Theory of Yarn Structure
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Lecture – 13
Mass Irregularity of Yarns (contd.,)

Welcome to this course, theory of yarn structure. In the last time we started module 5, mass irregularity of yarns. So in that lecture we talked about Martindale's model. Martindale's model was based on 4 assumptions. So Martindale considered a sliver which was prepared from many fibers, all fibers were straight parallel to sliver axis, all fibers had same length, the fibers deposited individually and randomly to create the sliver.

And the number of fibers present in the cross section of the sliver follows Poisson distribution. Under these 4 assumptions we derived a relationship of sliver fineness.

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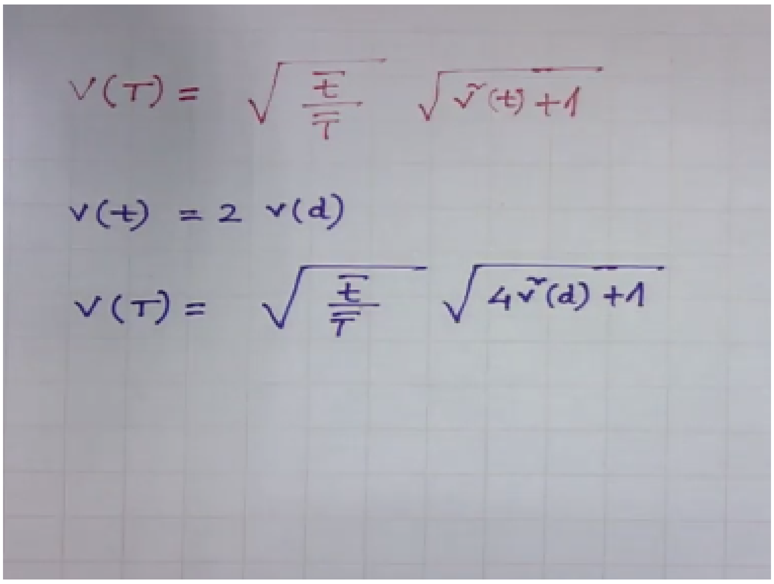

Which was equal to mean fiber fineness by means sliver fineness * square of CV of fiber fineness 1. So this derivation we completed in our last lecture. Also we established a relation between CV of fiber fineness and CV of fiber diameter which is often valid in case of wool fiber. So in that case the relation was CV of fiber fineness was equal to 2 times CV of fiber diameter.

Then if we substitute then we obtained +1. So often you will see in many articles and also books this expression you used for to find out the CV of sliver prepared from wool fibers. So in this lecture we will first discuss about the effect of doubling on mass irregularity. So what is the effect of doubling on mass irregularity of sliver. Almost all of us knows about doubling.

Doubling is basically combination of slivers. So when you combine or when we put one sliver beside another sliver we call doubling is = 2. When we place 3 slivers side-by-side we say doubling = 3. So doubling basically refers to combination of slivers. Now often we see in draw film in roller drawing machine we feed 6 to 8 slivers that is doubling = 6 to 8 and we obtain a drawn sliver. Why do we do that?

In a simpler form to say doubling reduces mass irregularity of sliver. How it is done? So in this lecture we are going to learn about that. So what is the theoretical basis that doubling reduces mass irregularity of sliver that is what we would like to learn today.

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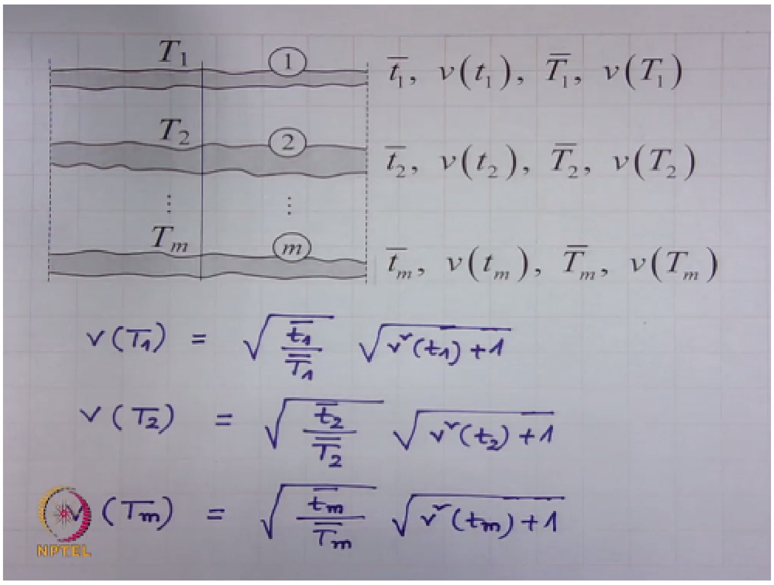

What we see here is m number of slivers. This is sliver number 1, sliver number 2 and likewise till mth sliver. This sliver fineness is denoted by T subscript 1 of course T1 is random variable. Second sliver, the sliver fineness T subscript 2 is also another random variable, amid sliver fineness T subscript m which is another random variable. Now each sliver is different.

They are different in terms of fibers, fiber parameters. They are different in terms of fineness. They are different in terms of their irregularity. Sliver 1 has a mean fiber fineness T1 bar. Coefficient of variation of fiber fineness v t1. Mean sliver fineness capital T1 bar and coefficient of variation of sliver fineness v T1, v T1 can be obtained by using Martindale's model, v T1 = t1 bar by this v squared t1 + 1 right.

Now we come to second sliver. The mean fiber fineness t2 bar cv of fiber fineness vt2, mean sliver fineness capital T2 bar and cv of sliver fineness vT2, this expression also can be obtained similarly by using Martindale's model. Now we come to m'th sliver. Mean fiber fineness small t subscript m bar, cv of fiber fineness v t subscript m, mean sliver fineness capital T subscript m bar and cv of sliver fineness v capital T subscript m.

This can also be found out by using Martindale's model in a similar manner right. Now so if we double then we will obtain 1 sliver that is called double sliver okay. So let us first characterize these individual slivers. So we can now consider one particular sliver say j'th sliver.

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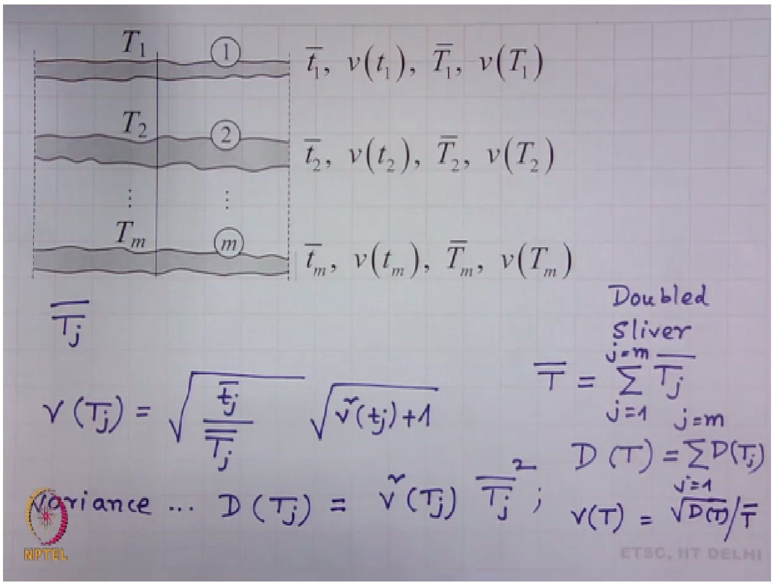

So what will be the mean value? Mean sliver fineness is this and the CV will be this. So these are the and also the variance will be is equal to this right, because CV square = variance/mean square. So variance = CV square * mean square, okay. Now what about the doubled sliver? For the double sliver, so if we combine all these individual slivers, we will form a double sliver.

That double sliver will also have a mean of sliver fineness that mean T bar will be = summation of all means right and it will also have a variance DT. These variances will be all additive, also it will have a coefficient of variation, this. So left hand side are the characteristics of individual slivers. Right hand side are the characteristics of double sliver. We need to find out the expression for CV of fineness of double sliver.

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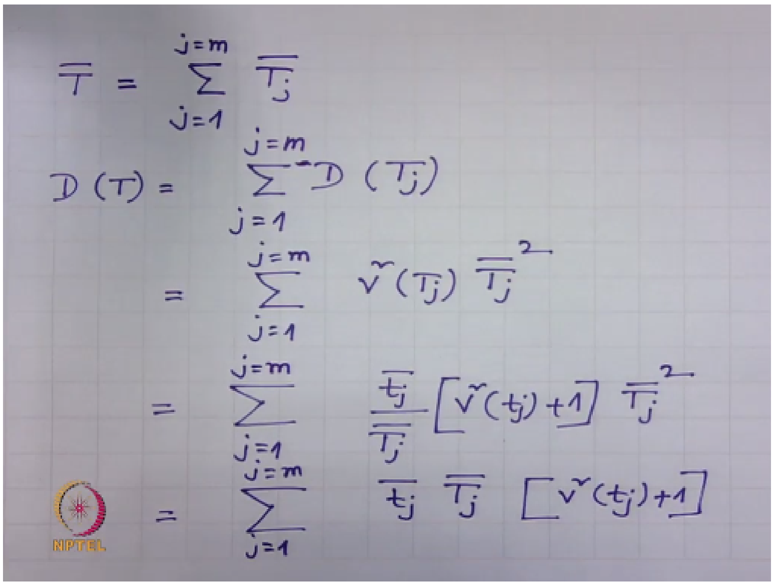

So T bar = this j = 1 to m, okay, and DT = summation D Tj, all variances are additive in nature. Now V square Tj * Tj bar square. We will substitute this from Martindale's model right. This 1 will cancel so the final expression for variance is this * this okay. So if this is the expression for variance of the double sliver then the CV of fineness of double sliver will be.

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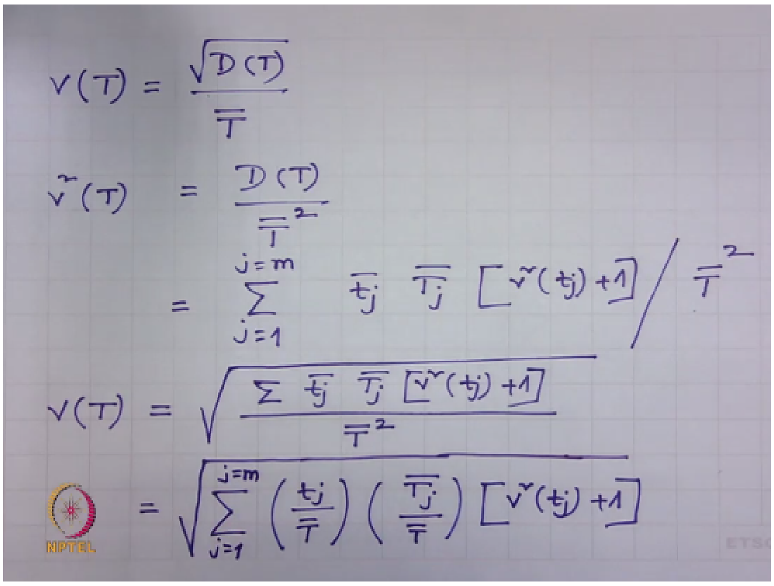

VT = DT/T bar or we can square it, DT/T bar square right. So we can write summation j = 1, j = m, tj, this v squared tj + 1/T bar square and if we take the root then square. Also we can write this expression in this manner and j = 1 to m right. So this is the most general expression for the CV of fineness of double sliver summation this is the fineness of individual, fiber fineness of j'th sliver.

This is the mean fineness of the double sliver. This is the fineness of j'th sliver. This is the mean fineness of double sliver. This is the square of CV of fiber fineness of j'th sliver +1. Now look at this fraction. Mean fineness of j sliver / mean fineness of double sliver. So this is the mass fraction of j'th sliver. So this is the mass fraction of j'th sliver right. So this is the most general expression. Now we will consider a very special case.

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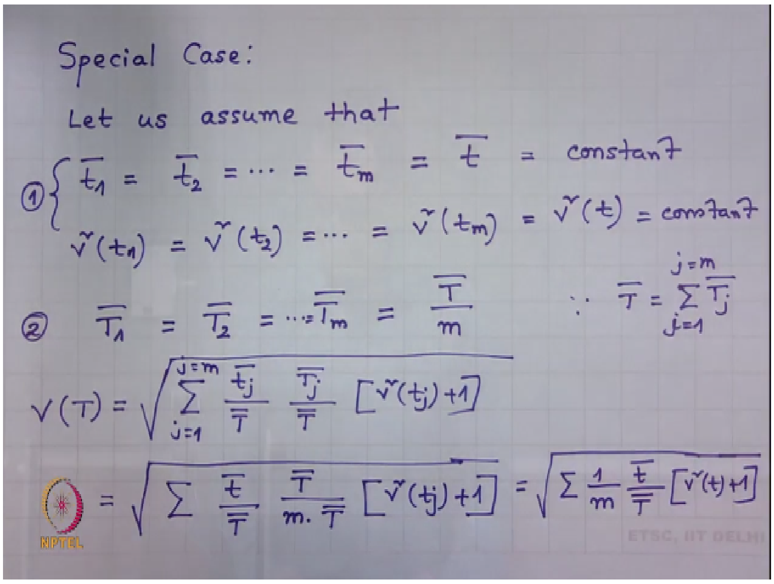

Let us assume that all fibers are same that means t1 bar = t2 bar = t bar. Then their CV will also be same. So they become constant right also we consider that the fineness of all slivers are same that means T1 bar, T2 bar, Tm bar, which is = capital T bar/m right why because capital T bar was this okay. Then what we obtain is let us rewrite. Here we will write T bar/m, so this is your t bar, this is your this, this is T bar by m and T bar is there already okay. We can write this as, so 1/m t bar by this v square t +1 okay.

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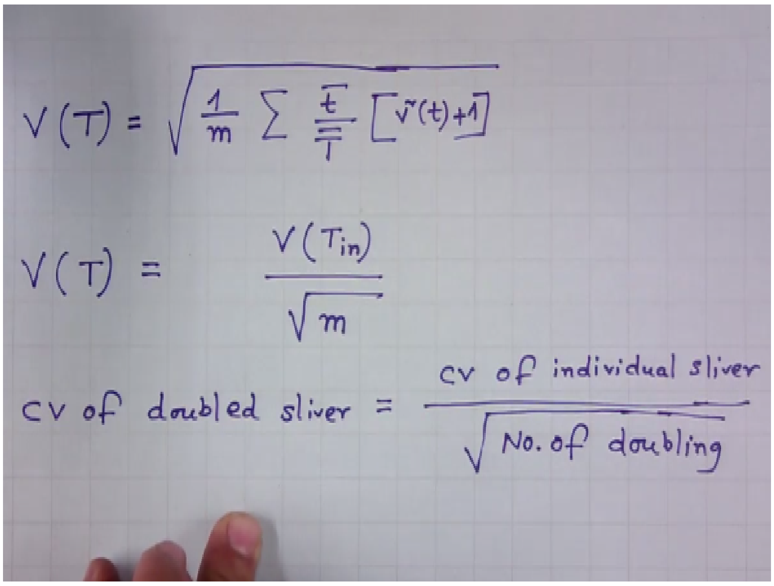

So further we can write it as VT = 1/m t bar by this * V square t + 1. So this becomes constant. So we can write VT double sliver is the CV of individual sliver by root over m right. So you have often seen this expression CV of double sliver = CV of individual sliver / number of doubling. So this expression is a very special case, not true always. It holds to only in one case when all fibers are same, their mean fineness same.

CV coefficient of variation of fiber fineness same, and also all individual slivers have same mean fineness, then only this expression is true. Otherwise if all individual slivers are very different then you will find out the CV of double sliver by using the most general form that we derived few minutes before right. So this was about effect of doubling on mass irregularity.

Now we will consider very interesting situation. Suppose fibers are not parallel to the axis of the sliver. Suppose the fibers are inclined at an angle from the axis of this sliver, what will be about the regularity of that sliver. Why we think about this it is a very special abstract case. We would like to learn the effect of fiber inclination on mass irregularity of sliver. So what is the effect of fiber inclination on the mass irregularity of sliver.

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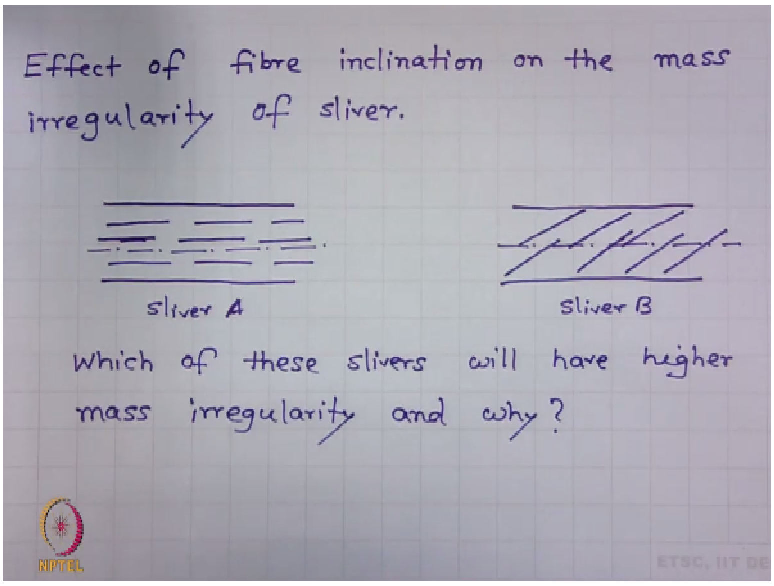

This we would like to learn now. Effect of fiber inclination on the mass irregularity of sliver. Suppose let me draw 2 slivers. This is one, second is this, this is the axis and here also this is the axis. This is suppose sliver A, this is suppose sliver B, which of these 2 slivers will have higher mass irregularity? and why? So this is our basic question. What do you think so? Sliver A will have higher mass irregularity than sliver B or the mass irregularity of sliver B is higher than that of sliver A. So let us solve this abstract case now.

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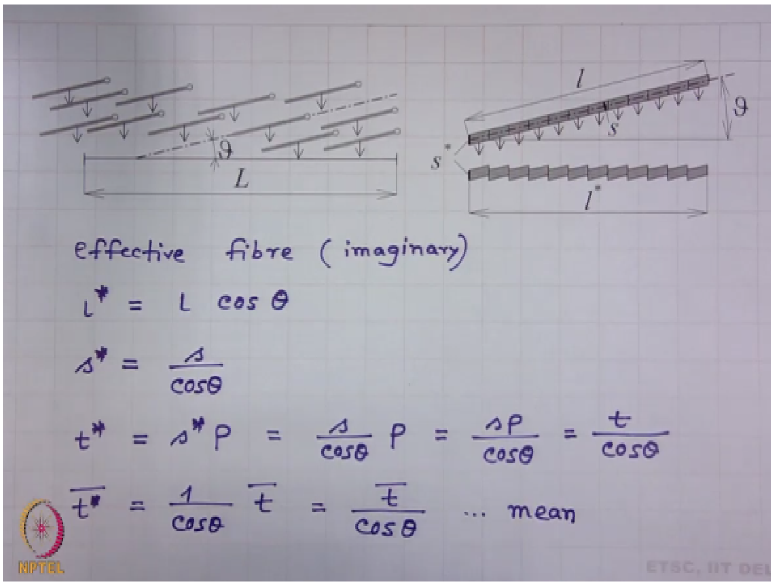

So what do you see is that, this is a diagram of a sliver with oblique fibers. So all fibers are inclined at an angle theta from the axis of sliver. In that case what will be the CV of fineness of this sliver. Now in order to solve this problem let us imagine a simple situation. What do you see here a fiber of length L which is also inclined at an angle theta from the axis. What do we do, the cross-sectional area of this fiber is S.

But the sectional area perpendicular to the axis of this sliver is S star. Obviously S star is greater than S. Now what we do, imaginatively we divide this l into many segments, many infinitesimally small segments, then we drop each segment along the axis of the sliver. So we obtain a fiber what we call as obviously an imaginary fiber. So this is called as effective fiber, as a result we obtain an effective fiber, of course this is imaginative.

This fiber will have a length l star. So the inclined fiber has a length l and this effective fiber has a length l star, what is the relation between l star and l, * cos theta right. This oblique fiber has a cross-sectional area S. This effective fiber has a sectional area S star. We refer to our module 1 where we defined and derived this relation. What is the relation between cross-sectional A of a fiber and sectional area of a fiber which is inclined at an angle theta.

So this relationship we derived in module 1 then what is the fineness of this effective fiber t star. We know from again module 1 this is the expression for fineness s/cos theta * rho. So s * rho/cos theta. What is s * rho? s * rho is t, t is the fineness of oblique fiber * cos theta right. So fineness of the effective fiber or imaginative fiber is = the fineness of oblique fiber / cos of the angle of inclination.

When theta = 0, cos theta = 1, t star = t, when theta is > 0, then cos theta is < 1, so t star is > t. So the effective fiber will be coarser than the oblique fiber when they are inclined at an angle theta. What will be the mean? This mean will be cos theta is the constant and expectation of t will be t bar right. So t bar by cos theta, this is the mean. What will be the variance?

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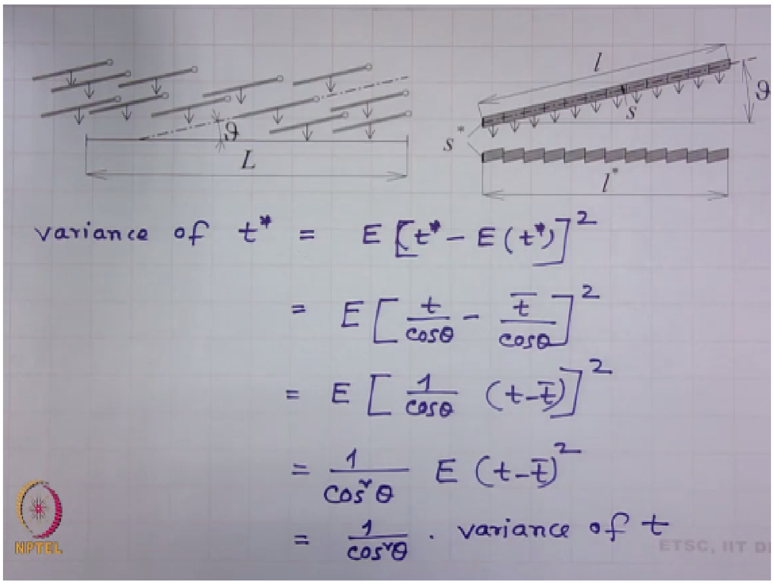

Variance of this is, this is the definition of variance t star, t star is t/cos theta, expectation of t start that is t stat bar, t start bar is by cos theta square. Now cos theta is the constant, so the constant will come this. So what is this? this is the variance of t, right. Then what will be the CV of t star.

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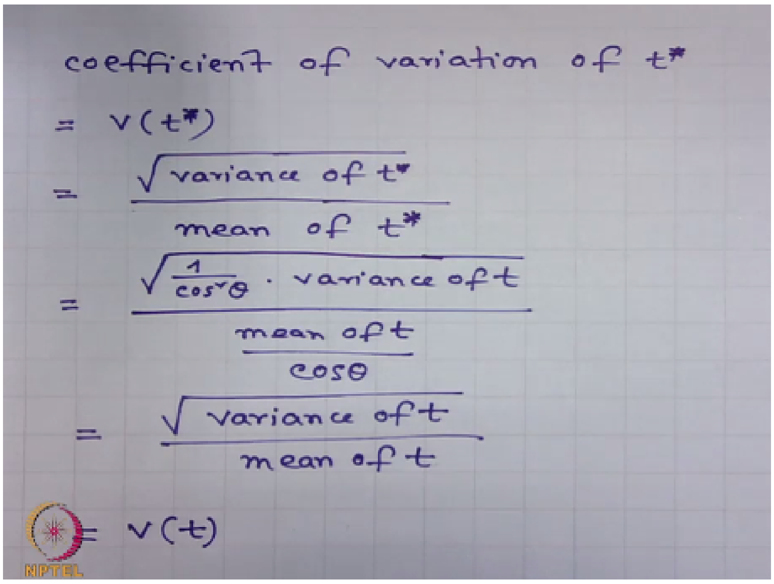

So coefficient of variation, that is = this. What is this? This is variance of this / mean of this. Variance we have just now derived 1/cos square theta * variance of t, mean of t star, mean of t/cos theta. So this cos theta, cos theta will cancel out as a result what is left is variance of t / mean of t that is = coefficient of variation of t. So this is a very interesting result. What we see is that whether a fiber is inclined it does not matter so far, CV of fiber fineness is concerned.

So CV of an incline fiber and CV of fineness of an inclined fiber and CV of fineness of a straight fiber is same that is what we obtain from here right. This we substitute in original Martindale's equation.

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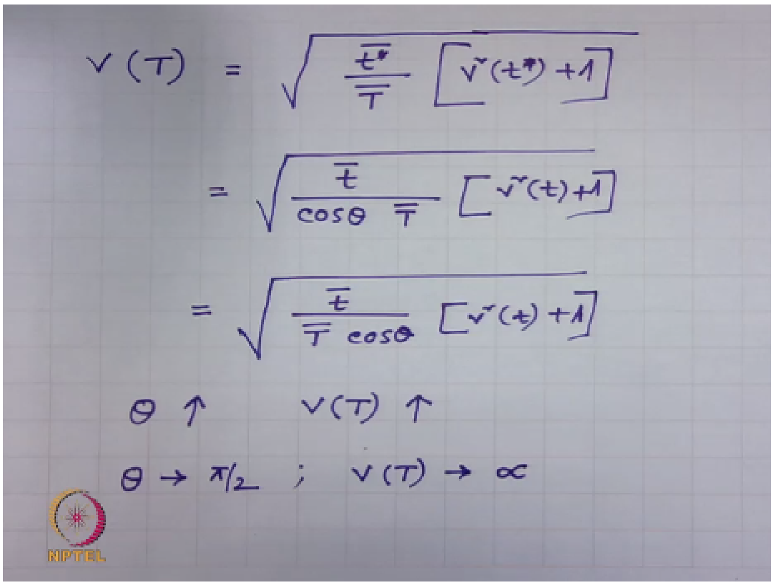

So v T = t star this * v square t * + 1. This will be the CV of fineness of sliver with inclined fibers. Now we substitute cos theta t bar. This does not change this so what we see is that 1 right. So what we see is that because of the inclination the sliver, the CV of sliver fineness is higher. So if theta increases V T increases. Higher is the inclination of fibers higher is the mass irregularity of the sliver.

So if we come back to our starting question which of these slivers will have higher mass irregularity? Of course sliver B is the answer and why this complete derivation answers this question why, right. Further what we see is that when theta tends to pi/2 that means fibers are perpendicular to the axis of the sliver, right. In that case cos theta, theta tends to pi/2, this tends to 0.

This is very small when this is very small this fraction is very high. So V T tends to infinity. So the mass irregularity of this sliver theoretically tends to infinity when the angle of inclination tends to pi/2. So when all the fibers are parallel to the axis of the sliver, the sliver will exhibit enormously high mass CV. So this very interesting however, abstract situation gives us some interesting information. Now we will discuss a few important attributes of mass irregularity of sliver.

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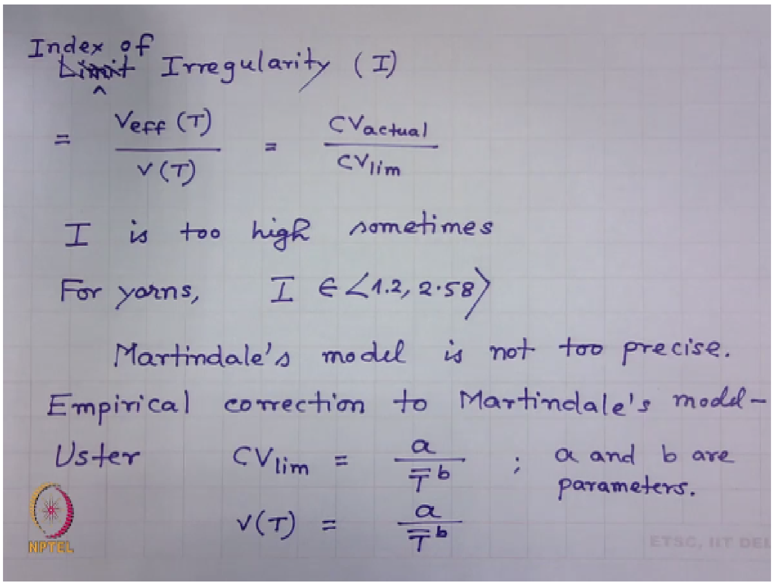

The first of it is limit irregularity. There exists a concept of limit irregularity. This limit irregularity is often expressed by I, is defined by actual experimental mass CV/limit CV that means CV actual / CV limit. Now there is a testing instrument so called Uster tester, Uster evenness tester, which measures the mass irregularity of sliver, roving, yarn and it actually measures, so we obtain actual CV from experiments.

CV limit we can calculate using Martindale's model. The CV limit and if we divide CV actual by CV limit we obtain limit irregularity. So we do not limit irregularity, it is index of irregularity I am sorry. This is index of irregularity. So index of irregularity I is defined by actual CV / limit CV. Uster tester measures actual CV, limit CV can be obtained from Martindale's equation. If we divide, then we obtain index of irregularity.

What we observed is that index of irregularity I is too high sometimes. For yarn this index of irregularity found to be this range. What infers is, if we consider index of irregularity is say 2.58 which is often found, then actual CV is 2.5 times higher than limit CV. This clearly indicates that Martindale's model is not too precise, had it been precise the index of irregularity would have come close to 1.

But it is 2.58 times higher, actual CV is 2.58 times higher than limit CV that means there is something imprecise in Martindale's model. There exist a few corrections in literature. One of the corrections empirical has been given by the company Uster. They have given this collection as CV limit = this a and b are parameters. So this empirical correction was suggested by the company Uster. If we write this so V T = a/b okay. Then is equal to a by b okay. Then what happens is that.

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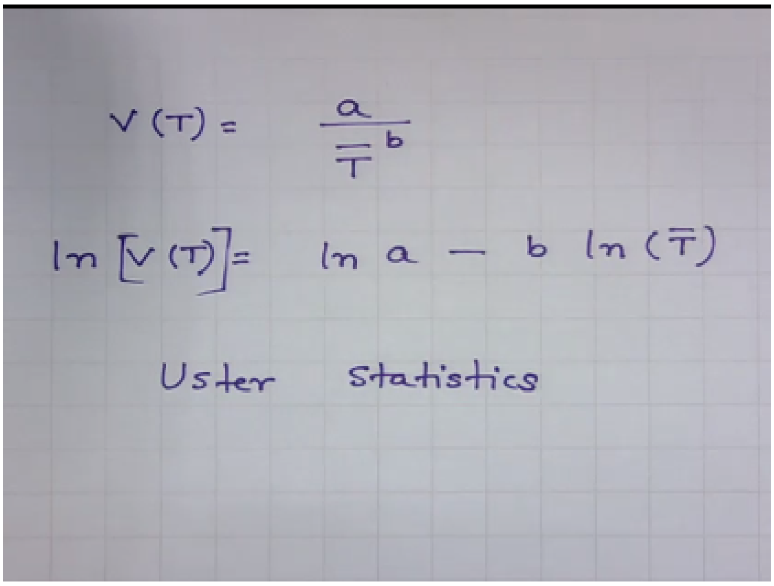

If we take logarithm both side right, so if we plot this along y-axis, this along x-axis, we obtained a linear curve. This so called Uster statistics. How does this curve looks like?

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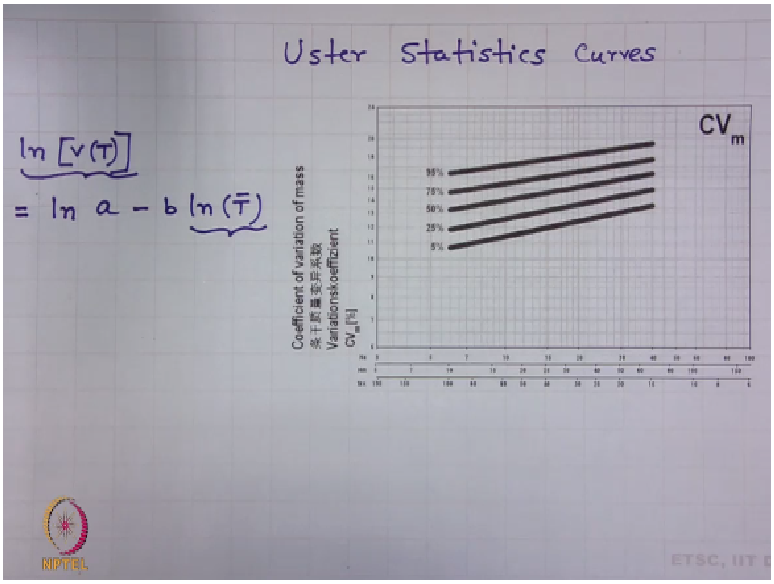

This curve looks like this Uster statistics. So VT = a – b ln T bar. This is y, this is x. We obtain a linear curve. So if T bar is this, it transforms here to here. So from here to here and we look from this side to this side it is basically any and 5% of the textile companies considered by Uster and shows this trend, 25% of the companies in the world surveyed by Uster shows this trend, 50% of the companies worldwide surveyed by Uster shows this trend, 75% of the companies surveyed by Uster shows this trend.

And 95% of the companies worldwide surveyed by Uster showed this trend. So this is how you should read Uster statistics curves. So this is linear because of this function. So this is one of the empirical corrections of Martindale's model; however, it is completely empirical, it does not give any insight of what is not to correct in Martindale's model. Probably a better formulation was given by Bornet.

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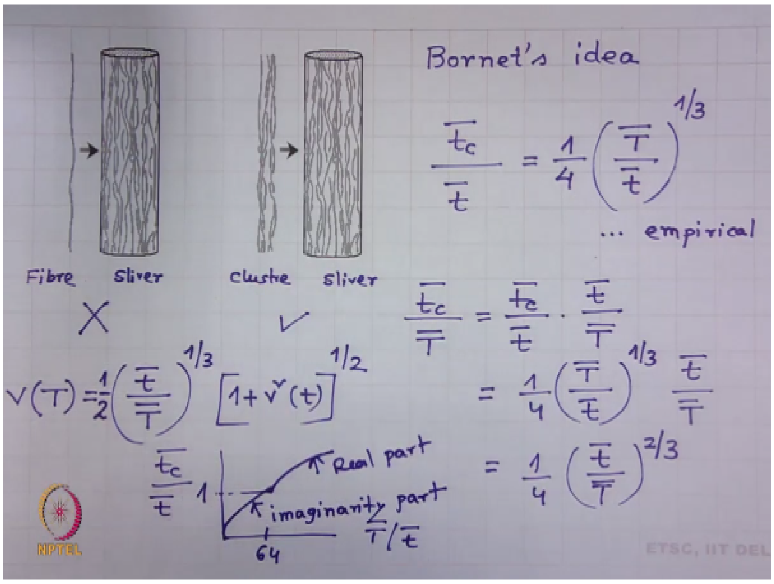

Bornet’s idea was also empirical; however, he tried to analyze Martindale's assumptions little deeply. What he thought is that Martindale’s model had 4 assumptions. Fibers have same length, it is acceptable, all fibers are parallel (()) (55:30) to sliver axis and they are straight, also to some extent acceptable. Fiber numbers in the cross section follows Poisson distribution is okay.

Fibers deposit individually to form a sliver, this assumption was questioned by Bornet. What Bornet thought that when fibers in a drafting system, when fibers move they move together in clusters. So some fibers agglomerate, they form a cluster, that cluster moves to form a sliver. So that was Bornet’s idea. So what you see in this image is, it is a Martindale’s model, 1 fiber and then sliver, here it is cluster and then sliver.

So he probably, Bornet probably thought this is not correct; however, this is correct that is what Bornet thought. Then he empirically suggested one relation. This is the mean fineness of cluster by mean fineness of fiber 1/4. Mean fineness of sliver, fineness of fiber to the power 1/3, this relation was empirical. So he proposed this relation. This is the mean fineness of cluster, mean fineness of fiber, mean fineness of sliver, mean fineness of fiber.

Then we can write by bar = tc/t bar * t bar / this, this is = 1/4 T bar by this to the power 1/3 t bar/ this show 1/4 2/3. Then we can write irregularity of this sliver will be this/2. In Martindale’s expression if you substitute this, if you substitute small t bar by capital T bar by this then you will get this from. So in Martindale's model small t bar/capital T bar this expression you substitute by this form.

Then you will obtain this expression and this value is numerically higher than that obtained from Martindale. So this empirical correction he suggested. So these 2 empirical corrections are existing in textile literature; however, Bornet’s idea of this expression has a serious problem. If I try to obtain this graph say this value is 64, then what will obtain this value will be = 1.

If this is < 64 then this is not possible because this will be imaginary part. The fineness of cluster cannot be < the fineness of fiber; however, this is the real part. So if one has to use this expression you need to be sure that the cross section of the fibre, in the cross section of sliver or yarn, number of fiber must be > 64, then only you should be able to use this expression.

If a yarn is having < 64 numbers in the cross section then this expression cannot be used because this expression has a serious problem, it does not define when number of fibers is < 64. If it is more than 64 you can use this expression. So we stop here, in the next class we will further discuss about mass irregularity of sliver. Thank you very much for your attention.