Orientation of Fibers

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Model No. # 01

Lecture No. # 07

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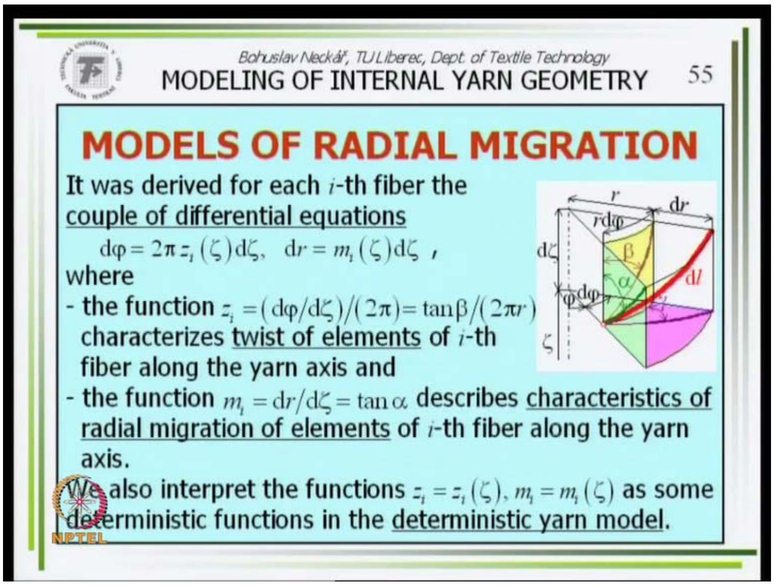

Today's lecture is oriented to one direction of modeling of yarn, internal yarn structure and their migration models. It is a little more difficult than the helical model and its application. Therefore, I will speak slowly as possible and please you to imagine as possible the geometric relations, which are valid there.

In our starting part to the theme of a structure, internal structure of the yarn, we spoke about a fiber element, about the relations, which are valid there; such element was like these here, is it not?

And we define two differential equations, this is the first and this is the second. We said that the Z i, which have to sense as a twist, the local twist of our assembly. Sorry, the local twist of fiber element is given by such equation and the second quantity m, is practically tangents alpha. What is alpha here? Alpha is the angle on our green wall between vertical direction and a project of our element to our green wall on our picture. It characterizes the radial migration of element. These functions must be some deterministic function when we want to speak about deterministic models of yarn structure.

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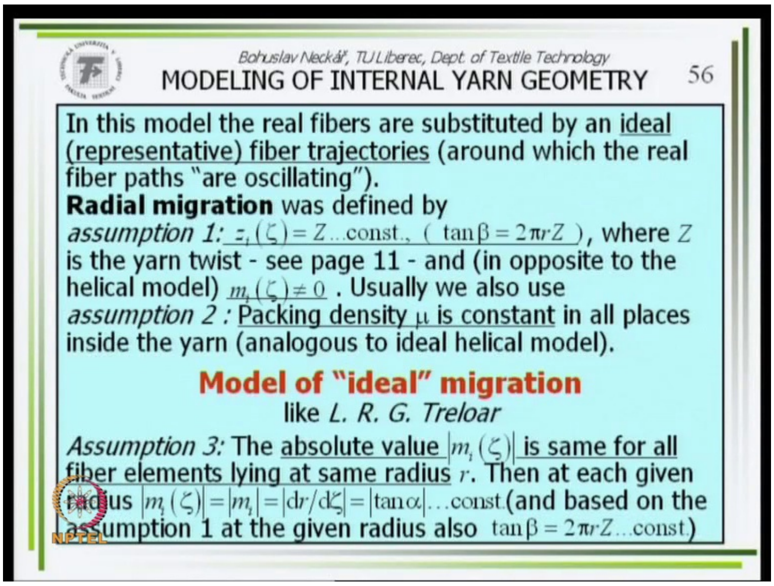

In such model the real fiber are substituted by an ideal means representative fiber trajectories around which the real fiber path outside it. Radial Migration was defined by a set of assumptions; assumption one is the function z i zeta, is Z, and it is constant. The same was assumed by helical model. nothing new for us. So, that or. So, the the the. The equation tangents beta is two pi r Z and is valid it is known to you from the last lecture.

Well the function m i zeta in opposite to helical model is now not equal 0. It is some value which is not equal to 0. What does it mean? This quantity m is tangents alpha. So, the tangents alpha is not equal to 0. Also, D r, which is the change of radius by the zeta incremental increase of vertical coordinate. So, this ratio is not equal to 0, it means the fibers change its radius; the radius is increasing or decreasing, that is changing it radius. Because the fiber changes it radius, but radius only is constant and as we say here Z is constant. Therefore, we speak about a radial migration.

We also will assume that a packing density mu is constant in all places inside the yarn. It is analogous idea to an ideal helical model. Well in years during 1955, I do not know at the moment clearly, one professor from U K, from Manchester, from (( )), his name was Professor Treloar, the name is mentioned here, for the first time constructed such model which is not based on the helical concept, such a migrating model.

He used assumption three. It was assumption one and assumption two. Now, it is a general assumption three. The absolute value m i is same for all fiber elements lying at the same radius, r. Then at each given radius, also the tangents alpha absolute value, we speak about absolute values; absolute value of tangents alpha must be constant. What does it means? Let us imagine some radius inside the yarn body, some hypothetical cylinder of some general radius r. In our migration model; the fibers now are going to change its radius. So, they are going also through our cylinder of the radius r; from inside to outside then from outside to inside and so on. Do you imagine it?

On our radius r, intersected this surface of our cylinder, with some angle alpha, and Treloar's assumption say that each intersection, from each fiber of our radius have same angle alpha on a given radius. On another radius, the angle alpha is another, but in given radius, all sections have the same angle alpha.

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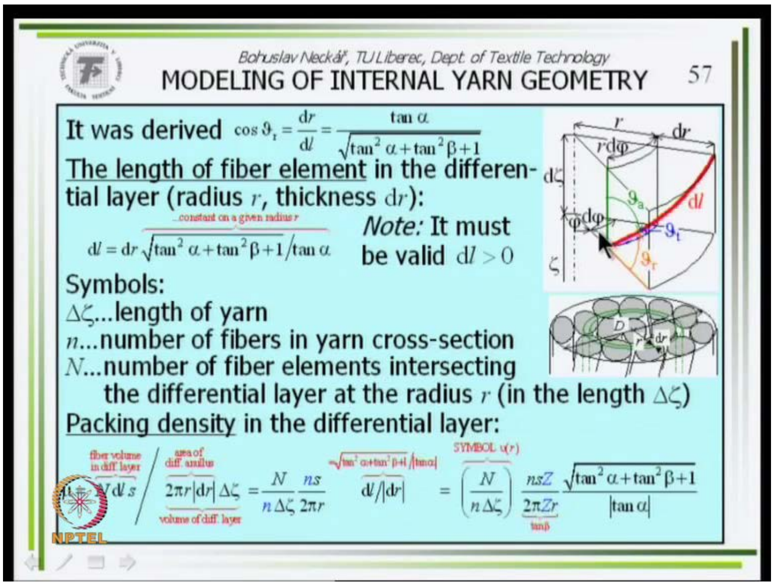

Earlier, we derived cosines of angle theta i, angle theta i, this green angle between the vertical direction of yarn axis and the right element of fiber. Is it not? We derived it. So, it is defined d r by d l; it was shown and we derived that it is tangent alpha by square root of tangent square alpha plus tangent square beta plus one.

The length of fiber element in the differential layer; what is differential layer? Last time we speak about a differential annulus and differential annulus is a cross section of our differential layer. Differential layer is the space between two cylinders, one have the direction r and the second have the direction r plus d r. So, differentially thin layer on the radius r between two cylinders, it is the differential layer and through this differential layer is going the fiber.

The lengths which such fiber have inside of our differential layer is dl, and we derived that or it is possible obtained from here that d l is d r times the square root by tangents alpha. This d l must be constant for each fiber in our radius only, because beta is constant; for each fiber beta is constant, for each fiber alpha is constant too, because of the assumption. So, we have these equation and now the symbols which we will use.

Let us imagine the yarn lengths as delta zeta; delta zeta it is the length of our yarn, and n is known value and is the number of fibers in yarn cross-section; number of this green ir here. Capital N, it is number of fiber elements intersecting the differential layer at the radius r in yarn in the length delta zeta. This differential layer have the length delta zeta and throughout lot of times, the fiber intersected from inside to outside, from outside to inside and so on. How many times all fibers intersect the differential layer? Capital N times, inside of our differential layer is lying capital N fiber portions, elemental fiber portions, each have length dl, is it imaginable?

Now, how is the packing density in the differential layer? So, packing density is the ratio of fiber volume by total volume and it was the general definition. How the fiber volume in our differential layer is is an elemental part of fiber? Each elemental part have the lengths d l and cross section of area of fiber is small s in our set of our lectures, permanently.

So, this is the fiber volume (Refer Slide Time: 12:39). What is in the denominator? Evidently, it must be our volume of our differential layer; the volume between these two cylinders. What is it? 2 pi r d r, it is the area of elemental annulus as you know it from last lecture and this times the zeta, the length of our yarn and we have a volume of differential layer; the total volume. The absolute value here is because based from the track theory of fiber, radius can increase as well as decrease, but for volume, I need a positive value of thickness of our differential layer. Therefore, symbolically we use absolute value here.

After arranging, we obtained here the black alphabets here (Refer Slide Time: 13:50) staying on the left hand side only graphically in other form. We divide it by n and multiplied by n too. We can do it; divide and multiply by the same quantity unless the quantity is not equal to 0, isn't it? So, we divide it into n and multiply by the quantity n and then we are here and now what we obtained a fourth black here (Refer Slide Time: 14:28). But on the place of d l by d r we used this here, why because of this one here.

We can also multiply and divide by capital Z that is the yarn twist. So, we obtained is the two pi r Z, is traditionally we know it tangents beta. For this ratio capital N by small n times delta zeta, we will use one symbol; nu r, because it based on r on the radius it can be changed this radius, so nu r (Refer Slide Time: 15:17).

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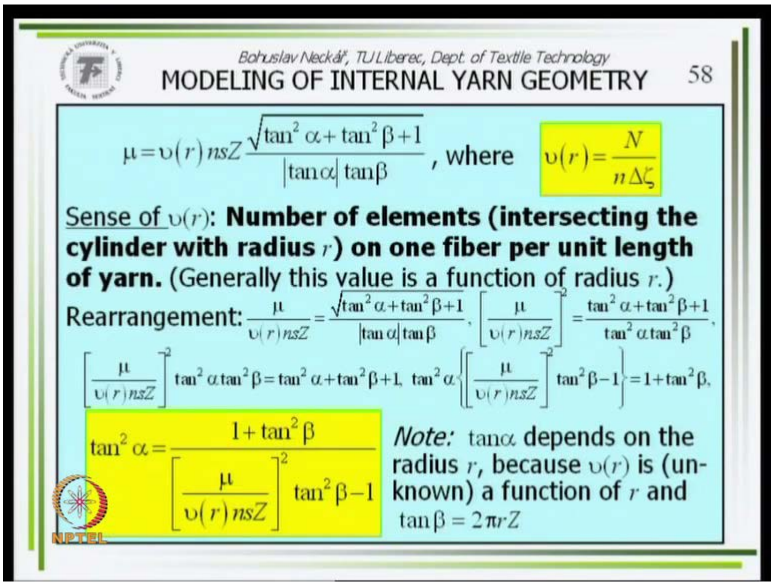

Using this we can write mu, which is given by such equation, such expression where the ratio nu r is given by this expression. Is it clear with what we are doing? The question is what the sense of the quantity nu r is or what is the logical sense of the quantity nu r? See, what is capital N? It is total number of intersections inside our differential layer of lengths delta zeta, but in our yarn of n fibers. Let us imagine please because it is easier our yarn from filament fibers. We have n fibers inside in our yarn, so that per one fiber we obtained capital N by small n, it is number of intersections per one fiber on to yarn lengths delta zeta and when I divide this value by delta zeta more, I obtain number of intersections of our differential layer per one fiber on the yarn lengths equal one. Is the logical sense is clear?

So, therefore, here it is written that nu r is number of elements intersecting the cylinder with radius r on one fiber per unit length of yarn. Generally, this value is a function of radius, is changed to its on radius on another radius, where we seem this quantity and other, may be it can be in the middle or on the middle fiber is often times near to axis or near to periphery and so on. Therefore, this quantity generally say is function of radius.

We derived this equation. You know, what is nu? This equation we can write also in this form, that is only rearranging this part is denominator on left hand side and nothing more here, on right hand side, then square root of both sides of this equation. We obtained this here, after a small rearranging, you want to have tangents alpha or with a tangents alpha square explicitly, so that we rearrange our equation to this form to the final rearranging.

We are rearranging only because to obtain tangents alpha in explicit form, in the form tangents alpha is equal to or here tangents square alpha is equal to. Tangents alpha depends on radius r, because of two functions, because nu r is in the moment unknown function of r, and tangents beta too, because tangents beta is 2 pi r Z and its known function, but also function of r. Therefore, tangents alpha must be the function of r.

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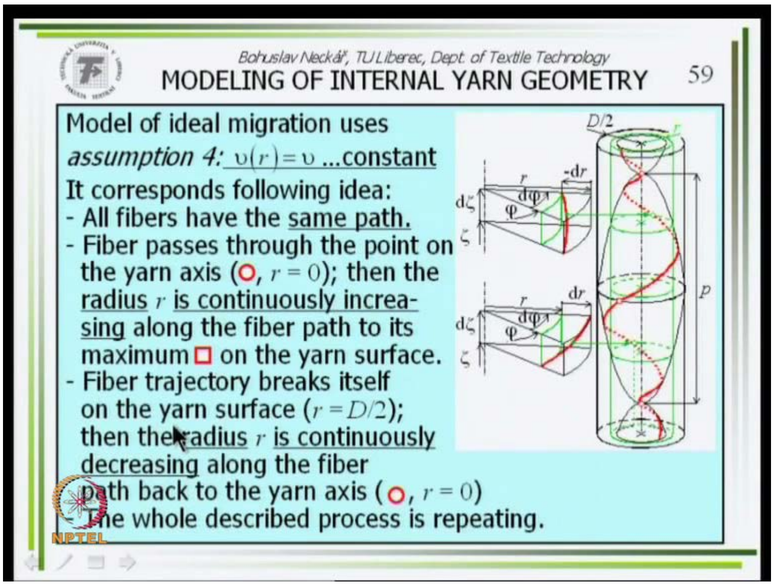

Postulate a fourth assumption; he said that we assume that the quantity nu r is constant; that is this nu and this nu is constant; independent of radius, same value in each radius, so that in each radius, number of intersections of radius, in each radius per one fiber and per lengths unit of the yarn, is same. For example, I do not know six. So, one fiber intersects six times this radius, small radius, very small radius, so that each radius, the fiber is intersecting six times, for example. This is the assumption; I can say ad hoc assumption or prior assumption of Treloar's model.

This assumption corresponds to following idea. All fibers have the same path, fiber passes through the point on the yarn axis, from this point here where radius r equal 0; and then the radius r is continuously increasing along the fiber path. You see that the red fiber increase in its radius to its maximum; the maximum is on the yarn surface. Then fiber trajectory breaks and (( )) breaks itself on the yarn surface, then the radius r is continuously decreasing along the fiber, that is symmetrical of way to smaller and smaller, smaller, smaller, to the yarn axis, the second red circle small, the point here (Refer Slide Time: 22:31).

So, the whole described process is repeated and so on from here. So, it is a geometrical interpretation of the trajectory, of deterministic trajectory of fiber in Treloar's migration model. You can see that the elements of fibers are going from inside to outside like this element and the radius is increasing. In this part, the radius is decreasing and they are going from outside to inside. Is it imaginable?

The yarn length between two axial points of fiber, in this idea, the yarn lengths that is these lengths, from this point to this point (Refer Slide Time: 23:41), we call it anterior chord, called as a period of migration. It is in one period it is repeated.

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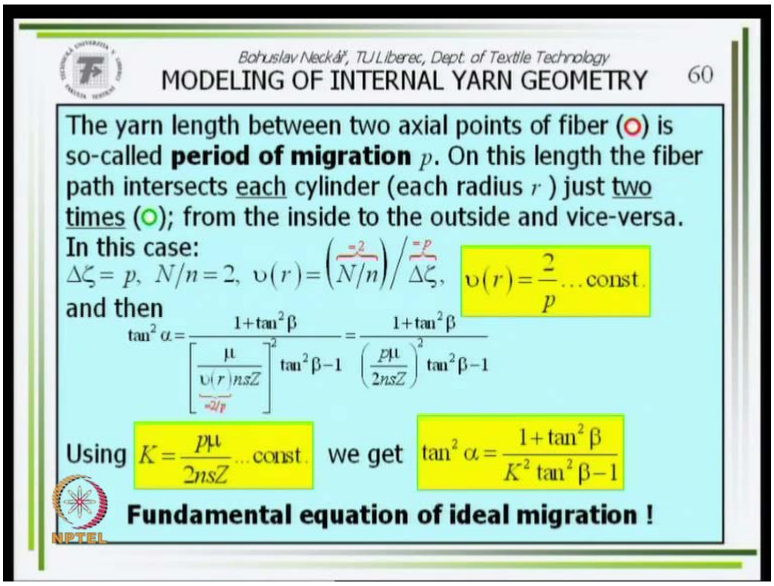

So, it is a period of migration, P. On these lengths the fiber path increases each cylinder; sorry intersects each cylinder, each radius r, just two times. On the yarn lengths P, yarn lengths P, for example, the green cylinder is intersected one time from inside to outside. Here, is the intersecting point and from outside to inside, here is the intersecting point, two times.

So, just imagine that the yarn length, delta zeta is now equal P, the period of migration. Then number of intersections per one fiber, capital N by small n, is equal to one times from inside to outside, one times from outside to inside. So, nu r, our quantity nu r, which is now constant, is capital N by n by delta zeta, then this is two and this is p. So, nu is now two by p. The quantity is two by p or one half value or reciprocal of one half of period.

Our equation, our earlier equation, this equation (Refer Slide Time: 25:42) can be rearranged using this knowledge, nu is two by pi; on the nu, we put two by pi. So, we obtain this expression, this quantity we can also put for shorter writing call K. Here some parameter K and this parameter is independent of radius; only nu can have sometimes and you can have some problems with mu, but by value, by constant value of mu, it is some parameter. Well using the symbol K some parameters, some constant, we obtain our equation in this form and these equation, it is a fundamental equation of ideal migration model according to Treloar. This is the fundamental equation.

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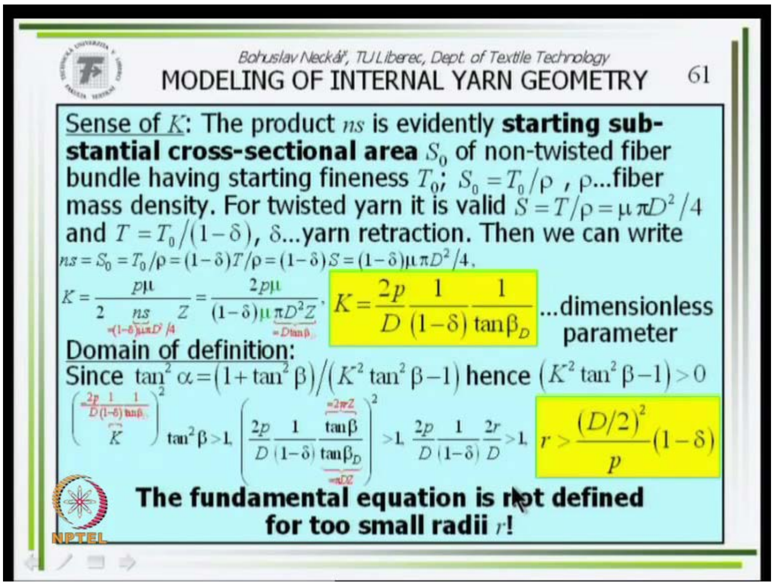

What is the sign of our parameter K in this fundamental equation? It is derived here, the product and times as is evidently starting substantial cross sectional area; substantial cross sectional area S 0. For a fiber bundle, cross sectional area of all fibers, its n times cross sectional area per one fiber and in per one fiber bundle it is S. So, that S 0 is T 0, starting fineness of per one fiber starting from which we create the yarn by rho; rho is fiber mass density as every times.

For twisted yarn it is valid as cross sectional area is T by rho, we cross in first lecture about the yarn structure and it can be also explained as a pi d square by four, the total area of yarn cross section times packing density mu. So, we can write and then we know that T is T 0 by one minus delta. It was from the chapter of yarn retraction, where delta is yarn retraction. All this equations we know, then we can write that ns is S 0 is T 0 by rho and is equal to one minus delta by T times T by rho. So, one minus delta time S and one minus delta times mu times pi times D square by 4. Using this in the formula for K, we obtain this here, this path it is D times tangents beta D, evidently. So, we can also rearrange K to the form K is 2 pi by D by one minus delta and by tangents beta D. You can see there this quantity is dimensionless (Refer Slide Time: 29:46).

Now, to the domain of definition, our equation, our fundamental equation, this one (Refer Slide Time: 30:00) is written here. On left hand side also, tangents square must be every times either 0 or positive value, it must not be negative value. This is positive, so that this value must be positive and higher than 0, of course, because the denominator. So, we must write K square tangent square beta minus one is higher than 0.

So, K square times tangent square beta from this equation must be higher one. Using this earlier expression for K here, we obtained this must be, this square must be higher than one. So, this must be higher than one. So, r from this, r must be higher than some positive value here, one half of yarn diameters or yarn radius, the radius of the yarn or yarn half of yarn diameter by period of migration times one minus yarn retraction. It is may be usually very small value; p, period of migration is in relation to yarn diameter usually very high, but it is some positive value. So, you can see that our fundamental equation is not defined for very small radius; r must be higher than some positive value.

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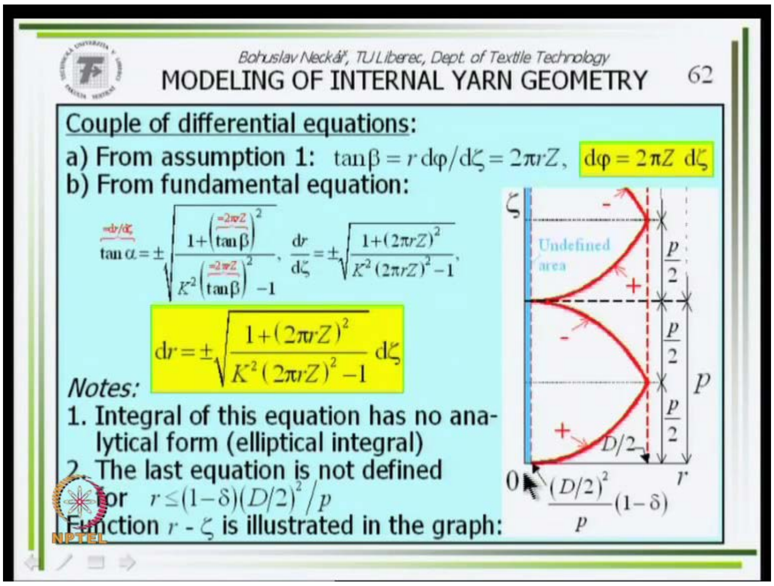

This is our fundamental square root from our fundamental equation, is shown here. Square root from tangent, square is tangent, square root of this expression is plus minus this here, because tangents alpha is d r by d zeta. We can write this equation. So, we can write this differential equation which is valid for Treloar's model. This is some function of radius d r. So, d r by this square root, on left hand side is only r, right hand side is d zeta. It is possible, principally it is possible to obtain to solve this differential equation, integrate both sides and we obtain the relation between r and zeta.

Sorry, this integral d r by this square root (Refer Slide Time: 33:41) integral do not exist in analytical form. May be you heard about some of such integrals, one group is an elliptical integrals and some think about this type of integral that it exist, it has a graphical interpretation as well, but such integral do not have an analytical form. So, it brings problems evidently. Nevertheless it exists and therefore, I can show you the relation between radiuses of a fiber on abscissa, which is scale of radius; on the ordinate is axial dimension of per point, so zeta coordinates.

The fiber points cannot be in this blue region, because r is too small and we said that radius must be higher than some value. So, it started here, then the radius is zeta and the radius is increasing from starting point to the periphery of the yarn, to the radius capital D by two and then symmetrically, because it changed the sign here back to the minimum radius, which is so and so. This is one half of period and this is lengths of period of migration. It is good and it is possible to obtain using the numerical mathematic, because this elliptical integral we must calculate numerically, but is a question if it is not enough good as some approximation.

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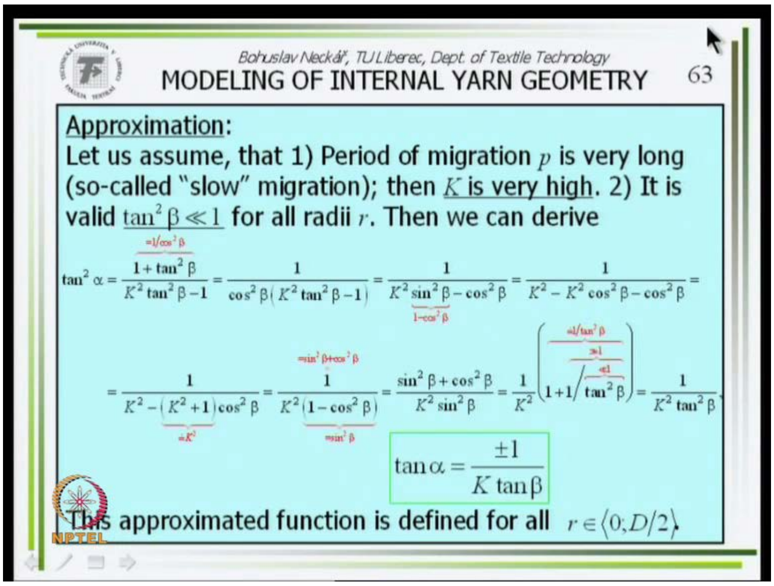

Therefore, Treloar start it with means of approximation; approximation formula. He mentioned period of migration P is a very long period. So, intensive migration of fiber is a very long period of migration. So, called slow migration and then K is very high value.

And second it is valid the tangents beta square is much smaller than quantity one because maximum is around 20 to 25 degree in yarn. Therefore, in a common yarn such quantity is very small in relation to value one. Then we can derive. This is our first and is our traditional fundamental equation (Refer Slide Time: 37:03). We can rearrange it; this is rearranging using some geometrical functions of these two, so we obtained this here. Then we obtain this expression (Refer Slide Time: 37:22) because K is very high, then K square plus one is approximately K square. K is high, and square of this is very high quantity, million plus minus one is practically million. So, it is possible to write it. Now, here is one minus cosine square, it is sine square. We obtained this expression and then we obtained this expression and because tangent square is a very small in relation to one.

Then this is a very small value, and then there one by tangent square is very high. It is much higher than one. This is very high in relation to one quantity. So, that one plus something very high is roughly this. This is one by tangent square beta is approximately only and we can write what tangent square alpha as approximation tangent square alpha is equal to one by k square times tangent square beta, using the set of approximation equations. Such equation is evident is defined for all radii, from 0 to D by 2. Now, only this blue area does not exist any more when we use an approximation equation.

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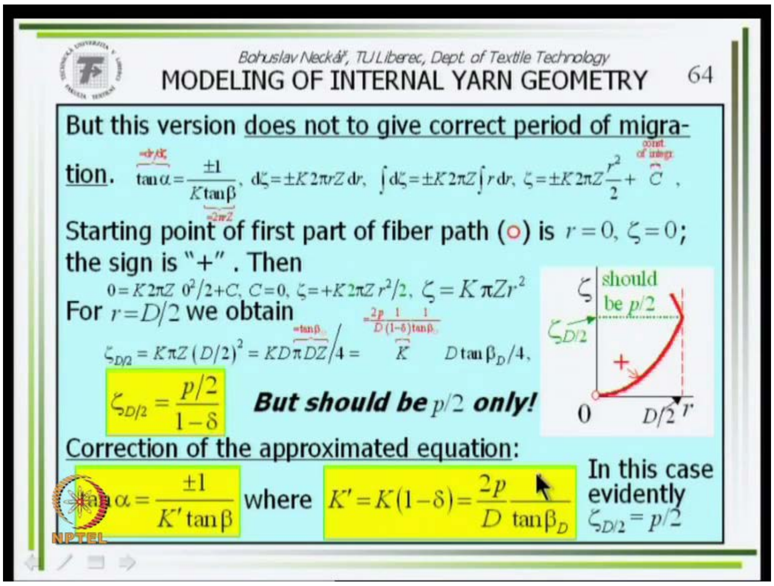

We can use the approximation equation, but it brings another problem, because each approximation bring some errors which is not totally identically original equation, and this bring some problem all through, which of this version does not give correct period of migration. Well, tangents alpha as it is, we said d r by d zeta. Now, approximation equation it is this one that is tangent beta is two pi r Z. So, we can write zeta is plus minus K times two pi r Z d r. Integral D zeta must be integral of right hand side over r over radius. After integration we can get zeta is equal plus minus this and plus C, where C is the constant of integration.

Well starting point of first part of fiber path is at d equal to 0. So, that r is 0, zeta is 0. We start in point r 0, zeta 0. Then we can derive that the C must be this integrating constant must be 0 and so, zeta is K times pi times Z times r square. Now, the integration process was possible. It was not elliptical integral and what we obtained is each of curves, it is parabola, it is here. So, this is the function and on the radius D by two, it must be or it should be it one half of period, period of migration and of course, it should be.

How is zeta at the radius D by 2? It is K times pi times Z times D by 2 square, it is this here. So, it is this here using on the place of K, our known expression from earlier slides we obtained that zeta is P by 2 by 1 minus delta and it is not correct, it is not one half, it should be P by 2 only; not divided by 1 minus delta. It is because you do not use original function than our easier approximation. So, it should be P by 2 only.

Therefore, Treloar, you have an idea to change the quantity K to another modified quantity K to the value K dash and tried the tangents alpha is plus minus one by K dash times tangents beta, where K dash is our original K, but multiplied by one Y minus yarn retraction. Using this you can check it, you obtain also for approximation equation one half of period of migration on the radius for zeta D by 2 (Refer Slide Time: 43:15). (No audio between: 43:16-43:26) The approximate differential equation is now this one, which is tangent beta, is D r by zeta. So, we obtain this here and then we can integrate it, set it after integrating starting from point zeta is 0 by r is 0 we obtain this equation.

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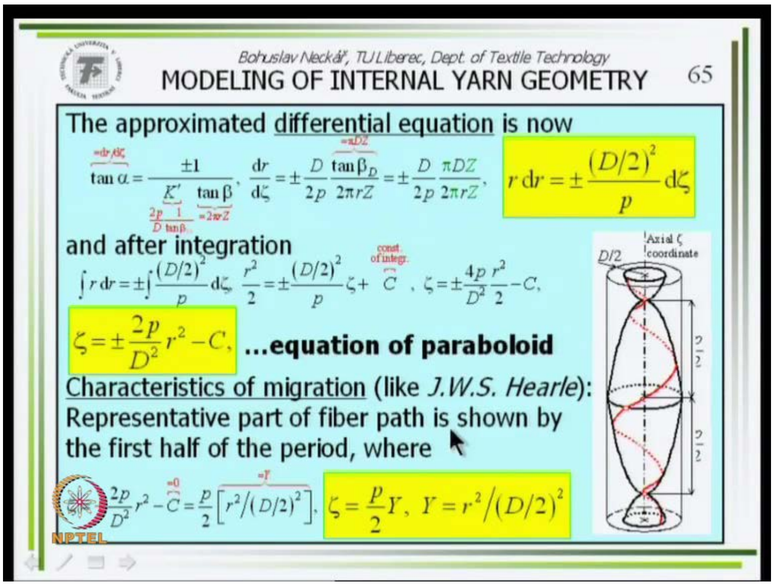

So, we obtain finally this relation. What is this equation? For a parabolic zeta and radius, on which is laying the trajectory of fiber. This parabolic, this approximation equation used by Professor Hearle; Hearle maybe is here and he must be very old man. I was a little boy and he was a top man of yarn structure in my young years. He was a professor on UMAST, Manchester, on the university Manchester, and author of lot of special models.

This equation Treloar equation used here for definition of characteristics of migration. Well, I think this characteristic will be the starting part for our next lecture. So, at the moment I thank you for your attention and I will be happy to see you in our next lecture.