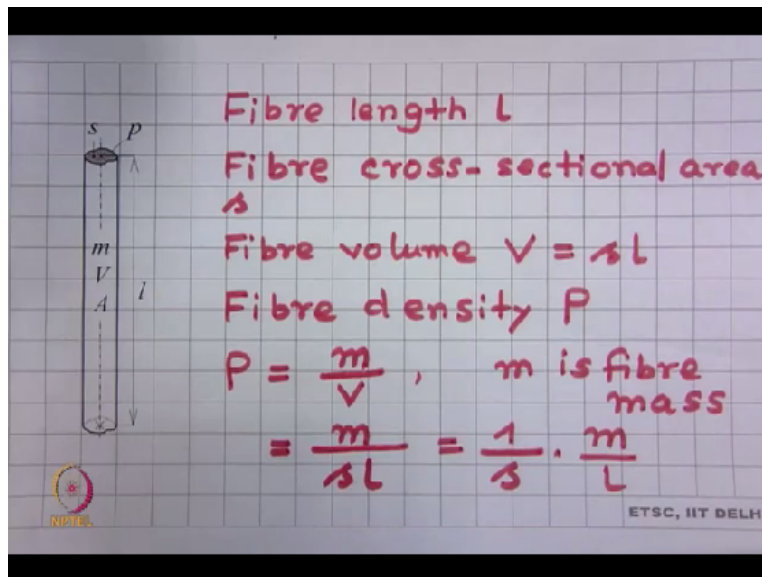


Theory of Yarn Structure
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Lecture – 01
The Building Block of Yarns

Welcome to this MOOCS online video course, Theory of Yarn Structure. Today, we will start module 1, Fibre, The Building Block of Yarns. In this module, we will define certain characteristics of fibre and we will establish the relationships among those characteristics. Specifically, we will talk about geometrical characteristics and mechanical characteristics of a single fibre.

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What you see here is a single fibre, is a scheme of a single fibre. This fibre has a length L . It also has a cross-sectional area s . So the volume of this fibre is cross-sectional area*length. Let us assume that this fibre has a density, we use symbol rho to denote the density of this fibre. From definition, we know density=mass/volume, where m is fibre mass. Well, if we substitute volume in terms of $s*L$, then we obtain this expression. If we little rearrange this expression, then we obtain this expression.

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$$\text{Fibre fineness } t = \frac{m}{L}$$

$$\rho = \frac{1}{s} \cdot \frac{m}{L} = \frac{1}{s} \cdot t = \frac{t}{s}$$

$$\rho = \frac{t}{s} ; t = s\rho$$

$$\text{Fibre size } s = \frac{t}{\rho}$$

Now we introduced one more terminology, that is fibre fineness. Fibre fineness is t which is equal to mass per unit length of the fibre. If we remember our earlier expression fibre density = $1/\text{cross-sectional area} \cdot \text{mass per unit length}$. Now if we substitute this mass per unit length by fineness, then we obtain this expression. If we little rearrange, then we finally obtain this expression.

So what it this expression? Rho fibre density = fibre fineness / fibre cross-sectional area. This is a very interesting expression. That means, t , fibre fineness = fibre cross-sectional area * fibre density. Now what is interesting here you see that fibre fineness, that we often use to characterize the fineness or coarseness of a fibre if not equal to the cross-sectional area only. However, fibre fineness = cross-sectional area * density.

That means if we talk about fibre size or size of any geometrical object, we often use cross-sectional area to denote the size of a material. Whether a material is thick or a material is thin, so we use cross-sectional area to characterize the size of a fibre. Now this cross-sectional area = fibre fineness / rho. It may so happen that a fibre being heavier may have higher numerical value of t , that means this fibre fineness is not a very suitable expression to denote the size of a fibre.

The fibre density is involved into that. We will come to this expression when we will solve some numerical problem. There we will be able to find out that fineness of 2 fibres may be different

but their cross-sectional size may be practically same because their densities are different. So geometrically 2 fibres having different values of fineness, may have same size. So the traditional expression of fibre fineness is not able to recognize this fact.

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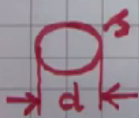
Fibre	Fineness
Micro fibre	< 1 dtex
Cotton or compatible synthetic fibre	1.6 dtex
Wool or compatible fibre	2.3 dtex

Well, Now typically if we look at the values of say fibre fineness, then what we see that there are micro fibres. Typically, the fineness is less than 1 decitex. There are also cotton or compatible synthetic fibres, their fineness range typically 1.6 decitex. There can be wool or compatible fibre, the fineness will be little high, say 2.3 decitex.

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Coarse (carpet) fibre > 2.7 dtex

Fibre cross-sectional area s



$$s = \frac{\pi d^2}{4}; d = \sqrt{\frac{4s}{\pi}}$$

d ... fibre diameter

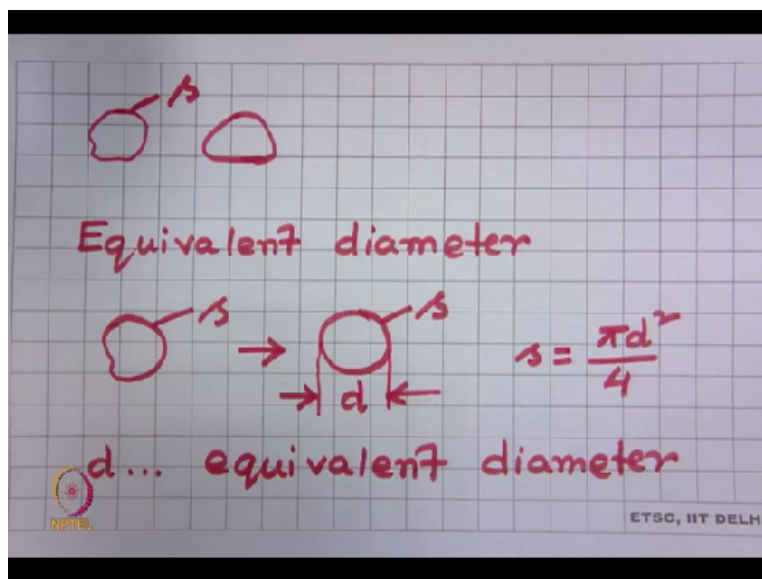
And there could be coarse fibre which are sometimes called as carpet fibre. Their fineness can be

greater than 2.7 decitex. So below the very practical values of fibre fineness, we typically deal with such fibres. Now if you have probably noticed that so far we talked about a typical fibre. We did not tell about the cross-section of this fibre. We have never mentioned that the scheme of the fibre was considered to be cylindrical.

So it was a very general fibre. Now these general fibre has a cross-sectional area what we consider s . Now in practice, a fibre cross-section may be circular, may not be circular. If it is circular, like this, if this fibre cross-section has an area s , then we can simply write this cross-sectional area $= \pi d^2 / 4$ where this is d . Here d stands for fibre diameter. So practically how could you calculate fibre diameter?

You will measure the cross-sectional area, then you will use this expression to determine diameter of a fibre. As I told you, practically there can be many fibres which do not have circular cross-section.

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Suppose a fibre may have non-circular cross-section or say a fibre may have this type of cross-section. How would you find out diameter? For such object, we introduce a term called equivalent diameter. Let me define this term, what is equivalent diameter? Suppose this cross-section has an area s , then what we imagine, let us have a circle which has the same cross-sectional area s .

Then diameter of this circle, if it is d , then we can write $s = \pi d^2/4$. Then d is called equivalent diameter. That means if a fibre cross-section is circular, we use the term diameter and if a fibre has a non-circular cross-section, then we use the term equivalent diameter. And this is the way we practically determine equivalent diameter, right.

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Fibre shape
 Fibre perimeter p

$$\frac{p}{\pi d} \geq 1, \quad \frac{p}{\pi d} - 1 \geq 0$$

$= q$

$$q = \frac{p}{\pi d} - 1, \quad q \dots \text{fibre shape factor}$$

$q \geq 0$

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Now we talk about fibre shape. How would you characterize the cross-sectional shape of a fibre? Now it is well known that the perimeter of a non-cylindrical fibre is always greater than the perimeter of a cylindrical fibre. Suppose we introduce a term fibre perimeter here p . So the perimeter of a real fibre/perimeter of a circular fibre which is equal to $\pi \cdot d$ is greater than equal to 1. This equal to sign will come if it is cylindrical fibre; the greater than sign will come if it is non-cylindrical fibre.

Then we can further write $p/\pi \cdot d - 1$ is greater than equal to 0. This expression, let us use a symbol to denote this expression that is q . So this q is $p/\pi \cdot d - 1$ where this q is called fibre shape factor. This means this q can be used to characterize the cross-sectional shape of a fibre. How would you find out q ? $q = \text{perimeter of a real fibre} / \text{perimeter of a circular fibre having same cross-sectional area} - 1$. So evidently, the value of q will lie from 0 onwards. When it is 0, let us explain that.

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Fibre shape	q [-]	q [1]
circular (ideal)	0	
circular (real)	0 - 0.6	
Δ Triangular (ideal)	0.29	
\triangle Triangular (practical)	0.12 - 0.2	

$$q = \frac{P}{\pi d} - 1 = \frac{P [\text{mm}]}{\pi d [\text{mm}]} - 1$$

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Let us consider a few shape and the value of q . If it is circular, ideal circular cross-section, then the value of $q=0$. By the way, what is the dimension of q ? $q=$, this. Now if we use p in millimeter and d also in millimeter, then this ratio is dimensionless, then q becomes dimensionless. Often in SI system, a dimensionless quantity is either expressed by this or sometimes they are also expressed by this.

Now one thing I must tell you that so far we have learnt a few expressions. All the expressions can be expressed in terms of suitable units. For example, this particular expression we have expressed in terms of practical units. You must practice all these expressions in terms of units. We generally use this style to write a variable and its unit together. All the expressions we have derived so far can be written in terms of units.

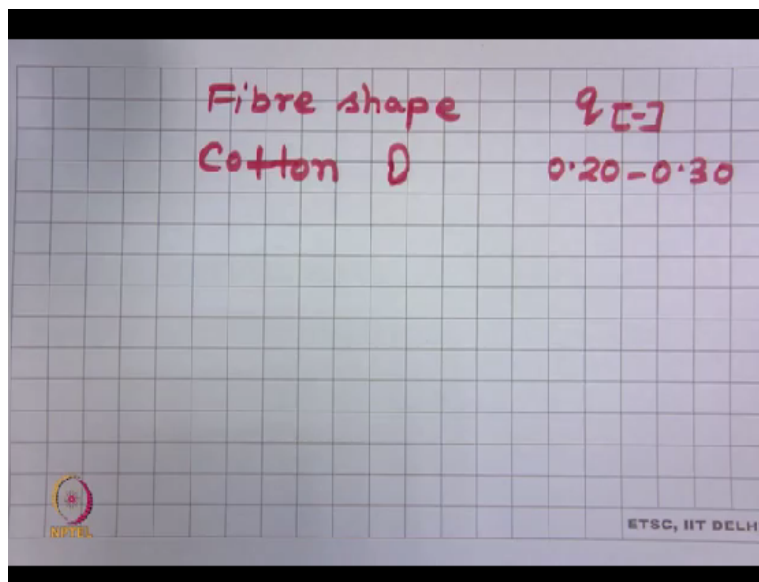
So sometimes we write while solving numerical problems, sometimes we do not write in terms of units. If you practice writing all expressions in terms of units, it will help you while solving numerical problems, right. So we go back to our fibre shape. If a fibre is ideally circular, its shape factor is 0. But often in practice, we hardly see a fibre cross-section is circular. We see little deviation from circularity.

For example, polyester fibres, man-made fibres, synthetic fibres. They are not ideally circular. But we often say they are practically circular. So if you cut the cross-section of such real circular

fibre, you will see their value of q will lie from 0 to say 0.6, typically. Similarly, if a fibre is triangular, ideally, that means if we consider an equilateral triangle, then you can calculate the shape factor will come approximately equal to 0.29.

In practice, though we do not encounter with ideal triangular fibre, but there are triangular fibres, practically triangular fibres. For example, trilobal fibre. So say practical, so the fibre typically looks like this. The shape factor typically ranges from 0.12 to 0.2. What about cotton? What is the value of fibre shape factor for cotton fibre?

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Fibre shape	$q [-]$
Cotton D	0.20-0.30

The image shows a handwritten table on a grid background. The table has two columns: 'Fibre shape' and ' $q [-]$ '. The first row contains 'Cotton D' and '0.20-0.30'. In the bottom left corner, there is a logo for 'RIPTEL' and in the bottom right corner, it says 'ETSC, IIT DELHI'.

Fully mature cotton fibre, you remember the shape of a cotton fibre. The typically shape factor you can find out 0.2 to 0.3 in that range. Also there are many other fibres which are now, it is available, deep groove fibres, 4 DG fibres, their shape factor is very large. As per this definition, shape factor will come for 4 digit fibre around 1.5.

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Fibre perimeter

$$q = \frac{P}{\pi d} - 1$$

$$P = \pi d (1 + q)$$

Fibre surface area A

$$A = PL = \pi d (1 + q) L$$

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Now we talked about fibre perimeter. So what we derived $q = \frac{p}{\pi d} - 1$. So we can write $\text{perimeter} = \pi d (1 + q)$. If it is circular, then $q = 0$, we obtain $p = \pi d$. If it is non-circular, q will have a value, accordingly we can obtain the value of p , okay. Now there are lot of properties of textile materials which are dependent on the available surfaces of fibres. So fibre surface is often required to be characterized.

How we characterize fibre surface? Fibre surface is typically characterized by fibre surface area, say we use a symbol A to denote fibre surface area. You look at this scheme of a fibre. How will you find out surface area of this fibre? Now surface area typically means this area + the top most surface area + the bottom most surface area. But the top most surface area and the bottom most surface area are much less than this surface area.

Therefore, we often neglect the top most surface area and the bottom most surface area while calculating the surface area of a fibre. Then we say that fibre surface area $A = p * \text{length}$. Now what is perimeter? $\pi d (1 + q) L$, right.

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$$\checkmark \bar{A} = \pi d l (1+q)$$

Specific surface area
Fibre surface area per unit mass

$$a = \frac{A}{m} = \frac{\pi d l (1+q)}{\underbrace{V \rho}_{= \pi \frac{d^2}{4} \cdot l}}$$

So if we rearrange, we obtain $A = \pi d l (1+q)$. Length of a fibre is very easy to determine. In any standard textile laboratory, it is possible to measure the length of a fibre. Diameter of a fibre is also possible to practically determine. Fibre shape factor is also possible to determine practically. Any advanced structure laboratory will be, where the cross-section, cutting, microtome, image processing techniques are available, you can easily find out diameter and also shape factor.

Then you can find out the value of A . Now so this is the expression for fibre surface area. Often you will see a term called specific surface area. What is specific surface area? Specific surface area = fibre surface area per unit mass. Let us use a symbol a to denote fibre surface area per unit mass. So what is a ? $a = A/m$, fibre surface area/mass, we used earlier symbol m to denote mass.

So a , fibre surface area per unit mass = fibre surface area, A ,/fibre mass. Now what is A ? A is $\pi d l (1+q)$. We have just now derived it. What is mass? Mass is volume*density, right. And what is this volume? This volume = cross-sectional area*length. If it is cylindrical, then this equal to $\pi d^2/4 * \text{length}$.

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$$a = \frac{\pi d l (1+q)}{\frac{\pi d^2}{4} L \rho} = \frac{4(1+q)}{d \rho}$$

$\underbrace{\frac{\pi d^2}{4} L \rho}_{=V}$

Fibre surface area per unit volume

$$\lambda = \frac{A}{V} = \frac{\pi d l (1+q)}{\frac{\pi d^2}{4} L} = \frac{4(1+q)}{d}$$

$$\lambda = a \rho$$

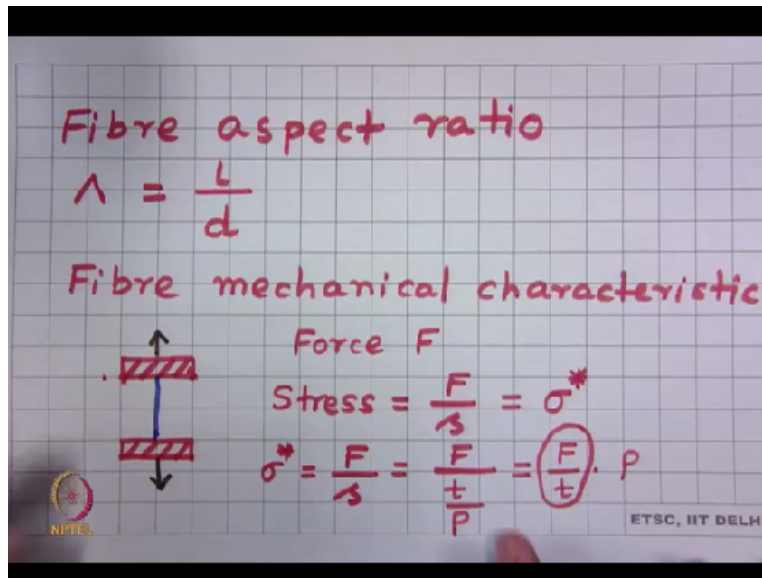
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So if we substitute, what we obtain is a $\pi \cdot d \cdot L \cdot (1+q)/V$. V is $s \cdot L$. s is $\pi d^2/4$. So $\pi d^2/4 \cdot L$, this is your $V \cdot \rho$, right. So this is your V , $V \cdot \rho$. So what we obtain is $4 \cdot (1+q)/d \cdot \rho$. Let us look at this expression little deeply. Fibre surface area per unit mass depends on fibre shape, fibre diameter and fibre density. That means because of the involvement of density, fibre surface area per unit mass is not a purely geometrical variable.

But when we talk about fibre surface area, we imagine geometry. However, fibre surface area per unit mass is not a geometrical variable because of the involvement of ρ , fibre density. That is why another terminology is often used to characterize fibre surface area that is fibre surface area per unit volume. How this becomes a purely geometrical variable? Let us explain. We use a symbol λ to denote fibre surface area per unit volume.

Now $\lambda = \text{fibre surface area}/\text{volume}$. What is fibre surface area? This. And what is volume? Cross-sectional area \cdot length. So $\pi d^2/4 \cdot L$. What we obtain? $4(1+q)/d$. See here fibre shape factor and fibre diameter, that is how λ becomes a purely geometrical variable. Now, so $\lambda = a \cdot \rho$. What is λ ? λ is fibre surface area per unit volume. What is a ? a is fibre surface area per unit mass and ρ is fibre density, right.

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There is one more term regarding the geometry of fibre which you will use in this course, is called fibre aspect ratio. What is fibre aspect ratio? Fibre aspect ratio=fibre length/fibre diameter. Typically, for textile fibre, the aspect ratio is quite large in the order of 10 to the power 3. Alright, so the basic geometrical characteristics of a fibre are explained. Now we will go to talk about the basic mechanical characteristics of a fibre.

Fibre mechanical characteristic. Now how do we test the mechanical properties of a single fibre? We have the jaws of a tensile tester and we mount a single fibre, right. Then we apply force. Suppose, this force is F. Then what is the stress? Stress is force/cross-sectional area. So that is called engineering stress. Let us use a symbol sigma* to denote engineering stress, okay. So sigma*=F/s.

The problem applying this expression in textile is, it is very difficult to practically determine cross-sectional area of a fibre because it is very small. So that is why we do not use this expression to calculate stress. What we use is called some other expression for stress. So let us come to that expression and we will show you how it comes. Now how cross-sectional area is related to fibre fineness? t/rho, right. We have derived a few minutes earlier. Then we can write F/t*rho, rho is fibre density. In textile, we often use this expression for stress, F/t.

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Textile stress $\sigma = \frac{F}{t}$

Engg. stress $\sigma^* = \frac{F}{s}$

$\sigma^* = \frac{F}{t} \cdot \rho = \sigma \rho$

Breaking length R

$F = \underbrace{R \cdot t}_{=m} g$, $\frac{F}{t} = Rg$, $\sigma = Rg$

$R = \frac{\sigma}{g}$

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So let us call this stress as textile stress. We use a symbol sigma which is equal to F/t and there is called engineering stress which is very popular term, $\sigma^* = F/s$. We have just now derived that $\sigma^* = F/t \cdot \rho$. So F/t is σ^* / ρ . So it is not difficult to determine engineering stress for textile fibre. This quantity we often determine practically if we multiply by density of fibre, we can find out engineering stress of a fibre, right.

To denote the stress, also we use a few other quantities. We will come to that but this stress is a general stress. When the stress at which a fibre breaks is called breaking stress which is related to a term called tenacity of a fibre, right. Alright. We often use a term called breaking length to characterize tensile stress of a fibre. What is breaking length? Breaking length is that length at which a fibre breaks under its own weight.

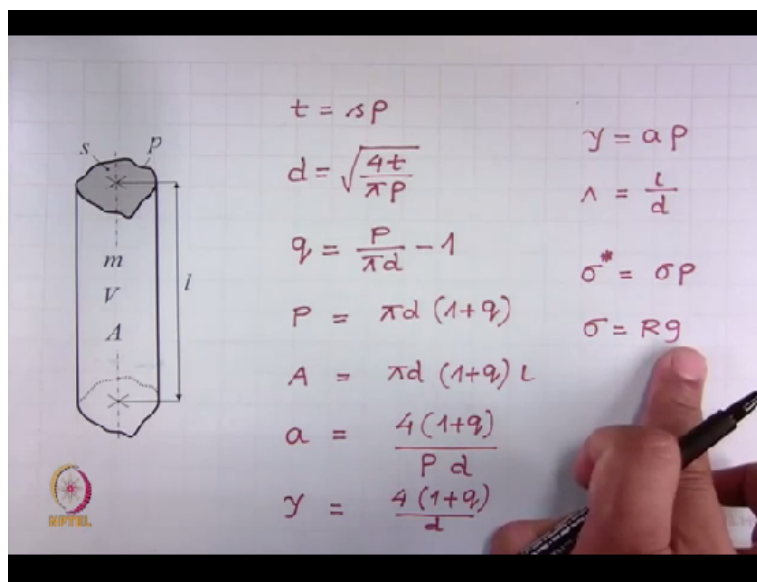
Suppose the breaking length of a fibre is R , right. Then what is its mass? $R \cdot t$ becomes its mass. Because t is mass per unit length. Length is R . So the mass is $R \cdot t$. If we multiply this mass by g , acceleration due to gravity, then we obtain force, F . Then we can write $F/t = R \cdot g$. What is F/t ? F/t is sigma. So we can write $\sigma = R \cdot g$. So finally we can write $R = \sigma / g$. So if we know sigma, we can find out breaking length and vice versa.

If we know R , we can find out sigma. So what do we see is that, all these terminologies, textile stress, engineering stress, breaking length, what we often use to characterize mechanical

behaviour of a fibre are interlinked, right, okay. So these are the basic physical characteristics of a single fibre which you will see often we will use in many modules in this course. Well in the last class, we established relationships on geometrical and mechanical characteristics of a single fibre.

Today, we are going to solve a few numerical problems on those characteristics in order to understand them in a better manner. But before going there, let us recapitulate what we learnt in the last class.

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If you remember in the last class, we started with this scheme of a single fibre, L denotes the length of the fibre, s denotes the cross-sectional area of the fibre, p denotes the perimeter of the fibre, m denotes the mass of the fibre, V denotes the volume of the fibre and A denotes the surface area of the fibre. Now the first important relationship what we established in the last class was $t=s*\rho$.

t denotes fibre fineness, s denotes fibre cross-sectional area, ρ denotes fibre density. Then the second important relation that we established in the last class was related to fibre diameter. Fibre diameter d was equal to square root of $4*\text{fibre fineness}/\pi*\rho$. d denotes fibre diameter, t denotes fibre fineness, ρ denotes fibre density. Afterward we established a relationship on fibre shape factor, $q=p$, fibre perimeter/perimeter of a circle-1.

So q denotes fibre shape factor, p denotes fibre perimeter, $\pi*d$ denotes the perimeter of an equivalent circle and if you remember well, q varies from 0 and onwards. 0 means circular fibre and non-circular fibre, then q will be greater than 0. Then from this relation, we can find out the expression for fibre perimeter, which is equal to this, fibre perimeter = $\pi*$ fibre diameter $*1+$ fibre shape factor.

Then we established an expression of fibre surface area, A which was equal to perimeter $*L$. You remember while deriving this relationship, we ignored the top most area and the bottom most area of the fibre because these 2 areas were negligibly smaller as compared to the area available on the surface. Then we established another relationship related to fibre surface area per unit mass.

So here a denotes fibre surface area per unit mass, q denotes fibre shape factor, ρ denotes fibre density, d denotes fibre diameter. Afterwards we derived another relationship on fibre surface area per unit volume which was equal to, γ , that means γ fibre surface area per unit volume, q is fibre shape factor and d denotes fibre diameter. So the relationship between γ and a is this.

So here γ denotes fibre surface area per unit volume, a denotes fibre surface area per unit mass and ρ denotes fibre density. Then we talked about aspect ratio of a fibre which is equal to fibre length/fibre diameter. Then we established a few mechanical characteristics of a single fibre. The first one was $\sigma^* = \sigma*\rho$. What is σ^* ? σ^* is the engineering stress of a fibre.

What is σ ? σ is textile stress, that is force/fineness of a fibre and ρ is fibre density. One more relationship we have established $\sigma = R*g$ where σ is breaking stress, force per unit fineness and R is the breaking length of the fibre, g is acceleration due to gravity. So these all relationships we have established in the last class. Today, we are going to use this relationship in order to solve a few numerical problems. Now let us start with the first numerical problem.

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Numerical Problem 1: State the expressions along with the physical dimensions (units) for the fiber characteristics given in Column A in terms of the fiber characteristics given in Column B.

Column A	Column B
Fineness	Mass and length
Fineness	Density and cross-sectional area
Diameter	Density and fineness
Shape factor	Perimeter and diameter
Specific surface area	Density, diameter, and shape factor
Surface area per unit volume	Diameter and shape factor
Surface area per unit volume	Density and specific surface area

$$t \text{ [tex]} = \frac{m \text{ [g]}}{L \text{ [m]}} \times 1000$$

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The very first numerical problem is this. You will see there are 2 columns written. Column A and column B. Under column A, certain characteristics of fibres are written. Under column B, certain characteristics of fibres are written. You have to express each fibre characteristics into the terms of the corresponding fibre characteristics along with units.

So this exercise is given to help you to write the correct expressions in terms of units. No doubt it is a very simple exercise. However, very simple things we do mistakes sometimes. So what is the first fineness? You have to express fineness in terms of mass and length along with suitable dimensions. So what is fineness? Fineness we have used symbol t , is often used in terms of $\text{tex} = \frac{\text{mass}}{\text{length}}$, let us express mass in terms of gram/length, let us express length in terms of meter.

So in order to balance both sides, you have to multiply by 1000. If you think length in terms of meter, is not a suitable dimension. No problem.

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$$t [\text{tex}] = \frac{m [\text{g}]}{l [\text{mm}]} \times 10^6$$

$$t [\text{tex}] = s [\text{m}^2] \rho [\text{kg/m}^3] \times 10^6$$

$$d [\text{mm}] = \sqrt{\frac{4t [\text{tex}]}{\pi \rho [\text{kg/m}^3]}}$$

Then you express length in terms of millimeter. So t tex mass in terms of gram/length in terms of millimeter. Then you write this. So by using this expression, you will be able to calculate t. Alright. What was the second expression? Second expression, fineness in terms of density and cross-sectional area of a fibre. So you have to express fineness, fibre fineness in terms of fibre density and fibre cross-sectional area.

How will you do? You have just now learnt fibre fineness often used in tex=fibre cross-sectional area, say meter square, *density, kg/meter cube. In order to balance, you have to write 10 to the power 6, okay. If you like to express in terms of millimeter square which is a more practical unit to express fibre cross-sectional area, you accordingly change this numerical value.

Alright, simple, okay. We go to the third one. What is the third one? Fibre diameter in terms of fibre density and fibre fineness. So we have to express fibre diameter in terms of fibre density and fibre fineness along with suitable units. So what is diameter? Diameter is often expressed in millimeter.

And we have just now learnt $4*t$, t let us express in tex, $\pi*\rho$, rho let us express in kg/meter cube. You need not to multiply by any factor here. So this is a balanced expression. Fibre diameter in millimeter= $\sqrt{4*t / \pi*\rho}$, rho in kg/meter cube, okay. What was the fourth one? Fourth one, fibre shape factor in terms of fibre perimeter and fibre diameter. This

exercise we have already done once. Let us repeat it.

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$$q [-] = \frac{P [mm]}{\pi d [mm]} - 1$$

$$a [m^2/kg] = \frac{4 (1 + q [-])}{d [mm] \rho [kg/m^3]} \times 10^3$$

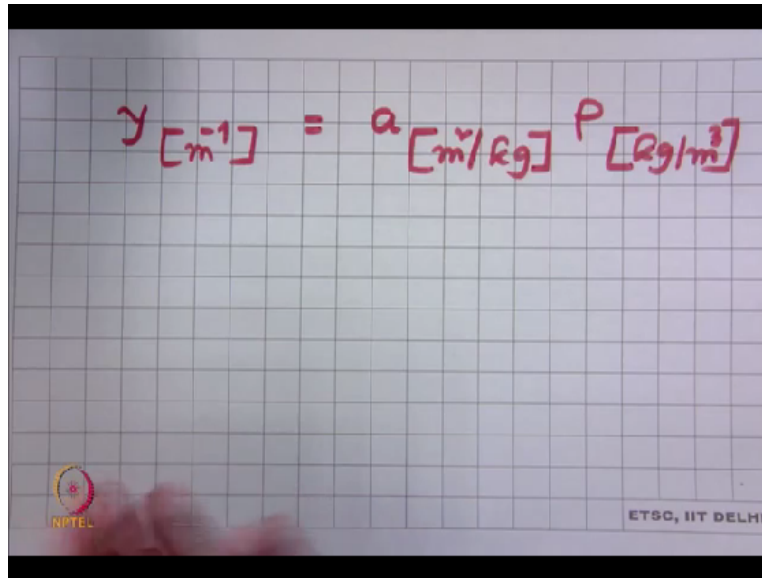
$$\gamma [m^{-1}] = \frac{4 (1 + q [-])}{d [mm]} \times 10^3$$

Fibre shape factor 1, dimensionless, =perimeter in terms of millimeter, -1. Fibre shape factor which is non-dimensional=fiber perimeter in millimeter/ π *d, d is fibre diameter in millimeter, -1, okay. Then we come to the next one. What was the next one? Next one was specific surface area in terms of density, diameter and shape factor.

So we have already derived the expression, specific surface area, a, suitable unit is meter square per kilogram= $4 \cdot 1 + q$, q is dimensionless, /d in millimeter and rho kilogram per meter cube. You have to balance. You have to multiply by 1000. So fibre specific surface area in terms of meter square per kg= $4 \cdot 1 + q$, q is fibre shape factor, dimensionless, /fibre diameter in millimeter and fibre density in kilogram per meter cube.

You have to balance. You have to multiply by 1000, okay. Then we come to the next one. Fibre surface area per unit volume. Diameter and shape factor. So we have already learnt meter inverse= $4 \cdot 1 + q/d$; d if you write in terms of millimeter, then you have to multiply by 1000, right. So gamma is a symbol for surface area per unit volume, q is fibre shape factor and d is fibre diameter. If we use millimeter which is a practical unit to express fibre diameter, then we have to multiply by 1000 to have a balanced expression, okay.

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$$\gamma \text{ [m}^{-1}\text{]} = a \text{ [m}^2\text{/kg]} \rho \text{ [kg/m}^3\text{]}$$


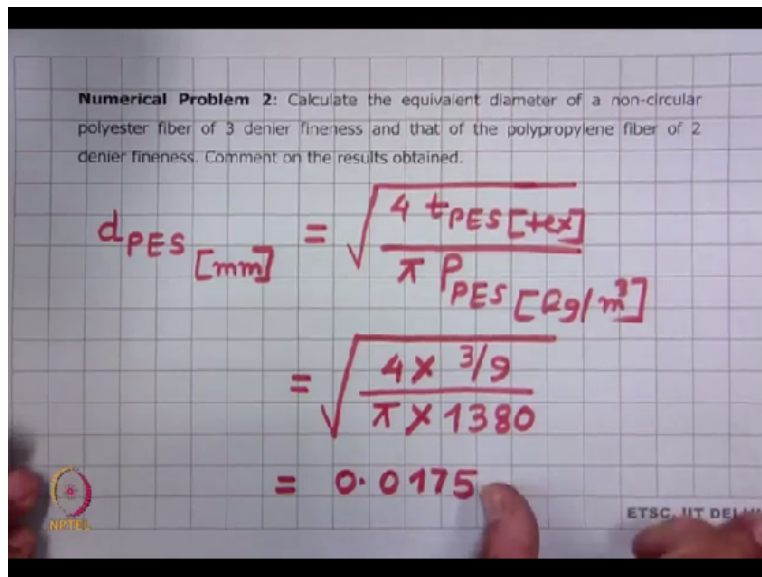
The last one is surface area per unit volume but in terms of density and specific surface area. So what is fibre surface area per unit volume? No problem. a meter square per kg rho kg per meter cube. It is already balanced, right. So this exercise was a simple exercise but you are supposed to express all expressions in this course along with units. So start doing that. What will help you? It will help you to solve numerical problems correctly and quickly. Now we will go to the second numerical problem.

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Numerical Problem 2: Calculate the equivalent diameter of a non-circular polyester fiber of 3 denier fineness and that of the polypropylene fiber of 2 denier fineness. Comment on the results obtained.

$$d_{PES} \text{ [mm]} = \sqrt{\frac{4 t_{PES} \text{ [tex]}}{\pi \rho_{PES} \text{ [kg/m}^3\text{]}}}$$

$$= \sqrt{\frac{4 \times 3/9}{\pi \times 1380}}$$

$$= 0.0175$$


Second numerical problem reads as follows. Calculate the equivalent diameter of a non-circular polyester fibre of 3 denier fineness and that of the polypropylene fibre of 2 denier fineness. Comment on the results obtained. So you have to basically calculate diameter, equivalent

diameter of polyester fibre and equivalent diameter of a polypropylene fibre. So let us do that.
Diameter of polyester.

We even use millimeter as its unit. What is the expression? Root over $4t$ of polyester in unit $\text{tex}/\pi \times \text{density}$ of polyester in kg per meter cube. Now what are given? $4 \times \text{fineness}$ of polyester is given, 3 denier. So in tex, it is $3/9$, $\pi \times \text{density}$, density of polyester fibre, is a well known value, 1380 kg per meter cube. So if you find out this value, you will probably see 175, 0.0175 millimeter. Similarly, we can calculate the diameter of polypropylene fibre.

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The image shows a handwritten derivation on a grid background. The formula for the diameter of polypropylene fibre (d_{PP}) in millimeters is given as:

$$d_{PP} [\text{mm}] = \sqrt{\frac{4 t_{PP} [\text{tex}]}{\pi \rho_{PP} [\text{kg/m}^3]}}$$

The derivation then substitutes the values for t_{PP} and ρ_{PP} :

$$= \sqrt{\frac{4 \times 2/9}{\pi \times 910}}$$

The final result is:

$$= 0.0176$$


In the bottom left corner, there is a logo for NPTEL. In the bottom right corner, it says "ETSC, IIT DELHI".

$4 \times \text{fineness}$ of polypropylene in $\text{tex}/\pi \times \text{density}$ of polypropylene in kg per meter cube, okay. Let us see what are given. $4 \times \text{fineness}$ of polypropylene, 2 denier. 2 denier means? This much of tex, $\pi \times \text{density}$ of polypropylene. Typically, density of polypropylene is considered to be 910 kg per meter cube. So if you calculate, you will find out this value will come around this.

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$t_{PES} = 3 \text{ denier}$
 $t_{PP} = 2 \text{ denier}$

$d_{PES} = 0.0175 \text{ mm} = 17.5 \mu\text{m}$
 $d_{PP} = 0.0176 \text{ mm} = 17.6 \mu\text{m}$

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Now what do we see is that the fineness of polyester fibre was given 3 denier. Fineness of polypropylene fibre was given as 2 denier and we calculated diameter of polyester fibre as 0.0175 mm that is probably is equal to micrometer and diameter of polypropylene 0.0176 mm which is equal to 17.6 micrometer. What we see is that, there is a significant difference in terms of fibre fineness.

So based on these 2 values we will say that polyester fibre is coarser than polypropylene fibre. But what we see is that their diameters are practically same. That means although there was a significant difference in fineness, but their diameters are practically same. So fibre fineness does not truly express fibre size. This is our comment on the results. Alright, now we will proceed to problem number 3.

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Numerical Problem 3: A shirt of 100 g weight is made up of non-cylindrical cotton fibers of 25.4 mm length, 3 ctex fineness, and 0.05 shape factor. Calculate the total surface area occupied by the fibers in the shirt.

$l = 25.4 \text{ mm}; t = 0.3 \text{ tex}; q = 0.05$

$$m [g] = \frac{t [\text{tex}] L [\text{mm}]}{1000 \times 1000} = 0.3 \times 25.4 \times 10^{-6}$$

$$N = \frac{M [g]}{m [g]} = \frac{100}{0.3 \times 25.4 \times 10^{-6}}$$

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Numerical problem 3. It reads as follows. A shirt of 100 gram weight is made up of non-cylindrical cotton fibres of 25.4 millimeter length, 3 decitex fineness and 0.05 shape factor. Calculate the total surface area occupied by the fibres in the shirt? Okay. So what is given here L 25.4, t 0.3 tex, q 0.05 and mass of the whole shirt is given 100 gram, okay. You have to calculate total surface area occupied by fibres in the shirt.

Now how will you find out the mass of 1 fibre, mass. Mass of a single fibre we need to find out. How we will find out? t in tex, L in millimeter/1000. So if we substitute these values, what we will see, $0.3 \times 25.4 \times 10^{-6}$, okay. Now how many fibres are present in the shirt? Suppose N we use to denote number of fibres present in the shirt. So mass of the shirt say we use M/Mass of a single fibre. So $100/\text{this expression}$, 6, okay.

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$$\begin{aligned}
 &\text{Total surface area (m}^2\text{)} \\
 &= \pi d [\text{mm}] (1+q [\text{--}]) L [\text{mm}] \times N \times 10^{-6} \\
 &= 3.14 \times \sqrt{\frac{4 \times 0.3}{3.14 \times 1520}} \times (1+0.05) \times 25.4 \times \\
 &\quad \times \frac{100}{0.3 \times 25.4 \times 10^{-6}} \times 10^{-6} \\
 &= 17.4262
 \end{aligned}$$

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Now total surface area, let us express in terms of meter square= surface area of 1 fibre*number of fibres. So how do we find out surface area of 1 fibre? Right, *number of fibres*10 to, in order to balance, you will have this factor, okay. So 3.14*diameter, diameter is $4t/\pi \rho$. What is t? t is 0.3, $\pi \rho$, density of cotton fibre. Density of cotton fibre we can take as 1520 kg per meter cube, *1+q.

What is q? q is given $0.05 \cdot L$. What is the length of the fibre? $25.4 \cdot \text{number}$. We have obtained this, $0.3 \cdot 25.4 \cdot 10$ to the power -6 * this 10 to the power -6. So if you calculate, you will obtain this value as 17.4262 meter square. So the total surface area occupied by the fibres in the shirt is 17.4262 meter square. Alright, okay. Let us proceed to the problem number 4. So now let us come to the fourth problem.

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Numerical Problem 4: The tenacity (tensile strength) of a polyester fiber is 0.43 N/tex. Calculate the mechanical (engineering) strength, expressed in MPa, of this fiber. Comment on whether this fiber is stronger than the ordinary steel which has engineering strength of 500 MPa.


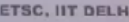
$$\sigma_{[N/tex]} = \frac{F [N]}{t [tex]}$$

$$\sigma^*_{[N/m^2]} = \frac{F [N]}{s [m^2]}$$



The fourth problem reads as follows. The tenacity of a polyester fibre is 0.43 Newton/tex. Calculate the mechanical engineering strength expressed in mega Pascal of this fibre. Comment on whether this fibre is stronger than the ordinary steel which has engineering strength of 500 mega Pascal, okay. So if you recall, we used symbol sigma to denote tenacity, tensile strength. Say Newton per tex=breaking force F, tex, okay.

And we used sigma* to denote engineering strength, Newton per meter square. Breaking force N, F and cross-sectional area meter square. So what, using these 2 expressions. Now what is s? s is related to fibre fineness and fibre density.

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$$\begin{aligned} \sigma^*_{[MPa]} &= \sigma_{[N/tex]} P_{[kg/m^3]} \\ &= 0.43 \times 1380 \\ &= 593.4 \end{aligned}$$



So if we use this expression, then finally, you will be able to write σ^* , engineering strength in mega Pascal, = σ Newton per tex* ρ density of fibre kilogram per meter cube. Now what are given? This value σ 0.43 Newton per tex given, density of polyester fibre. Density of polyester fibre we can assume as 1380 kg per meter cube. If we multiply these, we obtain 593.4. So engineering strength of this polyester fibre=593.4 mega Pascal.

It is well known that steel has an engineering strength of 500 mega Pascal. So this polyester fibre is more tenacious than ordinary steel. Alright, now we come to our last problem of this module. This problem reads as follows.

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Numerical Problem 5: The tensile strength of a cotton fiber is 0.32 N/tex. Find out the breaking length of this fiber.

$$\sigma [\text{CN/tex}] = 0.981 R [\text{km}]$$

$$R [\text{km}] = \frac{\sigma [\text{CN/tex}]}{0.981}$$

$$= \frac{32}{0.981} = 32.6198$$

The tensile strength of a cotton fibre is 0.32 Newton per tex. Find out the breaking length of this fibre. We will first write the relationship between tensile strength and breaking length of a fibre. Sigma is the tensile strength of a fibre. If we express it in centinewton per tex, then we write this expression where R is breaking length in kilometer. So this expression is balanced in both sides. So we need to find out R, breaking length in kilometer?

Sigma centinewton per tex/0.981. What is sigma? Sigma, 0.32 Newton per tex. So in centinewton per tex, 32 which is equal to little higher than 32.6198. So the breaking length of this fibre is 32.6198 kilometer, alright. So we have solved 5 numerical problems in this module. Thank you very much for your attention.