Orientation of Fibers
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Lecture No. # 05
Pores Among Fibers

We start as set of information, set of lectures about the yarns about the yarns. Yarns are very important objects in textile and very old objects in textile practices. Around 25000 years, the people of society know to produce some yarns. So, it is very old and very very original type of fibers assembly. In my speech, I want first to introduce some general vision about the possibility, how to model the structure of the yarn, then we will speak about the special type of models, so called helical models of the yarn, then something about the alternative to helical helical model, which is called as a migration model of the yarn.

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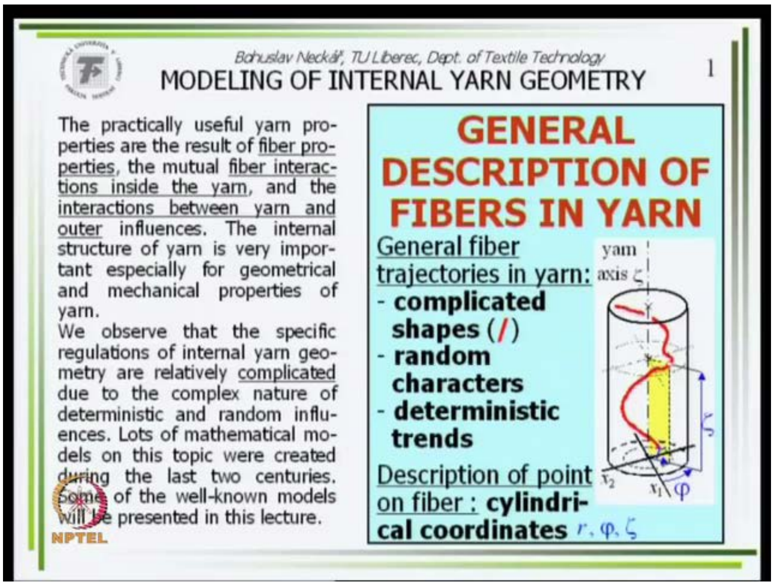

Let us start let us start this general introduction part, on your picture, you is a scheme of some some some general yarn is one that fiber inside. The fiber in the yarn is very, have the very complicated shape and we must use some coordinates for description of this of this shape, usually it is a cylindrical coordinates r, may be this is some general point on the fiber, r is a reduced and phi and the lengths zeta, the shape of the fiber in the yarn have the same moment random character as well as the general deterministic trend. You know intuitively that this, this the general trend is your to helical trend, is it not?

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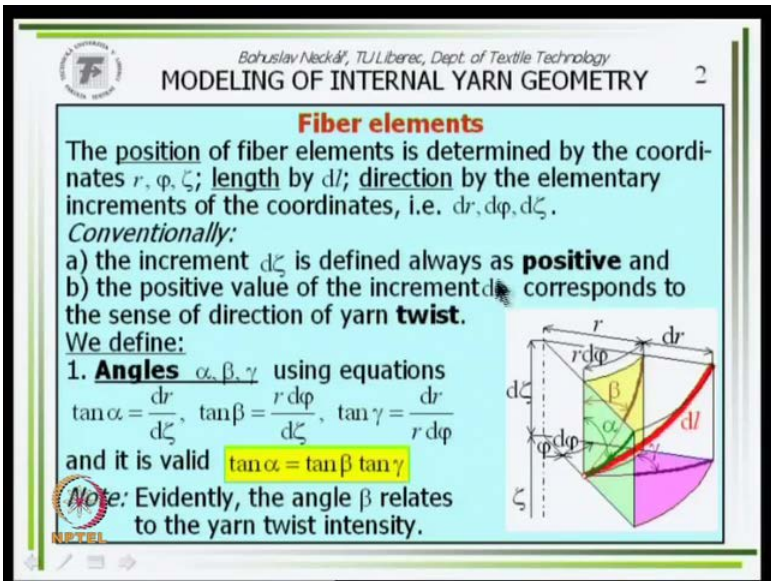

Let us think character about this one fiber element inside the yarn, this is yarn axis and this is a red part is some general fiber element, the distance is a here from r to r plus d r this angle is d phi so that these lines is r times d phi and they have this d zeta. Using this this analysis, we can define free angles which are here; angle alpha on the green wall angle beta on the yellow wall and angle gamma on this violet wall, it is evident that these three equations are valid and from this three, it is evident that also we saw that tangents alpha is tangents beta times tangents gamma yes sums more no it is evident that this angle beta relate it, relates to the yarn twist intensity, we will speak about it later.

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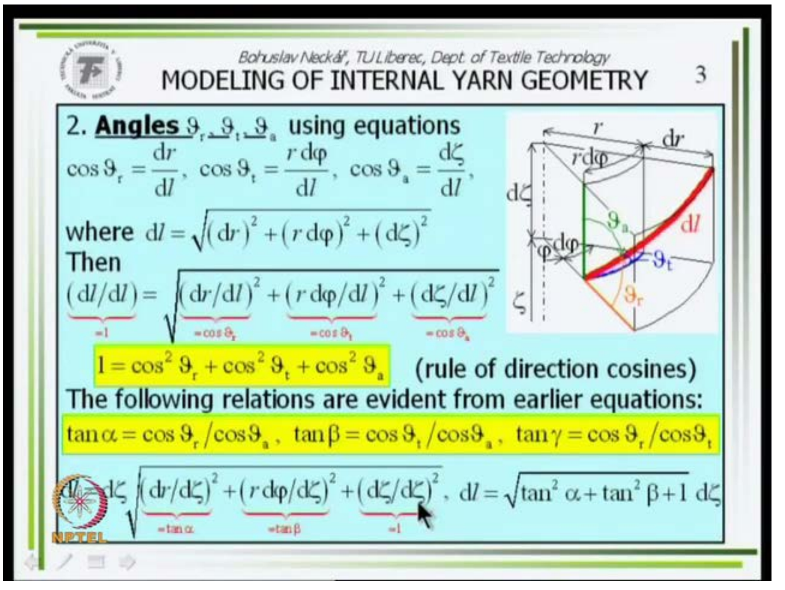

It takes it also our second possibility, second triplet of angles which can characterize our element there are the angles theta a to axial direction theta r to radial direction and theta t to some tangent direction. From this picture, you can see what is theta, theta r theta t and theta a and evidently based on the Pythagorean theorem in three dimensional space, this equation must be valid after dividing by d l we obtain this, so that we obtain this equation which is very known rule of direction cosines.

From definition of all angles, it is evident that following expressions are right, we can also refine lines of our red fiber element which is this here, d zeta times this here.

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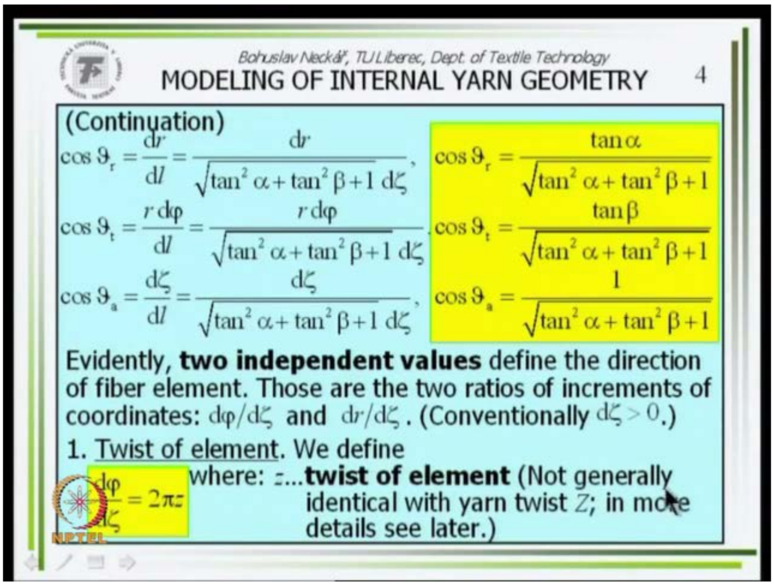

After you arrange, you obtain this equation derivation between, so that derivation between angles theta, theta t and theta a are given by this triplet of functions. Now, too difficult it is only the describing of geometry of fibers element. It is evident that two independent values they find the direction of fiber element, we use one value d phi by d zeta and the second d r by d zeta to, let us define d phi d phi by d zeta for one fiber element, let us call as a value two times pi times z where z is, we can call as a twist of element. We will explain it more precisely, why it is this twist of element can that need not be identical this yarn list which we know from technological terminology.

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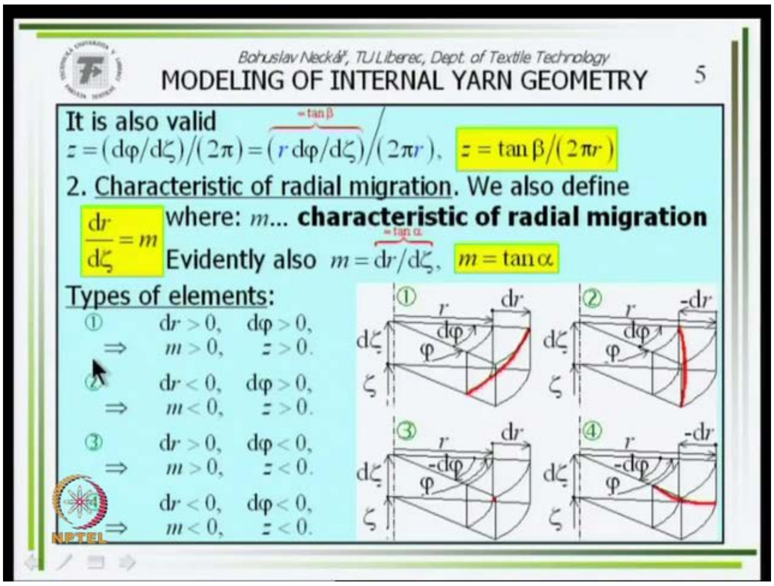

So, from so defined equation, z is d phi by d zeta by two times pi, we can multiply and divide by here blue value r, and this was tangents beta. So, z is tangents beta by 2 pi r. The second characteristic is the characteristic of the first one, is characteristic of twist for our fiber element, the second characteristic is the d r by d zeta which is m is a characteristic of radial migration.

Evidently going back to our equations about angles, we can also write about this m is sin than tangents alpha, this couple of two here, I must say that we conventionally use d zeta. So, increasing of vertical coordinates as a positive as a positive value, elemental but positive, we we assume that that our zeta coordinate is increasing when we go, when we have some like a microbe and when we go through this fiber path, inside of the yarn then our zeta coordinate is higher and higher and higher and higher. So, using these two parameters z and n for possible characters of fibers element can be can be defined, it is shown here.

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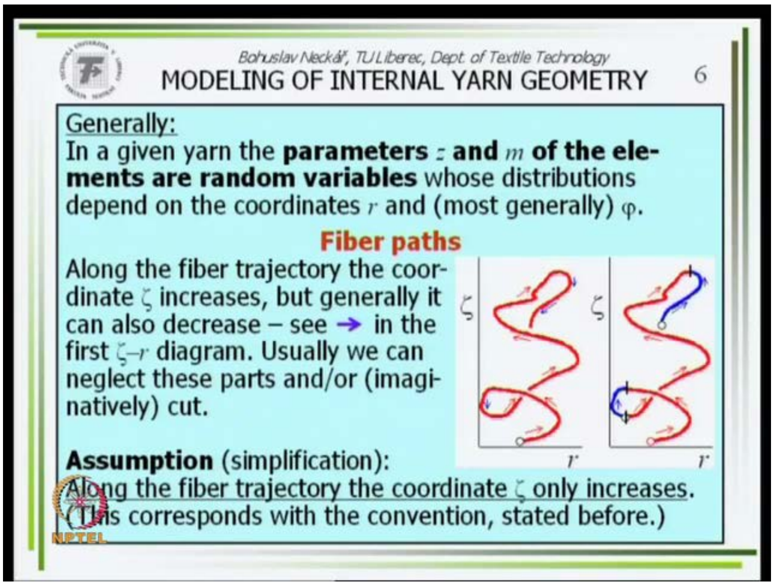

I say that we assume that all that that zeta coordinates must every times increase in our, this is the graph reduce of fiber for the points and zeta direction.

It is possible that the, generally that the fiber is can go also back some loop inside of the yarn. So, how to explain it when we say that we want zeta, where you have zeta increasing coordinate, it is easy, let us imagine that we divide our fiber so that we obtain this part, this blue part, this red part and that this blue part to some segments and on each such segments, we can imagine that the zeta coordinate is increasing so that that our assumption about the increasing value of coordinate zeta is possible to accept also for generally for each fibers.

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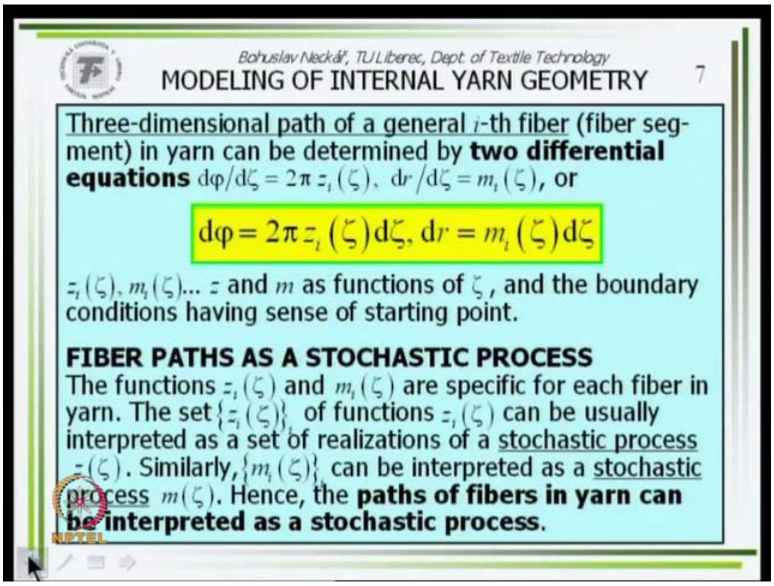

Free dimensional path of a general i th fiber, some fiber; one fiber in yarns rapture, let us call it i th fiber, general i th fiber we determine by two differential equations, we because d phi by d zeta is 2 pi z by d z for i th fiber is changed from point to point, from the fiber from element to element of our i th fiber. So, this z i is some function of zeta from this, of this we obtain d phi is 2 pi z i zeta times d zeta.

Similarly, also the quantity m i is changed on the path of our i th fiber therefore, it must be a function of zeta. So, from that d r is the function m i zeta times d zeta, this couple of two formally very easy, but both are a differential equations, is it not? This couple of two differential equations, they find three dimensional path of a general i th fiber and now we have two possibilities; how to create a model of the yarn, the first is to interpret a fiber path as a stochastic process.

Let us imagine to set a fibers inside of yarn, exist a set of functions z i zeta over o i for first fiber, for second fiber, for third fiber and so on. Each such function is (()) order, it is individual to fiber. So, with a set of such functions can be usually interpreted as a set of realization of a stochastic process. And similarly, also the set of functions and i zeta over o i can be interpreted as a stochastic process and the path of fibers in yarn can be interpreted as a stochastic process, so that each fiber is a not is a bit larger random oriented, now then now sin 1 as the order.

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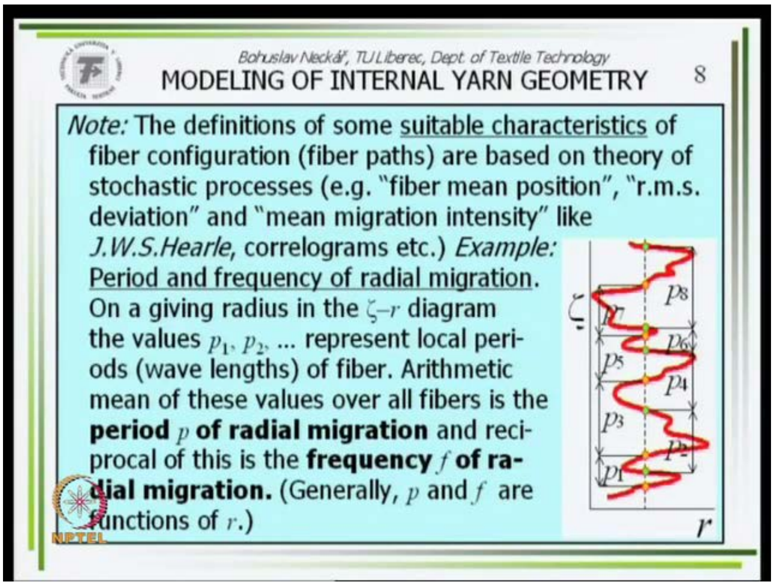

If it is so, then we need to use for evaluation of the yarns structure, the tools relate it to stochastic processes; it is for example, correlograms or maybe you heard about the parameters from professor Hero, fiber mean position deviation and mean migration in terms so on and so on. These all are tools how to evaluate the random process, also of (()) of migration and so on.

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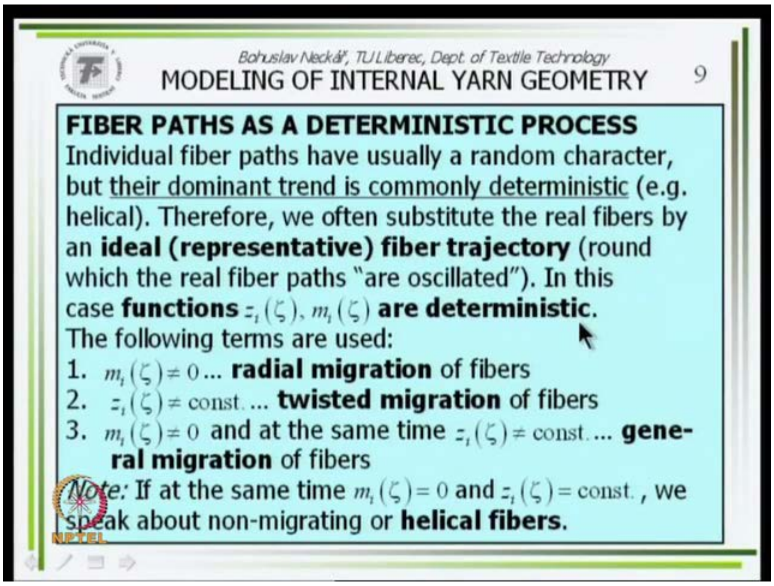

It is one way, one way now too easy in our lectures we were not to more discuss this direction; second, the second concept is to interpret a fiber fiber paths as a deterministic process. Individual fiber paths have usually a random character, but a dominant trend is commonly deterministic, for example, helical. Therefore, we often substitute the real fibers by an ideal, represent the fiber trajectory around which the real fiber paths are, we can say intuitively ask to write it, then this from on the, on our function z i zeta m i zeta and this this couple of this fibers must be deterministic, now probabilistic. If m i zeta is not equals 0 for all fibers, for all i, what is it m was d r 2 d zeta, if it is not equal 0, then the radials of fibers path is changed, is it not?

We speak about a radial migration of fibers, if zeta is different from constant z was d phi by d z. And now, increase of angle by zeta coordinates, if this value is not constant, then learn to twist, the twist of fibers elements is from point to point another so that we must speak about our twisted migration of fiber. And if both both (()), then we must speak about a general migration of fibers in yarn. In opposite, if each m i zeta is equal 0, if each zeta zeta is the same constant, we speak about non migrate non migrating or helical model.

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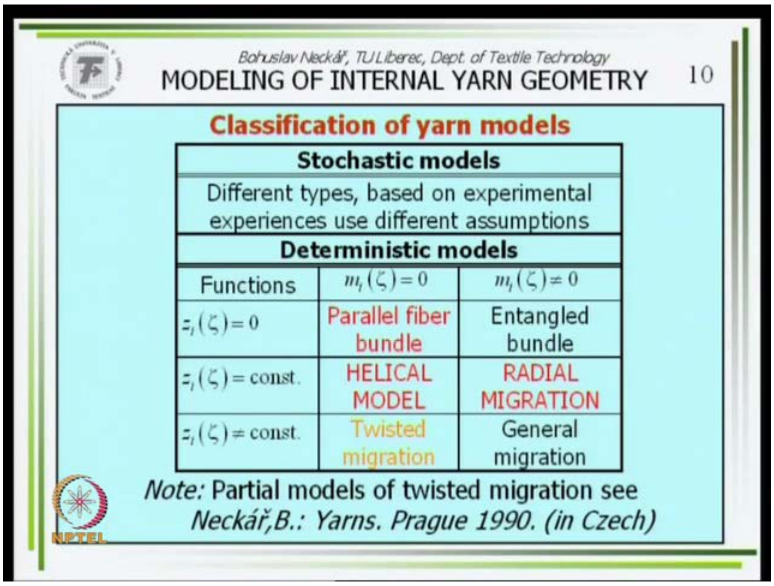

It brings us to this table which can in short to classify the lot of different types of models of yarn structure through some, in some system. For its group of this stochastic models, we do not want more to divide this possibility.

The second group is the deterministic models, and it base on the character of functions z i zeta and m i zeta. If both are equal, what it means reduce reduces stable for fiber, for each fiber d r is now different from 0 and d phi by d zeta is also 0. So, it is bundle of parallel fibers, parallel fiber bundle. If z function is equal 0, but m is different from 0 that fiber have not twist, but the fiber is on different rally then the (()).

If z is constant and m is 0, it is our non helical model; if z is constant and m is not 0, then it is the traditional radial migration model. For example, like if z if z is not constant and m is constant, then the fibers are lying on some imaginary cylinder every times. But on the cylinder, they are, do not create helices, then some other curve. We can speak about a twisted migration and the last position is if all is possible and this is the general migration.

We will speak, we will in more details are the speak about this free, about this is not necessary more speak bundle of parallel fibers was in our earlier lecture so that to important for helical model utmost, and when also the radial migration.

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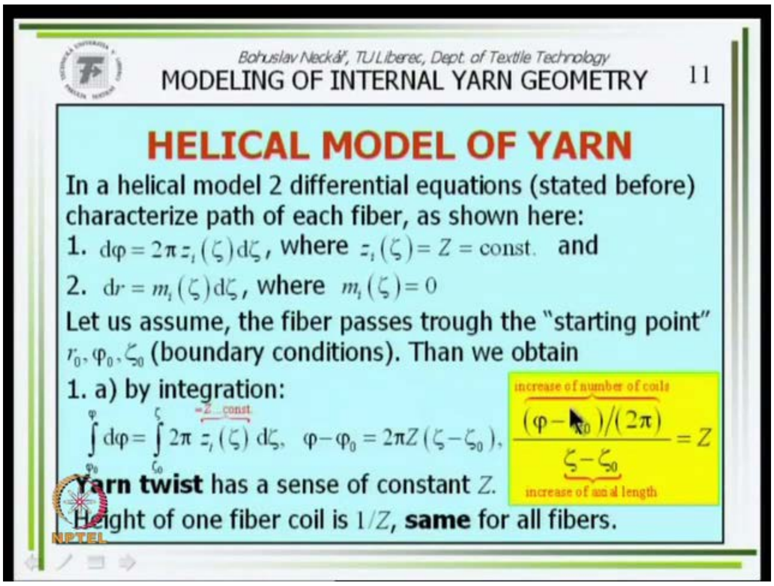

So, it was an slow introduction to how to general, in general to divide different types of models of yarns structure. Now, let us speak about the helical model of the yarn, we said the helical, each model is given by two differential equations; this is the first and this is the second. It is the same as first, in helical model, this functions z i zeta is constant for each fibers and each point, each segments, each elements on each fiber. This constant have named Z, it is constant and I have to some one or two slides to show that it is the twist of the yarn and the second and the second equation, this function m m zeta m i zeta is equal 0.

What we obtain, what we obtain from this first equation, when this function is equal Z. Let us integrate this equation so that integral over phi from some starting point coordinates phi 0 zeta 0 r 0 from phi 0 to phi, is equal to the integral from zeta 0 to zeta on right hand side that is function z is constant. Therefore, we obtain this here and then these here, what it there, what is it zeta minus zeta 0 is an increase of zeta coordinates, actual coordinated in the yarn, 5 5 minus 5 is 0, is increase of angle phi divided by 2 pi, 2 pi is one times round. So, it is number of coils, is it not? Number of coils number of coils by corresponding coils is definition of twist, is it so? So, the such constant Z have the sense as a yarn twist so that also the height height per one coil must be 1 by Z evidently.

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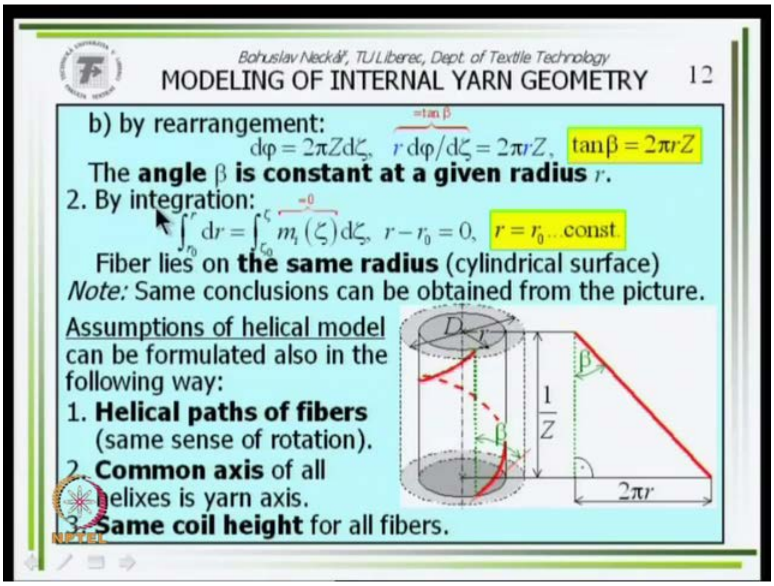

From the same equation, we can write also if this is what it my Z is constant, is not the function as a general it is constant after lot application if dividing by r, we obtain this here now this is tangents beta, so tangents beta is 2 pi r Z.

From the second equation, from the second differential equation after integrating from r 0 to r here from zeta 0 to zeta from starting point to some general point, because this is 0 then r minus r 0 must be equal 0. So, r is equal r 0, what it means, the fiber is lying on the same cylinder by constant, by the given fiber is lying on the cylinder is constant reduced of course, r is it each r is r 0.

There are this two this two equations are dizic equations for helical model that we can also to interpret this helical model very easy, we can say this is some scheme of the yarn according helical model on the general cylinder diameter r is lying one red fiber. Once this length is 1 by z, I know of between the tangent to our fiber and vertical direction is along a beta, is it not? And after unwiring of this surface of such imaginer cylinder, we obtain this, from this is also possible to calculate the tangents beta is 2 pi r by 1 by Z, so 2 pi r times Z.

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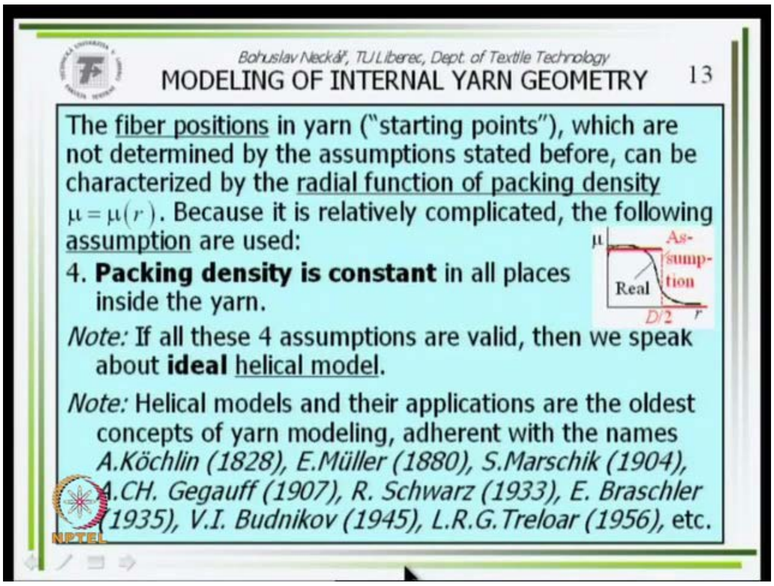

The, so we called differential of differential equation every times, some some starting point or something. So, our starting points from our fibers based on packing density, if some fiber is starting from some radius then this fiber is lying on this radius and have path to the packing density on this radius, how is the packing density, so much fibrous can be on this or that radius r.

Generally, the packing density is a function of radius, in a based on experiment to experience is, this is radius and this is packing density, then the real curve is something like this black curve on our small picture.

But it is difficult to obtain such function, we can obtain our experimentally based on now to easy experimental process or make some mechanical model of the yarn that is one of the easy. Therefore, we often use some, use a fourth assumption for simplification of our problems, we assume in that fourth packing density in all places inside of the yarn is constant like this red line (()).

Also, this assumption is valid, and then we can speak about the ideal helical model. Helical model is reserved this assumption ideal, in ideal helical model we accept also this assumption. The helical models and ideal helical models are in lot of publications, some of known authors, I mention here, from Kochlin I think 1828 to Treloar which which completed this helical model in a relatively complete concept well.

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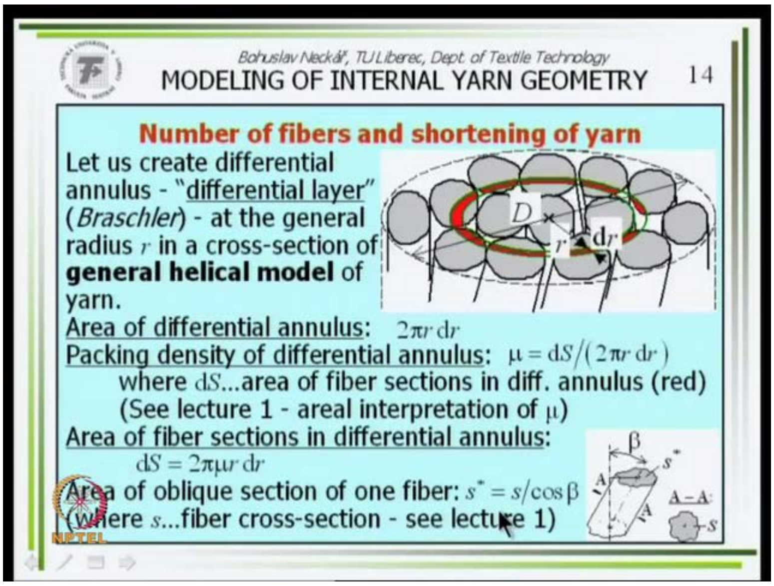

There are this is the, I can say definition of the, or description what is it a helical model. In our helical model, let us find the number of fibers and then also the shortening of the yarn due to twist. The first theme will be number of fibers in cross section of yarn, ideal yarn, yarn which corresponds to our helical model.

So, let us imagine some cross section of the yarn, the cross section of the yarn is over here, the grey islands here represents the section islands, islands of individual fibers. We create two circles, here two green circles here; the small one have the radius r and the higher one radius r plus d r plus differentially elemental increase of radius, so that we obtain a differential annulus, is it not?

Differential annulus is a how is the thought to area of this differential annulus its evident because its differential annulus its 2 pi r times thickness, when you cut it this you obtain something some long and its intuitively clear 2 pi r times d r is the r is inside of all annulus the area of fibers it is here as a red area

Now, how area of such annulus with a area of fiber section there are also some parts here isn't it well, but you know the how what is it a packing density so that the area of fibers d S must be 2 pi r d r times mu times packing density well and see here it also an order and for us known relation that the section of area there one fiber which is lying with z axis on the radius r of course, is a cross section area by cos inverse of this angle beta.

Now, is the question how is the number of fibers in differential annulus to this to solve this problem lets imagine one abstract situation lets imagine that I am nobody of us is here in this room and I am standing here one foot is inside of room the second my foot is outside of this room how many people is here may be one half because I am here only one half you understand it well.

You can see that number of fibers need not be only 1 2 3 4 natural number then it can be a real number for example, 1 half or something. So, second how many people is here we can say calculate 1 2 3 4 5, but I have another idea, let us go all together to some writing machine for tracks and. So, on and you will find a, our common right and then idea why this, is it possible this way logically is it possible?

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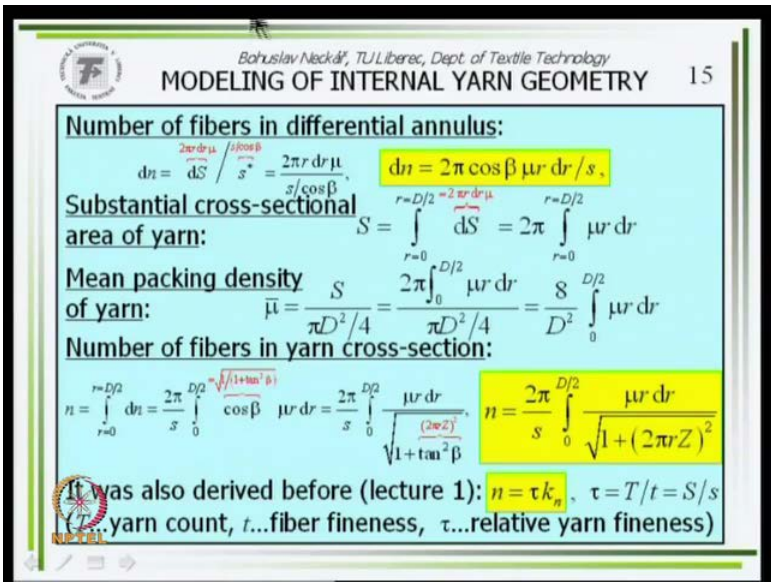

We may use this style of thinking fiber area in differential annulus is d S and area pair one fiber is a star a star so that d S by S star must be number of fibrous in all differential annulus, using equations derive we obtain for d and this, formula this expression. And now, how is the substantial cross sectional area of yarn substantial cross sectional area of yarn is this area in earlier, but integral of this, sum of this over all over all possible annulus, they are from r equal 0 to on the periphery r equal D by 2, about half of an yarn diameter so that it is this here; using s we create it, we obtain this equation. Mean packing density of the yarn, it is total substance cross section of the yarn by total area of cross section of the yarn pi D squared by 4, after we arranging we obtain this here, number of fibers in yarn cross section. Now it is integral from this the yarn of number pair one annulus from r equals 0 to r equal D by 2. After using, we use here some geometrical formula which we now 1 by cosine square equal to 1 plus tangents square and after slowly arranging we obtained this here, because tangents is 2 pi r Z squared, we said tangents beta.

Well, but it was also derived in our lecture one, starting lecture to this lecture that number of fibers is tau times k n where tau relative to 1 is the ratio yarn yarn count more precisely for example, in tax when your density a by fiber linear density fiber (()).

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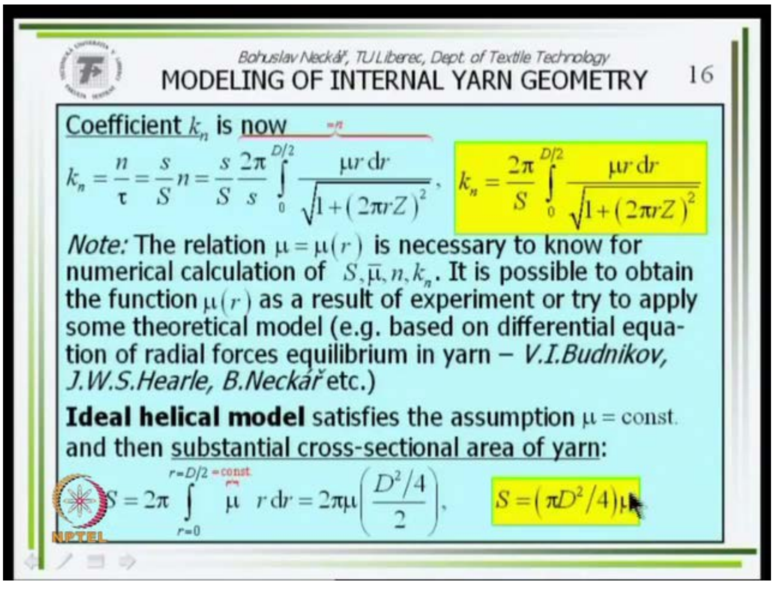

Coefficient k n, which is in this in this expression can be derive from this s n by tau, tau was 1 also cut it as by s times n, we now so that we obtain this expression for k n, this expression are valid for a helical model. It means therefore, I have mu insight of integrals because mu can be a function of radius.

I spoke about a difficult thing is done, so that let me now to make this very rearrange of our equations for the case of ideal helical model, it is what it is the model on which the packing density in all places inside out yarn is constant. Therefore, mu is possible to give B for integral as a constant.

How this, how is substantial cross section of area mu is going before, so we obtain this area formalize this here, now pi D square by 4 times mu, corresponds to our knowledge.

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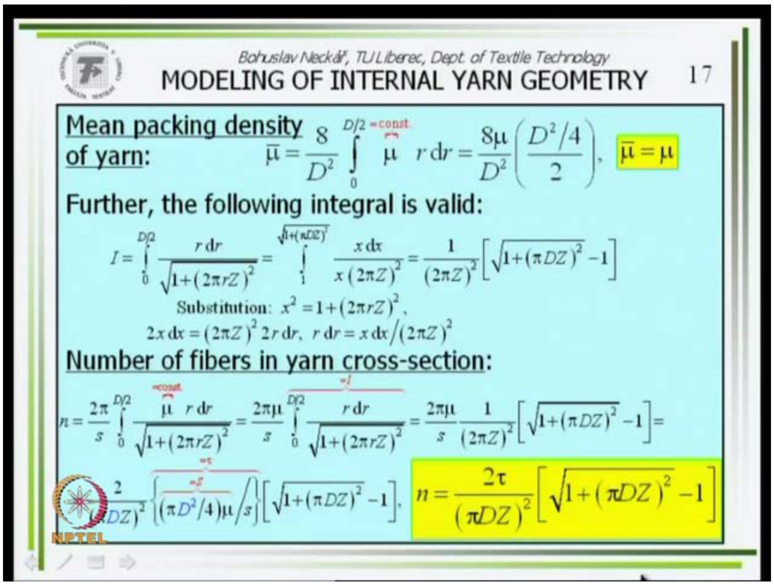

Mean packing density, we derive this equation times mu sorry, mu is constant. So, before integral for r d r its trivial so that you obtain final in final position, mu bar equal mu, evident if mu is constant then each all mu must be there for all yarn. For future, we are arranging we need to solve one integral, this integral is shown here, is shown also to why how to obtain it is not too difficult to use in such such substitution as we obtain this result. So, I think do not want to comment integration.

A number of fibers in yarn cross section, for this we had this expression. Now, mu is constant can go before integral we obtain this expression, but this is our early integral this one on the place of this, we can give this expression in a in a brackets and after. So, rearranging, where we use here is we multiply and divide by D square, then we understand that this is s capital S so that this is tau, you know it is earlier equations to the r equations, we obtain number of fibers in this form.

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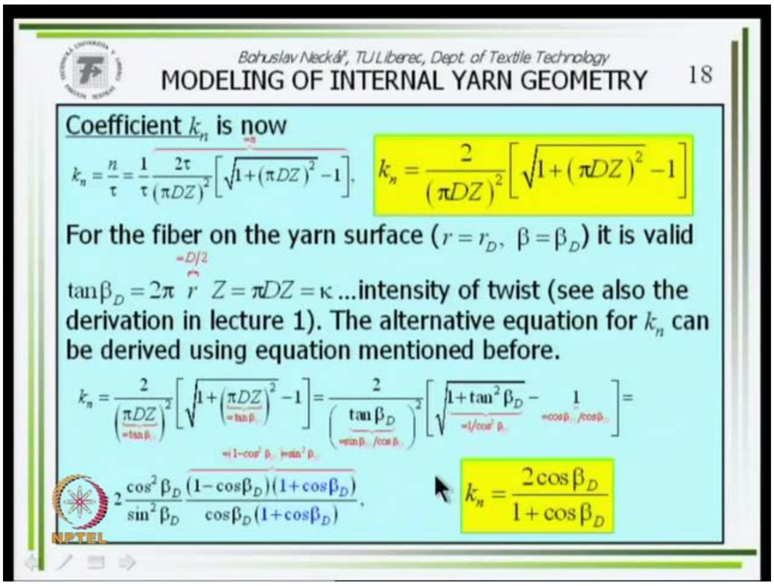

And k n k n because it is n by tau, k n is given by this equation. We can also we can also use another rearranging this is, this expression is identical is this expression, but we know for us along the pi D Z is tangents beta D of tangents of peripheral angle beta on the yarn.

So, tangents beta D, after rearranging multiply and divide by 1 plus cos beta D. Here, we obtain finally k n in this form, all rearranging pure trivial mathematical, rearranging which you know know from university, from high school, no difficultly. How the, how is k n how is k n graphically? How is k n when we use this last formula for k n?

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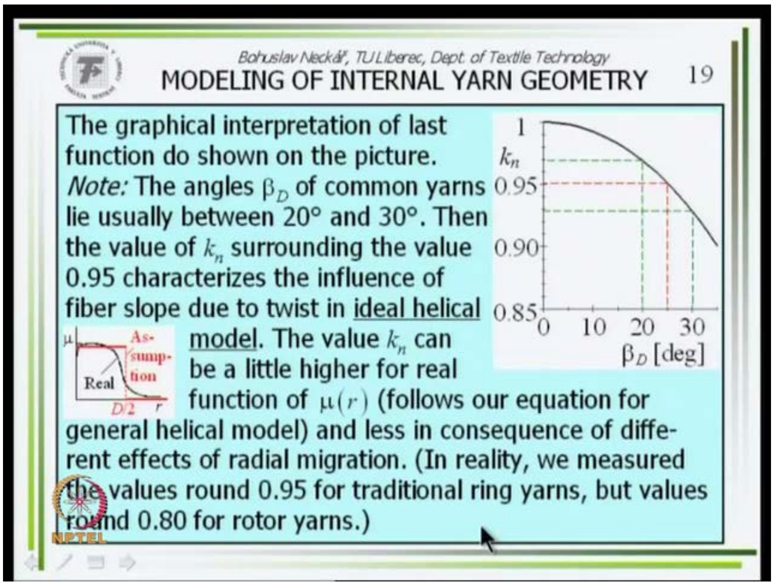

In or express it in graphical form, we obtain such graph here, this is axis of peripheral angle of beta D, and this is axis of value of k n, we obtain this thick curve, in the textile is usually usually we twist the yarn so that the peripheral angle is something between 20 and 30 degrees.

So, let us imagine average average value 25 degree, this is this red dotted line to this angle corresponds the coefficient 0.95. When you experimentally measured coefficient k n, are evaluated based on cross sectional microscopic triplets, cross sections of yarns, then really we obtain the value 0 point, around 0.95 for yarns, ring spun yarns.

Now for rotor yarns, for rotor yarns we obtain much more smaller value, because the angles of fibers are not in dominant effect, create it to twist that important is also the intensive unparallelity of ribbon in rotor, and so called birch fibers on the periphery of rotor yarn. You know this term, you know this problem so that in rotor yarn the coefficient k n is smaller.

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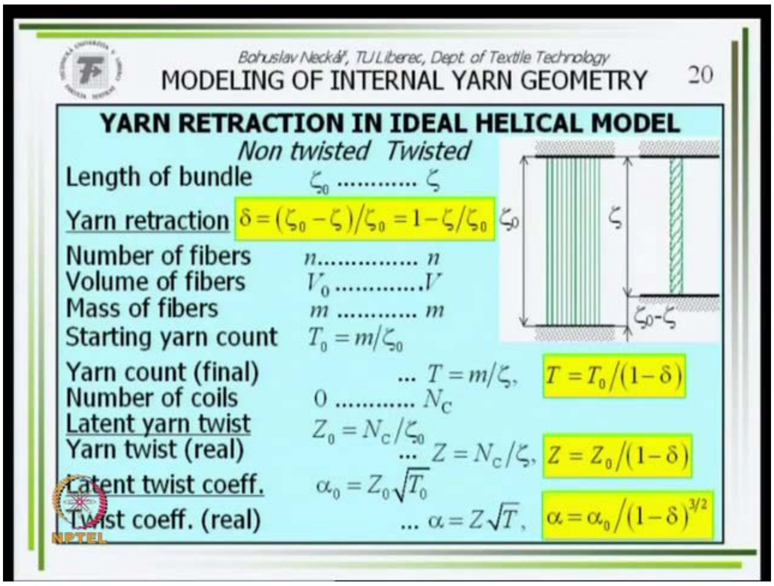

Now this is, this is all for coefficient k n, the theoretical value of this coefficient based on ideal helical model. In reality, it can be a bit larger why, because the structure is not perfectly ideal helical model. Well, to the problem of number of fibers and coefficient k n, rewrite it also the theme about the yarn retraction. You have found an individual experience, institutive experience, when you twist it something, some bundle of something may be fibers, may be also a bundle of yarns or something. So, then when you twist it this bundle, the bundle is shorter and shorter and shorter and shorter and shorter, is it not? It is not possible more twist inside; it means by twisting the fiber bundle is shorter and shorter.

Let us imagine some bundle of parallel fibers which is here, runs at zeta 0 after twisting the lengths of resulting yarn is zeta smaller than zeta 0, the zeta 0 minus zeta, it is the difference of ones between non twisted and twisted form of our bundle. This column here represents non twisted structure; this structure the second column here represents this structure. So, once a bundle non twisted bundle is zeta 0 twisted is zeta.

Yarn retraction, we define as ratio zeta 0 minus zeta, these lengths, the starting lengths zeta 0. So, we can write it 1 minus zeta by zeta 0, number of fibers by twisting is not changed, so here is n and here is also n, volume of fibers generally, we can say that it can be different volume of fibers can be different in this bundle, and in this bundle. Therefore, starting value is 0, final value is V, mass of fibers must be same, non twisted as well as in twisted structure, starting count starting count count of parallel fiber bundle is mass by lengths; mass is m, lengths is zeta 0. So, starting yarn count T 0 is m by zeta 0, is it not? After twisting the yarn count, I mean linear density that is a, the m count is now m by another lengths, lengths zeta.

The ratio between T 0 and T, there is under definition of yarn retraction is this one, number of coils in parallel fiber bundle is 0, here number of coils is N c. And we can construct two, or we were construct two quantities for twist; the first, which is here will be latent yarn twist, it is number of coils per lengths per lengths of starting non twisted structure clear.

How many coils I were give to 1 meter of non twisted parallel fiber bundle, do you understand this term? In opposite to start yarn twist which is which is the same number of coils, but by length of yarn how many coils is in on 1 meter of yarn, final yarn, so that it is N c by zeta between latent yarn twist, zeta 0 z 0 and yarn twist is Z, related to this expression is valid.

Latent twist coefficient, we can also construct the latent twist confident which is Z 0 times square root of T 0, latent twist and starting yarn count in opposite to this coefficient real which is Z times square root of T clear. So, this latent quantity related to starting lengths, now to final lengths, there is a difference here. It is starting quantities, now in a set of our of our slides are free variations of model for yarn retraction. We will comment only the second one, we will jump the first one and the third one, it is not too necessary to sign it here, when somebody will study deeper the problem of yarn retraction, he can use my my slides and immediately for this, from this slides to understand also the first and the second variation.

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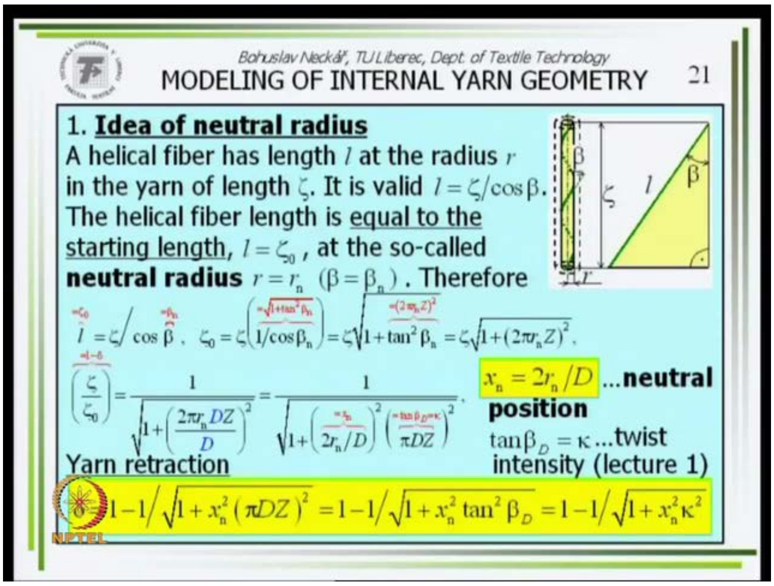

So, the first variation it is idea of neutral radius according a book, thus at we will not to to to do it.

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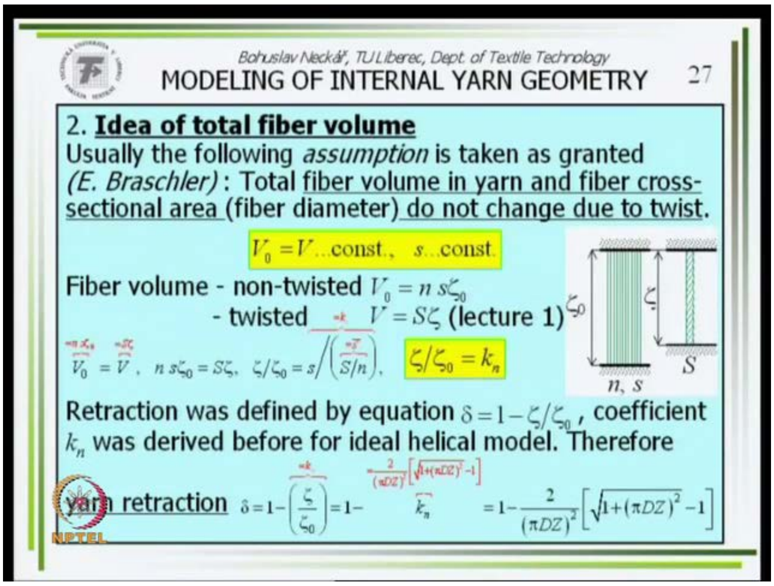

We will start this variation 2, the second variant of model idea of total fiber volume which is, which was created from Brasher around 1935. It was some special set of textile on the university in Switzerland, but how is this theoretical concept; I want to show you in the next lecture. So, in the moment, thank you for your attention.