Orientation of Fibers

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Lecture No. # 09

Mechanics of Parallel Fiber Bundles

In earlier lectures, we spoke about the terms like yarn count, yarn twist, packing density; also, we mentioned yarn diameter.

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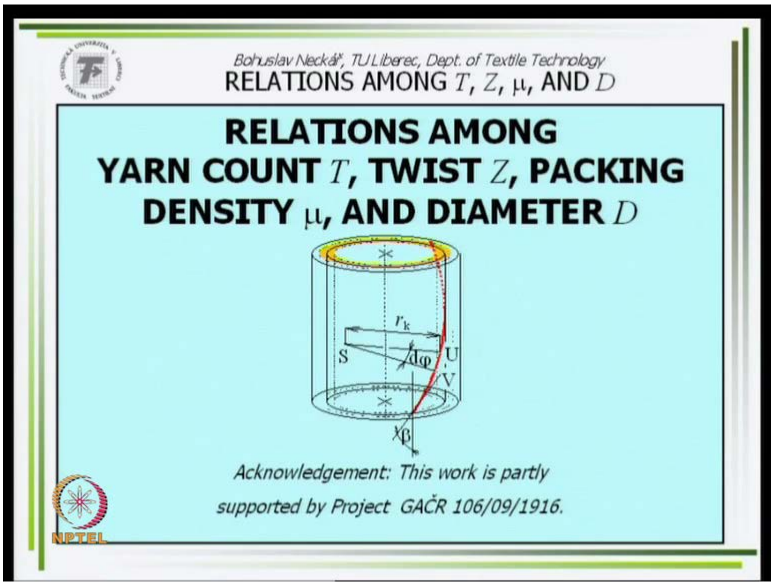

How is the relation among these often used quantities? It will be theme of today's lecture.

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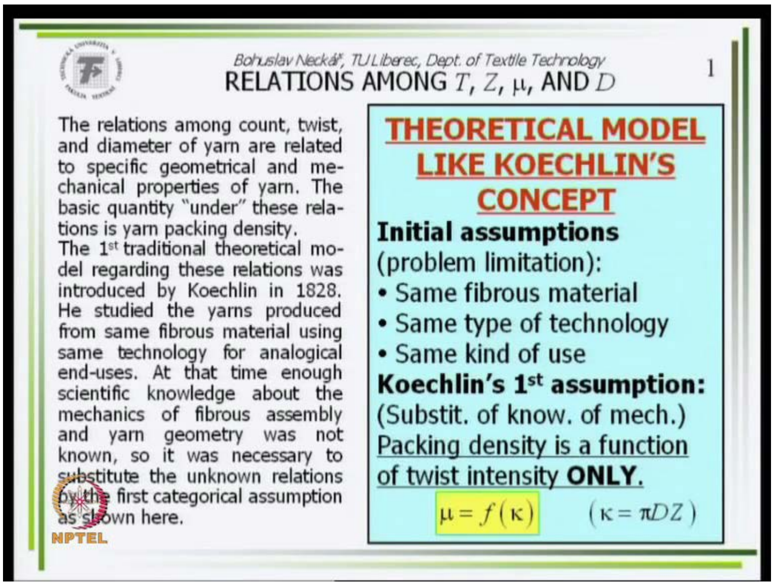

It is nothing new. 200 years ago, mister Koechlin in one French town Mews, presented his first model about the relation among these quantities. So, we start a very alternatical concept, which is roughly 200 years old. This model is usually quoted as a model-like Koechlin. Let us accept initial assumptions, which limited our program. Let us think that our yarns are produced from same fibrous material, from same type of technology and from same kind of use or similar kind of use. I will not repeat these assumptions, but we automatically will think about this limit of yarns.

The Koechlin's first assumption substitutes our knowledge of mechanics. We discussed earlier about the possibility how to calculate the relation between pressure and packing density. The fibrous material is compressed due to twist; is not it, in the yarn. But, in the time of mister Koechlin, this relation was not known, so that he must use some assumption. This assumption is here. Let us assume that packing density is a function of twist intensity only. Is in reality packing density the function of twist intensity? Evidently, yes. When I have higher twist, the twist intensity is increasing and the packing density is increasing too; it is evident. But, the assumption is that the packing density is the function of intensity of twist only. Later, we will show that packing density more precisely is the function of intensity of this, but also, it is a function of other quantities.

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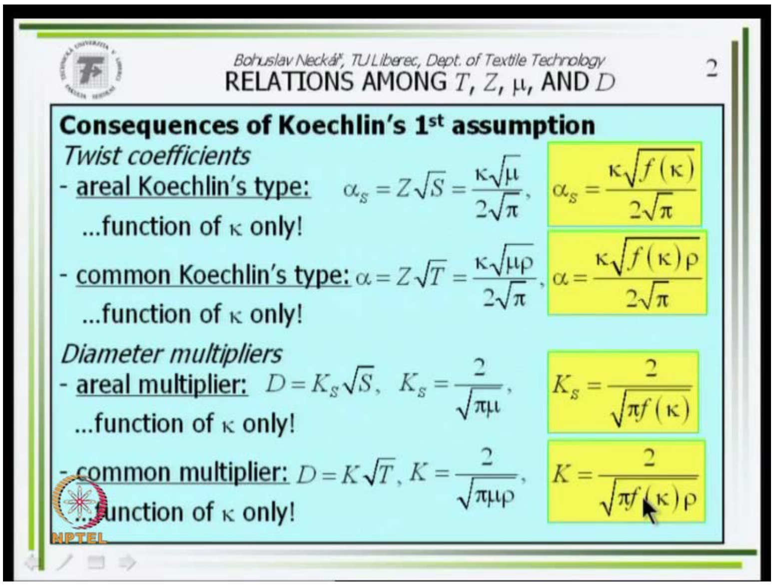

In Koechlin, mu is a function of kappa; where kappa was pi D Z – twist intensity. You know it from our lesson 1. How are the consequences of Koechlin's first assumption? We will use equations known from our lesson 1 based now in the form, which accepts first assumption of Koechlin. We derived areal Koechlin's type of twist coefficient; we called it as alpha s. It was Z times square root of S; yarn twist times substance cross sectional area of the yarn. And, after arranging in lesson 1, it was also this expression, where kappa is twist intensity and mu is packing density; is not it? So, generally, alpha is a function of two variables: kappa and mu. But, first assumption of Koechlin says that the mu is function of kappa, so that now, alpha is a function of kappa only. Similarly, this areal Koechlin's type of twist factor is using the theory.

In textile practice, use some common Koechlin type, which is this here. This expression was derived in lesson 1 too; more easier rho – specific mass of fibrous material. And similarly, the alpha is given by such equation. You can see that alpha as well as alpha s as well as alpha are functions of kappa of intensity of this only; (Refer Slide Time: 06:13) only one variable on the right-hand side of these two equations. And, how it is these diameter multipliers? Areal diameter multiplier for D was K s; and K s was 2 by square root of pi mu; back to our lesson 1. Now, because mu is function of intensity of twist only, K s is a function of kappa only. And similarly, the common multiplier K, because this is K times square root of T, is after such arranging function of intensity of twist only. These four quantities are now based on the first assumption; the function of only one variable – it is kappa; it is intensity of twist.

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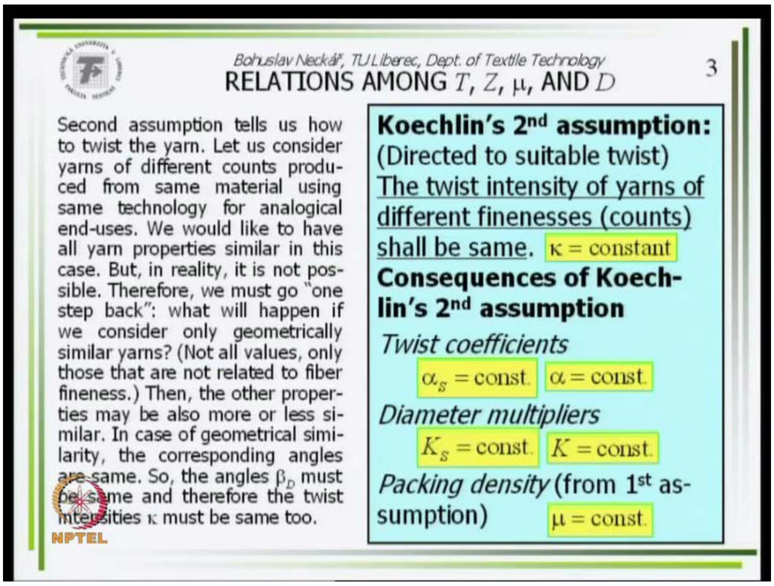

The second Koechlin's assumption is directed to suitable twist, said that the twist intensity of yarns of different finenesses, different counts, shall be same; kappa shall be constant. What is the logical root of this assumption? This logical root based on geometrical similarity – you know that when we have different geometrical objects, which are similar means geometrically similar, then corresponding angles are same; is not it? And, Koechlin thought that the yarn – some course yarn, some fine yarn; both will have same possibility; for application, we will have some similar properties; especially mechanical properties, geometrical properties when they are same from point of view of geometrical similarity are similar. Therefore, if these ideas we accept as logical root, therefore, also, the angle beta d – the angle of peripheral fiber in our idealized yarn, must be same in each yarn for the same use and so on.

And, what is the same angle? Tangent of peripheral angle, tangent of beta d is intensity of twist kappa, so that kappa shall be constant. What is now with these four equations and the first assumption when we accept that kappa is constant? In these four expressions, you do not know this function f, but we know that it is a common function for each equation; by the way, monotone increasing function it must be. So, when we use kappa is constant, then evidently, alpha is alpha K s (Refer Slide Time: 10:05) as well as K, must be constant; and, mu is constant too; is not it? Yes.

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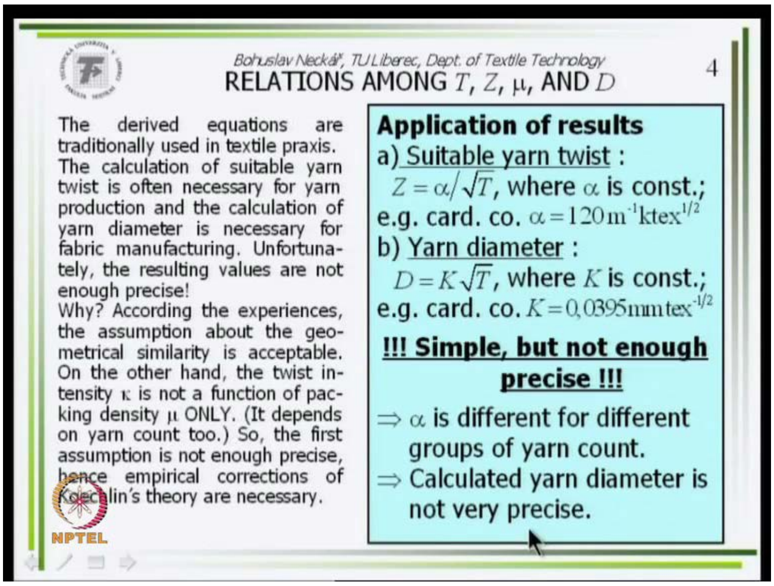

We said that good idea based on Koechlin's model, Koechlin's concept, is to have the same angle – peripheral angle of fiber, because geometrical similarity on each yarn. Now, this idea we can say to the people in spinning mill, you must measure the angle of peripheral fiber in your yarns. You can imagine what they can answer to you when you give this idea to your spinning mill. Nevertheless, it is possible so much for a region, because we said that the result alpha s, alpha, K s and K must be constant. So, we do not need to measure the angle; we can say, for example, for practice, in spinning mill, alpha must be constant. What is alpha? From definition of alpha of twist factor, twist coefficient, we can say that twist is alpha by square root of yarn count, means finenesses.

Then, we can say to working people in spinning mill, yes, you must twist each yarns. So, then, alpha is… I do not know what based on your experience in such spinning mill; for example, 120 meters to the power minus 1 kilo tex to the power one half. This is dimension – physical dimension of alpha; this is metrical alpha (Refer Slide Time: 12:05). And, when the people will produce another yarn count, then they use the same alpha. And, using this equation, they can very easily calculate in spinning mill, which of twist is necessary for this or that yarn count. This is the first result of Koechlin's theory.

The second result is, write it to our specialist in weaving technology. In weaving technology, you know, is necessary to know the yarn diameter, because covering cover factors, maybe covering can similar quantities, which defined the density of woven fabrics and so on. How to obtain the yarn? How to obtain the yarn diameter? Koechlin's theory said it is easy. Yarn diameter D is parameter K times square root of T and K must be constant. Diameter multiplier (Refer Slide Time: 13:27) must be constant for given material, given technology and given type of use of our yarns. Maybe based on our experiences, we can say, it must be 0.0395 for example. And, people can calculate it; very easy, very elegant theoretical model, 200 years old, but not too enough precise. It is very often used, because roughly, it is possible to apply it, but when you want to work more precisely, then these results are not enough. Why? The practical experiences say that such alpha must be a little different for different groups of yarn count, so that in textile hand books, you can read that from count, these two counts, usual use of this alpha; then, from count these two counts, this interval of counts and a little larger alpha and so on and so on. It is typical for hand books for spinning practice. This is one way.

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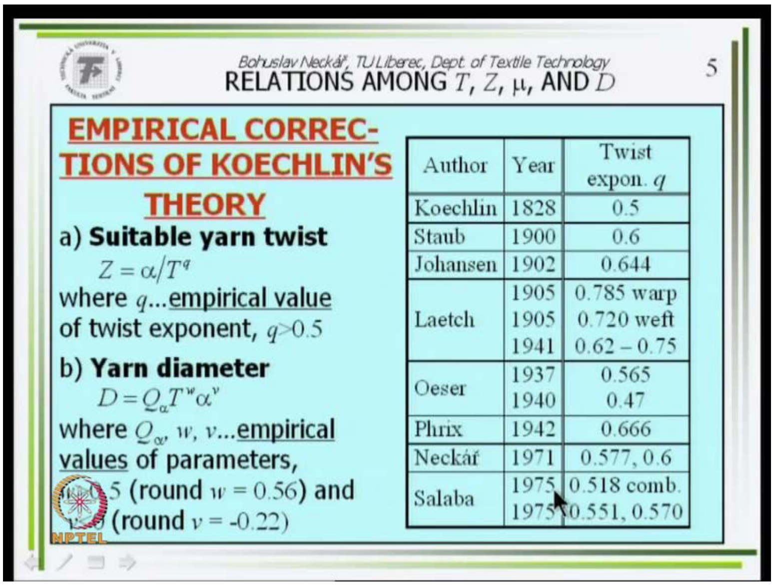

Second empirical way is to empirically change the Koechlin's equations. On the place square root, it means power to one half. It is possible for yarn twist to use the ratio alpha by T power to some exponent q; where, q exponent of twist is an empirical value; a little different from 0.5. Ruther fortress studied a problem in relation to this equation, which of exponent is the best; Koechlin's at 0.5; is not it square root? Then, lots of others have different ideas based on this or that experimental experiences. The yarn diameter can be empirically generalized to such form, D is some parameter Q alpha times yarn count power to some exponent times alpha power to another exponent.

An example for these values may be good for carbon coated yarns is here. This exponent w is usually something around 0.56 in this equation and the exponent by alpha is usually minus 0.22. But, my experiences may be another (( )) can see another world based on pipe of cotton beds or pipe of technology; based of lot local influences.

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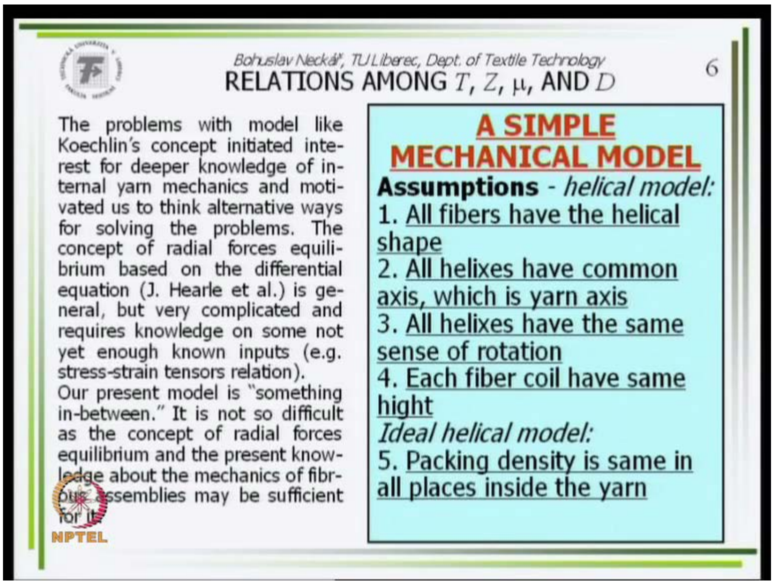

Why the model of Koechlin is not enough precise? His second assumption is very good; and, it is the earlier after Koechlin's experiences show that the geometrical similarity is very good idea. What is not too good is the first assumption (Refer Slide Time: 17:55) that the packing density is a function of twist intensity only. I mentioned it; Koechlin in 1828, had a chance to use some models, which respect the physical relation between pressure and compression of fibers inside of the twisted yarn. It exists some second way how to solve it. This way is in my check book, which professor Ishteyak gave; he can show you, it will be especially interesting for you.

The second way is go out from some differential equation of equilibrium of radial forces inside of the yarn body. And, based on tools of continue mechanics solve this problem for you as a problem of continue mechanic. I proved it earlier, lot years ago. But, there is a problem here. To this time, we do not know the relation between the stress tensor and the strain tensor. Stress tensor and strain tensor are some structures very popular to say; to these days, we do not understand enough general the relation between stress and strain in multidimensional, three dimensional case especially for fibrous assembly. Therefore, we can calculate, we can derive the differential equation, but, we have not enough well input to this equation. We must make some assumptions, some simplifications and so on. All these are very difficult from point of view of mathematical tools; you must solve some differential equation and so on.

Nice theme – it was lot years ago; the theme of my PhD thesis. But, to these days, this way is not… and the position to be practical tool for application. Therefore, we derived something in between, which is very easy, but not too precise theory of Koechlin. And physically, the best version differential equation of radial equilibrium solving of this one; something in between, which is easier, not so precise from point of view of Physics, but better than Koechlin's type. Let us assume the non-assumptions from ideal helical model. All fibers have the helical shape; all helixes have common axis, which is yarn axis; all helixes have the same sense of rotation; each fiber coil have same height. We mentioned these assumptions when we analyzed the helical model. And, fifth – packing density is same in each place in the yarn.

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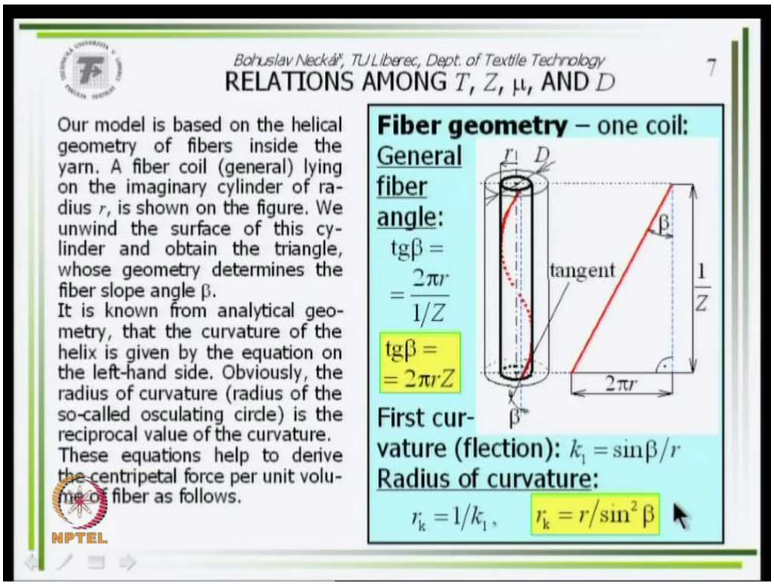

Then, it is more repetition for a helical model. Let us imagine some general fiber inside the yarn body; yarn body is this. Here schematically, the cylinder having diameter D.

And, inside on some general radius r. This thick black cylinder is like one fiber – red fiber on the general radius r helix shape. After unwind of this cylinder, we obtain such triangle, which is possible to the tangent beta, which is 2 pi r Z; tangent beta is 2 pi r Z is known for you from our earlier lecture. When you open some hand book about the mathematic and when you find some properties of different curves, also for 3D curves, space curves, you can read also what is so-called first curvature – also, means flection is used. This first curvature of three dimensional curves in the space in the case of helix is constant, independent to body to points on which you measure it. And, it is k 1 – sine of beta by r; you can read it in each hand book.

Reciprocal value of first curvature – it is radius of curvature; it is the radius of some rings, which can approximate our curve in a very theoretically infinitesimal path. You know what this radius of curvature is, so that the radius of curvature of such helix is r by sine square beta; is not it?

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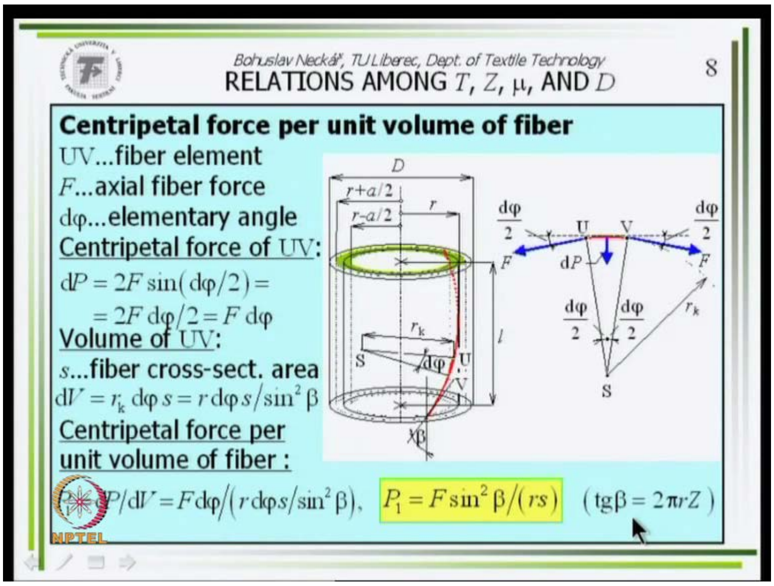

Let us think now about a fiber lying in yarn body on a hypothetical cylinder of harass. It is not from metal; it is only imagination. On a radius r, this is the red fiber. Let us think about the elemental part of this fiber – part UV. This is the radius of curvature, r k. On the angle here is differentially small; it is angle D phi. In the fiber, let us be some axial force – capital F – axial fiber force. So, when we make the picture of this part especially here, that you can see UV – our elemental part of infinitesimal small part of fiber on which is tangential force F – axial force in fiber. This is the angle phi, one half; it is no wider the straight line is not to see here. This is so-called angle d phi; and, this is one half and this is second half of angle d phi; so, d phi by 2 and d phi by 2.

The projection of force F to the vertical direction is F times sine of d phi by 2. And, we have two forces; then, the resulting radial force is two times F sine of d phi by 2. If angle is very small, we can write that value of sine is same that the angle in radial in each theoretical work, we shall calculate or think about the angles with radials. So, the sine of d phi by 2 is d phi by 2, because it is elementally small. Then, we obtain F times d phi. Volume of such differentially small part of the fiber is which of fiber cross section is s. So, the volume of this d V is length of the fiber times cross section; length of the fiber is a part of ring on the radius r; it is radius r k; is not it? And, angle is d phi. So, r k times d phi is length of fiber times fiber cross section; it is this. This is on the r k, we use r by sine square. Once more – length of part UV of fiber is which? Radius times angle. Radius is r k; angle is d phi. So, r k times d phi is the length UV. Now, it is clear. This is the length; length times r k times D phi; length times fiber cross section is volume of the red fiber segment or elemental segment. It is easy; is not it? And, use now on the place of r k; our earlier expression we obtained is here; I think now it is clear.

Let us calculate centripetal force per unit volume of fiber. (( )) is dP here by the volume of fiber. So, dP by dv. Using our equations after rearranging, we obtain this force; P 1 is given by F times sine square beta by r s; beta is given by equation tangent beta is 2 pi r Z. Back to this picture (Refer Slide Time: 30:10) – Theoretically, each fiber each fiber compressed the material or can bring some (( )) compression, but really, it cannot be too real. They are fibers around yarn surface. In reality, the packing density in vicinity of yarn surface is in reality small. The fiber to fiber contacts are not so intensive; different silly pitch fiber is possible, so that the friction is not fully used; so that the radial force from such fibers is not too high; you can imagine. Second – the fibers around yarn axis – there is lot of fibers. They have good normal forces for friction, but they are need to straight line; the angle beta is very small; the radius of curvature is very high. Therefore, the radial component from such fiber is extremely small. Result – these fibers also do not influence significantly for compression of fibrous material inside the yarn. So, this is the simplification.

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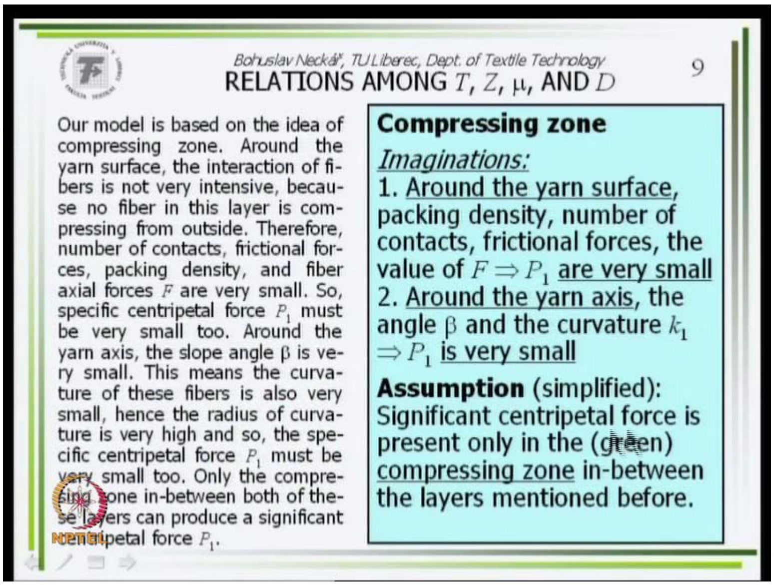

Significance centripetal force is present only in the green compressing zone in-between the layers mentioned before. So, now, whole material, but only the material in some (Refer Slide Time: 32:13) schematically green layer has significant influence to compression of fibrous material.

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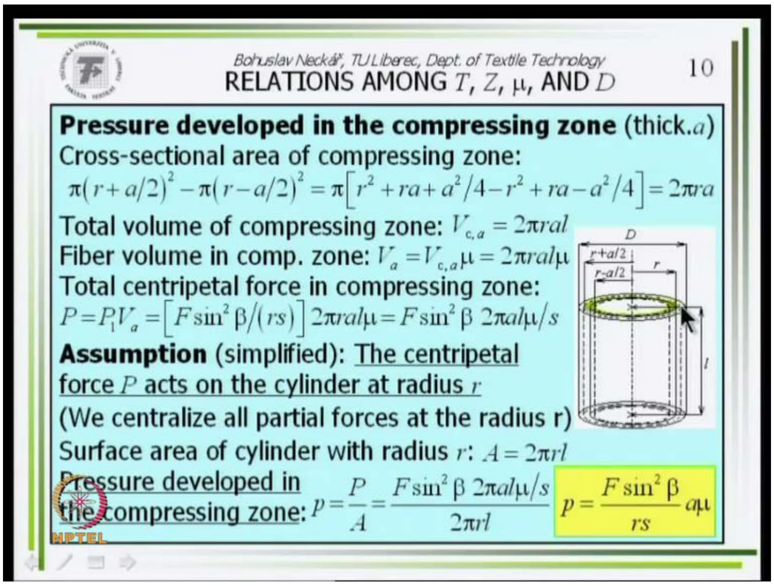

The thickness of this green zone go under the symbol a. This a – middle radius of this green layer is called r, so that the radii of this green layer are going from r minus a by 2 to r plus a by 2. The area of this green annulus is shown here is 2 pi r a evidently. Total volume of compressing zone is green zone. It is V c, a – 2 pi r a times l; volume area times r. Fiber volume in compressing zone is (( )) volume times mu from definition of packing density. So, 2 pi r a l times mu. And, the total centripetal force in compressing zone, P is P 1 times… P 1 we know; we know P 1 times V a. Using this equation after more rearranging, we obtain such equation.

Our assumption for simplification is the centripetal force P acts on the cylinder at radius r. They are fibrous like on the smaller radii in our green zone as well as some other fibers, which are lying on higher radius than r – the average layer of our zone. But, we make it easier and we all affects concentrate to some average radius, our radius r. Do understand this assumption. Then, how is the surface area of the cylinder on the radius r, that is, 2 pi r times… And, how is the pressure, which creates our yarn, which compressed our yarn. The pressure is force by A. We calculate this A on the average radius of our green zone; yes, using our equations for P and for A – this is for P; this is for A – (Refer Slide Time: 35:39) we obtain this after small rearranging. The pressure is given by such equation; this yellow equation.

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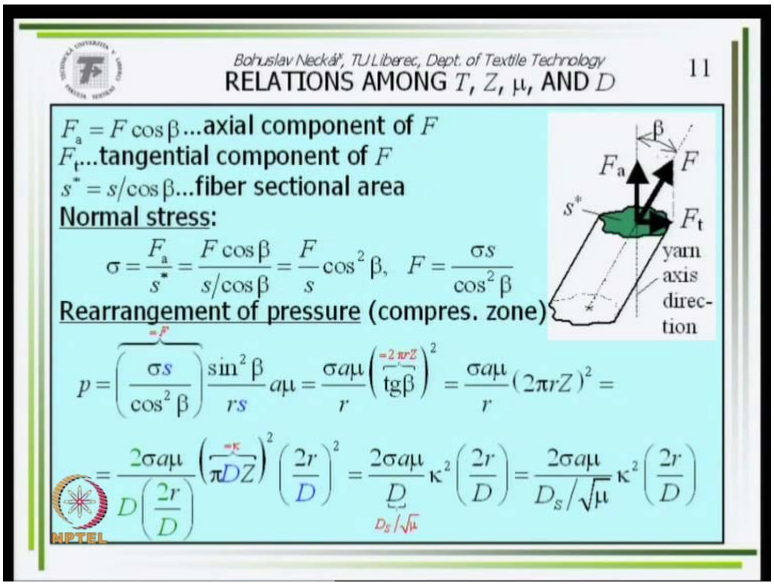

You see here is some part of some fiber. In such fiber, the axial for F exist. The axial component of this data – the force F – the direction of fiber axis; the component of this force in direction to yarn axis is a component F a; is not it? Axial means the direction to yarn axis. It is evident that F a is F times cosine of beta. By the way, it exists also such tangential force, which from all fibers together give some thousand moments in yarn, but, it is other (( )). The green section area of the fiber s star is – we mentioned it lot of times earlier – it is s by cosine of beta, so that the normal stress on the green area – it is normal force to green area F a by area s star; F a by s star. F times cosine of beta by s by cosine of beta when we use expressions derived earlier. So, it is F by s times cosine square beta. So, F therefore, is sigma s by cosine square beta. That is rearrangement.

Now, our formula for pressure – using this expression, p is possible to write also the black symbols here; identical is our earlier equation (Refer Slide Time: 38:29). When I multiply and divide by s, I obtain this expression (Refer Slide Time: 38:37). And, in brackets, – what is it in brackets? Now, this is the normal stress sigma, so that from this equation, we can say that this is sigma s by cosine square beta; and, this is force F. We can write this equation – tangents beta – it is 2 pi r Z; use 2 pi r Z on the place of tangents beta. Now, we black symbols are the same expressions as here. Here we multiply and divide by blue D, yarn diameter; we also multiply and divide by green D, yarn diameter; then, pi D z is kappa intensity of twist. So, we can write this expression (Refer Slide Time: 39:52). And, D yarn diameter here is D s by square root of mu. D s was in our lecture 1 – substance diameter – diameter of hypothetic yarn, which has not added inside of body. Yes. So, for pressure, we obtain now this expression after such rearrangement.

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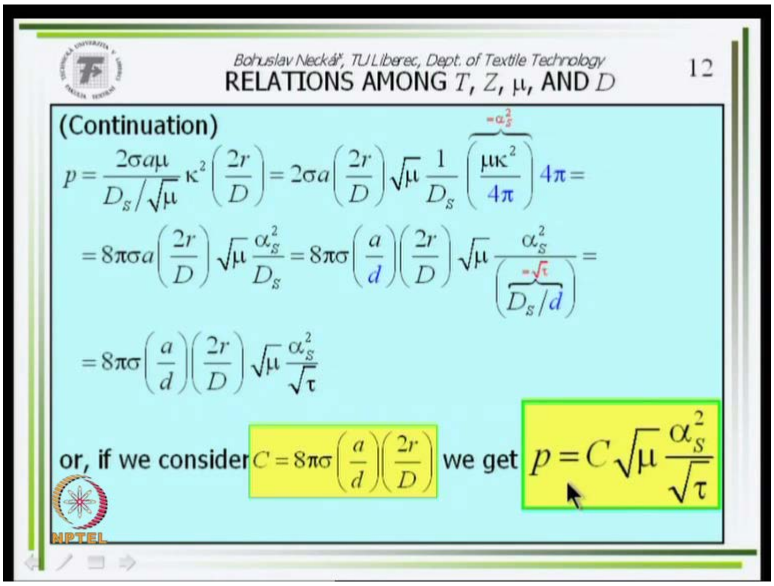

Let us continue our work with rearranging. This is repetition from last slide. We can graphically calculate; write it also – these black symbols. And, we multiply and divide by 4 times pi. And, we do it. Then, this expression when you compare it with equation for alpha s in our lecture 1, you can see if alpha square… So, we can write it in such form. Last, multiply and divide blue diameter of fiber; we divide by fiber diameter here; and, divide it in denominator; it mean multiply; so, we can, but here it is square root of Tau of relative count, relative finenesses of the yarn – also, from lecture 1. So, we obtain for pressure, this equation (Refer Slide Time: 41:38) – this expression for pressure.

We can call on the symbol C – this part (Refer Slide Time: 41:53). And so, we obtain what is here; we obtain the formula P is C times square root of mu times alpha s square by square root of Tau. What is this symbol? Here sigma is the normal pressure and on fiber area in yarn cross section; a is the thickness of the green zone or compressing zone; d is fiber diameter; r is average radius of the green zone; and, D is yarn diameter. So, we obtain this equation.

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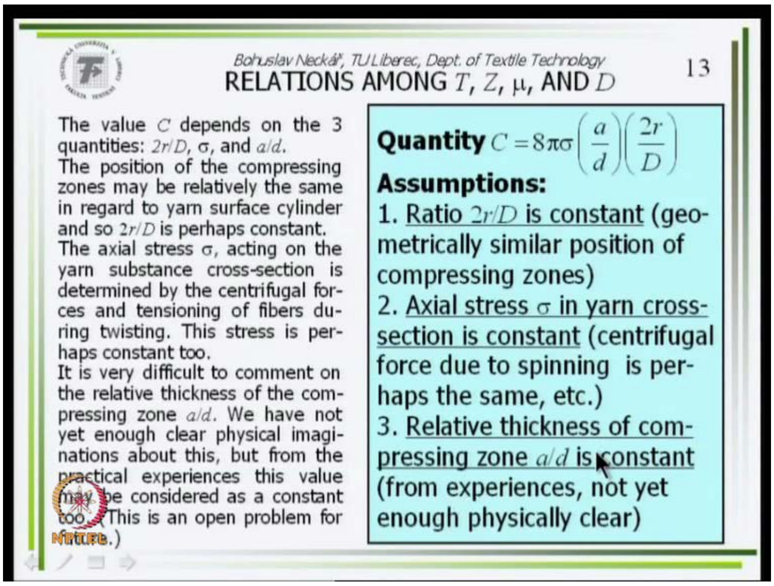

Let us discuss the quantity C; 2 r by D – it is said the position of the green zone – more precisely, average radius in the ring of yarn cross section. We can assume that this position is ratio, because the geometric similarity is a constant for yarn of given technological material and so on. Axial stress sigma in yarn cross-section – we assume is constant too, for example, centrifugal force due to spinning is perhaps the same.

Now, the centrifugal force – this stress from the centrifugal effect, so that we can imagine that also the sigma is constant. Relative thickness of compressing zone is the ratio a by d. It is difficult to explain. All experiments say that the least ratio shall be also constant value in the yarn. Why? We have some semi hypothesis for this, but often say the assumption that this ratio is constant, is not fully theoretically analyzed and based at most on the experimental results. Nevertheless, we will use it. We will assume that a by d is constant too. Then, hold this parameter C, is some characteristic constant in the yarn.

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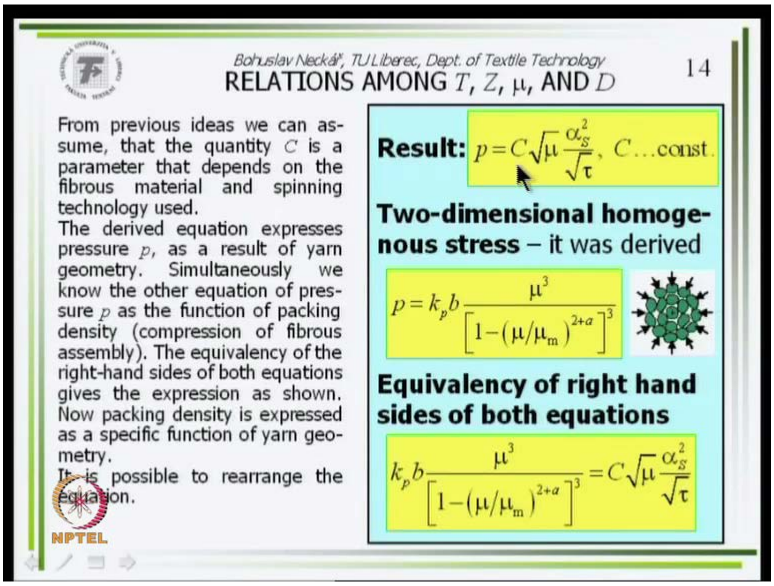

And, we can write p is C times square root of mu times alpha s square by square root of Tau, where C is some constant.

Our hour is out. Thank you for your attention. In next lecture, we will show how to apply (( )) derived pressure. This pressure is derived from geometrical relations inside of the yarn structure for the yarn structure. In next lecture, we will use also our known equation for pressure from our lecture about the compressibility of fibrous material. Thank you for your attention.