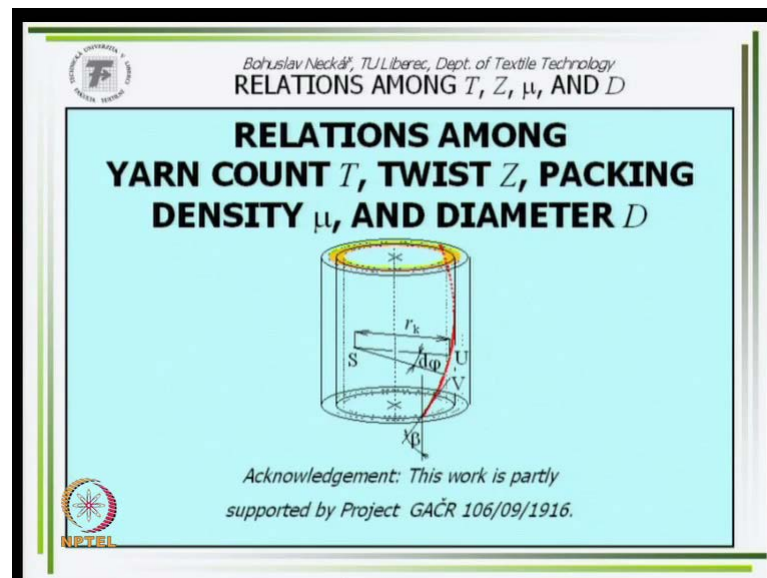


Orientation of Fibers
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Lecture No. # 09
Mechanics of Parallel Fiber Bundles

In earlier lectures, we spoke about the terms like yarn count, yarn twist, packing density; also, we mentioned yarn diameter.

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How is the relation among these often used quantities? It will be theme of today's lecture.

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RELATIONS AMONG T , Z , μ , AND D

The relations among count, twist, and diameter of yarn are related to specific geometrical and mechanical properties of yarn. The basic quantity "under" these relations is yarn packing density. The 1st traditional theoretical model regarding these relations was introduced by Koechlin in 1828. He studied the yarns produced from same fibrous material using same technology for analogical end-uses. At that time enough scientific knowledge about the mechanics of fibrous assembly and yarn geometry was not known, so it was necessary to substitute the unknown relations by the first categorical assumption as shown here.

THEORETICAL MODEL LIKE KOECHLIN'S CONCEPT

Initial assumptions
(problem limitation):

- Same fibrous material
- Same type of technology
- Same kind of use

Koechlin's 1st assumption:
(Substit. of know. of mech.)
Packing density is a function of twist intensity ONLY.

$\mu = f(\kappa)$ ($\kappa = \pi D Z$)

NPTEL

It is nothing new. 200 years ago, mister Koechlin in one French town **Mews**, presented his first model about the relation among these quantities. So, we start a very **alternatical** concept, which is roughly 200 years old. This model is usually quoted as a model-like Koechlin. Let us accept initial assumptions, which **limited** our program. Let us think that our yarns are produced from same fibrous material, from same type of technology and from same kind of use or similar kind of use. I will not repeat these assumptions, but we automatically will think about this limit of yarns.

The Koechlin's first assumption substitutes our knowledge of mechanics. We discussed earlier about the possibility how to calculate the relation between pressure and packing density. The fibrous material is compressed due to twist; is not it, in the yarn. But, in the time of mister Koechlin, this relation was not known, so that he must use some assumption. This assumption is here. Let us assume that packing density is a function of twist intensity only. Is in reality packing density the function of twist intensity? Evidently, yes. When I have higher twist, the twist intensity is increasing and the packing density is increasing too; it is evident. But, the assumption is that the packing density is the function of intensity of twist only. Later, we will show that packing density more precisely is the function of intensity of this, but also, it is a function of other quantities.

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Consequences of Koechlin's 1st assumption

Twist coefficients

- **areal Koechlin's type:** $\alpha_s = Z\sqrt{S} = \frac{\kappa\sqrt{\mu}}{2\sqrt{\pi}}$, $\alpha_s = \frac{\kappa\sqrt{f(\kappa)}}{2\sqrt{\pi}}$
...function of κ only!
- **common Koechlin's type:** $\alpha = Z\sqrt{T} = \frac{\kappa\sqrt{\mu\rho}}{2\sqrt{\pi}}$, $\alpha = \frac{\kappa\sqrt{f(\kappa)\rho}}{2\sqrt{\pi}}$
...function of κ only!

Diameter multipliers

- **areal multiplier:** $D = K_s\sqrt{S}$, $K_s = \frac{2}{\sqrt{\pi\mu}}$, $K_s = \frac{2}{\sqrt{\pi f(\kappa)}}$
...function of κ only!
- **common multiplier:** $D = K\sqrt{T}$, $K = \frac{2}{\sqrt{\pi\mu\rho}}$, $K = \frac{2}{\sqrt{\pi f(\kappa)\rho}}$
...function of κ only!

In Koechlin, μ is a function of κ ; where κ was $\pi D Z$ – twist intensity. You know it from our lesson 1. How are the consequences of Koechlin's first assumption? We will use equations known from our lesson 1 based now in the form, which accepts first assumption of Koechlin. We derived areal Koechlin's type of twist coefficient; we called it as α_s . It was Z times square root of S ; yarn twist times substance cross sectional area of the yarn. And, after arranging in lesson 1, it was also this expression, where κ is twist intensity and μ is packing density; is not it? So, generally, α is a function of two variables: κ and μ . But, first assumption of Koechlin says that the μ is function of κ , so that now, α is a function of κ only. Similarly, this areal Koechlin's type of twist factor is using the theory.

In textile practice, use some common Koechlin type, which is this here. This expression was derived in lesson 1 too; more easier ρ – specific mass of fibrous material. And similarly, the α is given by such equation. You can see that α as well as α_s as well as α are functions of κ of intensity of this only; (Refer Slide Time: 06:13) only one variable on the right-hand side of these two equations. And, how it is these diameter multipliers? Areal diameter multiplier for D was K_s ; and K_s was 2 by square root of $\pi\mu$; back to our lesson 1. Now, because μ is function of intensity of twist only, K_s is a function of κ only. And similarly, the common multiplier K , because this is K times square root of T , is after such arranging function of intensity of

twist only. These four quantities are now based on the first assumption; the function of only one variable – it is kappa; it is intensity of twist.

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Second assumption tells us how to twist the yarn. Let us consider yarns of different counts produced from same material using same technology for analogical end-uses. We would like to have all yarn properties similar in this case. But, in reality, it is not possible. Therefore, we must go "one step back": what will happen if we consider only geometrically similar yarns? (Not all values, only those that are not related to fiber fineness.) Then, the other properties may be also more or less similar. In case of geometrical similarity, the corresponding angles are same. So, the angles β_D must be same and therefore the twist intensities κ must be same too.

Koechlin's 2nd assumption:
(Directed to suitable twist)
The twist intensity of yarns of different finenesses (counts) shall be same. $\kappa = \text{constant}$

Consequences of Koechlin's 2nd assumption

Twist coefficients
 $\alpha_s = \text{const}$ $\alpha = \text{const}$

Diameter multipliers
 $K_s = \text{const}$ $K = \text{const}$

Packing density (from 1st assumption) $\mu = \text{const}$

The second Koechlin's assumption is directed to suitable twist, said that the twist intensity of yarns of different finenesses, **different** counts, shall be same; kappa shall be constant. What is the logical root of this assumption? This logical root based on geometrical similarity – you know that when we have different geometrical objects, which are similar means geometrically similar, then corresponding angles are same; is not it? And, Koechlin thought that the yarn – some course yarn, some fine yarn; both will have same possibility; for application, we will have some similar properties; especially mechanical properties, geometrical properties when they are same from point of view of geometrical similarity are similar. Therefore, if these ideas we accept as logical root, therefore, also, the angle **beta d** – the angle of peripheral fiber in our idealized yarn, must be same in each yarn for the same use and so on.

And, what is the same angle? Tangent of peripheral angle, tangent of **beta d** is intensity of twist kappa, so that kappa shall be constant. What is now with these four equations and the first assumption when we accept that kappa is constant? In these four expressions, you do not know this function f, but we know that it is a common function for each equation; by the way, monotone increasing function it must be. So, when we use

kappa is constant, then evidently, alpha is alpha K s (Refer Slide Time: 10:05) as well as K, must be constant; and, mu is constant too; is not it? Yes.

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RELATIONS AMONG T , Z , μ , AND D 4

The derived equations are traditionally used in textile praxis. The calculation of suitable yarn twist is often necessary for yarn production and the calculation of yarn diameter is necessary for fabric manufacturing. Unfortunately, the resulting values are not enough precise!
Why? According to the experiences, the assumption about the geometrical similarity is acceptable. On the other hand, the twist intensity κ is not a function of packing density μ ONLY. (It depends on yarn count too.) So, the first assumption is not enough precise, hence empirical corrections of Koechlin's theory are necessary.


Application of results
a) **Suitable yarn twist :**
 $Z = \alpha / \sqrt{T}$, where α is const.;
e.g. card. co. $\alpha = 120 \text{ m}^{-1} \text{ ktex}^{1/2}$
b) **Yarn diameter :**
 $D = K \sqrt{T}$, where K is const.;
e.g. card. co. $K = 0.0395 \text{ mm tex}^{-1/2}$
!!! Simple, but not enough precise !!!
 $\Rightarrow \alpha$ is different for different groups of yarn count.
 \Rightarrow Calculated yarn diameter is not very precise.

We said that good idea based on Koechlin's model, Koechlin's concept, is to have the same angle – peripheral angle of fiber, because geometrical similarity on each yarn. Now, this idea we can say to the people in spinning mill, you must measure the angle of peripheral fiber in your yarns. You can imagine what they can answer to you when you give this idea to your spinning mill. Nevertheless, it is possible so much for a region, because we said that the result alpha s, alpha, K s and K must be constant. So, we do not need to measure the angle; we can say, for example, for practice, in spinning mill, alpha must be constant. What is alpha? From definition of alpha of twist factor, twist coefficient, we can say that twist is alpha by square root of yarn count, means finenesses.

Then, we can say to working people in spinning mill, yes, you must twist each yarns. So, then, alpha is... I do not know what based on your experience in such spinning mill; for example, 120 meters to the power minus 1 kilo tex to the power one half. This is dimension – physical dimension of alpha; this is metrical alpha (Refer Slide Time: 12:05). And, when the people will produce another yarn count, then they use the same alpha. And, using this equation, they can very easily calculate in spinning mill, which of twist is necessary for this or that yarn count. This is the first result of Koechlin's theory.

The second result is, write it to our specialist in weaving technology. In weaving technology, you know, is necessary to know the yarn diameter, because covering cover factors, maybe covering can similar quantities, which defined the density of woven fabrics and so on. How to obtain the yarn? How to obtain the yarn diameter? Koechlin's theory said it is easy. Yarn diameter D is parameter K times square root of T and K must be constant. Diameter multiplier (Refer Slide Time: 13:27) must be constant for given material, given technology and given type of use of our yarns. Maybe based on our experiences, we can say, it must be 0.0395 for example. And, people can calculate it; very easy, very elegant theoretical model, 200 years old, but not too enough precise. It is very often used, because roughly, it is possible to apply it, but when you want to work more precisely, then these results are not enough. Why? The practical experiences say that such alpha must be a little different for different groups of yarn count, so that in textile hand books, you can read that from count, these two counts, usual use of this alpha; then, from count these two counts, this interval of counts and a little larger alpha and so on and so on. It is typical for hand books for spinning practice. This is one way.

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EMPIRICAL CORRECTIONS OF KOECHLIN'S THEORY

a) Suitable yarn twist
 $Z = \alpha / T^q$
 where q ...empirical value of twist exponent, $q > 0.5$

b) Yarn diameter
 $D = Q_\alpha T^w \alpha^v$
 where Q_α , w , v ...empirical values of parameters,
 5 (round $w = 0.56$) and
 5 (round $v = -0.22$)

Author	Year	Twist expon. q
Koechlin	1828	0.5
Staub	1900	0.6
Johansen	1902	0.644
Laetch	1905	0.785 warp
	1905	0.720 weft
	1941	0.62 – 0.75
Oeser	1937	0.565
	1940	0.47
Pluix	1942	0.666
Neckář	1971	0.577, 0.6
Salaba	1975	0.518 comb.
	1975	0.551, 0.570

Second empirical way is to empirically change the Koechlin's equations. On the place square root, it means power to one half. It is possible for yarn twist to use the ratio alpha by T power to some exponent q ; where, q exponent of twist is an empirical value; a little different from 0.5. Ruther fortress studied a problem in relation to this equation, which of exponent is the best; Koechlin's at 0.5; is not it square root? Then, lots of others have

different ideas based on this or that experimental experiences. The yarn diameter can be empirically generalized to such form, D is some parameter Q alpha times yarn count power to some exponent times alpha power to another exponent.

An example for these values may be good for carbon coated yarns is here. This exponent w is usually something around 0.56 in this equation and the exponent by alpha is usually minus 0.22. But, my experiences may be another (()) can see another world based on pipe of cotton beds or pipe of technology; based of lot local influences.

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The problems with model like Koechlin's concept initiated interest for deeper knowledge of internal yarn mechanics and motivated us to think alternative ways for solving the problems. The concept of radial forces equilibrium based on the differential equation (J. Hearle et al.) is general, but very complicated and requires knowledge on some not yet enough known inputs (e.g. stress-strain tensors relation). Our present model is "something in-between." It is not so difficult as the concept of radial forces equilibrium and the present knowledge about the mechanics of fibrous assemblies may be sufficient for it.

A SIMPLE MECHANICAL MODEL

Assumptions - helical model:

1. All fibers have the helical shape
2. All helixes have common axis, which is yarn axis
3. All helixes have the same sense of rotation
4. Each fiber coil have same height

Ideal helical model:

5. Packing density is same in all places inside the yarn


Why the model of Koechlin is not enough precise? His second assumption is very good; and, it is the earlier after Koechlin's experiences show that the geometrical similarity is very good idea. What is not too good is the first assumption (Refer Slide Time: 17:55) that the packing density is a function of twist intensity only. I mentioned it; Koechlin in 1828, had a chance to use some models, which respect the physical relation between pressure and compression of fibers inside of the twisted yarn. It exists some second way how to solve it. This way is in my check book, which professor Ishteyak gave; he can show you, it will be especially interesting for you.

The second way is go out from some differential equation of equilibrium of radial forces inside of the yarn body. And, based on tools of continue mechanics solve this problem for you as a problem of continue mechanic. I proved it earlier, lot years ago. But, there is a problem here. To this time, we do not know the relation between the stress tensor and

the strain tensor. Stress tensor and strain tensor are some structures very popular to say; to these days, we do not understand enough general the relation between stress and strain in multidimensional, three dimensional case especially for fibrous assembly. Therefore, we can calculate, we can derive the differential equation, but, we have not enough well input to this equation. We must make some assumptions, some simplifications and so on. All these are very difficult from point of view of mathematical tools; you must solve some differential equation and so on.

Nice theme – it was lot years ago; the theme of my PhD thesis. But, to these days, this way is not... and the position to be practical tool for application. Therefore, we derived something in between, which is very easy, but not too precise theory of Koechlin. And physically, the best version differential equation of radial equilibrium solving of this one; something in between, which is easier, not so precise from point of view of Physics, but better than Koechlin's type. Let us assume the non-assumptions from ideal helical model. All fibers have the helical shape; all helixes have common axis, which is yarn axis; all helixes have the same sense of rotation; each fiber coil have same height. We mentioned these assumptions when we analyzed the helical model. And, fifth – packing density is same in each place in the yarn.

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RELATIONS AMONG T , Z , μ , AND D

Our model is based on the helical geometry of fibers inside the yarn. A fiber coil (general) lying on the imaginary cylinder of radius r , is shown on the figure. We unwind the surface of this cylinder and obtain the triangle, whose geometry determines the fiber slope angle β .

It is known from analytical geometry, that the curvature of the helix is given by the equation on the left-hand side. Obviously, the radius of curvature (radius of the so-called osculating circle) is the reciprocal value of the curvature. These equations help to derive the centripetal force per unit volume of fiber as follows.

Fiber geometry – one coil:

General fiber angle:

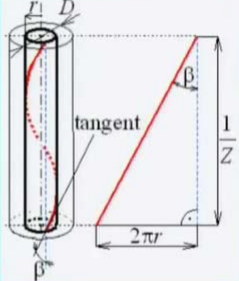
$$\operatorname{tg} \beta = \frac{2\pi r}{1/Z}$$

$$\operatorname{tg} \beta = 2\pi r Z$$

First curvature (flection): $k_1 = \sin \beta / r$

Radius of curvature:

$$r_k = 1/k_1, \quad r_k = r / \sin^2 \beta$$



Then, it is more repetition for a helical model. Let us imagine some general fiber inside the yarn body; yarn body is this. Here schematically, the cylinder having diameter D .

And, inside on some general radius r . This thick black cylinder is like one fiber – red fiber **on** the general radius r helix shape. After **unwind** of this cylinder, we obtain such triangle, which is possible to the tangent beta, which is $2\pi r Z$; tangent beta is $2\pi r Z$ is known for you from our earlier lecture. When you open some hand book about the mathematic and when you find some properties of different curves, also for 3D curves, space curves, you can read also what is so-called first curvature – also, means flection is used. This first curvature of three dimensional curves in the space in the case of helix is constant, **independent to body to points** on which you measure it. And, it is $k = 1/r \sin \beta$ – sine of beta by r ; you can read it in each hand book.

Reciprocal value of first curvature – it is radius of curvature; it is the radius of some rings, which can approximate our curve in a very theoretically infinitesimal path. You know what this radius of curvature is, so that the radius of curvature of such helix is $r \sin^2 \beta$; is not it?

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Centripetal force per unit volume of fiber

UV...fiber element
 F ...axial fiber force
 $d\varphi$...elementary angle
Centripetal force of UV:
 $dP = 2F \sin(d\varphi/2) = 2F d\varphi/2 = F d\varphi$
Volume of UV:
 s ...fiber cross-sect. area
 $dV = r_k d\varphi s = r d\varphi s / \sin^2 \beta$
Centripetal force per unit volume of fiber :

$P/dV = F d\varphi / (r d\varphi s / \sin^2 \beta)$, $P_1 = F \sin^2 \beta / (rs)$ ($\tan \beta = 2\pi r Z$)

Let us think now about a fiber lying in yarn body on a hypothetical cylinder of **harass**. It is not from metal; it is only imagination. On a radius r , this is the red fiber. Let us think about the elemental part of this fiber – part UV. This is the radius of curvature, r_k . On the angle here is differentially small; it is angle $d\varphi$. In the fiber, let us be some axial force – capital F – axial fiber force. So, when we make the picture of this part especially here, that you can see UV – our elemental part of infinitesimal small part of fiber on

which is tangential force F – axial force in fiber. This is the angle ϕ , one half; it is no wider the straight line is not to see here. This is so-called angle $d\phi$; and, this is one half and this is second half of angle $d\phi$; so, $d\phi/2$ and $d\phi/2$.

The projection of force F to the vertical direction is F times $\sin(d\phi/2)$. And, we have two forces; then, the resulting radial force is two times $F \sin(d\phi/2)$. If angle is very small, we can write that value of sine is same that the angle in radial in each theoretical work, we shall calculate or think about the angles with radials. So, the sine of $d\phi/2$ is $d\phi/2$, because it is elementally small. Then, we obtain F times $d\phi$. Volume of such differentially small part of the fiber is which of fiber cross section is s . So, the volume of this dV is length of the fiber times cross section; length of the fiber is a part of ring on the radius r ; it is radius r ; is not it? And, angle is $d\phi$. So, r times $d\phi$ is length of fiber times fiber cross section; it is this. This is on the r , we use r by sine square. Once more – length of part UV of fiber is which? Radius times angle. Radius is r ; angle is $d\phi$. So, r times $d\phi$ is the length UV. Now, it is clear. This is the length; length times r times $d\phi$; length times fiber cross section is volume of the red fiber segment or elemental segment. It is easy; is not it? And, use now on the place of r ; our earlier expression we obtained is here; I think now it is clear.

Let us calculate centripetal force per unit volume of fiber. $(\)$ is dP here by the volume of fiber. So, dP by dv . Using our equations after rearranging, we obtain this force; P is given by F times $\sin^2 \beta$ by r ; β is given by equation $\tan \beta = 2\pi r Z$. Back to this picture (Refer Slide Time: 30:10) – Theoretically, each fiber each fiber compressed the material or can bring some $(\)$ compression, but really, it cannot be too real. They are fibers around yarn surface. In reality, the packing density in vicinity of yarn surface is in reality small. The fiber to fiber contacts are not so intensive; different silly pitch fiber is possible, so that the friction is not fully used; so that the radial force from such fibers is not too high; you can imagine. Second – the fibers around yarn axis – there is lot of fibers. They have good normal forces for friction, but they are need to straight line; the angle β is very small; the radius of curvature is very high. Therefore, the radial component from such fiber is extremely small. Result – these fibers also do not influence significantly for compression of fibrous material inside the yarn. So, this is the simplification.

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Our model is based on the idea of compressing zone. Around the yarn surface, the interaction of fibers is not very intensive, because no fiber in this layer is compressing from outside. Therefore, number of contacts, frictional forces, packing density, and fiber axial forces F are very small. So, specific centripetal force P_1 must be very small too. Around the yarn axis, the slope angle β is very small. This means the curvature of these fibers is also very small, hence the radius of curvature is very high and so, the specific centripetal force P_1 must be very small too. Only the compressing zone in-between both of these layers can produce a significant centripetal force P_1 .

Compressing zone
Imaginations:

1. Around the yarn surface, packing density, number of contacts, frictional forces, the value of $F \Rightarrow P_1$ are very small
2. Around the yarn axis, the angle β and the curvature $k_1 \Rightarrow P_1$ is very small

Assumption (simplified): Significant centripetal force is present only in the (green) compressing zone in-between the layers mentioned before.

Significance centripetal force is present only in the green compressing zone in-between the layers mentioned before. So, now, whole material, but only the material in some (Refer Slide Time: 32:13) schematically green layer has significant influence to compression of fibrous material.

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Pressure developed in the compressing zone (thick, a)

Cross-sectional area of compressing zone:

$$\pi(r+a/2)^2 - \pi(r-a/2)^2 = \pi[r^2 + ra + a^2/4 - r^2 + ra - a^2/4] = 2\pi ra$$

Total volume of compressing zone: $V_{c,a} = 2\pi ral$

Fiber volume in comp. zone: $V_a = V_{c,a}\mu = 2\pi ral\mu$

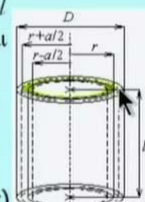
Total centripetal force in compressing zone:

$$P = P_1 V_a = [F \sin^2 \beta / (rs)] 2\pi ral\mu = F \sin^2 \beta 2\pi al\mu / s$$

Assumption (simplified): The centripetal force P acts on the cylinder at radius r
 (We centralize all partial forces at the radius r)

Surface area of cylinder with radius r : $A = 2\pi rl$

Pressure developed in the compressing zone:
$$p = \frac{P}{A} = \frac{F \sin^2 \beta 2\pi al\mu / s}{2\pi rl} \quad p = \frac{F \sin^2 \beta}{rs} a\mu$$




The thickness of this green zone go under the symbol a. This a – middle radius of this green layer is called r, so that the radii of this green layer are going from r minus a by 2 to r plus a by 2. The area of this green annulus is shown here is 2 pi r a evidently. Total

volume of compressing zone is green zone. It is $V_c = 2\pi r a l$; volume area times r . Fiber volume in compressing zone is $(())$ volume times μ from definition of packing density. So, $2\pi r a l$ times μ . And, the total centripetal force in compressing zone, P is P_1 times... P_1 we know; we know P_1 times V_a . Using this equation after more rearranging, we obtain such equation.

Our assumption for simplification is the centripetal force P acts on the cylinder at radius r . They are fibrous like on the smaller radii in our green zone as well as some other fibers, which are lying on higher radius than r – the average layer of our zone. But, we make it easier and we all affects concentrate to some average radius, our radius r . Do understand this assumption. Then, how is the surface area of the cylinder on the radius r , that is, $2\pi r$ times... And, how is the pressure, which creates our yarn, which compressed our yarn. The pressure is force by A . We calculate this A on the average radius of our green zone; yes, using our equations for P and for A – this is for P ; this is for A – (Refer Slide Time: 35:39) we obtain this after small rearranging. The pressure is given by such equation; this yellow equation.

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$F_a = F \cos \beta$...axial component of F
 F_t ...tangential component of F
 $s^* = s / \cos \beta$...fiber sectional area
Normal stress:

$$\sigma = \frac{F_a}{s^*} = \frac{F \cos \beta}{s / \cos \beta} = \frac{F}{s} \cos^2 \beta, \quad F = \frac{\sigma s}{\cos^2 \beta}$$

Rearrangement of pressure (compres. zone)

$$p = \left(\frac{\sigma s}{\cos^2 \beta} \right) \frac{\sin^2 \beta}{rs} a \mu = \frac{\sigma a \mu}{r} \left(\frac{-2\pi r Z}{\tan \beta} \right)^2 = \frac{\sigma a \mu}{r} (2\pi r Z)^2 =$$

$$= \frac{2\sigma a \mu}{D} \left(\frac{2r}{D} \right)^2 \left(\frac{2r}{D} \right)^2 = \frac{2\sigma a \mu}{D_s \sqrt{\mu}} \kappa^2 \left(\frac{2r}{D} \right)$$

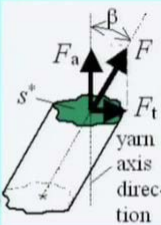



Diagram illustrating the forces F_a and F_t acting on a fiber cross-section at an angle β to the yarn axis direction. The fiber sectional area is s^* .



You see here is some part of some fiber. In such fiber, the axial for F exist. The axial component of this data – the force F – the direction of fiber axis; the component of this force in direction to yarn axis is a component F_a ; is not it? Axial means the direction to yarn axis. It is evident that F_a is F times cosine of beta. By the way, it exists also such

tangential force, which from all fibers together give some thousand moments in yarn, but, it is other (\cos^2) . The green section area of the fiber star is – we mentioned it lot of times earlier – it is s by cosine of beta, so that the normal stress on the green area – it is normal force to green area F_a by area s star; F_a by s star. F times cosine of beta by s by cosine of beta when we use expressions derived earlier. So, it is F by s times cosine square beta. So, F therefore, is σs by cosine square beta. **That** is rearrangement.

Now, our formula for pressure – using this expression, p is possible to write also the black symbols here; identical is our earlier equation (Refer Slide Time: 38:29). When I multiply and divide by s , I obtain this expression (Refer Slide Time: 38:37). And, in brackets, – what is it in brackets? Now, this is the normal stress σ , so that from this equation, we can say that this is σs by cosine square beta; and, this is force F . We can write this equation – tangents beta – it is $2 \pi r Z$; use $2 \pi r Z$ on the place of tangents beta. Now, we **black** symbols are the same expressions as here. Here we multiply and divide by blue D , yarn diameter; we also multiply and divide by green D , yarn diameter; then, $\pi D z$ is kappa intensity of twist. So, we can write this expression (Refer Slide Time: 39:52). And, D yarn diameter here is D_s by square root of μ . D_s was in our lecture 1 – substance diameter – diameter of hypothetic yarn, which has not added inside of body. **Yes**. So, for pressure, we obtain now this expression after such rearrangement.

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(Continuation)

$$p = \frac{2\sigma a \mu}{D_s \sqrt{\mu}} \kappa^2 \left(\frac{2r}{D}\right) = 2\sigma a \left(\frac{2r}{D}\right) \sqrt{\mu} \frac{1}{D_s} \left(\frac{\mu \kappa^2}{4\pi}\right) 4\pi =$$

$$= 8\pi \sigma a \left(\frac{2r}{D}\right) \sqrt{\mu} \frac{\alpha_s^2}{D_s} = 8\pi \sigma \left(\frac{a}{d}\right) \left(\frac{2r}{D}\right) \sqrt{\mu} \frac{\alpha_s^2}{\left(\frac{D_s}{d}\right)} =$$

$$= 8\pi \sigma \left(\frac{a}{d}\right) \left(\frac{2r}{D}\right) \sqrt{\mu} \frac{\alpha_s^2}{\sqrt{\tau}}$$

or, if we consider $C = 8\pi \sigma \left(\frac{a}{d}\right) \left(\frac{2r}{D}\right)$ we get $p = C \sqrt{\mu} \frac{\alpha_s^2}{\sqrt{\tau}}$

Let us continue our work with rearranging. This is repetition from last slide. We can graphically calculate; write it also – these black symbols. And, we multiply and divide by 4 times pi. And, we do it. Then, this expression when you compare it with equation for alpha s in our lecture 1, you can see **if** alpha square... So, we can write it in such form. Last, multiply and divide blue diameter of fiber; we divide by fiber diameter here; and, divide it in denominator; it mean multiply; so, we can, but here it is square root of **Tau** of relative count, relative finenesses of the yarn – also, from lecture 1. So, we obtain for pressure, this equation (Refer Slide Time: 41:38) – this expression for pressure.

We can call on the symbol C – this part (Refer Slide Time: 41:53). And so, we obtain what is here; we obtain the formula P is C times square root of mu times alpha s square by square root of Tau. What is this symbol? Here sigma is the normal pressure and on fiber area in yarn cross section; a is the thickness of the green zone or compressing zone; d is fiber diameter; r is average radius of the green zone; and, D is yarn diameter. So, we obtain this equation.

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The value C depends on the 3 quantities: $2r/D$, σ , and a/d . The position of the compressing zones may be relatively the same in regard to yarn surface cylinder and so $2r/D$ is perhaps constant. The axial stress σ , acting on the yarn substance cross-section is determined by the centrifugal forces and tensioning of fibers during twisting. This stress is perhaps constant too. It is very difficult to comment on the relative thickness of the compressing zone a/d . We have not yet enough clear physical imaginations about this, but from the practical experiences this value may be considered as a constant too. (This is an open problem for future.)

Quantity $C = 8\pi\sigma \left(\frac{a}{d}\right) \left(\frac{2r}{D}\right)$

Assumptions:

1. Ratio $2r/D$ is constant (geometrically similar position of compressing zones)
2. Axial stress σ in yarn cross-section is constant (centrifugal force due to spinning is perhaps the same, etc.)
3. Relative thickness of compressing zone a/d is constant (from experiences, not yet enough physically clear)

Let us discuss the quantity C; $2r$ by D – it is said the position of the green zone – more precisely, average radius in the ring of yarn cross section. We can assume that this position is ratio, because the geometric similarity is a constant for yarn of given technological material and so on. Axial stress sigma in yarn cross-section – we assume is constant too, for example, centrifugal force due to spinning is perhaps the same.

Now, the centrifugal force – this stress from the centrifugal effect, so that we can imagine that also the sigma is constant. Relative thickness of compressing zone is the ratio a by d. It is difficult to explain. All experiments say that the **least** ratio shall be also constant value in the yarn. Why? We have some semi hypothesis for this, but often say the assumption that this ratio is constant, is not fully theoretically analyzed and based at most on the experimental results. Nevertheless, we will use it. We will assume that a by d is constant too. Then, **hold** this parameter C, is some characteristic constant in the yarn.

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The slide, titled "RELATIONS AMONG T , Z , μ , AND D ", is presented by Bohuslav Neckář, TU Liberec, Dept. of Textile Technology. It contains the following text and equations:

From previous ideas we can assume, that the quantity C is a parameter that depends on the fibrous material and spinning technology used. The derived equation expresses pressure p , as a result of yarn geometry. Simultaneously we know the other equation of pressure p as the function of packing density (compression of fibrous assembly). The equivalency of the right-hand sides of both equations gives the expression as shown. Now packing density is expressed as a specific function of yarn geometry. It is possible to rearrange the equation.

Result:
$$p = C \sqrt{\mu} \frac{\alpha_s^2}{\sqrt{\tau}}, C \dots \text{const.}$$

Two-dimensional homogeneous stress – it was derived

$$p = k_p b \frac{\mu^3}{[1 - (\mu/\mu_m)^{2+a}]^3}$$

Equivalency of right hand sides of both equations

$$k_p b \frac{\mu^3}{[1 - (\mu/\mu_m)^{2+a}]^3} = C \sqrt{\mu} \frac{\alpha_s^2}{\sqrt{\tau}}$$

And, we can write p is C times square root of μ times α_s square by square root of τ , where C is some constant.

Our hour is out. Thank you for your attention. In next lecture, we will show how to apply **(())** derived pressure. This pressure is derived from geometrical relations inside of the **yarn structure for the yarn structure**. In next lecture, we will use also our known equation for pressure from our lecture about the compressibility of fibrous material. Thank you for your attention.