Orientation of Fibers

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Lecture No. # 06

Pores among Fibers

Let us continue our theme about a yarn retraction,based on the ideal helical model.We spoke about the starting position offiber bundle, parallel fiber bundle and the twisted yarn.In idea of Braschler,we assume that the total fiber volume in yarn and fiber cross-sectional area, means fiber diameter do not change due to twist.

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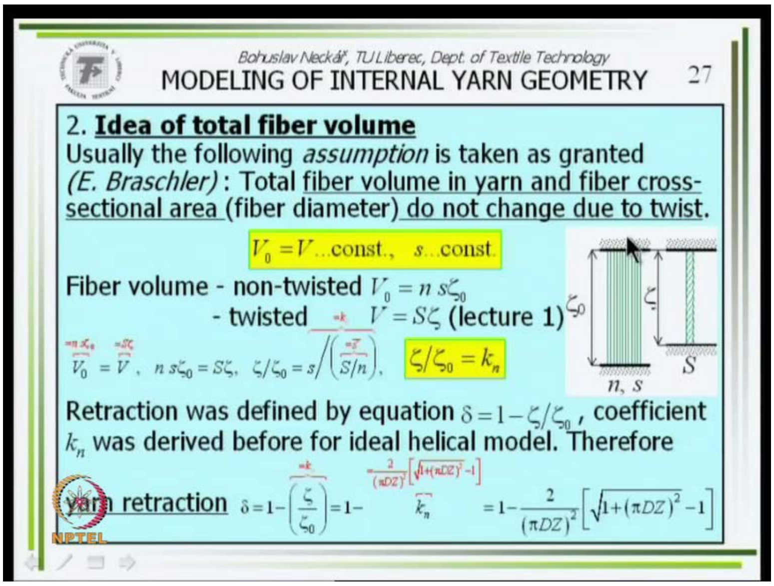

May be, that on the periphery fiberis a little longer than thevolume of such fiber is a little, may be small, may be in the center is a little pressed. So, it can be opposite, but these differences are so small thatBraschler mentioned that,this assumption is possible to use.

When we use this, we can write V0 is equal V,so that it is a constant yeah and also fiber cross section S is constant, now change through the process of twisting.Fiber volumein non-twisted V0 is n times because in bundle is infibers, n times S fiber cross section times lengths.Length is zeta 0fiber cross section is S times zeta 0is volume of parallelfiber times, numbers of fibers.In twisted form, the volume offibers, all fibersmaterial in this portion is S times zeta.S is the substance cross section of our yarn times, final zeta because V equal V0 equal.We using thisformula, we obtain this here.Then, we obtain zeta by zeta 0, which is S bySby nand capital S by n substance cross section by n.It is in the bean area,sectional area pairone fiber s star bar and it was definition ofK n.It was K n. So, zeta by zeta 0 is equal to K n.How it is now in ideal helical model with yarn retraction?Yarn retraction deltais 1 minus zeta by zeta 0.It was starting slight to retraction problem, but this is K n,soit is 1 minus K n.

We know K n fromearlier analysis of ideal helical model number of fibers in cross section yarn corresponding to ideal helical model.Isn't it? So, we can use our equation derived on theplace of K n and we obtained that delta ishere.A lot is only rearranging.Let us rearrange this equation using this way to this equation rearranging only common denominator andso on andso forth.

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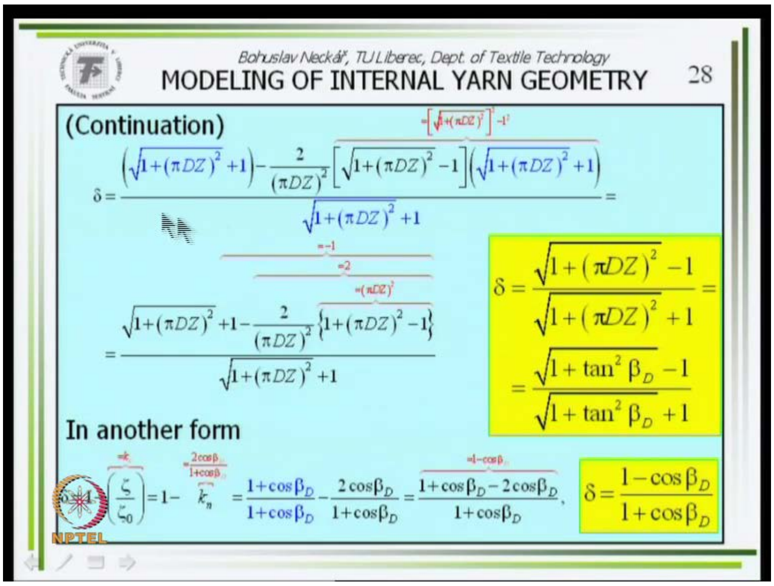

Well,I think youcan make it own self.No special routine is here.Well,so on the end we obtain delta squareroot from one plus pi DZ square minus 1by 1 plus pi DZ square plus 1 or because pi DZ is tangents beta D.We can write it also in that form.

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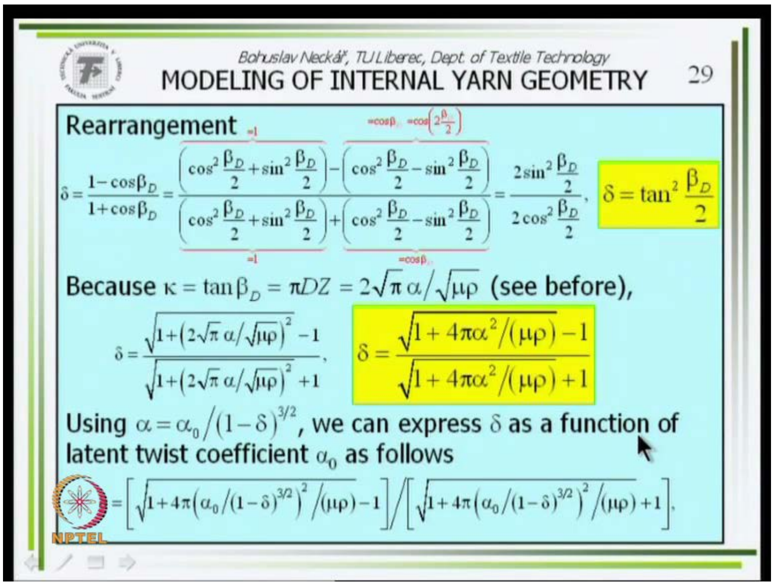

Secondexpression for K n, second version of rearranging of K n for helical model was this here. K nas a function of,it is this here. So, that after rearranging, we can write the same delta also in such form.The fact isrearranging according Braschler, when delta is this here, we write it here.Then, after rearranging, we obtain delta is equal tangent square beta D by 2. I think it is this.This is the easiest formally easiest.All isidentical all is the same it is only all different shapes ofinterpretation.

Well,now we wantalso to know this delta yarn retraction as a function of this coefficient alpha.We know thatintensity of this intensity is tangents beta BD is pi DZ andit was derived that it is also this here.Using this, we obtain for delta on the place oftangentsbeta D here or pi DZ here.

We obtain this equation onso thatafter small rearranging this.This may be fourth or fifth identicalequation for yarn retraction. More interesting will be toexpress yarn retraction as a function of latent twist coefficient related to starting lengths where Z twist and the count,I relate it to starting values.Let us remember it.

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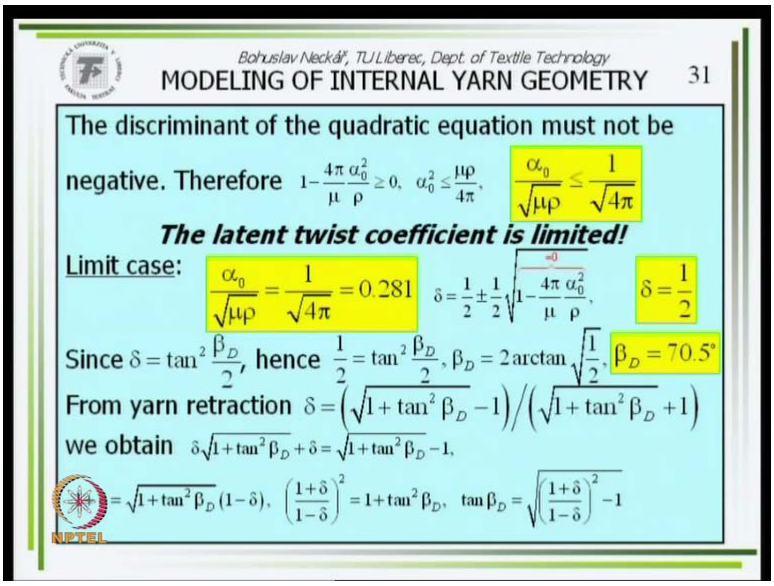

We know thatit was shownlot pages back.It was shown that alpha is alpha 0 by 1 minus delta power3 by 2. Sorry, I was here. So, we use this.We use this equation on the place of alpha here.We obtain this equation.The same is the here,this equation.The problem is that, now we have delta on left hand side as earlier, but also here and here. So, we must rearrange; its only mathematical operation.

We must rearrange this equation because to obtain the delta explicitly, it means in the formbeta delta is equals some function on righthand sidemust not be delta.It is possible the way it is shown here.What isyou canquietly home step by step to see it.It is goodin first step.For first step, call this ratio as a, then delta is this here rearranging.Then, I make square of left hand side as well as right hand siderearranging here.Yes, this,this, this and here. I give back to the helping quantity a,thisratio here.After rearranging, we obtain such resulting equation.

It ispermanently same delta, but now the delta is functionis explained as afunction oflatent twist coefficient alpha 0.See itfrom a one point of view.It isinteresting.Why?Under square root mustnot be a(()) value.Isn't it? So, this value must be higher equal 0.

So, that this value must be higher equal 0 because under square root is not possible.We speak about the realvalues, notcomplex values.Well, it must be this here. So, Z is here. So, that this here is very important.It is very important because it saymu is some quantity from 0 to 1.Some think it is rho specificmassdensity of fibers.It is some value,some constant.It shown here that this alpha 0 by this constant square root must be smaller than some parameter 1 by square root of 4 pi.

So, that alpha 0 must have some border, must be smaller than some border value.Isn't it?It is not possible to givenumber of horse to run unit for starting such a starting fiber bundle.How you want?It exist some momentin which is not possible more to give the coil inside.When you prove it, then you obtain something.Other, I will show you later. Do understand what represent this equation here?

Let uscalculate the parameters by maximum possible twist of the yarn.The maximum twist is, if this equivalency here is valid.So, that its valid alpha 0 by square root of mu, rho is equal 1 by square root 4 times pi, by the way 0.28 and something.Using this to the equation forretraction yarn retraction on this place, we obtain 0.We obtaindelta is 1 half50 percentage of this theoretical maximum retraction of the yarn.

Since, delta equal tangent beta square by 2, hence 1half13peripheral angle in this point of maximum twist beta D is 70 degree. Now,using equations derived, we obtain also tangents beta D, which is2 times square root of 2, roughly 2.8.So, otherequations or results, the border results for maximum twist.As a root fromquadratic equation which was here, we obtain plus minus.The question is what is real?Automatically, we have two roots.Physically, only one is real, one is not.The second is not possiblein reality to obtain.It is plus or minus C.

Plussomebody said plus.Plus do notbe possible.Yes.You are right, you are right because let us imaginethe structure without twist, then alpha 0 is equal 0 square root, this 0square root from1 is 1. So, it is 1 half plus minus 1 half.You can obtain 0or 1.Whenyou have parallel fiber bundlewithout twist, you cannot have the value of retraction 1to 0.You must havethe valueto 0 once. I mean, you must be having this value; this retraction must be equal to 0.Therefore, this minus is real,real symbol here.

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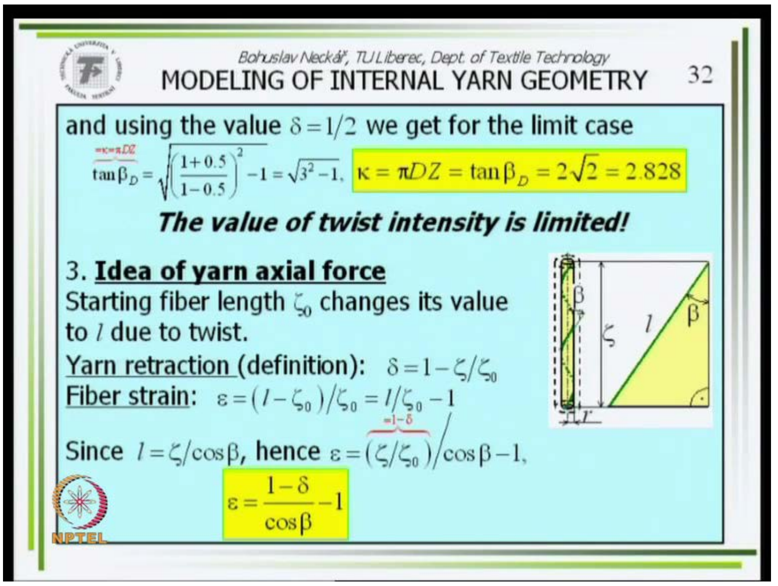

The third idea basedon axial forces is also now important for us.This version according toBraschler is enough good.Yes and pleaseany minutes for change of yarn 2.Let us continue our theme about a yarn retraction.

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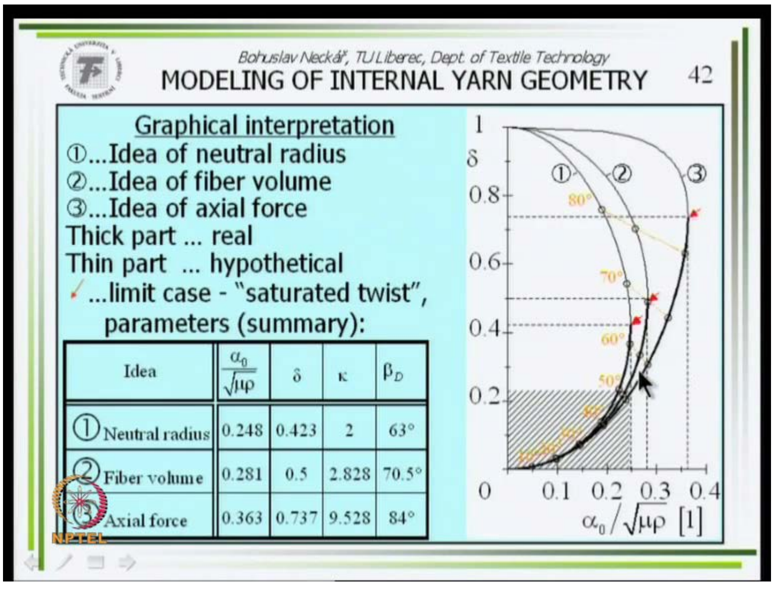

It is asummary for all three ideas.We analyze only the idea number 2, according to Braschler and we obtained this expression foryarn retraction.How it is graphically interpreted?Here is alpha 0 latent twist co-efficient by square root of mu rho in relation to yarn retraction delta.Ourcurve is the curve number 2.It is here.What we obtainedby first course?Now,significant changein lengths, then the speed with increasing,with shortening increase and increase and here is the maximum value. Here is the maximum possible value, the border value and now more is possible.

Theoretically, when we use minus than plus in our equation, you obtain the second thin curve.It is only for completeness.The real is this thin line.Real is the thick line here.Usually, we havethe chance to go to this theoretically derived end point andpractical maximum twistiscomingearlier, may be in this moment.How is this graph, this part of our graph is shown here?

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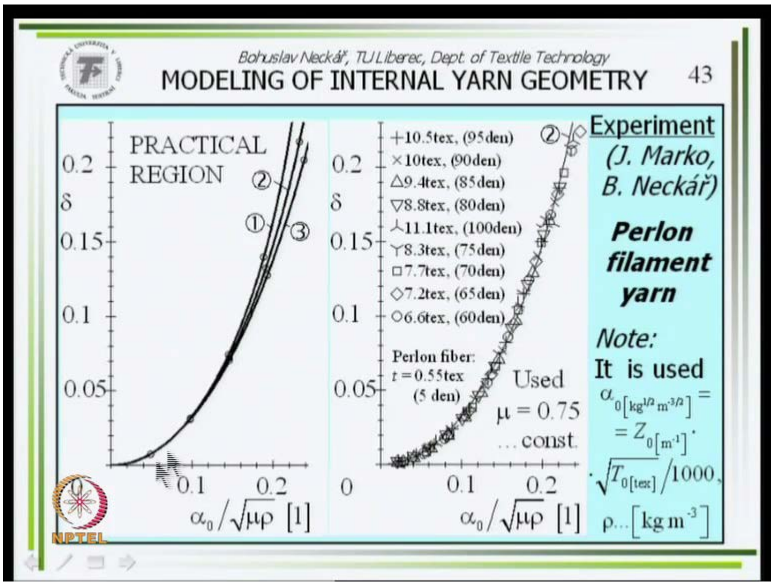

The second curve is the middle curve among thesethree. So, you seethatall three hypotheses are very similar.Is it well? Is not well?In relation to the reality because we applyideal helical model,evidentlyto this model nearest structure is the structure of twistedfilament yarn.Lot years ago, we measured together with my older colleague,Professor Marco,we measuredlot offilament yarnson a special instrument which we created and we measured the yarn retraction.

You can see that alldifferent counts, all different twists together are lying on only onecurve.Using peck intensity 0.75 in our equation,thiscontinual line show the curve in our, according our theoretical model.The curve Perkinis excellent.It is the best in my life.The comparison betweenexperimental research and theoretical curve,so perfect it is, but around 0.25,around this point, stop the possibility in practice in laboratory, stop the possibility totest filament yarn wasnot more possible.In relation to the theoretical model, it isroughly here in this moment,now here.

From point of view ofyarn retraction, the difference is high, but from point of view of alpha 0, the difference is not too high.The question is, why coming thisborder situation earlier?It is evident, because now our expressions are related to the very complicated structure of the real yarn.Real yarn is not axially perfectly symmetric.Therefore, one bending mechanism also is coming,when the peripheral fibers press the central andso on because the yarn is not absolutely perfect in symmetric axial symmetric.Therefore, this critical moment is coming a little earlier than our results from theory, but it is coming. It is coming.

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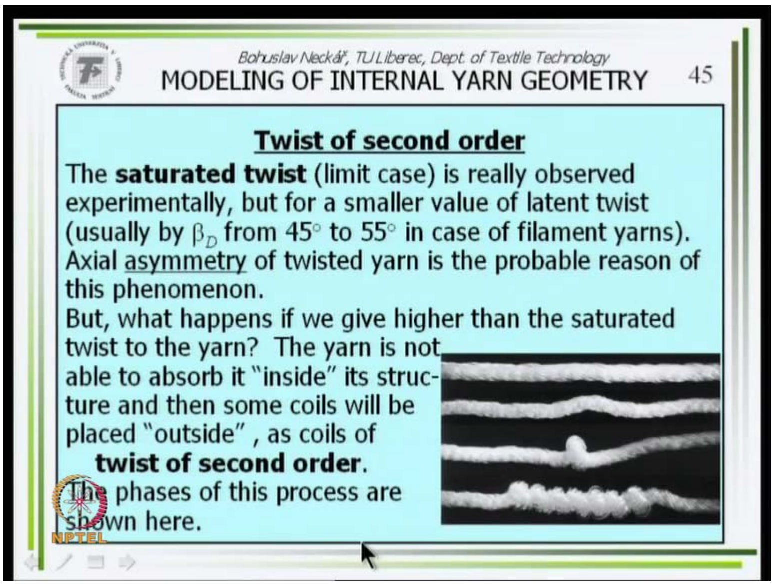

What is twisted filament yarn?When wegive more coils inside this one, the maximum of possible twist, we call it as a saturated twist,that terminology saturated twist.Then,it is not possible more to give the twist to our filament yarn.I start it first,and then, more and morehelicosof twist of second order.It is important, be under this border in the technology of texturising because when you have in machine, you use too high twist.Then, such moments like this or this can break your yarn and destroy this.The process is not continual.

More difficult for filament yarn, this model based onideal helical model can be applied without problem,nevertheless forstaple yarn, it brink complicate.It is more complicated.Why?Staple yarns are from staple fibers,so that the ends,fiber ends and somethingcan slip,especially on the periphery of the yarn by twisting.Isn't it?

Therefore,not so high force for retraction as by filament yarn,therefore the yarn retraction in staple yarn is usually smaller.Not too much, but a little smaller than the retraction by filament yarn.It is difficult to measure it, very difficult, retraction by staple yarn.Now,some experimental results exist.Therefore, I can recommend you.When you have not something better, then I can recommend you, use it.To use ourequation for,the shape of our equation for yarn retraction but on the place of real yarn diameter D gives some more defined diameter D delta.Some smaller, a little smaller value, then real yarn diameter D andthis D delta is related to D.Using this pure empirical form, it is pure empiric expression based on some set of experimental results.

So, that D yarn diameter, staple yarn diameter times this expression, where mu specking density and S is substantial cross section ofthe yarn.It is better than nothing,built now totoo good, but it is difficult to say what is good and what is bad because it is very difficult to measure yarn retraction by yarn, experimentally.When we are speaking about different applications of helical model, let us mention also yarn stress strain relation model, according a generalize theory of Gegauff and others.By the way,Gegauff was one of us and of 19th century, second half of 19th century.

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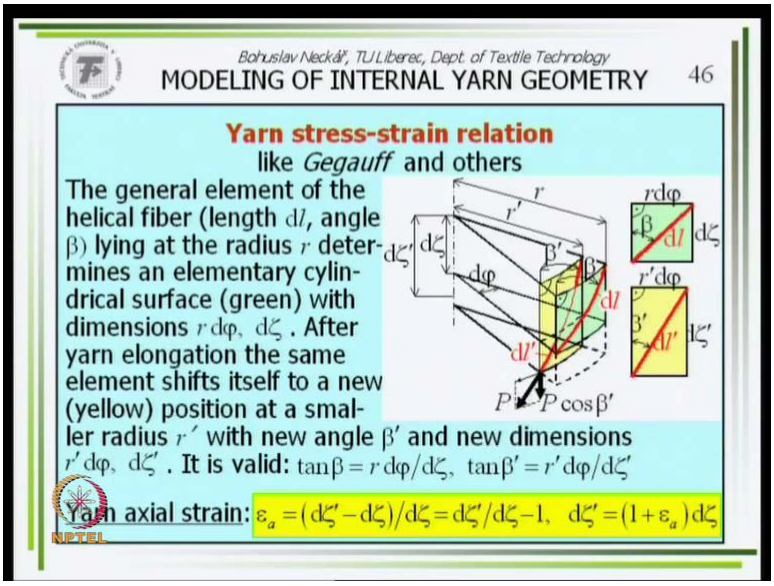

In the picture, we have two fiber elements,2 red fiber elements.1 red fiber element is laying on the green surfaces of some cylinder and elemental part of this cylinder surface is shown.This height is D Zeta. This ends must bethis is radius r. So, this ends is r times D Zeta.Well, the angle, the element of angle is Dphi.Such element, this green element we said red fiber is also here. So, this length is RD phi.This isD zeta and this angle, this angle,this angle beta our known angle beta.

Well, now,let ustake the yarn and load this yarn in some breaking machine or so.Then, youobtain a new position, a new geometry of our fiber element.By theway,do you know how it isin English?((FL)).You know it.Did you use as ayes and when you use it make also no.

Whatdid you sawthat the thickness of this(())is decreasing, some contraction.Isn't it?The same effect is in the yarn by itsholding,so that our element which started on the radius R is changed his position to another smaller radius r dash.Of course, because we elongatethe yarn, balance the zeta startinglength D Zeta which also elongate to new length D Zeta dash.Sorry,it is my mistake here.This arrow shall be here.It is my mistake in this in this picture.Sorry.Nobody is perfect.Well,to mu lengths d dash.So, the lengths of the fiber DL is now longer, have the lengths DL dashand the angle beta is changed, B may be smaller, mu angle is B dash.Isn't it?

Let us, now define some relative quantities.Epsilon L,epsilon in axial direction, epsilon,thereforea.It is D Zeta dash minus the zeta by D Zeta,eachstrain axial strain. So, that it is this one or D Zeta dash is 1 plus epsilon a times D Zeta.Well,it is a repetition fromlastpicture.

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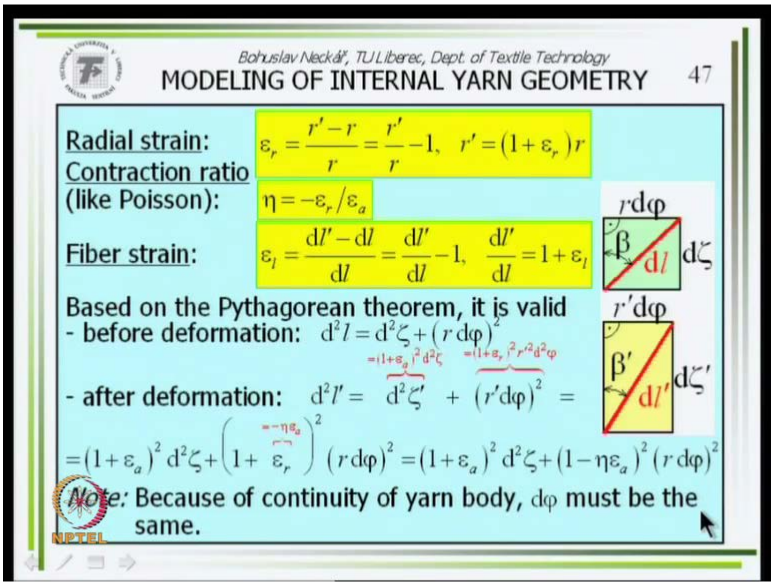

Radial strain is final radius minus starting radius by starting radius. So, it is this one. So, it is this here.Let usthink thatepsilon r must be major difference because r dash is smaller than starting r.The chewing gum effectcontraction,isn't it?

Contractionratio you know, it may be under thePoisson contraction ratio. It is defined as a minus epsilon r by epsilon a. So, it is positive value because minus epsilon r is positive value.Well, fiber strain epsilon L is DL dashminus DL by D L evidently. So, it is this one. DL dash by DL is 1 plus epsilon L.

Now, based on PythagoreanTheorem,we can write D square L dash is DL dash. So, square of this is equal D Zetadash square D square zeta dash this length square plus this length square.It is this here. Nevertheless,according the reality quantities which we derived, we can write on the place of this here, such expression and here, this expression and after rearranging using also the quantity contraction ratio, we obtainD square L dash in such form, rearranging trivial rearranging.

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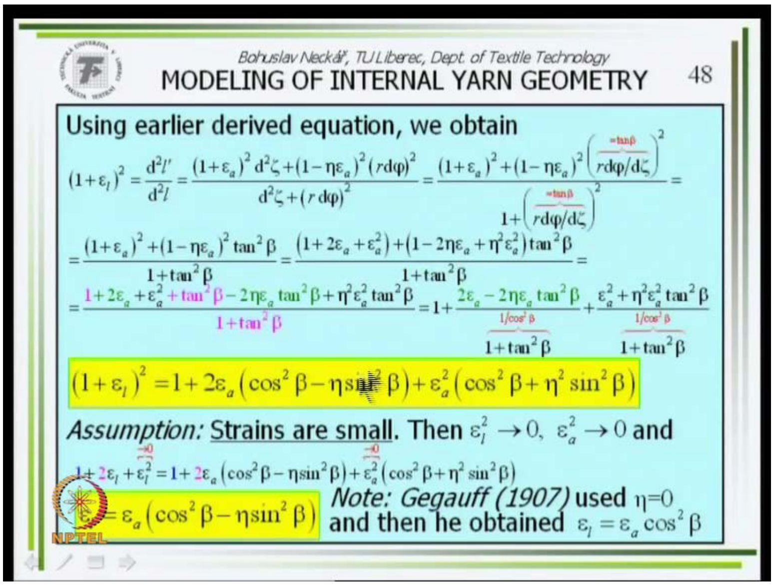

Now, all geometric quantities arechanged by yarn elongation, but the angle D phi must stay the same.Why?We have not only one element, but lot of elements around and sum of all D phi around our yarn is cross section give two phi the 160 degree before as well as after elongation.If D phi for example, will be smaller than by elongation on the yarn, it will be some whole some.Somethingso, thereforeD phimustnot be changed.

Well, now your D square, it is here.Fromthis triangle, the Pythagorean Theorem is easier is D square L is D square zeta plus R D phi square from this triangle. So, it is d square L and D square L dash.The ratio D square L dash by D square L is equal to 1 plus epsilon L.We said we used what?We derived this and this.We arranged this, we know what that for example, r d phi by D Zeta is tangents beta andso on andso on.

After re-arranging, we obtain this ratio wherethe green numbers have linear epsilon a,the black this and this, has epsilon r square andviolet are result epsilon a.Some of these violet are same than the denominator here. So, that we can write our expression which start 1 plus epsilon L square is equal tosuch form and because 1 plus tangent beta square is on B by square is here too. So, we obtain this expression.

In our lecture, we usually solve the easiest cases. So, let us imaginebecause be easier and sometimes, it is real.Inlot cases,it is possible toimagine that the formation, the elongation of the yarn is small.

If it is small, thenepsilon L square is very small and epsilon a,sorry epsilon a square is also very small.It is limited to 0,so that 1plus 2 time epsilon l plus epsilon l square.We canwrite roughly 0 on the place of you have1 plus 2 times epsilon l for epsilon r.We use those, we derived and then after rearrangingformation, only we obtain epsilon l is epsilon a times square beta minus eta times sine square beta.Eta is a contraction ratio,Poisson's contraction ratio.Some think between 0 and 0.5 may be.Without this member which is the influence of contraction, this equation was derived by Gegauff in 1907.

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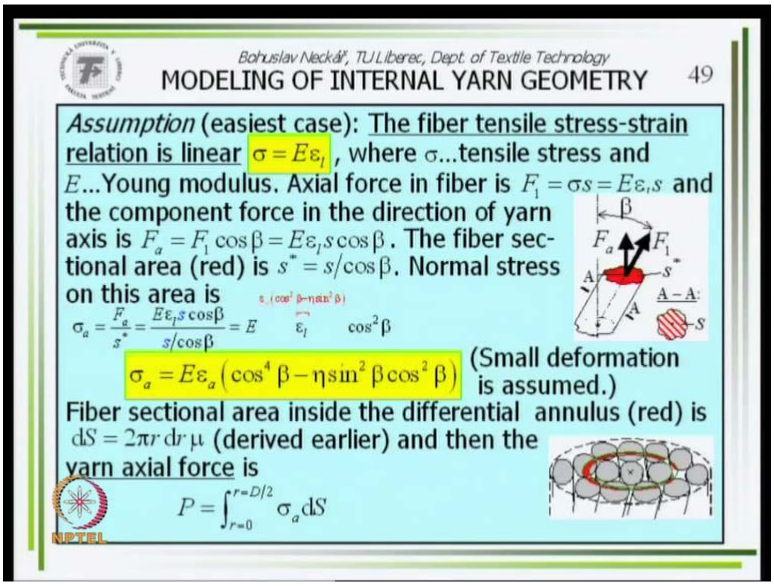

You see that Ido not present you some quite a new theoretical model.It is 100 year old model.Well, now it is which forces?In a fiber,the axial force is called as F1. F1 because pair one fiber.The fiber tensile stress-strain relation, let us imagine I said easiest case that its linear, then thestress sigmais proportional to epsilon l.In Hooks law, then E is young modulus, but it is a constantin our case.Axial force F1 is to the area perpendicular to fiber axis, of course is sigma times s. So, an epsilon l times s.The vertical component FA is F1times cos beta. So, that it is this here.

The fiber sectional area, this red is s star which is s by cos beta.In today, we said it.Normal stress on this area, on thisred area is sigma a, which is normal force FAby areas star.It is this here.Then,epsilon l, for epsilon l we use the expression for small deformation of course.We obtain sigma a, is E times epsilon a times this expression for small deformation I said.

Now back to our scheme which we used for number of fibers in yarn cross section.We know that in elemental annulus, the area of fibers is Ds.You know it was derived.Now,the sigma a is given by this equation,so that we can integrate, we can say what is the total axial force as a sum from all areas of fiber sections.It is shown here.

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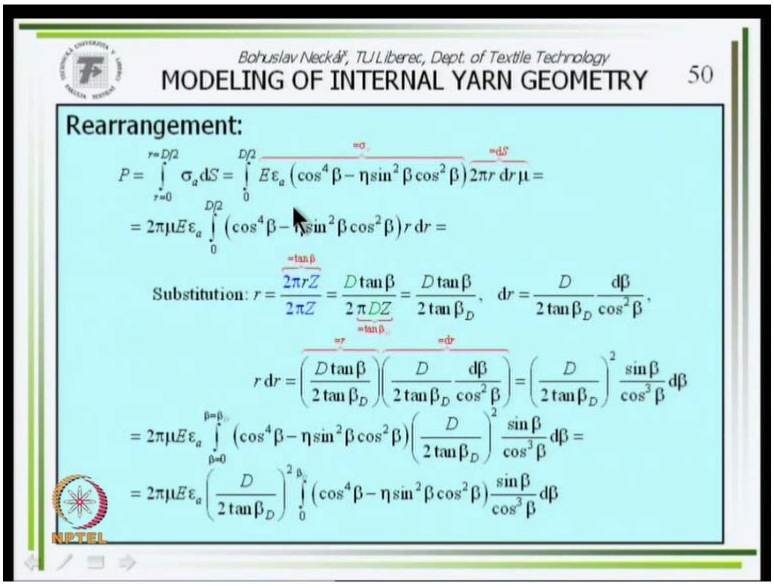

All other substitution, only one substitution rearranging.Then,it is a waywe assume.Yes in one moment,we assume that etacontraction ratio is constant is same in v in each point in yarn.Hence, we integrate and integrate and integrate.Nothing more traditional integration which you must absorb in your first semesters by mathematic,isn’t it?

Well,thenon the final form, we obtain this equation,nothing more than integration of this one.Logically, it is clear. We obtained this equation for force P force. P is given by this pi times mu pecking density times module,may be young modulus times epsilon a axial strain of our yarn 1 half ofdiameter squareand here is angle beta D peripheral angle of fiber in yarn on the yarn surfaceand eta and eta.It is nice, but more interesting can be the tensile force utilization coefficient.

It was one side.We know the force which we need for elongation of yarnusing strain epsilon a axial strain.In other case, let us imagine another yarn without twist parallel fiber bundle or something. So, let us ask how we must, how is the force for the same strain in such parallel fiber bundlestrain?Yeah same count.

Well,sigma a,was e times epsilon a, because the fibers areperpendicular to cross section parallel fiber bundle.So, it is equal and S is equal to PD square by 4 times mu. Isn’t it?Using this, we obtain such expression forforce P star,for force in the same yarn, but without twist.Then, the effect of twist to mechanical properties, we can do tensile force.We canexpress as a ratio axialforce in twisted yarn by axial force in non-twisted yarn.

Using our equations, you canobtain this.Here, you can see that this expression is same thanwhich is before brackets.Therefore, it is this here.Well, how it is theoretically and how is thecomparison to some experiments?This is the utilization coefficientphi is called phi here.It is utilization coefficient and it based on angle beta D peripheral angle and also,(())is playing eta contractionratio.This is angle beta d.This is our utilization coefficient and there are functions for different values of eta.If eta is equal 0,so that thetraditional Gegauff's idea, we obtain such function.Here higher is eta,then the function is smaller.If smaller and smaller values of phi, but (())is same.The theoretical curves are here based on our equation, which is this here.

How this practically?John Heiro, a very known professor from mist from Manchester,now he is old man, but he was really the top person among his years. As hisPHD student studied the strength of different types offilament yarns, and they obtained suchpoints for these yarns.It is from the book of Heiro and co-workers.Now, bookon mechanics.

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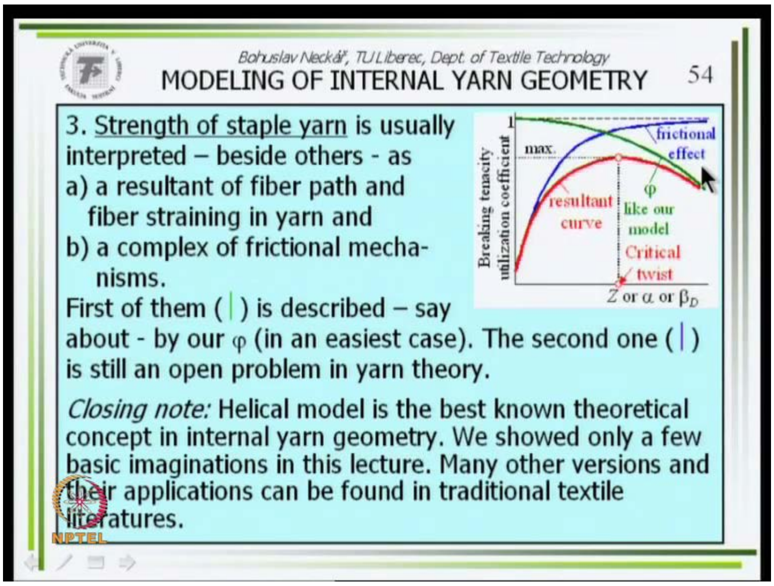

Well, this is the axis of our phi.This is our beta d and using eta equal 0.5, we obtain this function. So, you see that,only here is not it is little order for, but from this moment, it corresponds to our result.So, you can say that from filament yarn, this result can be roughlyused for estimation of strength.Having higher twist,this quantity phi is decreasing.It means, when I want elongate, Ido not know 5 percentage parallel fibers bundle and twisted yarns and counts,then the twisted,both filament yarn. I mean, then inparallel fiber bundle, I need a higher force than by twisted yarn.Twisted yarn is not, filament yarn is notso tough thanparallel fiber bundles.

Why because the forces,it will be a variant I think fromequations which I have and not to the strength of staple yarn.You know that the staple yarn among others, for strength of staple yarn influences two important phenomenons.One is twist andsecond isour friction among fibers,then the internal geometry of the yarn.The internal geometrybrings some decreasingcurve,this green decreasing curvefor yarn tenacity or something.In similar way,then our model of course,our model is based on lot of assumptions, but principally this is same effect.

The second effect is the blue curve here, which shows symbolically the effect of friction.When we havevery small twist or 0 twist, then infilament yarn, for elongation of filament yarn,we needhighest force.Therefore, also easier, say the strength ofparallel fiber bundle is highest, but we cannot use it by staple yarn because friction fiber wasso smallas a blast in air. So, the friction effect is increasing principally according to this blue curve and two such influences together with energy give something like this, a red curve.Therefore, this curve has some maximum point, point of critical twist point in which the strength of the yarn is maximum.Isn’t it?

Now, to the future of such modeling, I said thatthis curve, we principally are able to create.The easiest version of it, I present it here now.Fromothers eye, the blue curve is in whole world, the secret to these days.Theproblem is friction among fibers.We only know that the traditional equations like Coulomb equation andEuler’s equations and soon are not enough well foruse inmodeling of yarn internal structure.How it is?That is a question.Hope, somebody of you in future will be successfuland will solve these speciallaws which arevalid for friction among fibers in yarn and other fibers as well.I think that is all for today.