Theory of Yarn Structure
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Lecture - 16
Radial Migration of Fibres in Yarns

Welcome to you all to this MOOC's online video course theory of yarn structure. Today, we will start module 6. This module deals with radial migration of fibres in yarn. Radial migration of fibres discusses about the movements of fibres radially in order to develop the structure of a yarn. This module is very important because due to radial migration of fibres yarn structure is developed.

So this phenomenon decides many physical as well as mechanical behaviors of yarn and radial migration of fibres generally happens during manufacturing of yarn particularly in a ring spinning frame for example at the neap of the (()) (01:26). There is a significant radial migration of fibres happens because of that finally yarn structure is developed. Now before going to this phenomenon, we would like to discuss first about the general fibre path in yarn.

How we can discuss about the general fibre path in yarn, then we will discuss about the fibre elements, we will talk about some specific angles. Those angles, some of those angles, some of those functions are basically related to radial migration of fibres. Then, we will discuss about this phenomenon. So first let us talk about the general fibre path in yarn.

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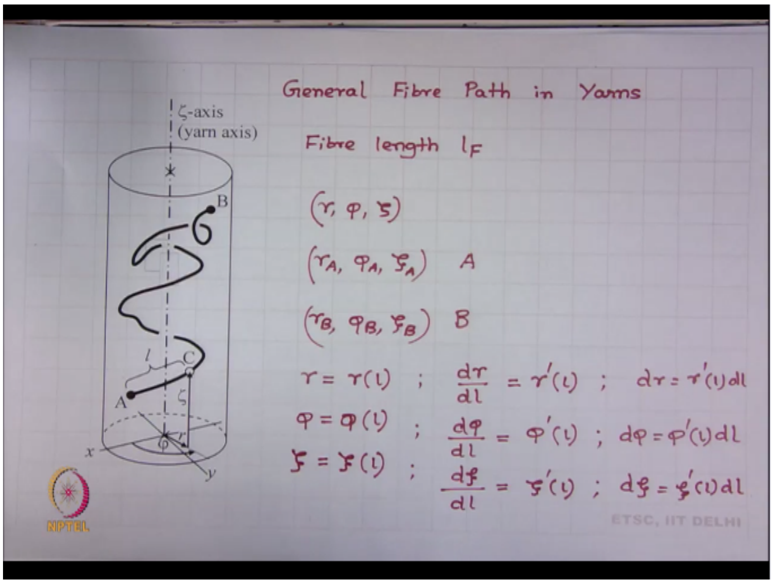

So this picture talks about general fibre path in yarns. Fibre path means fibre axis. In this diagram what you see is that there is a thick curve which starts from A and ends at B. This thick curve denotes a fibre. So A to B this distance basically represents fibre length. We denote it by l subscript F, F stands for fibre okay. This fibre housed inside the yarn. This yarn resembles a cylinder.

So this is a cylinder, basically it indicates the yarn body and the yarn axis is basically the zeta axis is the yarn axis. Now there are two more perpendicular axis x and y along with zeta represents a Cartesian system. So the fibre path inside the yarn can be represented by Cartesian system; however, we prefer to describe the fibre path using a cylindrical coordinate system.

This cylindrical coordinate system is often used to describe the fibre path or a specific point along the fibre. Let us take a specific point C. This point the distance from the starting position this point has a length zeta along the zeta axis. From yarn axis, we obtain radius r and from x axis we obtain an angle phi. So r phi and zeta represents the coordinate of a point along the fibre path.

Now at point A it will be rA, phi A, zeta A at point A and the coordinates at point B will be rB, phi B and zeta B. So this three, this triplet represents any point along the fibre path inside a yarn body. Let us think that the distance between A to C is l, so l is a length along the yarn body. So this length l is always increasing, so this point rA is basically is a function of length. Similarly, phi is also a function of length and zeta is also a function of length.

Now if we differentiate dr with respect to l, we obtained the first derivative. Similarly, d phi/dl we obtained the first derivative, d zeta/dl we obtain the first derivative. Now so dl dr is into dl. Similarly, d phi is this. Similarly, d zeta is dl. So these 3 differentials or these 3 differential functions can be used to obtain the fibre path. If we know the starting point as a boundary condition, then by using these differentials will be obtained the fibre path along the fibre inside a yarn body.

So in order to create, in order to know the fibre path we must know about these differentials and also the starting position, here it is A. Then, we discussed about one fibre element, one small fibre element.

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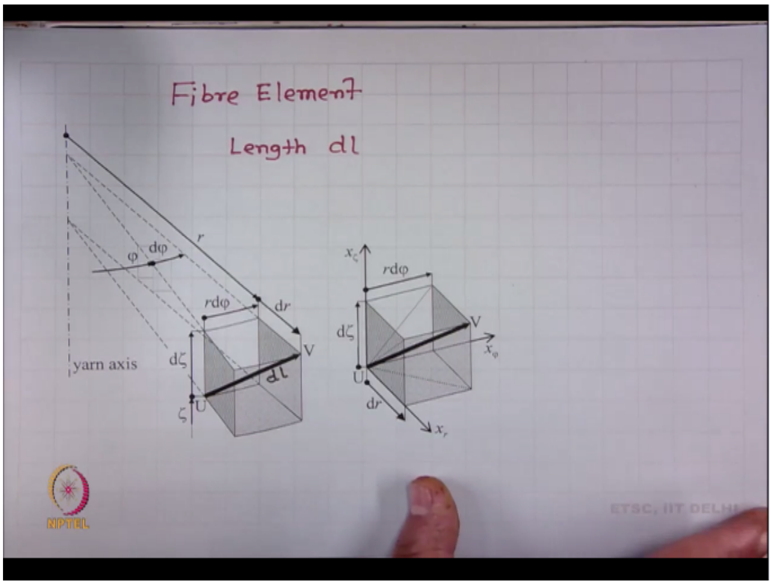

Dl is the length of the fibre element and d zeta is the increment along the yarn axis. Similarly, d phi is the angular increment and dr is the radial increment. So r*d phi is the width of this dimension. So the starting position of this fibre element U and end position is V. So this three d phi, d zeta and dr are basically those differentials. They denote the increment. These increments are so small that they form an elementary prism which is shown here.

So there are three axes, x subscript r, x subscript phi and x subscript zeta. So this is the radial increment dr, this is the axial increment d zeta and this is the width increment in width r d phi. Now also the fibre path can be characterized by using 3 important angles.

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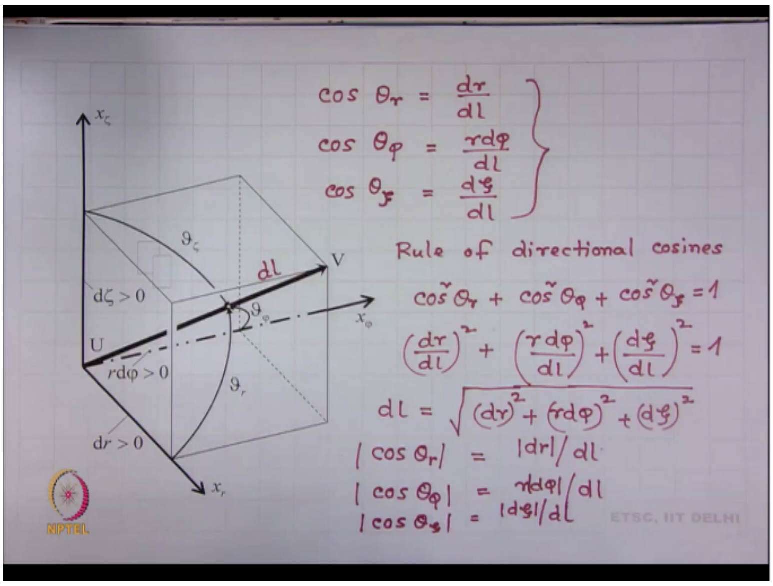

These 3 angles are given as theta r is this angle, so this is again the fibre element UV, length is dl and this angle theta r and theta phi is this angle from this axis and from this axis the angle is theta zeta. So these 3 angles also characterize the position of this fibre element. So now if we use basic trigonometric relations then we will be able to find out this 3 cos functions.

Now so if we use now rule of directional cosines what is that rule? That rule is cos square theta r+cos square theta phi+cos square theta zeta is=1. So this is the rule of directional cosines. If we substitute these 3 expressions here, then what we obtain dr/dl square+r d phi/dl square+d zeta/dl square is=1. So what is the length of this fibre element dl? dl is the square root of dr square+r d phi square+d zeta square.

Now these 3 functions can have negative values as well. Negative means actually decrement, positive means real increment with respect to certain directions. These directions we generally take positive when the direction is towards the yarn twisting direction and we take negative when it is in the opposite side. So we can now dl is always positive; however, dr, r d phi, d zeta can also be negative because of this.

They can be decremented also as well. Therefore, it is also possible to write the negative values. So it is always better to write in terms of absolute value. So we write in terms of absolute value cos theta r dr/dl, cos theta phi d phi/dl and absolute value of this is equal to absolute value of this/dl, dl always stands for positive right. These expressions we will use later on.

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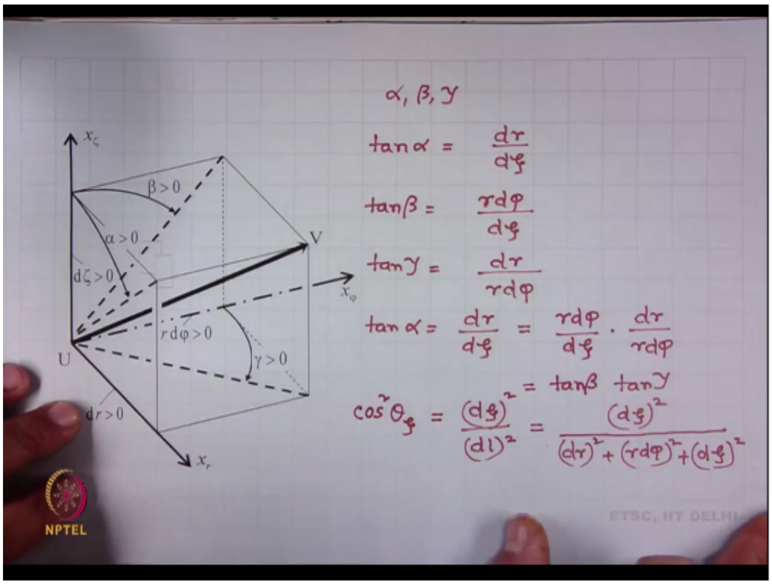

Now sometimes also the projection of the fibres element onto the walls of the elementary prism are also used to denote the path of the fibre. So 3 angles are very important in this case, one is this alpha, alpha is here from this axis alpha, beta and gamma. These 3 angles are very important when these projections are considered. If we apply trigonometric relations, then we will see tan alpha will be dr/d zeta, tan beta will be r d phi/d zeta and then gamma will be dr/r d phi.

So these 3 relations can be obtained from this. Now we see that tan alpha is dr/d zeta, dr/d zeta can also be written as r d phi/d zeta*dr/r d phi. This r d phi and this r d phi will cancel out, so it will remain dr/d zeta, dr/d zeta. So what is this? This is your tangent of beta and this is tangent of gamma. So these angles are also related. In the earlier slide, what we observed is cos square theta zeta is=d zeta square/dl square, now d zeta square/dl square.

What was dl square in the last slide you derive? dl square+r d phi square+d zeta square. So dr square+r d phi square+d zeta square.

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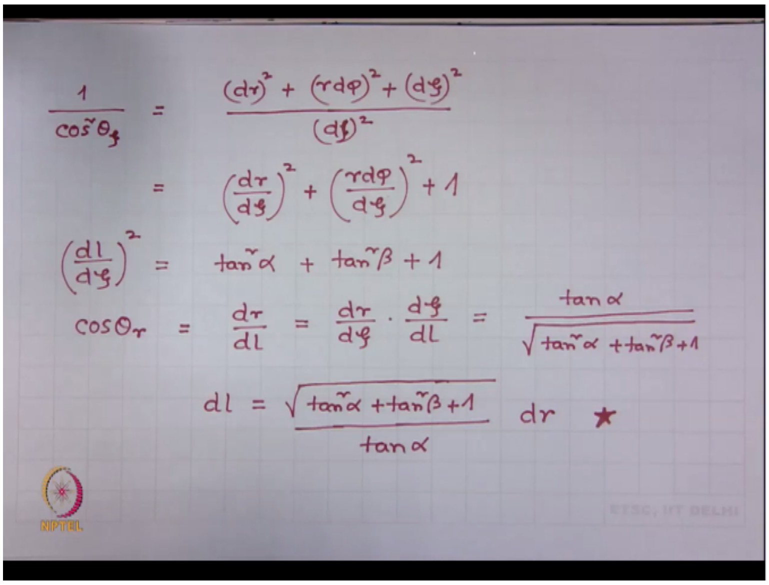

So 1/cos square theta is=dr square+r d phi square+d zeta square/dl square right and so dl d zeta square, so we can write this dr/d zeta square+r d phi/d zeta square+1 right. Now what is dr/d zeta? This is your tan alpha. So this is your tan square alpha and what is r d phi/d zeta is tangent beta, so tan square beta+1 right and what was your cos theta zeta, that was your dl/d zeta square right.

So this expression can also be obtained. Similarly, in the earlier slide what we wrote cos theta r is dr/dl. Now dr/dl can also be written as dr/d zeta*d zeta/dl right. Then, what is your dr/d zeta? Tangent of alpha and what is your dl/d zeta is tan square alpha+tan square beta+1. So dl is=square root of tan square alpha+tan square beta+1/tangent alpha*dr. This relation is important. We will use this relation when we will discuss the theory of radial fibre migration in yarn right.

Now there are two functions which are very important. So far the path of fibre inside the yarn is concerned.

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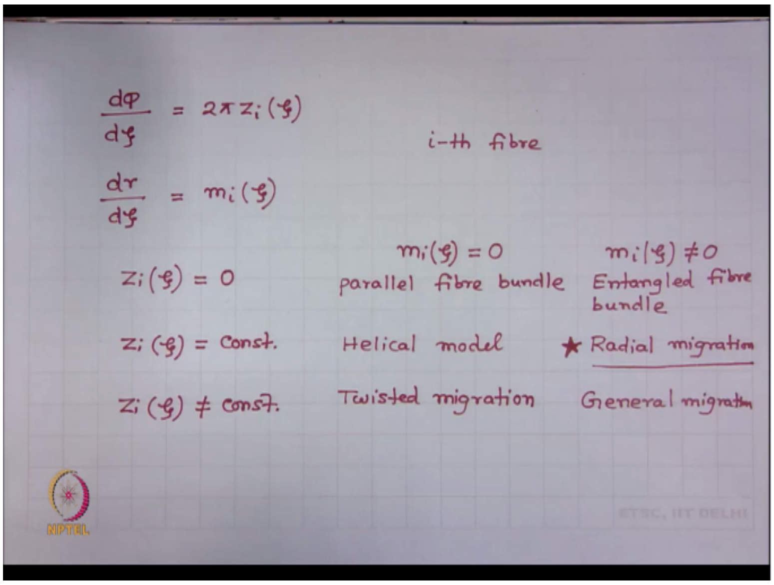

One function is d phi/d zeta. It is basically related to twist. Second function is dr/d zeta which is equal to this. This i represents i-th fibre. There are many fibres inside the yarn. Let us talk one general fibre say i-th fibre. So this i-th fibre will have these two functions in order to describe its path. So these two functions are very important to describe the path of fibre inside the yarn.

Even using these two functions very interestingly we can classify the models of yarn structure. How we do that? So these two functions, this can be equal to 0, this can be equal to constant, this function cannot be constant, three possible situations. Similarly, this function for i-th fibre can be equal to 0, may not be equal to 0. So if these two situations happen, so there is no twist in the fibrous assembly and there is no radial movement.

There is no twist, there is no radial movement. That means all fibres are straight. That means we talk about parallel fibre bundle. So when these two situations are valid, these two conditions are valid, we talk about parallel fibre bundle. Now we talk about these two situations. There is no twist; however, there is a radial movement of fibres.

That means fibres are, there is no twist and there is radial movement of fibre that means fibres are probably entangled. In that case, we talk about entangled fibre bundle. Now we come to these two situations, zi constant and mi 0, what is the meaning of this? So there is a twist and the constant amount of twist is given; however, there is no radial movement of fibres 0. That means we talk about helical fibre model.

This model we have already discussed in probably module 4. Now we come to this situation, constant amount of twist is given in the yarn and there is a possibility of radial movement of fibres. That we talked about radial migration of fibre. Radial migration, this is basically the theme of this module radial migration. This we will discuss in detail in this model. Fifth situation, twist is not constant.

Different fibres can have different amount of twist but there is no radial movement. This is called as twisted migration and the last situation, different fibres will have different amount of twist and there is a possibility of radial movement of fibres. These we talked about general migration, so general migration is this. Just to give you a note, there is lot of knowledge available in yarn structure regarding parallel fibre bundle, helical model, radial migration.

In some literature also, certain ideas are available on entangled fibre bundles and twisted migration. In this particular module, we focus on radial migration of fibres in yarn. So in order to fibres move radially, two conditions are very important. One twist at i-th fibre is constant and this twist is of course a function of zeta and radial movement must be there. So mi so this for i-th fibre is not equal to 0, so tangent of alpha.

This is equal to dr/d zeta that is equal to tangent of alpha. So tangent of alpha is not equal to 0. So if these two situations happen, then there is a possibility of radial migration of fibres. So this is basically the theme of this module. Now we will discuss about radial migration of fibres.

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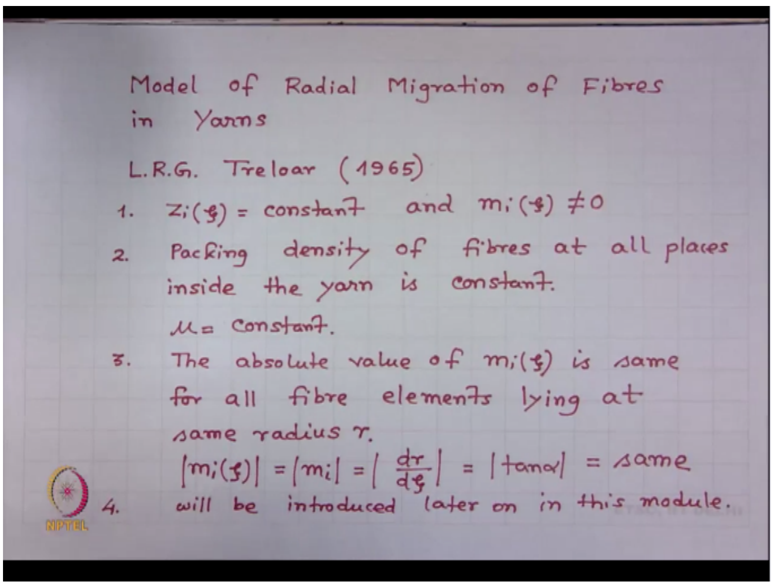

Model of radial migration of fibres in yarn, probably the first theoretical concept on radial migration of fibres in yarn was given by L.R.G. Treloar. This model was published in the year of 1965. We will first discuss about Treloar's model. This model is based on 4 important assumptions. Out of these 4 assumptions, first assumption is basically related to the definition of radial migration.

So the first what is the definition, the definition of radial migration is z this is equal to constant and this dr/d zeta m is not equal to 0, so this was the first assumption which is correct because it is the definition of radial migration. Second assumption of Treloar is packing density of fibres at all places inside the yarn is constant. That means mu is constant. This was the second assumption in Treloar's model of radial migration of fibres in yarns.

Third assumption, a little critical, the absolute value of this function which denotes the radial migration is same for all fibre elements lying at same radius r. What does that mean? Inside a cylindrical yarn, take any radius, at that radius there are many fibre elements available. The absolute value of this dr/d zeta for all fibre elements which are lying at that particular radius is same.

Mathematically, this function its absolute value this is equal to this is same for all fibres in the yarn. So this is a very crucial assumption. The fourth assumption of Treloar's model, we will introduce it later on in this module, when it will be required we will introduce it. Now based on these 3 assumptions, it is possible to derive certain theoretical relations. We are interested to find out those relations at this moment of time.

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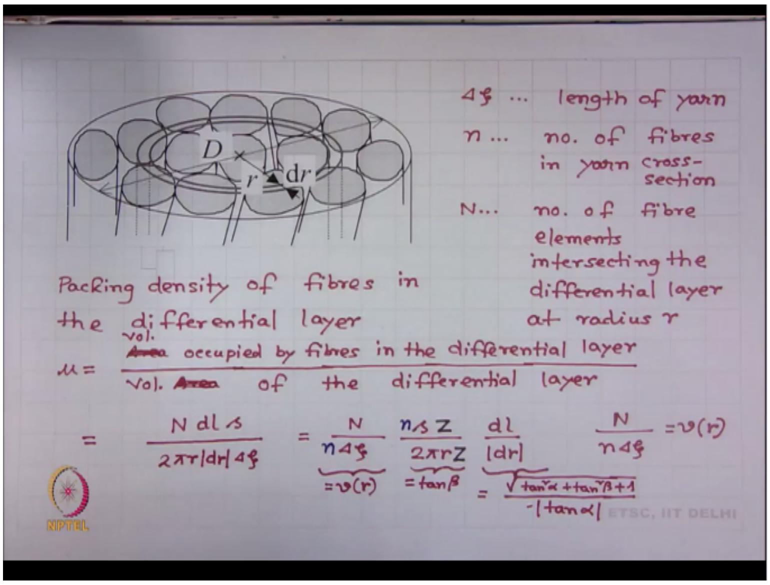

For that purpose, let us think about the cross section of a yarn. It is a typical cross section of a yarn cylindrical body. There are many fibres present. The diameter of the yarn is D. D is the diameter of the yarn. We consider a small length of yarn; say elementary length delta zeta is the length of yarn, small length of yarn delta zeta. Now how many fibres are present in the cross section of yarn?

Let us think n, small n denotes number of fibres in yarn cross-section and capital N denotes number of fibre elements intersecting this differential layer at a particular radius say r. What does that mean? Inside the yarn, we think about a differential layer, it is kind of ring. There could be many fibre segments which will intersect this differential layer. This differential layer is situated at a radius r and its thickness is dr.

So there is a differential layer inside the yarn which is situated at a radius r and this differential layer has a thickness small dr. Now there could be many fibre segments which will intersect this differential layer. For example, you see here this segment, this segment, likewise there could be many segments large number which will intersect this differential layer, that number we think about capital N.

Now we need to find out the packing density of fibres in this differential layer. So what is the packing density layer? This we need to find out. Now based on assumption to this packing density is same at all places inside this yarn. How to find out this packing density? Packing density is defined by the ratio of fibre volume to yarn volume. If we consider fibre length and yarn length same, then packing density can be interpreted as area occupied by fibres/area occupied by the yarn.

We are talking about this differential layer, so here area occupied by yarn that means area occupied by this differential layer and within this differential layer what is the area occupied by the fibres. So area occupied by fibres in the differential layer/area of the differential layer that is the packing density. Area occupied by fibres in the differential layer/area of the differential layer.

Let us find out first the numerator. Area occupied by fibres in the differential layer, how many fibre elements are present here? Capital N. What is the length of each segment? Dl. What is the cross-sectional area of fibre? Small s. So what is the total area occupied by the fibres in the differential layer N number of fibre segments, each segment has a length dl and s. So this is basically the volume and this is volume.

So U is the definition of mu, volume occupied by fibres in the differential layer, volume of the differential layer. So this N, capital N is the number of fibre elements available in the differential layer. Each element has a thickness, elementary thickness, elementary length dl and cross-section area of each element is small s, so this gives the total volume occupied by fibres in the differential layer.

Now we will come to the denominator. Volume of the differential layer, area of the differential*length. What is the length? Length is given and what is the area? 2 pi r dr. So 2 pi r let us write in terms of absolute value, dr*length. So this is the volume of the differential layer, so N this*s/2 pi r*dl/dr right. Now we multiply and divide small n, so we multiply by small n, we divide by small n. What is n? Small n is number of fibres in yarn cross-section.

Also we multiply and divide by yarn twist z here and z here right. Then, 2 pi r z is tangent of beta, so this is equal to tangent of beta. Look at this expression, the whole expression capital N/small n*delta zeta. What is this? Number of fibre elements intersecting the differential layer at radius r/number of fibres/per unit length of yarn. That means this ratio gives you the number of fibre elements intersecting the differential layer of differential layer at radius r per one fibre per unit length of yarn.

This is let us say a function nu of radius r. This we consider as a function of r because if r will change, this number will of course change. So this we will substitute by nu*r right and what is your dl/dr, dl/dr we have already derived, tan square alpha+tan square beta+1/absolute value of tangent of alpha right. So we substitute this all relations to find out another form of mu. So what will be that form?

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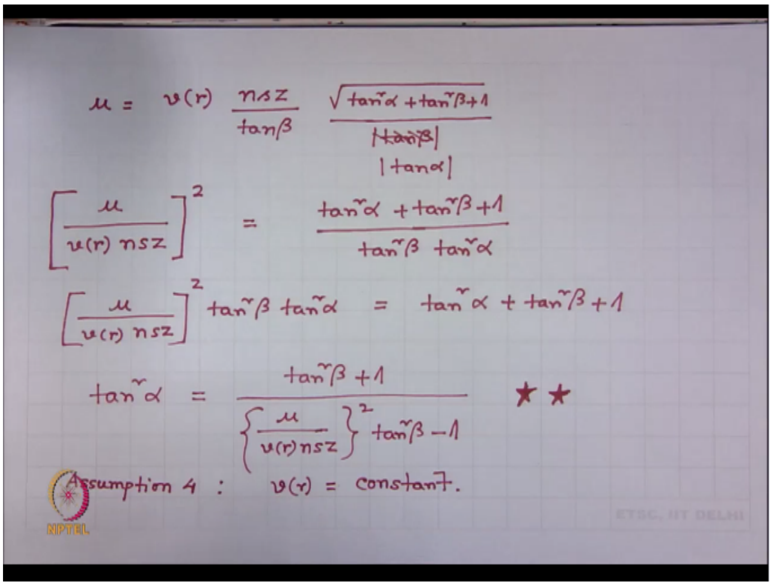

Mu is=nu r n s z/tangent of beta*dl/dr tan square alpha+tan square beta+1/tangent of alpha right. So mu/nu r n s z square is=tan square alpha+tan square beta+1/tan square beta, absolute value always will be a positive quantity, when it is positive tan square alpha. Now radial migration, which parameter is very important? Tangent of alpha. So we need to express these in terms of tangent of alpha.

That means tan square alpha we have to take one side left hand side, rest all in other side right hand side. So let us do that. So mu*n s z square beta tan square alpha tan square alpha+tan square beta+1. Then, tan square alpha=tan square beta+1/mu nu r n s z square tan square beta-1. So tan square alpha is=tan square beta+1/mu packing density nu r n s z square tan square beta-1.

This is a very important equation because it characterizes radial migration of fibre in yarn. What we can learn from this equation is tan alpha dr/d zeta m is a function of radius r because tangent beta is also a function of radius, tangent beta is=2 pi r z and nu r is also a function of radius, so tan alpha radial migration is a function of radius, it must be that is why it is called radial migration right.

Now this is the very important equation for radial migration of fibres. Now we introduce Treloar's fourth assumption. You remember I already told you there were 4 assumptions. First 3 we have already talked about. The fourth one we did not say. Now we are introducing fourth assumption which will be required now. This nu r is a constant. What does that mean and what is its physical significance?

Meaning is very simple, number of fibre segments intersecting a cylinder at radius r per fibre per unit length of yarn is same at all places inside the yarn. Number of fibre elements intersecting a cylinder of radius r per fibre per unit length of yarn is same at all places inside the yarn. This is the simple meaning of this statement. What is the significance of this? This can be understood from this diagram.

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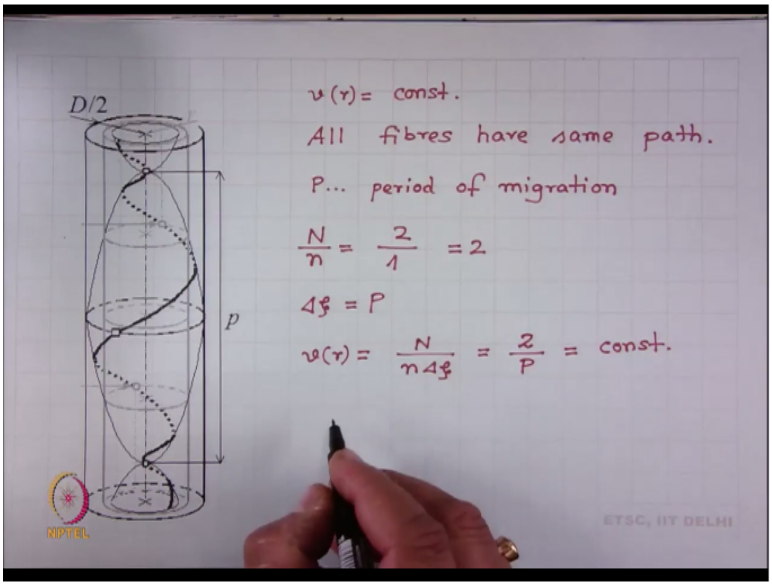

What you see is that a fibre and this is a cylindrical body, diameter is D. Now this fibre starts from here, it intersects the center at this point, then it goes outside the yarn, it touches the surface here, then again it goes inside the yarn, it touches the center here, then it goes here. That means this path of fibres is same for all fibres. So nu r is a constant, this means all fibres have same path.

Typically, what is the path? Imaginably, let us say fibre starts from here, it touches the axis here, then it goes out toward it touches the surface here, then it goes inside it touches the axis here, then it goes. So if we imagine this is the path of fibres inside the yarn, then all fibres must have the same path, then only this assumption is valid. So this was Treloar's fourth assumption.

Now in this diagram, you see a small p is written. What is the small p? Small p stands for period of migration. This period is basically the distance between two points touching the, two adjacent points touching the yarn axis. So this is the definition of period of migration p. Now how many fibre segments intersect at these two points? Two fibres. If we consider one fibre, then capital N is 2, small n is 1.

So capital N/n if you think about 1 fibre then what is capital N? Capital N is 2 because two times it will intersect, then only the definition of period of migration will come into play. So N is 2 right. Delta zeta is p okay, so what is your nu r? Nu r was capital N small n*this. So that is equal to 2/p. This value is constant. All fibres will have same period. This is the interpretation of this assumption.

Of course, it will be because we assume all fibres have same path. Then, we come back to our equation.

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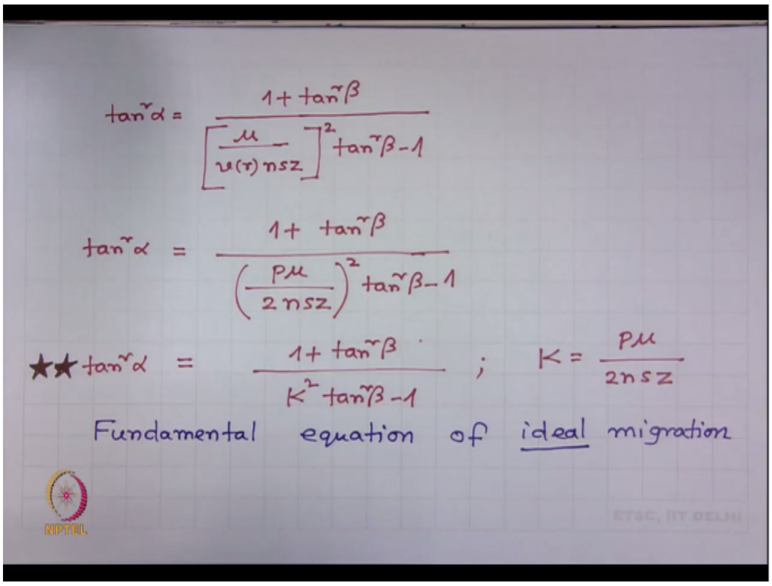

Tan square alpha is=1+tan square beta*mu/nu r n s z square tan square beta-1. We substitute nu r/2/p, so 1+tan square beta is 2/p so p mu/2 n s z square tan square beta-1. This can be further written as where K is 2 n s z. This expression is very important expression. This expression is known as fundamental equation of ideal migration or Treloar's vision. Why this ideal is there because of fibre path.

We have idealized a fibre path. All fibres have same path, so the fibre path is ideal and it will intersect with same frequency. Then only nu r will be constant. So the fibre path is idealized in Treloar's model and this is the fundamental equation of ideal migration. We will stop here. In the next lecture, we will first start with this equation and we will discuss about the meaning of this parameter K and then we will proceed to establish to know more about this radial migration. Thank you very much for your attention.