# Theory of Yarn Structure

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## Lecture – 14
## Mass Irregularity of Yarns (contd.,)

Welcome to this MOOCS online video course theory of yarn structure. In the last two lectures we discussed about module 5 mass regularity of yarns we started with Martindales basic assumptions then through mathematical derivations we reach to Martindales formula which is a well known to us then we found that Martindales formula would not explain the mass irregularity of sliver or yarn very well.

Two empirical corrections we discussed one was given by Worcester company which was purely empirical. Second empirical correction was given by GM bonnet which was also empirical however it was based on a very nice idea of fibres forming clusters those clusters forming sliver. However, GM bonnet model has a problem it could not explain the sliver or yarn formation when the number of fibres in the cross section is < 64.

That was the problem with bonnets empirical correction. Today we are going to critically review Martindales assumptions and then we need to discuss a mathematical model on mass irregularity of sliver or yarn after incorporating corrections to Martindales Luther model. So, we will now discuss about Martin dale's assumption.

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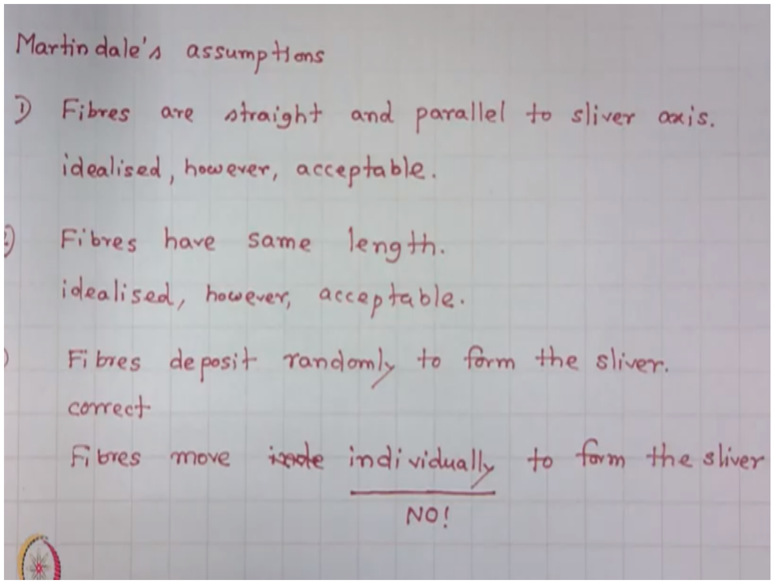


First he thought that fibres forming slivers are straight and parallel to sliver axis. Of course this is an idealized one however it is acceptable but dominant tendency of fibres in the sliver are parallel to silver axis. Second assumption of Martin rule was fibres have same length again this is an idealized one however acceptable though we know that in case of cotton fibre have a significant variation in terms of length.

However, in case of synthetic fibres we can assume that fibres have same length third assumption of Martin dale was fibres deposit randomly do form the sliver which is also correct. This assumption has an extension also that is fibres move independently move individually fibres move individually to form the sliver. This is not correct GM bonnet already proposed that fibres form clusters they have a tendency to form clusters.

And those clusters are ultimately form sliver. So, this assumption we will modify today and we will discuss about a new theory Neckar's theory.

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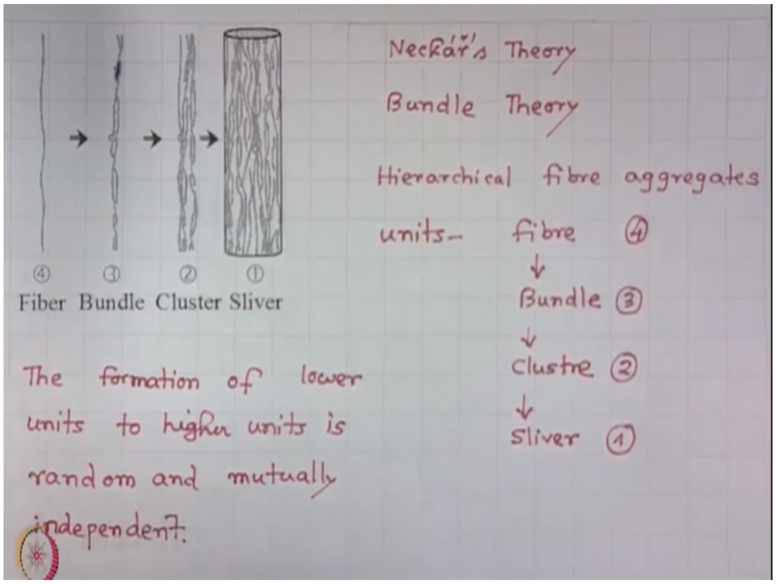


This is also known as bundle theory sure this theory modifies the individual assumption of fibres in this sliver and it imagines that a sliver is formed from aggregates of several units typically these aggregates are hierarchical fibre aggregates the lowest unit is fibre which is shown here a single fibre this unit is given a number 4 this lowest unit fibre form bundle.

So, 2 or 3 or 4 fibres aggregate and they form a bundle this bundle is given a number 3 unit number 3 several bundles form a cluster. So, these clusters are given unit number 2 several clusters form the ultimate sliver. So, the sliver unit is given 1 so fibre unit number 4 bundle unit number 3 cluster unit number 2 sliver unit number 1. So, fibres form bundles form cluster clusters form sliver.

Now in a bundle a few fibres are present it is almost impossible to open the bundles into single fibre stage. Because you can imagine imaginatively fibres are having glued or they are forming some knots to form the bundle. We know that cotton fibres have honeydew so they have a tendency to stick to each other right and also during processing during mechanical processing several fibres agglomerate and they form a bundle.

It is almost impossible to separate individual fibres from a bundle. Those bundles form a cluster. So, cluster consist of several bundles and it is possible by technological ways to separate those bundles from cluster. By opening process, it is possible to separate those bundles from a cluster and ultimately several clusters form a sliver So, this is the hierarchical fibre agreement that this theory considers.

Now the creation of higher units from lower unit suppose from fibre to bundle the lower unit is fibre higher unit is bundle. So, fibres create bundle this creation is random and independent so similarly bundles from cluster lower unit is bundle higher unit is cluster. So, the formation of clusters from bundle is random and independent similarly clusters from sliver here the lower unit is cluster higher unit is sliver.

So, the conversion from lower unit to higher unit that is from cluster to sliver is random and independent. So, we write that the formation of lower units to higher unit is random and mutually independent. Now we would like to know about the numbers of lower unit in a higher unit.

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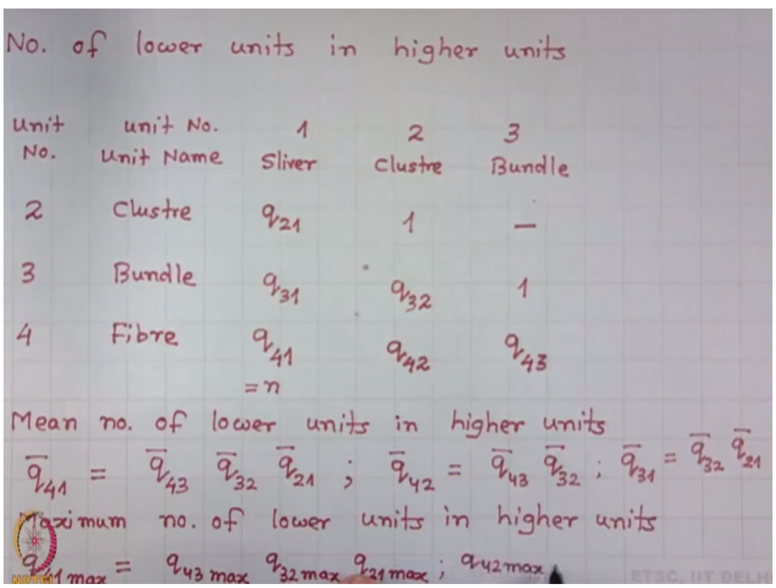


So, number of lower units in higher units how many fibres form a bundles how many bundles form a cluster how many clusters form a sliver right? So let us write in this column unit number and also in this row unit number now unit name so 2 unit number 2 means cluster 3 bundle 4 fibres. So we write here cluster, we write here bundle, we write here fibre and here we write 1 2 3 1 is silver 2 is cluster 3 is bundle.

Now how many cluster form silver these we denote body by symbol q 2 1 this 2 stand for cluster 1 stand for silver. So, number of clusters in silver is denoted by the symbol q subscript 2 1 here the 2 denotes the unit of cluster that is 2 and a 1 denotes the unit of silver that is 1. Similarly, how many cluster form a cluster it must be 1 how many clusters form a bundle from higher unit to lower unit not possible then how many bundles are present in a silver q first subscript is for stands for bundle.

Second is stand for the higher rate q31 so q31 is the number of bundles in sliver. Similarly, how many bundles form cluster q first subscript lower unit second subscript higher unit and how many bundles form the bundle must be 1 then the lowest unit fibre how many fibres are present in a sliver q first lowest unit 4 then 1 so q4 1 denotes the number of fibres in a sliver.

That is basically in number of fibres in a sliver? now how many fibres from a cluster q first subscript is lower unit 4 second subscript is higher unit 2 q 4 2 how many fibres form a bundle q the first subscript lower unit the second subscript higher unit so this is how we denote the number of lower units in higher units. Now we will discuss about the mean numbers of lower units in higher units what is the mean number of fibre in sliver?

What is the mean number of fibre in sliver? this must be =mean number of fibres in bundle multiplied by mean number of bundles in cluster multiplied by mean number of clusters in sliver so mean number of fibres in bundle multiplied by mean number of bundles in cluster multiplied by mean number of clusters in sliver right? Because these units consist only of fibrous material similarly what is the mean number of fibres in cluster.

So, fibres form bundle bundling form clusters so many number of fibres in cluster = the mean number of fibres in bundle and multiplied by mean number of bundles in cluster right? Similarly, what is the mean number of bundles in sliver mean q 3 1 bar mean number of bundles in silver this bundle must be=mean number of bundles in clusters multiplied by mean number of clusters in sliver.

Right similarly there could be maximum number of lower units in higher units what is the maximum number of fibres in silver? this must be =maximum of 3 units that is maximum of fibres in bundle multiplied by maximum of bundles in cluster multiplied by maximum of clusters in sliver. Similarly, what is the maximum of fibres in cluster. So, this must be =to maximum of fibres in bundle multiplied by maximum of bundles in cluster q4 3 max*q 2max.

Similarly, what will be q let us write here maximum of bundles in sliver so maximum of bundles in cluster* maximum number of clusters in sliver right? So this is about the number of lower units in higher units and maximum number of lower units in higher units right now we will discuss about fineness of these different units what is the fineness of bundle? What is the fineness of cluster? What is the findings of silver? These we are now going to discuss.

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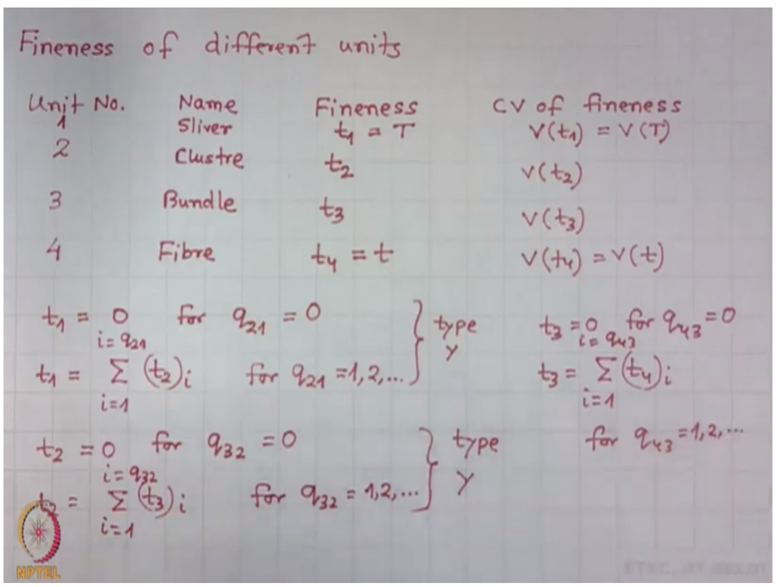


So, fineness of different units let us first write unit number 2 3 4 then unit name must be 1 1 2 3 4. So, the first unit is sliver this is the highest unit second cluster third bundle fourth fibre fineness so finest of sliver let us write t subscript 1, t stands for finest and 1 stands for unit. So, basically finest of sliver we generally denote it by capital T so t1 is capital T here. Fineness of cluster how to denote t stands for finest and unit number is 2 so t2.

T2 denotes the fineness of cluster similarly how to denote fineness a bundle t is a symbol for fineness and 3 is the same unit number for bundles. So, t subscript 3 similarly fibre the lowest unit t is the fineness 4 is the unit number so t subscript 4 t subscript 4 denotes the fineness of fibre what is t substitute 4 that is = t because we generally denote by fineness of fibre right. Then we will talk about CV of fineness.

We generally denote it by vt1=VT same symbol then t2 is the fineness of cluster. So, CV vt2 similarly vt3 is CV of fineness of bundle and vt 4 which is =vt which is the fineness of CV of fineness of fibre right? Now let us define them t1 what is the expression for sliver fineness? local sliver fineness it can be 0 locally if there is no fibre or no bundle no cluster so it is 0 for q 2 1 number of cluster in sliver=0.

If there is no cluster fineness is 0 if there are clusters, then it must be = summation of fineness of clusters. So, i=1 i= number of clusters in sliver for q 2 1=1 2 and right you remember in the last two last modules this type of random variable we discussed either type y variable y type of random variable and remember this is the definition of a random variable what will be the expression for its CV? We will discuss that later on.

Now we are interested to define t2. t2 can be 0 if there is no bundle t2 will be summation of t3 i if i=12 q 32 for again this is also y type random variable and we have discussed what will be the coefficient of variation of fineness of such random such quantity. Similarly, t3 bundle it can be 0 if there is no fibre in the bundle right? But if there are fibres then it will be summation of fibres right this is also type y random variable right?

So, this is how we define the fineness of the higher units. So, fineness of bundle fineness of cluster fineness of sliver these all are type y random variable. Now we considered that the number of lower units in the higher units follow binomial distribution.

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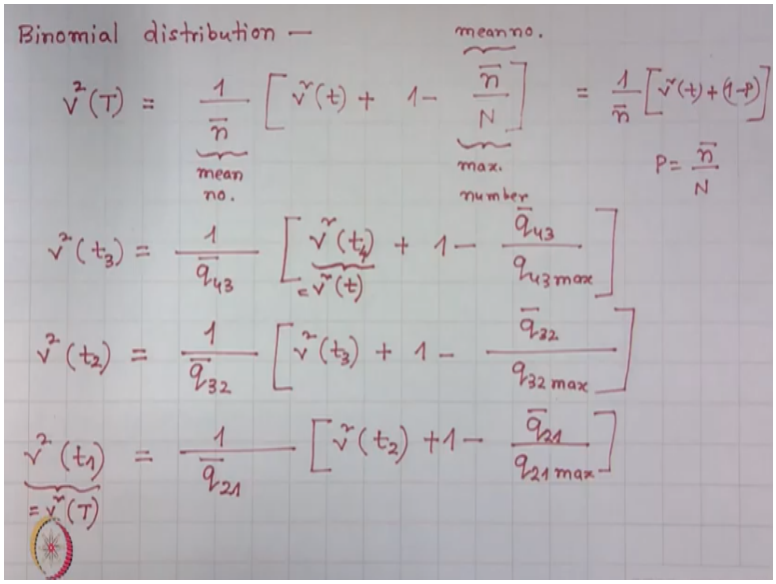


So, number of fibres in bundle follows binomial distribution number of bundles in cluster follows binomial distribution number of clusters in silver follows binomial distribution so there are 2 binomial distributions earlier we have discussed that in case of binomial distribution. So, number of fibres in bundles follows binomial distribution number of bundles in cluster follows binomial distribution number of clusters in sliver follows binomial distribution.

So, there are 3 binomial distributions. Earlier we have discussed that in case of binomial distribution the fineness of sliver = mean number of fineness in sliver+1-n bar/n you remember this formula we have discussed in the first lecture of this module so what is this. This basically mean number similarly this is also mean number and n is the maximum number remember we have discussed this formula v squared t +1-p this p is basically probability small n/capital N n bar/N right now this we will use in case of bundle cluster and sliver.

So, we can write v square t3 square of CV of fineness of bundle=1/mean number of bundles. Mean number of fibres in bundle q43*v square t t1 t4 =t you know +1- mean q43 q43 max right. This is basically t4 and t4 is 2 okay we can write it t4 so that is basically T. Similarly, for cluster mean number of what mean number of bundle in cluster q32 right mean number of bundle in cluster co efficient square of co efficient of variation of bundle+1- mean number of bundles in cluster /maximum number of bundles in cluster Okay then square of CV of 1 square of CV of sliver 1/mean number of clusters in sliver.

Square of CV of cluster +1 -mean number of clusters in sliver/maximum number of clusters in sliver. So, this we can also write as v square t and this we can substitute by v square capital T is it not. Now what we will we will now write it together so here we will substitute this expression then this will be substituted by this expression so it will be a quite long expression. So, let us do that

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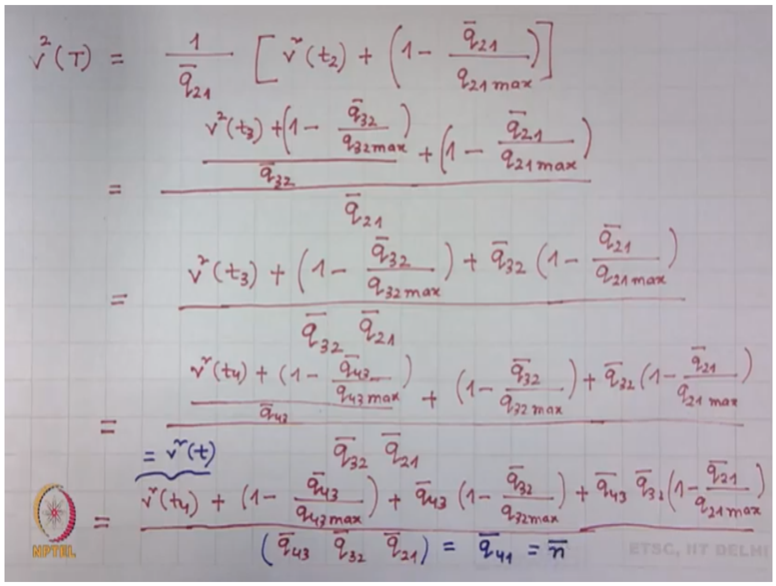


So, we write it again v square capital T what is this square of CV of fineness of sliver 1/this v square t2+1-this/this right so we rewrite the same expression. Now we will substitute here in terms of t3 right so if we substitute v square t2 right +1- let us put a bracket for convenience okay so what do we get q32 q21 t3+1-this max+q32 1-alright this expression will be longer further because we are now going to substitute here.

In terms of t2 t4 so q32 q21 v square t3 is v square t4+1-q43 bar q43 max/q43 bar + what is remaining 1-q32 2 max +2 right. Then finally we write this expression q43 q32 q21 v square t4 v square t4 is v square t+1-q43 bar/q43 max+q43 bar 1-q32 bar max +q43 bar q32 bar 1-q21/max right this is quite long expression. Now we can substitute here v square t square of CV of fibre fineness.

And look at this q43 bar mean number of fibres in bundle* mean number of bundles in cluster*mean number of clusters in sliver which is = mean number of fibres in sliver which is further =n bar right then we can rewrite this expression by changing this.

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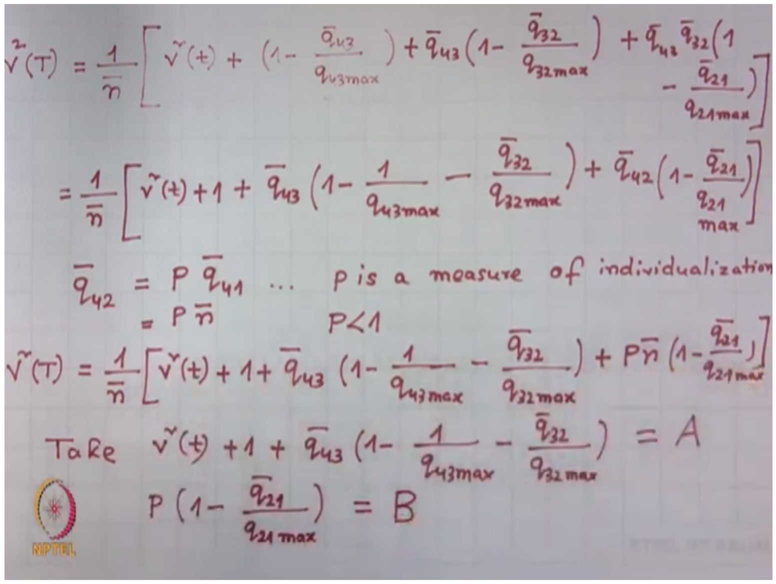


So, v square t=1/n bar square bracket starts v square t+1-q43 max+q43 bar 1-q32 max+q43 bar32 bar 1-/and we close the square bracket okay. Now what we do we rewrite this expression in a little different manner n bar this one we write here v square t+1 this 1 is here right -instead of 1 you write it as +q43 this q43 let us take it common and q43 also here so we write 1-1/q43 max right – this bracket close.

Okay now what is this mean numbers of fibres in bundle* mean number of bundles in cluster. So, that is =mean number of fibres in cluster mean number of fibres in cluster right. We now assume that this mean number of fibres in cluster is =some factor*mean number of fibres in sliver. So, this p is a measure of fibre individualization p is a measure of individualization and of course p must be <1.

Then we substitute here this quantity and we would like to rewrite what it becomes 1/n bar*v square t+1+q43 1 max-this max bracket close +p and q41 bar =n bar so p times n bar. Okay let us consider now this whole expression v square t+1+this=A. So, this part we consider =A and the remaining except n bar we considered as B. So, p*1- we consider as B so this expression becomes v square t becomes 1/n bar A+ n bar *B. So, that is we write next.

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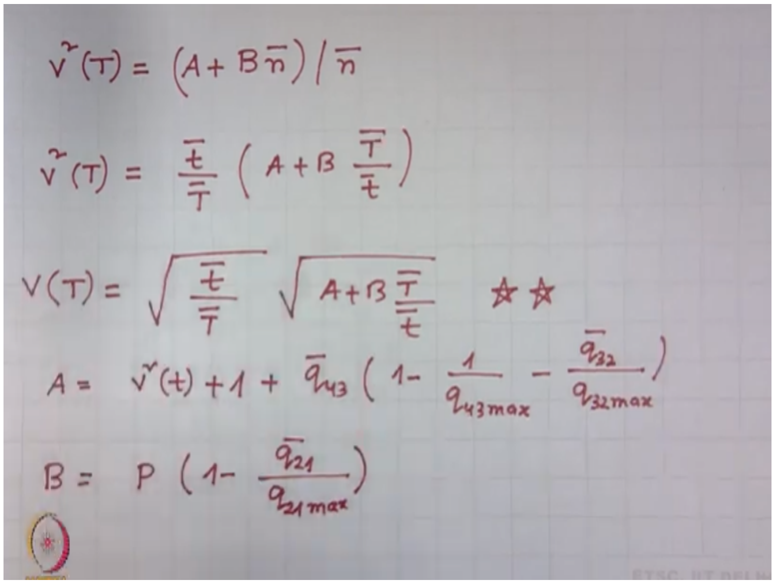


So this expression becomes v square t= A+B times n bar/ n bar. So, if we write in a standard form v square t is this A+B this/this or vt=this*A+B t bar/T bar. So this bundle theory under the assumption of binomial distribution leads to this expression A and B are two parameters right what is A? A is v square t+1+q43 1-1/q43 max-32/max and v=p times 1/divided by max right. Now a small note here if these two maximum quantities are very large in number.

Then this fraction becomes small this fraction becomes small so there is problem in A however, this is small then this fraction is very large if this is very small then this fraction is very large quantities is subtracted from 1 so this whole can be negative. So, v square t+1 can be <A right. Alright so this is the final expression for sliver irregularity based on bundle theory under the assumption of binomial distribution. What will be the form of this under Poisson distribution? So, now we will discuss about Poisson distribution.

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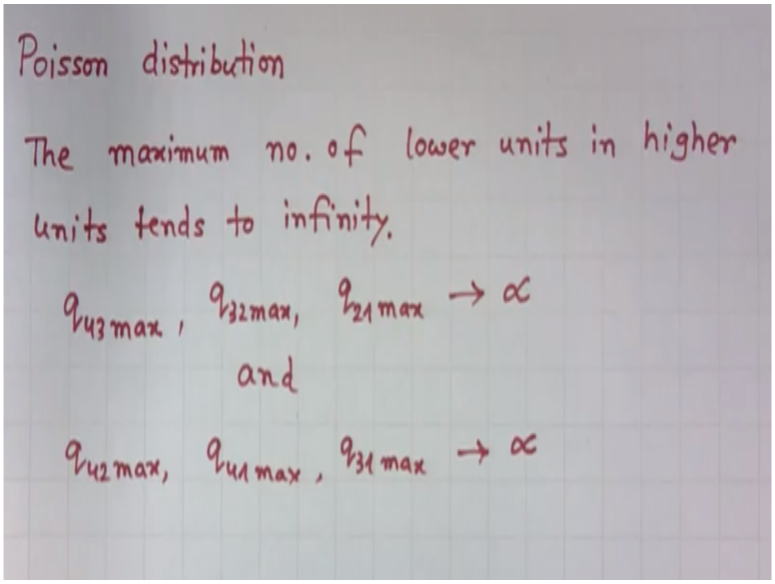


So, finally we come to Poisson distribution of our assumption is the number of lower units in higher units follow Poisson distribution. So, in Poisson distribution the maximum number of lower units in higher units tends to infinity what does that mean q43 max q32 max q21 max tends to infinity and q42 max q41 max q31 max tends to infinity right. Now we will come back to our previous one q43 max tends to infinity this then very small.

Similarly, q32 max tends to infinity this becomes very small so these two fractions we can neglect. Similarly, so A becomes v square t+1 + q43 bar similarly when q21 max tends to infinity this becomes very small so B becomes =p. So, in Poisson distribution this A becomes v square t+1+q43 bar and B becomes p. So the final equation is this but A is this and B=p right. So, we obtained two important expressions under bundle theory. One based on binomial distribution and the other based on Poissonian distribution.

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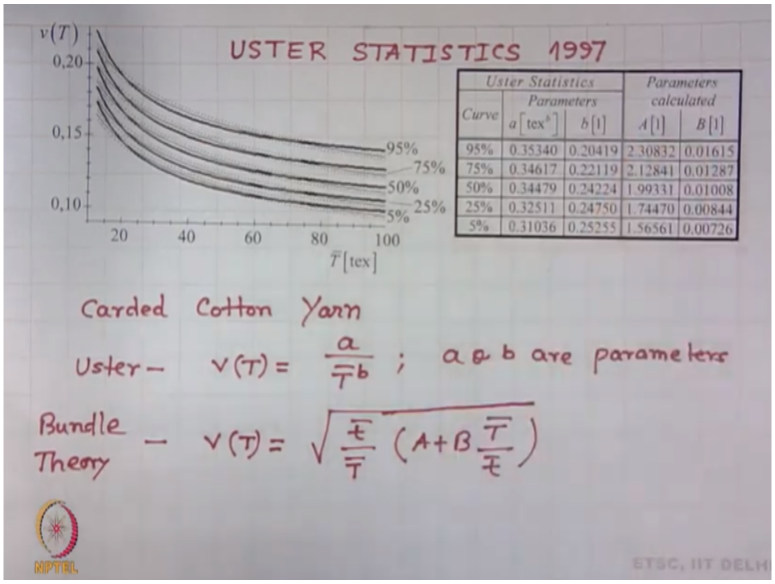


Now what we will discuss we will discuss this theory with the view of Uster statistics data. So, Uster statistics 1997 so you know Uster conducted a huge amount of trails in different spinning companies around the world and they collected the data of massive regularity versus count skull and the they plotted graphs called USTER statistics curve very similar curves are here. So, this was in the case of Carded cotton yarn.

So, along the x axis fineness yarn this was in case of yarn to along the yarn and along the x axis yarn fineness in Textile product along the y axis CV of yarn fineness dimensionless is plotted. Here you see there is one for each one there are 2 dotted lines. So, if you remember in Uster statistics curve is a thick red line? This thick red line has one top surface and also one bottom surface and one middle one.

The top and bottom are denoted by the dotted lines here and the fine line is the middle of Uster statistics curve and the thick line is obtained from bundle theory statistical regression technique was used to obtain the thick line that is from bundle theory and as you know Uster defines irregularity in this manner a and b are parameters and this bundle theory VT= t/T bar A+B this right for the bundled theory.

Then for different curves 5% of the of this spinning companies worldwide follows this curve 25% follows the next one 50% of followers the next one 75% of the spinning companies worldwide follow the next one and 95% follows this one. So, these curves were used to find out these parameters small a and small b for USTER. And capital A and Capital B for bundled theory.

So, you see that for 95% for this parameter a was 0.35340 and b was 0.0419 however A 2.30832 and B was 0.01615. Similarly, for different curves these values are given what we can all observe is that the value of B is very small it is in the order of 0.001 to 0.01 and the value of A is little more than 1 1.56 1.74 1.99 2.12 2.31 it is like that. So, this is what we obtained in case of Carded cotton yarn.

How this bundles theory behaves or how these bundled theory is compared with Martindales theory that we will discuss in the when we will talk about the numerical problems thank you, thank you for your attention.