models will be presented in this lecture.

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GENERAL DESCRIPTION OF FIBERS IN YARN

General fiber trajectories in yarn:

- **complicated shapes (/)**
- **random characters**
- **deterministic trends**

Description of point on fiber : **cylindrical coordinates** r, φ, ζ

The diagram shows a cylindrical yarn cross-section with a red wavy line representing a fiber trajectory. The vertical axis is labeled ζ (yarn axis). The horizontal axes are labeled r_2 and r_1 . The angle between the horizontal axis and the fiber trajectory is labeled φ . The radius of the yarn is labeled r .

Let us start let us start this general introduction part, on your picture, you is a scheme of some **some some** general yarn is one that fiber inside. The fiber in the yarn is very, have the very complicated shape and we must use some coordinates for description of this of

this shape, usually it is a cylindrical coordinates r , may be this is some general point on the fiber, r is a reduced and ϕ and the lengths ζ , the shape of the fiber in the yarn have the same moment random character as well as the general deterministic trend. You know intuitively that this, **this** the general trend is your to helical trend, is it not?

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Fiber elements

The position of fiber elements is determined by the coordinates r, ϕ, ζ ; length by d ; direction by the elementary increments of the coordinates, i.e. $dr, d\phi, d\zeta$.

Conventionally:

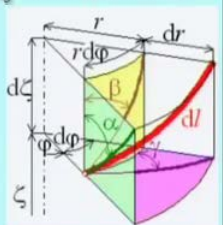
a) the increment $d\zeta$ is defined always as **positive** and
 b) the positive value of the increment $d\phi$ corresponds to the sense of direction of yarn **twist**.

We define:

1. **Angles** α, β, γ using equations
 $\tan \alpha = \frac{dr}{d\zeta}, \tan \beta = \frac{r d\phi}{d\zeta}, \tan \gamma = \frac{dr}{r d\phi}$

and it is valid **$\tan \alpha = \tan \beta \tan \gamma$**

Note: Evidently, the angle β relates to the yarn twist intensity.



Let us think character about this one fiber element inside the yarn, this is yarn axis and this is a red part is some general fiber element, the distance is a here from r to r plus d r this angle is $d\phi$ so that these lines is r times $d\phi$ and they have this $d\zeta$. Using this **this** analysis, we can define free angles which are here; angle α on the green wall angle β on the yellow wall and angle γ on this violet wall, it is evident that these three equations are valid and from this three, it is evident that also we saw that tangents α is tangents β times tangents γ **yes** sums more **no** it is evident that this angle β relate it, relates to the yarn twist intensity, we will speak about it later.

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2. Angles $\vartheta_r, \vartheta_t, \vartheta_a$ using equations
 $\cos \vartheta_r = \frac{dr}{dl}, \cos \vartheta_t = \frac{r d\varphi}{dl}, \cos \vartheta_a = \frac{d\zeta}{dl}$

where $dl = \sqrt{(dr)^2 + (r d\varphi)^2 + (d\zeta)^2}$
 Then $\left(\frac{dl}{dl}\right) = \sqrt{\underbrace{\left(\frac{dr}{dl}\right)^2}_{=\cos^2 \vartheta_r} + \underbrace{\left(r \frac{d\varphi}{dl}\right)^2}_{=\cos^2 \vartheta_t} + \underbrace{\left(\frac{d\zeta}{dl}\right)^2}_{=\cos^2 \vartheta_a}}$

$1 = \cos^2 \vartheta_r + \cos^2 \vartheta_t + \cos^2 \vartheta_a$ (rule of direction cosines)
 The following relations are evident from earlier equations:
 $\tan \alpha = \cos \vartheta_r / \cos \vartheta_a, \tan \beta = \cos \vartheta_t / \cos \vartheta_a, \tan \gamma = \cos \vartheta_r / \cos \vartheta_t$

$d\zeta = \sqrt{\underbrace{\left(\frac{dr}{d\zeta}\right)^2}_{=\tan^2 \alpha} + \underbrace{\left(r \frac{d\varphi}{d\zeta}\right)^2}_{=\tan^2 \beta} + \underbrace{(d\zeta/d\zeta)^2}_{=1}}, dl = \sqrt{\tan^2 \alpha + \tan^2 \beta + 1} d\zeta$

It takes it also our second possibility, second triplet of angles which can characterize our element there are the angles theta a to axial direction theta r to radial direction and theta t to some tangent direction. From this picture, you can see what is theta, theta r theta t and theta a and evidently based on the Pythagorean theorem in three dimensional space, this equation must be valid after dividing by d l we obtain this, so that we obtain this equation which is very known rule of direction cosines.

From definition of all angles, it is evident that following expressions are right, we can also refine lines of our red fiber element which is this here, d zeta times this here.

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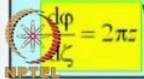
$$\cos \vartheta_r = \frac{dr}{dl} = \frac{dr}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1} d\zeta}, \quad \cos \vartheta_r = \frac{\tan \alpha}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1}}$$

$$\cos \vartheta_t = \frac{r d\varphi}{dl} = \frac{r d\varphi}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1} d\zeta}, \quad \cos \vartheta_t = \frac{\tan \beta}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1}}$$

$$\cos \vartheta_a = \frac{d\zeta}{dl} = \frac{d\zeta}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1} d\zeta}, \quad \cos \vartheta_a = \frac{1}{\sqrt{\tan^2 \alpha + \tan^2 \beta + 1}}$$

Evidently, **two independent values** define the direction of fiber element. Those are the two ratios of increments of coordinates: $d\varphi/d\zeta$ and $dr/d\zeta$. (Conventionally $d\zeta > 0$.)

1. **Twist of element.** We define $d\varphi = 2\pi z d\zeta$ where: z ... **twist of element** (Not generally identical with yarn twist Z ; in more details see later.)



After you arrange, you obtain this equation derivation between, so that derivation between angles theta, theta t and theta a are given by this triplet of functions. Now, too difficult it is only the describing of geometry of fibers element. It is evident that two independent values they find the direction of fiber element, we use one value d phi by d zeta and the second d r by d zeta to, let us define d phi **d phi** by d zeta for one fiber element, let us call as a value two times pi times z where z is, we can call as a twist of element. We will explain it more precisely, why it is this twist of element can that need not be identical this yarn list which we know from technological terminology.

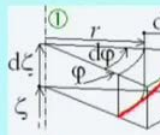
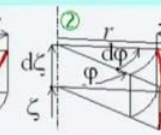
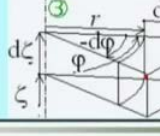
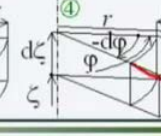
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It is also valid $z = \frac{d\varphi/d\zeta}{2\pi} = \frac{r d\varphi/d\zeta}{2\pi r}$, $z = \frac{\tan \beta}{2\pi r}$

2. **Characteristic of radial migration.** We also define $\frac{dr}{d\zeta} = m$ where: m ... **characteristic of radial migration**. Evidently also $m = \frac{dr}{d\zeta}$, $m = \tan \alpha$

Types of elements:

①	$dr > 0, d\varphi > 0,$ $\Rightarrow m > 0, z > 0.$		
②	$dr < 0, d\varphi > 0,$ $\Rightarrow m < 0, z > 0.$		
③	$dr > 0, d\varphi < 0,$ $\Rightarrow m > 0, z < 0.$		
④	$dr < 0, d\varphi < 0,$ $\Rightarrow m < 0, z < 0.$		

So, from so defined equation, z is $d\phi$ by $d\zeta$ by two times π , we can multiply and divide by here blue value r , and this was tangents β . So, z is tangents β by $2\pi r$. The second characteristic is the characteristic of the first one, is characteristic of twist for our fiber element, the second characteristic is the $d r$ by $d\zeta$ which is m is a characteristic of radial migration.

Evidently going back to our equations about angles, we can also write about this m is \sin than tangents α , this couple of two here, I must say that we conventionally use $d\zeta$. So, increasing of vertical coordinates **as a positive** as a positive value, elemental but positive, we **we** assume that **that** our ζ coordinate is increasing when we go, when we have some like a microbe and when we go through this fiber path, inside of the yarn then our ζ coordinate is higher and higher and higher and higher. So, using these two parameters z and n for possible characters of fibers element can be **can be** defined, it is shown here.

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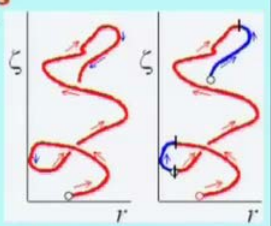
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Generally:
 In a given yarn the **parameters z and m of the elements are random variables** whose distributions depend on the coordinates r and (most generally) ϕ .

Fiber paths

Along the fiber trajectory the coordinate ζ increases, but generally it can also decrease – see \rightarrow in the first ζ - r diagram. Usually we can neglect these parts and/or (imaginatively) cut.

Assumption (simplification):
 Along the fiber trajectory the coordinate ζ only increases. (This corresponds with the convention, stated before.)



I say that we assume that all **that** that ζ coordinates must every times increase in our, this is the graph reduce of fiber for the points and ζ direction.

It is possible that the, generally that the fiber is can go also back some loop inside of the yarn. So, how to explain it when we say that we want ζ , where you have ζ increasing coordinate, it is easy, let us imagine that we divide our fiber so that we obtain this part, this blue part, this red part and that this blue part to some segments and on each

such segments, we can imagine that the zeta coordinate is increasing so that that our assumption about the increasing value of coordinate zeta is possible to accept also for generally for each fibers.

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Three-dimensional path of a general i -th fiber (fiber segment) in yarn can be determined by **two differential equations** $d\varphi/d\zeta = 2\pi z_i(\zeta)$, $dr/d\zeta = m_i(\zeta)$, or

$$d\varphi = 2\pi z_i(\zeta)d\zeta, dr = m_i(\zeta)d\zeta$$

$z_i(\zeta), m_i(\zeta), \dots$ z and m as functions of ζ , and the boundary conditions having sense of starting point.

FIBER PATHS AS A STOCHASTIC PROCESS
 The functions $z_i(\zeta)$ and $m_i(\zeta)$ are specific for each fiber in yarn. The set $\{z_i(\zeta)\}$ of functions $z_i(\zeta)$ can be usually interpreted as a set of realizations of a stochastic process $z(\zeta)$. Similarly, $\{m_i(\zeta)\}$ can be interpreted as a stochastic process $m(\zeta)$. Hence, the **paths of fibers in yarn can be interpreted as a stochastic process.**

Free dimensional path of a general i th fiber, some fiber; one fiber in yarns rapture, let us call it i th fiber, general i th fiber we determine by two differential equations, we because $d\varphi$ by $d\zeta$ is $2\pi z$ by $d\zeta$ for i th fiber is changed from point to point, from the fiber from element to element of our i th fiber. So, this z_i is some function of ζ from this, of this we obtain $d\varphi$ is $2\pi z_i \zeta$ times $d\zeta$.

Similarly, also the quantity m_i is changed on the path of our i th fiber therefore, it must be a function of ζ . So, from that dr is the function $m_i \zeta$ times $d\zeta$, this couple of two formally very easy, but both are a differential equations, is it not? This couple of two differential equations, they find three dimensional path of a general i th fiber and now we have two possibilities; how to create a model of the yarn, the first is to interpret a fiber path as a stochastic process.

Let us imagine to set a fibers inside of yarn, exist a set of functions $z_i \zeta$ over o_i for first fiber, for second fiber, for third fiber and so on. Each such function is (O) order, it is individual to fiber. So, with a set of such functions can be usually interpreted as a set of realization of a stochastic process. And similarly, also the set of functions and $i \zeta$ over o_i can be interpreted as a stochastic process and the path of fibers in yarn can be

interpreted as a stochastic process, so that each fiber is a not is a bit larger random oriented, now then now sin 1 as the order.

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Note: The definitions of some suitable characteristics of fiber configuration (fiber paths) are based on theory of stochastic processes (e.g. "fiber mean position", "r.m.s. deviation" and "mean migration intensity" like *J.W.S.Hearle*, correlograms etc.) *Example:* Period and frequency of radial migration.

On a giving radius in the ζ - r diagram the values p_1, p_2, \dots represent local periods (wave lengths) of fiber. Arithmetic mean of these values over all fibers is the **period p of radial migration** and reciprocal of this is the **frequency f of radial migration**. (Generally, p and f are functions of r .)

If it is so, then we need to use for evaluation of the yarns structure, the tools relate it to stochastic processes; it is for example, correlograms or maybe you heard about the parameters from professor Hero, fiber mean position deviation and mean migration in terms so on and so on. These all are tools how to evaluate the random process, also of (O) of migration and so on.

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FIBER PATHS AS A DETERMINISTIC PROCESS

Individual fiber paths have usually a random character, but their dominant trend is commonly deterministic (e.g. helical). Therefore, we often substitute the real fibers by an **ideal (representative) fiber trajectory** (round which the real fiber paths "are oscillated"). In this case **functions $z_i(\zeta), m_i(\zeta)$ are deterministic**.

The following terms are used:

1. $m_i(\zeta) = 0 \dots$ **radial migration** of fibers
2. $z_i(\zeta) \neq \text{const} \dots$ **twisted migration** of fibers
3. $m_i(\zeta) \neq 0$ and at the same time $z_i(\zeta) \neq \text{const} \dots$ **general migration** of fibers

Note: If at the same time $m_i(\zeta) = 0$ and $z_i(\zeta) = \text{const}$, we speak about non-migrating or **helical fibers**.

It is one way, **one way** now too easy in our lectures we were not to more discuss this direction; second, the second concept is to interpret a fiber **fiber** paths as a deterministic process. Individual fiber paths have usually a random character, but a dominant trend is commonly deterministic, for example, helical. Therefore, we often substitute the real fibers by an ideal, represent the fiber trajectory around which the real fiber paths are, we can say intuitively ask to write it, then this from on the, on our function z_i ζ m_i ζ and this **this** couple of this fibers must be deterministic, now probabilistic. If m_i ζ is not equals 0 for all fibers, for all i , what is it m was $d r$ $2 d$ ζ , if it is not equal 0, then the radials of fibers path is changed, is it not?

We speak about a radial migration of fibers, if ζ is different from constant z was d ϕ by $d z$. And now, increase of angle by ζ coordinates, if this value is not constant, then learn to twist, the twist of fibers elements is from point to point another so that we must speak about our twisted migration of fiber. And if both **both** **(())**, then we must speak about a general migration of fibers in yarn. In opposite, if each m_i ζ is equal 0, if each ζ **zeta** is the same constant, we speak about **non migrate** non migrating or helical model.

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Classification of yarn models

Stochastic models		
Different types, based on experimental experiences use different assumptions		
Deterministic models		
Functions	$m_i(\zeta) = 0$	$m_i(\zeta) \neq 0$
$z_i(\zeta) = 0$	Parallel fiber bundle	Entangled bundle
$z_i(\zeta) = \text{const.}$	HELICAL MODEL	RADIAL MIGRATION
$z_i(\zeta) \neq \text{const.}$	Twisted migration	General migration

Note: Partial models of twisted migration see Neckář, B.: Yarns. Prague 1990. (in Czech)

It brings us to this table which can in short to classify the lot of different types of models of yarn structure through some, in some system. For its group of this stochastic models, we do not want more to divide this possibility.

The second group is the deterministic models, and it base on the character of functions z i zeta and m i zeta. If both are equal, what it means **reduce** reduces stable for fiber, for each fiber $d r$ is now different from 0 and $d \phi$ by $d z$ is also 0. So, it is bundle of parallel fibers, parallel fiber bundle. If z function is equal 0, but m is different from 0 that fiber have not twist, but the fiber is on different rally then the **(0)**.

If z is constant and m is 0, it is our non helical model; if z is constant and m is not 0, then it is the traditional radial migration model. For example, like **if z** if z is not constant and m is constant, then the fibers are lying on some imaginary cylinder every times. But on the cylinder, they are, do not create helices, then some other curve. We can speak about a twisted migration and the last position is if all is possible and this is the general migration.

We will speak, we will in more details are the speak about this free, about this is not necessary more speak bundle of parallel fibers was in our earlier lecture so that to important for helical model utmost, and when also the radial migration.

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HELICAL MODEL OF YARN

In a helical model 2 differential equations (stated before) characterize path of each fiber, as shown here:

1. $d\phi = 2\pi z_i(\zeta) d\zeta$, where $z_i(\zeta) = Z = \text{const.}$ and
2. $dr = m_i(\zeta) d\zeta$, where $m_i(\zeta) = 0$

Let us assume, the fiber passes trough the "starting point" r_0, ϕ_0, ζ_0 (boundary conditions). Then we obtain

1. a) by integration:

$$\int_{\phi_0}^{\phi} d\phi = \int_{\zeta_0}^{\zeta} 2\pi z_i(\zeta) d\zeta, \quad \phi - \phi_0 = 2\pi Z (\zeta - \zeta_0)$$

Yarn twist has a sense of constant Z .

Height of one fiber coil is $1/Z$, **same** for all fibers.

increase of number of coils

$$\frac{(\phi - \phi_0) / (2\pi)}{\zeta - \zeta_0} = Z$$

increase of axial length

So, it was an slow introduction to how to general, in general to divide different types of models of yarns structure. Now, let us speak about the helical model of the yarn, we said the helical, each model is given by two differential equations; this is the first and this is the second. It is the same as first, in helical model, this functions z i zeta is constant for each fibers and each point, each segments, each elements on each fiber. This constant

have named Z, it is constant and I have to some one or two slides to show that it is the twist of the yarn and the second and the second equation, this function m m zeta m i zeta is equal 0.

What we obtain, what we obtain from this first equation, when this function is equal Z. Let us integrate this equation so that integral over phi from some starting point coordinates phi 0 zeta 0 r 0 from phi 0 to phi, is equal to the integral from zeta 0 to zeta on right hand side that is function z is constant. Therefore, we obtain this here and then these here, what it there, what is it zeta minus zeta 0 is an increase of zeta coordinates, actual coordinated in the yarn, 5 5 minus 5 is 0, is increase of angle phi divided by 2 pi, 2 pi is one times round. So, it is number of coils, is it not? Number of coils number of coils by corresponding coils is definition of twist, is it so? So, the such constant Z have the sense as a yarn twist so that also the height height per one coil must be 1 by Z evidently.

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b) by rearrangement:
 $d\varphi = 2\pi Z d\zeta$, $r \frac{d\varphi}{d\zeta} = 2\pi r Z$, $\tan\beta = 2\pi r Z$

The angle β is constant at a given radius r .

2. By integration:
 $\int_{r_0}^r dr = \int_{\zeta_0}^{\zeta} m_i(\zeta) d\zeta$, $r - r_0 = 0$, $r = r_0 \dots \text{const}$

Fiber lies on the same radius (cylindrical surface)
 Note: Same conclusions can be obtained from the picture.

Assumptions of helical model can be formulated also in the following way:

1. Helical paths of fibers (same sense of rotation).
2. Common axis of all helixes is yarn axis.
3. Same coil height for all fibers.

From the same equation, we can write also if this is what it my Z is constant, is not the function as a general it is constant after lot application if dividing by r, we obtain this here now this is tangents beta, so tangents beta is 2 pi r Z.

From the second equation, from the second differential equation after integrating from r 0 to r here from zeta 0 to zeta from starting point to some general point, because this is 0 then r minus r 0 must be equal 0. So, r is equal r 0, what it means, the fiber is lying on the

same cylinder by constant, by the given fiber is lying on the cylinder is constant reduced of course, r is it each r is r_0 .

There are **this two** this two equations are dicit equations for helical model that we can also to interpret this helical model very easy, we can say this is some scheme of the yarn according helical model on the general cylinder diameter r is lying one red fiber. Once this length is 1 by z , I know of between the tangent to our fiber and vertical direction is along a β , is it not? And after unwiring of this surface of such imaginer cylinder, we obtain this, from this is also possible to calculate the tangents β is $2\pi r$ by 1 by Z , so $2\pi r$ times Z .

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The fiber positions in yarn ("starting points"), which are not determined by the assumptions stated before, can be characterized by the radial function of packing density $\mu = \mu(r)$. Because it is relatively complicated, the following assumption are used:

4. Packing density is constant in all places inside the yarn.

Note: If all these 4 assumptions are valid, then we speak about **ideal helical model**.

Note: Helical models and their applications are the oldest concepts of yarn modeling, adherent with the names *A.Köchlin (1828), E.Müller (1880), S.Marschik (1904), A.CH. Gegauff (1907), R. Schwarz (1933), E. Braschler (1935), V.I. Budnikov (1945), L.R.G. Treloar (1956), etc.*

The slide includes a small graph on the right showing a curve labeled 'Real' and a dashed curve labeled 'Assumption' plotted against radius r . The radius r is marked from 0 to $D/2$.

The, so we called differential of differential equation every times, some **some** starting point or something. So, our starting points from our fibers based on packing density, if some fiber is starting from some radius then this fiber is lying on this radius and have path to the packing density on this radius, how is the packing density, so much fibrous can be on this or that radius r .

Generally, the packing density is a function of radius, in a based on experiment to experience is, this is radius and this is packing density, then the real curve is something like this black curve on our small picture.

But it is difficult to obtain such function, we can obtain our experimentally based on now to easy experimental process or make some mechanical model of the yarn that is one of the easy. Therefore, we often use some, use a fourth assumption for simplification of our problems, we assume in that fourth packing density in all places inside of the yarn is constant like this red line (○).

Also, this assumption is valid, and then we can speak about the ideal helical model. Helical model is reserved this assumption ideal, in ideal helical model we accept also this assumption. The helical models and ideal helical models are in lot of publications, some of known authors, I mention here, from Kochlin I think 1828 to Treloar which which completed this helical model in a relatively complete concept well.

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Number of fibers and shortening of yarn

Let us create differential annulus - "differential layer" (*Braschler*) - at the general radius r in a cross-section of **general helical model** of yarn.

Area of differential annulus: $2\pi r \, dr$

Packing density of differential annulus: $\mu = dS / (2\pi r \, dr)$
 where dS ...area of fiber sections in diff. annulus (red)
 (See lecture 1 - areal interpretation of μ)

Area of fiber sections in differential annulus:
 $dS = 2\pi\mu r \, dr$

Area of oblique section of one fiber: $s^* = s / \cos \beta$
 (where s ...fiber cross-section - see lecture 1)

There are this is the, I can say definition of the, or description what is it a helical model. In our helical model, let us find the number of fibers and then also the shortening of the yarn due to twist. The first theme will be number of fibers in cross section of yarn, ideal yarn, yarn which corresponds to our helical model.

So, let us imagine some cross section of the yarn, the cross section of the yarn is over here, the grey islands here represents the section islands, islands of individual fibers. We create two circles, here two green circles here; the small one have the radius r and the higher one radius r plus $d r$ plus differentially elemental increase of radius, so that we obtain a differential annulus, is it not?

Differential annulus is a how is the thought to area of this differential annulus its evident because its differential annulus its $2\pi r$ times thickness, when you cut it this you obtain something some long and its intuitively clear $2\pi r$ times dr is the r is inside of all annulus the area of fibers it is here as a red area

Now, how area of such annulus with a area of fiber section there are also some parts here isn't it well, but you know the how what is it a packing density so that the area of fibers dS must be $2\pi r dr$ times μ times packing density well and see here it also an order and for us known relation that the section of area there one fiber which is lying with z axis on the radius r of course, is a cross section area by \cos inverse of this angle β .

Now, is the question how is the number of fibers in differential annulus to this to solve this problem lets imagine one abstract situation lets imagine that I am nobody of us is here in this room and I am standing here one foot is inside of room the second my foot is outside of this room how many people is here may be one half because I am here only one half you understand it well.

You can see that number of fibers need not be only 1 2 3 4 natural number then it can be a real number for example, 1 half or something. So, second how many people is here we can say calculate 1 2 3 4 5, but I have another idea, let us go all together to some writing machine for tracks and. So, on and you will find a, our common **right** and then idea why this, is it possible this way logically is it possible?

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MODELING OF INTERNAL YARN GEOMETRY 15

Number of fibers in differential annulus:

$$dn = \frac{2\pi r dr \mu}{s \cos \beta} = \frac{2\pi r dr \mu}{s \cos \beta}, \quad dn = 2\pi \cos \beta \mu r dr / s,$$

Substantial cross-sectional area of yarn:

$$S = \int_{r=0}^{r=D/2} dS = 2\pi \int_{r=0}^{r=D/2} \mu r dr$$

Mean packing density of yarn:

$$\bar{\mu} = \frac{S}{\pi D^2 / 4} = \frac{2\pi \int_0^{D/2} \mu r dr}{\pi D^2 / 4} = \frac{8}{D^2} \int_0^{D/2} \mu r dr$$

Number of fibers in yarn cross-section:

$$n = \int_{r=0}^{r=D/2} dn = \frac{2\pi}{s} \int_0^{D/2} \frac{\mu r dr}{\cos \beta} = \frac{2\pi}{s} \int_0^{D/2} \frac{\mu r dr}{\sqrt{1 + \tan^2 \beta}}, \quad n = \frac{2\pi}{s} \int_0^{D/2} \frac{\mu r dr}{\sqrt{1 + (2\pi r Z)^2}}$$

It was also derived before (lecture 1): $n = \tau k_n$, $\tau = T/t = S/s$
 (...yarn count, t ...fiber fineness, τ ...relative yarn fineness)

We may use this style of thinking fiber area in differential annulus is dS and area pair one fiber is **a star** a star so that dS by S star must be number of fibrous in all differential annulus, using equations derive we obtain for d and this, formula this expression. And now, how is the **substantial cross sectional area of yarn** substantial cross sectional area of yarn is this area in earlier, but integral of this, sum of this over all **over all** possible annulus, they are from r equal 0 to on the periphery r equal D by 2, about half of an yarn diameter so that it is this here; using s we create it, we obtain this equation. Mean packing density of the yarn, it is total substance cross section of the yarn by total area of cross section of the yarn πD squared by 4, after we arranging we obtain this here, number of fibers in yarn cross section. Now it is integral from this the yarn of number pair one annulus from r equals 0 to r equal D by 2. After using, we use here some geometrical formula which we now 1 by cosine square equal to 1 plus tangents square and after slowly arranging we obtained this here, because tangents is $2\pi r Z$ squared, we said tangents beta.

Well, but it was also derived in our lecture one, starting lecture to this lecture that number of fibers is τ times k_n where τ relative to 1 is the ratio yarn **yarn** count more precisely for example, in tax when your density a by fiber linear density fiber **(0)**.

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MODELING OF INTERNAL YARN GEOMETRY 16

Coefficient k_n is now $\frac{n}{\tau}$

$$k_n = \frac{n}{\tau} = \frac{s}{S} n = \frac{s}{S} \frac{2\pi \int_0^{D/2} \mu r dr}{\int_0^{D/2} \sqrt{1 + (2\pi r Z)^2} dr}$$

$$k_n = \frac{2\pi \int_0^{D/2} \mu r dr}{S \int_0^{D/2} \sqrt{1 + (2\pi r Z)^2} dr}$$

Note: The relation $\mu = \mu(r)$ is necessary to know for numerical calculation of $S, \bar{\mu}, n, k_n$. It is possible to obtain the function $\mu(r)$ as a result of experiment or try to apply some theoretical model (e.g. based on differential equation of radial forces equilibrium in yarn – *V.I. Budnikov, J.W.S. Hearle, B. Neckář* etc.)

Ideal helical model satisfies the assumption $\mu = \text{const.}$ and then **substantial cross-sectional area of yarn:**

$$S = 2\pi \int_0^{r=D/2} \mu r dr = 2\pi\mu \left(\frac{D^2}{4} \right)$$

$$S = (\pi D^2 / 4) \mu$$

Coefficient k_n , which is **in this** in this expression can be derive from this s by τ , τ was 1 also cut it as by s times n , we now so that we obtain this expression for k_n , this

expression are valid for a helical model. It means therefore, I have mu insight of integrals because mu can be a function of radius.

I spoke about a difficult thing is done, so that let me now to make this very rearrange of our equations for the case of ideal helical model, it is what it is the model on which the packing density in all places inside out yarn is constant. Therefore, mu is possible to give B for integral as a constant.

How this, how is substantial cross section of area mu is going before, so we obtain this area formalize this here, now pi D square by 4 times mu, corresponds to our knowledge.

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Mean packing density of yarn: $\bar{\mu} = \frac{8}{D^2} \int_0^{D/2} \mu r dr = \frac{8\mu}{D^2} \left(\frac{D^2}{4} \right), \bar{\mu} = \mu$

Further, the following integral is valid:

$$I = \int_0^{D/2} \frac{r dr}{\sqrt{1+(2\pi rZ)^2}} = \int_1^{\sqrt{1+(\pi DZ)^2}} \frac{x dx}{x(2\pi Z)^2} = \frac{1}{(2\pi Z)^2} \left[\sqrt{1+(\pi DZ)^2} - 1 \right]$$

Substitution: $x^2 = 1+(2\pi rZ)^2$,
 $2x dx = (2\pi Z)^2 2r dr$, $r dr = x dx / (2\pi Z)^2$

Number of fibers in yarn cross-section:

$$n = \frac{2\pi}{s} \int_0^{D/2} \frac{\mu r dr}{\sqrt{1+(2\pi rZ)^2}} = \frac{2\pi\mu}{s} \int_0^{D/2} \frac{r dr}{\sqrt{1+(2\pi rZ)^2}} = \frac{2\pi\mu}{s} \frac{1}{(2\pi Z)^2} \left[\sqrt{1+(\pi DZ)^2} - 1 \right] =$$

$$\frac{2}{(DZ)^2} \left[\frac{\pi D^2}{4} \mu / s \right] \left[\sqrt{1+(\pi DZ)^2} - 1 \right], n = \frac{2\tau}{(\pi DZ)^2} \left[\sqrt{1+(\pi DZ)^2} - 1 \right]$$

Mean packing density, we derive this equation times mu sorry, mu is constant. So, before integral for r d r its trivial so that you obtain final in final position, mu bar equal mu, evident if mu is constant then each all mu must be there for all yarn. For future, we are arranging we need to solve one integral, this integral is shown here, is shown also to why how to obtain it is not too difficult to use in such such substitution as we obtain this result. So, I think do not want to comment integration.

A number of fibers in yarn cross section, for this we had this expression. Now, mu is constant can go before integral we obtain this expression, but this is our early integral this one on the place of this, we can give this expression in a in a brackets and after. So, rearranging, where we use here is we multiply and divide by D square, then we

understand that this is s capital S so that this is tau, you know it is earlier equations to the r equations, we obtain number of fibers in this form.

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MODELING OF INTERNAL YARN GEOMETRY 18

Coefficient k_n is now

$$k_n = \frac{n}{\tau} = \frac{1}{\tau} \frac{2\tau}{(\pi DZ)^2} \left[\sqrt{1 + (\pi DZ)^2} - 1 \right], \quad k_n = \frac{2}{(\pi DZ)^2} \left[\sqrt{1 + (\pi DZ)^2} - 1 \right]$$

For the fiber on the yarn surface ($r = r_D$, $\beta = \beta_D$) it is valid

$\tan \beta_D = 2\pi r Z = \pi DZ = \kappa$... intensity of twist (see also the derivation in lecture 1). The alternative equation for k_n can be derived using equation mentioned before.

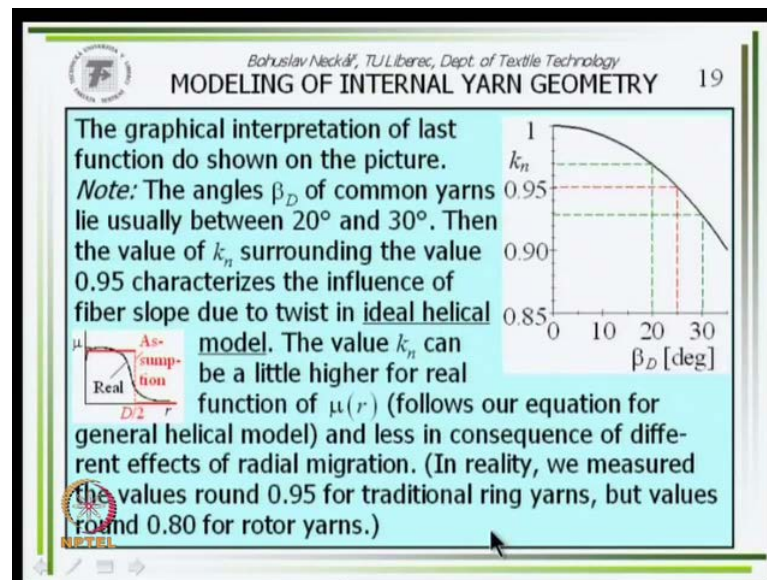
$$k_n = \frac{2}{\left(\frac{\pi DZ}{\tan \beta_D}\right)^2} \left[\sqrt{1 + \left(\frac{\pi DZ}{\tan \beta_D}\right)^2} - 1 \right] = \frac{2}{\left(\frac{\tan \beta_D}{\cos \beta_D}\right)^2} \left[\sqrt{1 + \frac{\tan^2 \beta_D}{\cos^2 \beta_D}} - 1 \right] =$$

$$2 \frac{\cos^2 \beta_D (1 - \cos \beta_D)(1 + \cos \beta_D)}{\sin^2 \beta_D \cos \beta_D (1 + \cos \beta_D)}, \quad k_n = \frac{2 \cos \beta_D}{1 + \cos \beta_D}$$

And k_n because it is n by τ , k_n is given by this equation. We can also use another rearranging this is, this expression is identical is this expression, but we know for us along the $\pi D Z$ is tangents β_D of tangents of peripheral angle β on the yarn.

So, tangents β_D , after rearranging multiply and divide by $1 + \cos \beta_D$. Here, we obtain finally k_n in this form, all rearranging pure trivial mathematical, rearranging which you know from university, from high school, no difficulty. How the, how is k_n graphically? How is k_n when we use this last formula for k_n ?

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In or express it in graphical form, we obtain such graph here, this is axis of peripheral angle of beta D, and this is axis of value of k n, we obtain this thick curve, in the textile is usually **usually** we twist the yarn so that the peripheral angle is something between 20 and 30 degrees.

So, let us imagine average **average** value 25 degree, this is this red dotted line to this angle corresponds the coefficient 0.95. When you experimentally measured coefficient k n, are evaluated based on cross sectional microscopic triplets, cross sections of yarns, then really we obtain the value 0 point, around 0.95 for yarns, ring spun yarns.

Now for rotor yarns, for rotor yarns we obtain much more smaller value, because the angles of fibers are not in dominant effect, create it to twist that important is also the intensive unparallelity of ribbon in rotor, and so called birch fibers on the periphery of rotor yarn. You know this term, you know this problem so that in rotor yarn the coefficient k n is smaller.

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MODELING OF INTERNAL YARN GEOMETRY 20

	Non twisted	Twisted	
Length of bundle	ζ_0	ζ	
Yarn retraction	$\delta = (\zeta_0 - \zeta) / \zeta_0 = 1 - \zeta / \zeta_0$		
Number of fibers	n	n	
Volume of fibers	V_0	V	
Mass of fibers	m	m	
Starting yarn count	$T_0 = m / \zeta_0$		
Yarn count (final)		$T = m / \zeta$	$T = T_0 / (1 - \delta)$
Number of coils	0	N_c	
Latent yarn twist	$Z_0 = N_c / \zeta_0$		
Yarn twist (real)		$Z = N_c / \zeta$	$Z = Z_0 / (1 - \delta)$
Latent twist coeff.	$\alpha_0 = Z_0 \sqrt{T_0}$		
Twist coeff. (real)		$\alpha = Z \sqrt{T}$	$\alpha = \alpha_0 / (1 - \delta)^{3/2}$

Now this is, this is all for coefficient k_n , the theoretical value of this coefficient based on ideal helical model. In reality, it can be a bit larger why, because the structure is not perfectly ideal helical model. Well, to the problem of number of fibers and coefficient k_n , rewrite it also the theme about the yarn retraction. You have found an individual experience, instinctive experience, when you twist it something, some bundle of something may be fibers, may be also a bundle of yarns or something. So, then when you twist it this bundle, the bundle is shorter and shorter and shorter and shorter and shorter, is it not? It is not possible more twist inside; it means by twisting the fiber bundle is shorter and shorter.

Let us imagine some bundle of parallel fibers which is here, runs at ζ_0 after twisting the lengths of resulting yarn is ζ smaller than ζ_0 , the ζ_0 minus ζ , it is the difference of ones between non twisted and twisted form of our bundle. This column here represents non twisted structure; this structure the second column here represents this structure. So, once a bundle non twisted bundle is ζ_0 twisted is ζ .

Yarn retraction, we define as ratio ζ_0 minus ζ , these lengths, the starting lengths ζ_0 . So, we can write it $1 - \zeta / \zeta_0$, number of fibers by twisting is not changed, so here is n and here is also n , volume of fibers generally, we can say that it can be different volume of fibers can be different in this bundle, and in this bundle. Therefore, starting value is 0 , final value is V , mass of fibers must be same, non twisted

as well as in twisted structure, starting count **starting count count** of parallel fiber bundle is mass by lengths; mass is m , lengths is $zeta_0$. So, starting yarn count T_0 is m by $zeta_0$, is it not? After twisting the yarn count, I mean linear density that is a , the m count is now m by another lengths, lengths $zeta$.

The ratio between T_0 and T , there is under definition of yarn retraction is this one, number of coils in parallel fiber bundle is 0 , here number of coils is N_c . And we can construct two, or we were construct two quantities for twist; the first, which is here will be latent yarn twist, it is number of coils per lengths **per lengths** of starting non twisted structure **clear**.

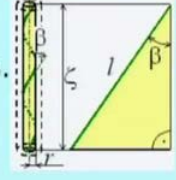
How many coils I were give to 1 meter of non twisted parallel fiber bundle, do you understand this term? In opposite to start yarn twist **which is** which is the same number of coils, but by length of yarn how many coils is in on 1 meter of yarn, final yarn, so that it is N_c by $zeta$ between latent yarn twist, $zeta_0 z_0$ and yarn twist is Z , related to this expression is valid.

Latent twist coefficient, we can also construct the latent twist confident which is Z_0 times square root of T_0 , latent twist and starting yarn count in opposite to this coefficient real which is Z times square root of T **clear**. So, this latent quantity related to starting lengths, now to final lengths, there is a difference here. It is starting quantities, now in a set of our **of our** slides are free variations of model for yarn retraction. We will comment only the second one, we will jump the first one and the third one, it is not too necessary to sign it here, when somebody will study deeper the problem of yarn retraction, he can use my **my** slides and immediately for this, from this slides to understand also the first and the second variation.

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MODELING OF INTERNAL YARN GEOMETRY 21

1. Idea of neutral radius
 A helical fiber has length l at the radius r in the yarn of length ζ . It is valid $l = \zeta / \cos \beta$. The helical fiber length is equal to the starting length, $l = \zeta_0$, at the so-called **neutral radius** $r = r_n$ ($\beta = \beta_n$). Therefore



$$l = \zeta / \cos \beta, \quad \zeta_0 = \zeta \left(\frac{1}{\cos \beta_n} \right) = \zeta \sqrt{1 + \tan^2 \beta_n} = \zeta \sqrt{1 + (2\pi r_n Z)^2}$$

$$\left(\frac{\zeta}{\zeta_0} \right) = \frac{1}{\sqrt{1 + \left(\frac{2\pi r_n DZ}{D} \right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{r_n}{2r_n/D} \right) \left(\frac{2\pi DZ}{\pi DZ} \right)^2}}$$

Yarn retraction $x_n = 2r_n/D$...neutral position $\tan \beta_D = \kappa$...twist intensity (lecture 1)

$$1 - 1/\sqrt{1 + x_n^2 (\pi DZ)^2} = 1 - 1/\sqrt{1 + x_n^2 \tan^2 \beta_D} = 1 - 1/\sqrt{1 + x_n^2 \kappa^2}$$

So, the first variation it is idea of neutral radius according a book, thus at we will not to do it.

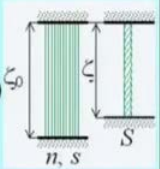
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MODELING OF INTERNAL YARN GEOMETRY 27

2. Idea of total fiber volume
 Usually the following *assumption* is taken as granted (E. Braschler): Total fiber volume in yarn and fiber cross-sectional area (fiber diameter) do not change due to twist.

$V_0 = V \dots \text{const.}, \quad s \dots \text{const.}$

Fiber volume - non-twisted $V_0 = n s \zeta_0$
 - twisted $V = S \zeta$ (lecture 1)



$$\frac{V_0}{V} = \frac{n s \zeta_0}{S \zeta} = \frac{\zeta_0}{\zeta} = s / \left(\frac{S}{n} \right), \quad \zeta / \zeta_0 = k_n$$

Retraction was defined by equation $\delta = 1 - \zeta / \zeta_0$, coefficient k_n was derived before for ideal helical model. Therefore

Yarn retraction $\delta = 1 - \left(\frac{\zeta}{\zeta_0} \right) = 1 - \frac{2}{(\pi DZ)^2} \left[\sqrt{1 + (\pi DZ)^2} - 1 \right]$

We will start this variation 2, the second variant of model idea of total fiber volume which is, which was created from Brasher around 1935. It was some special set of textile on the university in Switzerland, but how is this theoretical concept; I want to show you in the next lecture. So, in the moment, thank you for your attention.