Theory of Yarn Structure
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Lecture - 17
Radial Migration of Fibres in Yarns (contd.,)

Welcome to you all to this MOOC's online video course theory of yarn structure. In the last lecture, we started module 6 on radial migration of fibres in yarns. So we have talked about general fibre path in yarns, how to characterize this fibre path that we have discussed. Then, we talked about a fibre element, how to characterize this fibre element, we used cylindrical coordinates.

And also we derived certain basic relations among the angles to characterize the fibre path inside the yarn. Then, we classified yarn structural models based on 2 differential equations and then from there we defined what is radial migration of fibres and we started Treloar's ideal migration of fibre model. We introduced 4 assumptions of Treloar's model and based on those 4 assumptions, we derived the fundamental equation of ideal migration.

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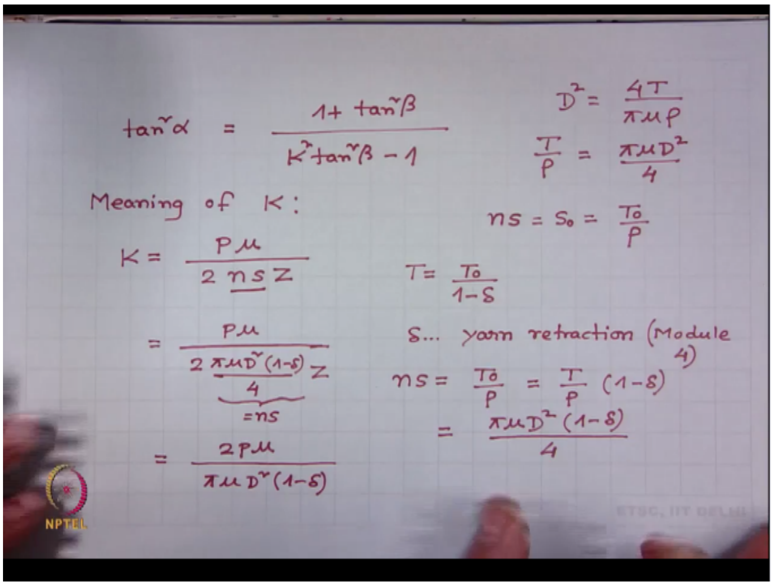

The fundamental equation of ideal migration took this form; tan square alpha is 1+tan square beta/K square tan square beta-1. This was the fundamental equation of ideal migration or fundamental equation of Treloar's model. Now we will discuss about K, what is the meaning of K? We would like to know more about this K. K was equal to p period of migration*packing density 2 times n number of fibres in yarn cross-section, s is the cross-sectional area of a fibre; z is twist which is constant.

So this was equal to K. Now let us think about this quantity n times s, this one. What is this quantity? Number of fibres in yarn cross-section*cross-sectional area of a fibre, so this is the cross-sectional area of all fibres when they are parallel. That means this is basically the substance cross-sectional area of a parallel fibre bundle. The concept of substance area we introduced in module 2.

So this n is we can write as substance cross-sectional area of the non-twisted fibre bundle. In module 2, we used capital S to denote substance cross-sectional area. Here we use the same symbol with subscript 0 to denote the initial non-twisted fibre bundle and also in module 2 we have introduced this relation where capital T0 denotes the fineness of the bundle, parallel fibre bundle and rho is fibre density.

This T0 is related to final yarn by this relation where this denotes yarn retraction. Yarn retraction was discussed in helical model of fibres in yarn. That was probably module 4 right. So this n is T0/rho that is=T/rho*1-retraction right. Capital T is yarn fineness. We know the basic relation between diameter and fineness of yarn. That is your D square is=4 times capital T/pi mu rho.

This is yarn diameter D capital D, T is yarn fineness, mu is packing density, rho is fibre density, so T/rho pi mu D square/4. If we substitute here T/rho then what we will obtain, pi mu D square 1- retraction/4, that is your ns right. This ns we substitute here, so what you obtain is p period of migration*packing density/2 pi mu D square 1-retraction/4. This is your ns and this is your z right, so 2 p mu/pi mu D square 1-retraction.

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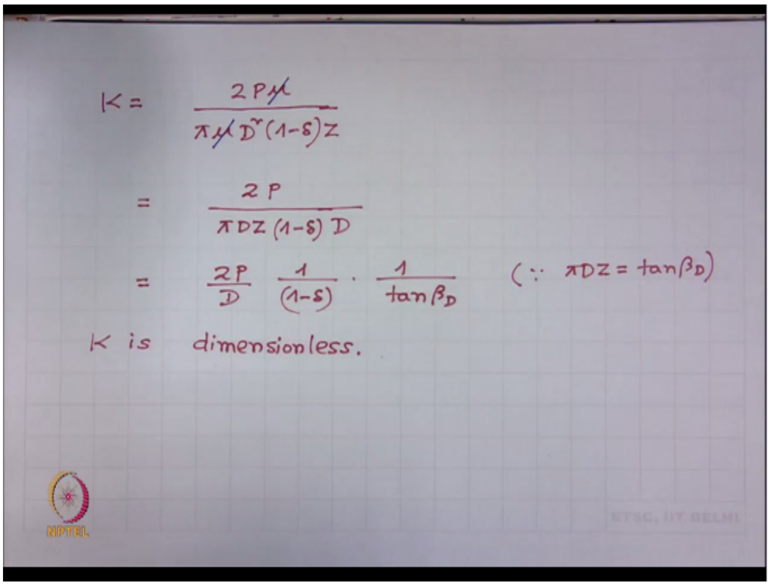

Let us write it further, K is=2 p mu/pi mu D square 1-retraction right. Now this mu mu cancel out right, where is z, z must be here. This z was missing from here. This z must be here, so 2 p mu/pi mu D square 1-retraction*z right okay. This we can further write 2 p/pi D z 1-retraction*D. In a nice form, 2 p/D 1/1-retraction*what is pi D z, yarn twist intensity, tangent of beta D module 2, tangent of beta D right.

So this is what is K period of migration has a dimension of length millimeter. Yarn diameter has also having length dimension millimeter. So these two length dimensions cancel out. Yarn retraction dimensionless, p D z kappa twist intensity is also dimensionless, so K is a dimensionless quantity. Often said in theoretical research work, dimensionless quantities are very important.

They signifies something about the structure right. Now we come back to the fundamental equation of radial migration and we would like to know the domain of that expression.

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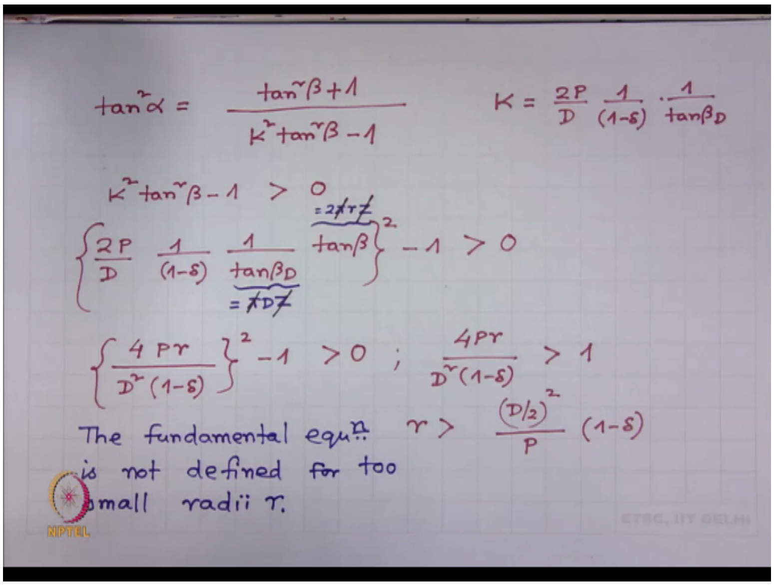

So fundamental equation was tan square alpha=tan square beta+1/K square tan square beta-1, where K is=2p/D 1/1-retraction*1/tangent of beta D right. Left hand side is a square quantity that means this denominator must be >0. So K square tan square beta-1 must be >0. What is K? 2p/D 1/1-retraction 1/tangent of beta D, here also tangent of beta square-1 must be >0. What is tangent of beta D? That is equal to pi D z and what is tangent of beta? 2 pi r z.

So what we see is that this pi cancel out, z cancel out, so 2r/D, so basically 4. So what we are left with, 4pr/D square 1-retraction-1>0 right. So 4pr D square 1-this must be >1. So r must be greater than this. So r must be greater than a small quantity, D is very small, p is small, retraction is also small. So this expression gives you a very small value; however, it is the value and r greater than this.

That means there is a problem. What is the problem? Problem is the fundamental equation is not defined for a small value of r. So there is some problem in this expression. What is the meaning of this?

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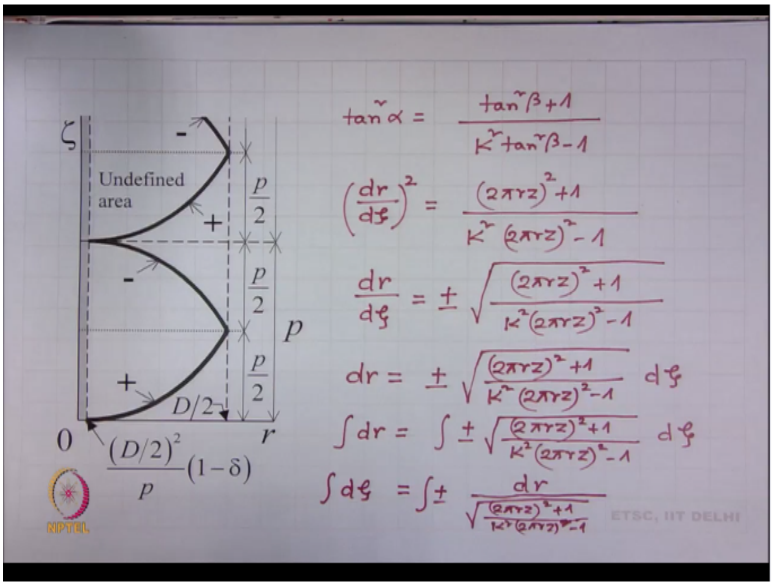

Meaning of this is something like this. This is the radius and this is the axial length of yarn r zeta. Fibre is not starting from 0 because r is greater than a small value. So probably the migration starts from here. So it goes there, come back, it is not touching the axis. A small distance is left; this distance is this D/2 square/p 1-retraction. Again, it goes, come back. So in this gray color region, migration is not taking place.

This is the meaning of this. So what is the fibre path? According to this, tan square alpha=tan square beta+1/K square tan square beta-1. Now what is tan alpha? dr/d zeta square. What is tan beta? 2 pi r z+1/k square 2 pi r z-1. So dr/d zeta is +-1, so dr is=+- 2 pi r z square+1/2 pi r z square-1*d zeta. So if we integrate dr or else d zeta +-dr/this expression 2 pi r z square+1 K square 2 pi r z square-1.

So if we integrate this, then we will be able to obtain the path of a migrating fibre; however, this integration does not have an analytical solution. It is elliptical integral. This is one problem. Problem number two, fibre path is not defined at a very small radius. So this so far the fundamental expression of radial fibre migration as per Treloar's model have two problems.

One, it is not possible to obtain the fibre path of a migrating fibre analytically. Second, fibre path is also not defined at a too small radius. That is why we introduced two approximations.

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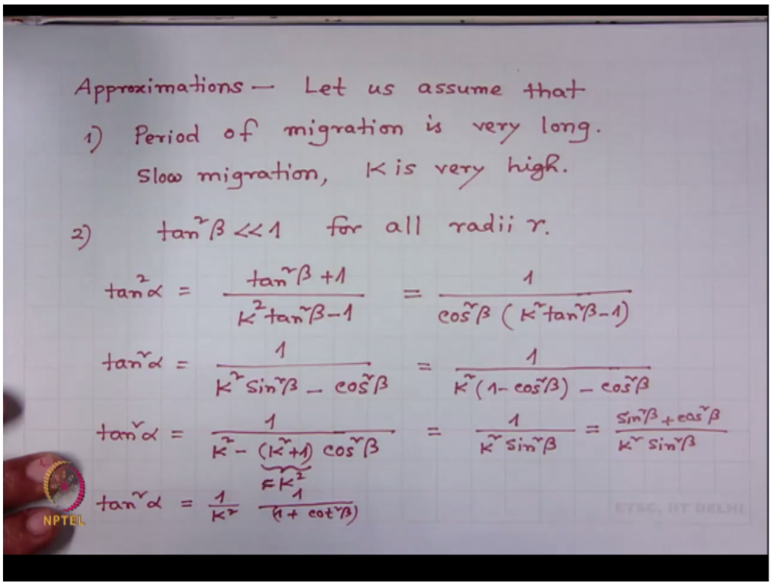

So we introduced two approximations so that there will be an analytically possible fibre path and it will be defined at all radius. So the first approximation, period of migration is very long. We think that so let us assume that period of migration is very long. What does that mean? Slow migration. If period of migration is very long, K is proportional to p, so K is very high, so this is our first assumption.

Second, we introduced we assumed that tan square beta is much<1, a very small value, much<1 for all radii r. So we here introduced two approximations and we will like to know what is the implication of these approximations. So we come back to the original fundamental equation, tan square alpha is=tan square beta+1/K square tan square beta-1 okay. Tan square beta+1 is sec square beta, sec square beta is 1/cos square beta.

So 1 so we can write 1/K square sin square beta-cos square beta. Sin square beta is=1-cos square beta right. So K square-K square+1 cos square beta. K is very high, so K square is very high, K square+1 very high quantity+1 remains very high approximately equal to K square. Then, it becomes K square*1-cos square beta, so K square sin square beta. So we used here first approximation.

Now we would like to use second approximation somewhere, let us write 1 as sin square beta+cos square beta/K square sin square beta. Then, you see tan square alpha is=1/K square 1/1+cos square beta. Let us go to the next page.

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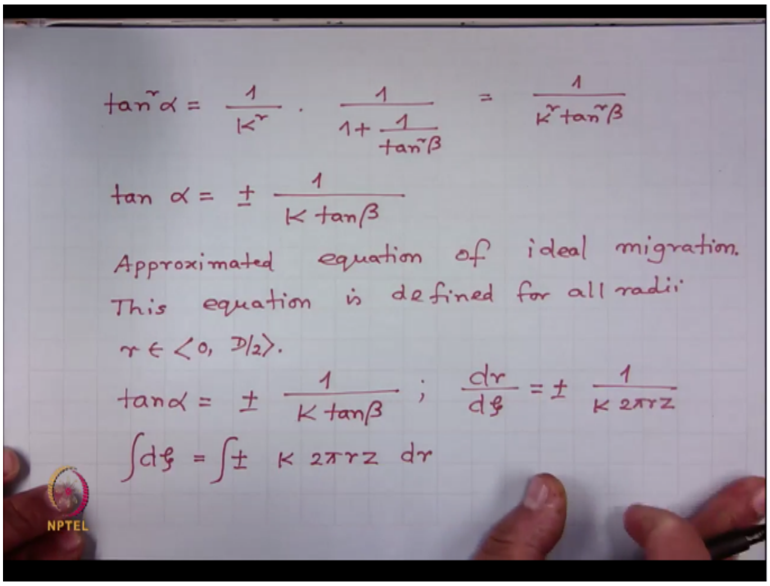

So tan square alpha is=1/K square*1 1+cos square beta, so 1+tan square beta. Now you see tan square beta is much <1, 1/tan square beta is much >1 right, very large quantity+1 remains that large quantity. So this becomes K square tan square beta, so tangent alpha is=+-1/K tan beta. This is known as approximated equation of ideal migration. This equation is defined for all radii, so one problem is over.

This expression is defined for all radii. What was the second problem, second problem of the fundamental equation? The analytical solution of the integral was not possible. Let us see if it is possible here. So tangent alpha is=+-1/K times tangent beta. What is tangent alpha? dr/d zeta and tangent beta is 2 pi r z. So we can write dr/d zeta is=1/K times 2 pi r z right. So d zeta is=+-K times 2 pi r z*dr right. Now if we integrate this what you will obtain?

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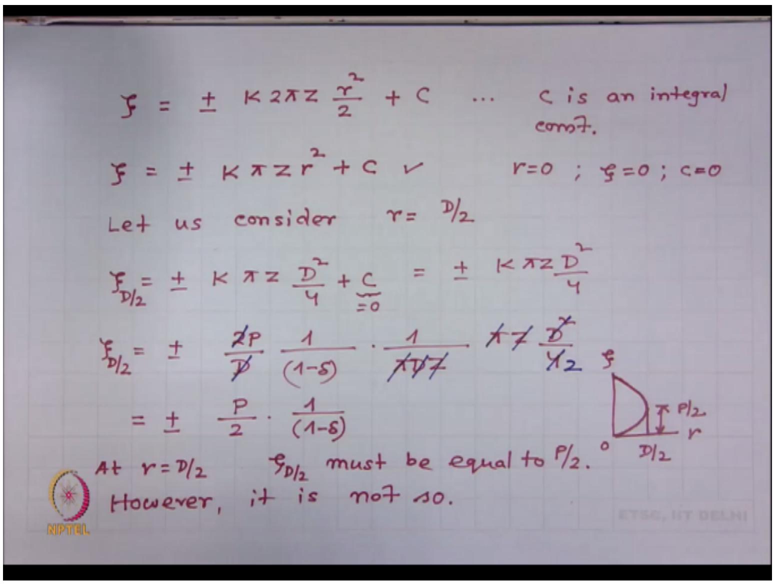

You will obtain zeta is=+-K 2 pi z*r square/2+c where c is an integral constant right. So zeta is=+-K pi z r square+c. So this integration is possible. So the two problems associated with the fundamental equation, one was analytical solution was not possible, second it was not defined for very small radius. So in order to avoid those two problems what we thought we considered two approximations.

One is K value is very high, slow migration, period of migration is very high. Second, tan square beta much <1. Based on these two, we obtained we found that it was defined the approximate equation was defined for small radius and also analytical solution is possible. So we are in this way by introducing two approximations we could solve the problem associated with the radial migration of fibres.

But this equation gives additional problem. What is that problem? Let us see. Let us consider r is=D/2, so +-K pi z D square/4+c. Now look at this equation, when r is=0 this must be equal to 0, so c is=0, so this is equal to 0. So what we find is that +-K times pi z D square/4. What is K? K was 2p/D*1/1-retraction*1/tangent of beta D, tangent of beta D is pi D z*pi z D square/4. Now this pi is cancel out, z cancel out, this D this D and D square cancel out.

This two this two, so what we obtain is that p/2*1-this. This is a problem because at D/2, this value must be equal to p/2. Why? Because if this is r, this is zeta, if this is your D/2, then this distance is p/2. So at r is=D/2, zeta D/2 must be=p/2; however, it is not so. So we need to correct the approximated equation.

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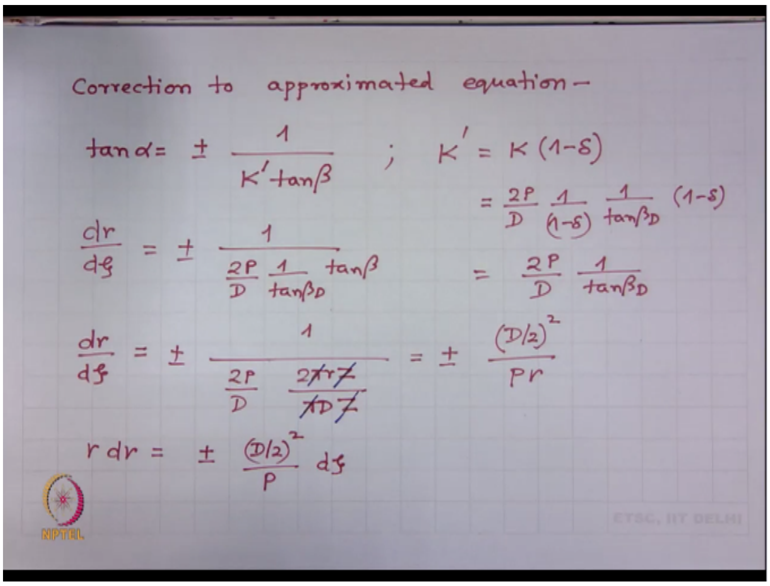

So correction to approximated equation, let us say tangent of alpha is=+-1/K prime tangent of beta, instead of K we consider K prime. How is the relation between K prime and K? K prime is=K*1-retraction. So K was 2 p/D 1-retraction 1/tangent of beta D*1-reraction, so 2p/D 1/tangent of beta D right. So then what will be your tangent alpha? So by using this, we are able to solve which is now defined at this p and this when r is=D/2, so let us say zeta value.

So dr/d zeta is=+-1/K times and we have to write it so 2p/D 1/tangent of beta D*tangent of beta okay. So let us see what is happening in the right hand side, 1/tangent of beta 2 pi r z tangent of beta D pi D z okay, that is equal to this. So now this z z cancel out, pi pi cancel out. So what we see is +-this D square will be in the numerator, so D/2 square/4, this p will be in the denominator, that is all.

D square/4p, so then this r will be there, yes so r dr is=+-D/2 square/p*d zeta. So let us integrate now.

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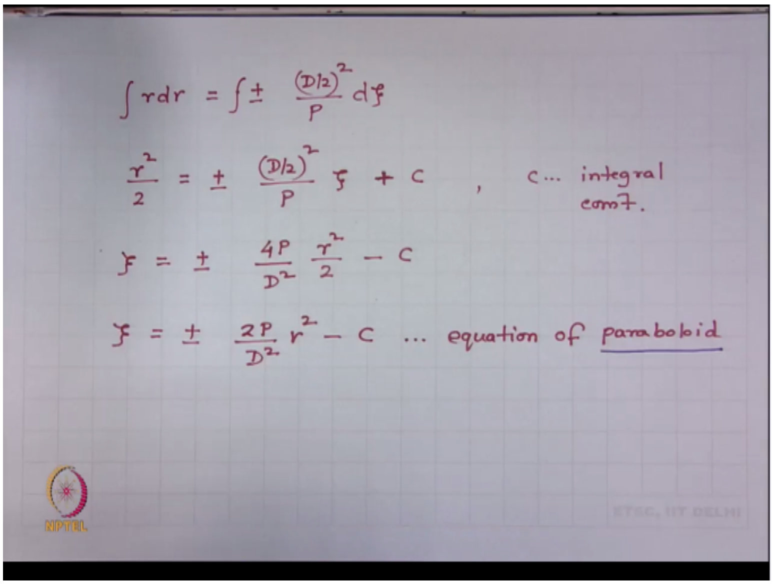

Integrate r dr, integrate +-D/2 square/p*d zeta right. So this integration will give you r square/2+-, this is constant. Integration of d zeta is zeta+say constant integral constant right. So what is zeta? Zeta is=+-4p/D square r square/2-c. Further we can write is equal to +- 2p/D square*r square-c. This is an equation of paraboloid. So the corrected form of approximated equation fibre path is parabolic inside the yarn.

So according to Treloar's vision, ideal fibre migration approximation and corrections after doing this we obtained that the fibre path inside the yarn is a parabola. How it looks like imagination, imaginatively this is what is happening as per Treloar's model.

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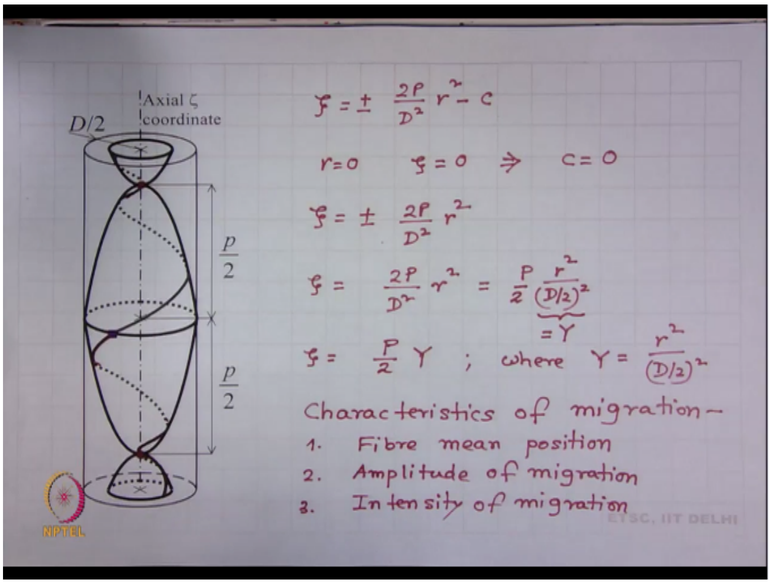

Suppose a fibre starts from here, it touches the axis red color, then it comes out to the surface dotted line you can see, from the surface it again goes inside, then it touches the surface, it comes to the surface, goes inside, then it touches the axis again. Then, it goes out so this path of fibre is parabolic one and this distance is p/2, this distance is p/2, so this is the total period of migration p.

So as per Treloar's model, you can imagine how is the fibre path inside the yarn. It basically resembles a parabola. So what was the form? Form was this=+-2p/D square r square-c. When r is=0, this must be equal to 0. This infers c is=0, so equal to +-2p/D square r square. Now as you can see, this is a very symmetric one. So that is why we can take for calculations either + or -, + is outward inward. As it is a symmetric, so probably in order to establish the characteristics of migration we can take one, any one positive.

So this equation let us take the positive quantity only because it is a symmetric one, r square. This we can further write p r square/D/2 square. This r square/D/2 square let us consider this as capital Y because you can measure this. D is measurable and r is also measurable by tracer fibre technique and all. So then what do you find zeta is=p/2*into Y, where Y is equal=r square/D/2 square.

Now we introduce characteristics of migration. How we can characterize migration experimentally? Generally, Treloar and his co-worker they used 3 characteristics for radial migration. One fibre mean position; second is the amplitude of migration, third is the intensity of migration. How intense is the migration, whether it is slow or fast how? We would like to discuss about these 3 characteristics now. First start with fibre mean position.

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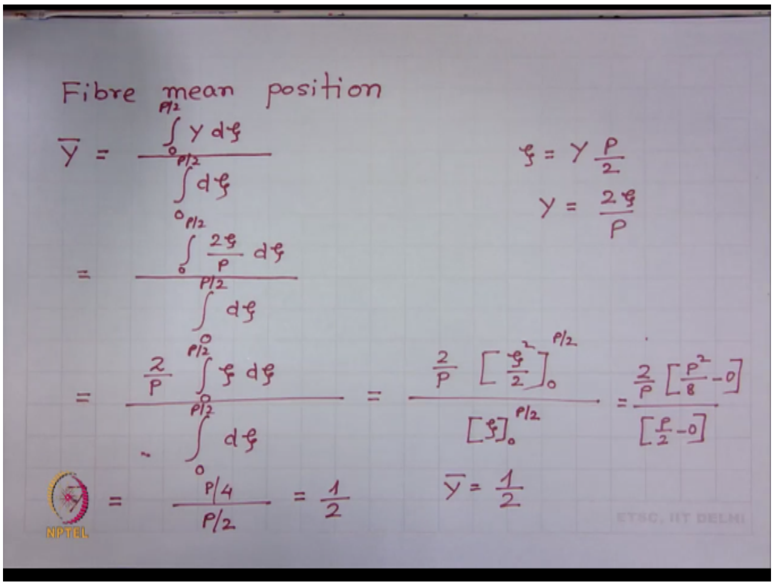

Fibre mean position is defined by Y bar is=Y times d zeta/this. This is 0 to p/2, this is 0 to p/2 and what is Y? Y was your zeta is=Y times p/2. So Y is=2 zeta/p. This is how fibre mean position is defined. If we substitute 0 to p/2 2 zeta/p zeta 0 to p/2, p is constant, 2 is constant, it will come out of integration. So 2/p zeta times d zeta 0 to p/2 zeta 0 to p/2. Integration zeta d zeta, zeta square/2, integration d zeta, zeta.

So 2/p*p square/8-0, here it is p/2-0, so Y bar is here it is p/2 and there it is p/4. So ½, Y bar is=1/2, mean fibre position is 1/2. As it is a symmetric migration, there is an ideal fibre path, regular fibre path, so mean fibre position is equal to 1/2 in Treloar's model ideal migration. Second characteristics is amplitude of migration.

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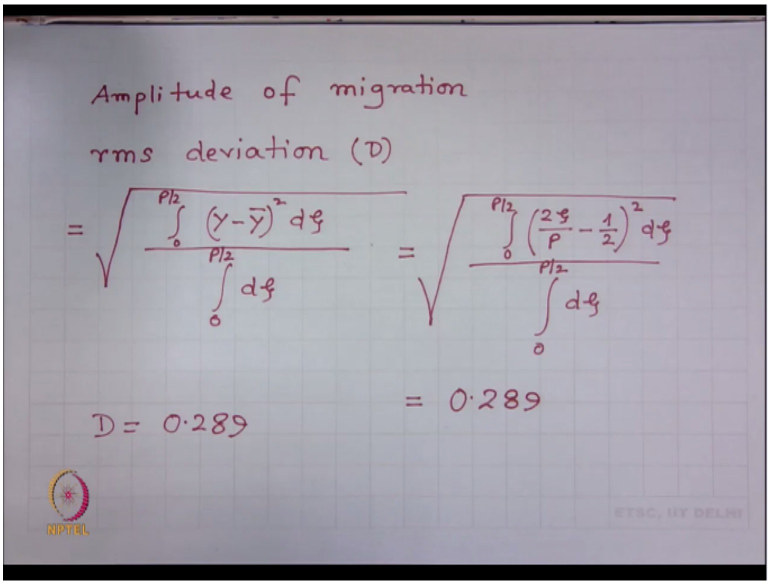

Amplitude of migration is discussed in terms of rms deviation. What is rms deviation? Rms deviation is defined by Y-Y bar square d zeta/d zeta. This is the definition of rms deviation. So what is your Y? Y is your 2 zeta/p. What is Y bar? 1/2 square d zeta/d zeta. So if you integrate this, you will get this value. D is=0.289 in case of this. There is amplitude of migration. The last characteristics is intensity of migration.

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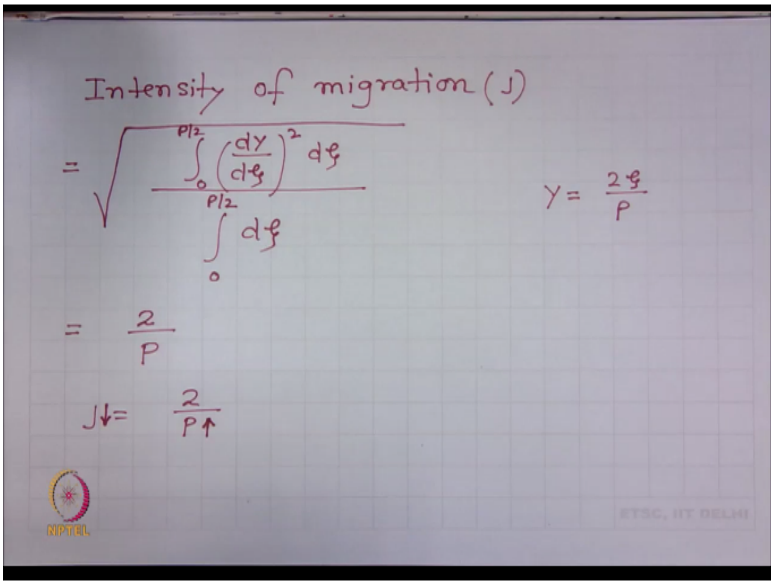

Intensity of migration, we use symbol this, dY/d zeta so dY/d zeta square d zeta 0 to p/2, 0 to p/2. So this is how we define intensity of migration. What was Y? Y was 2 zeta/p right. So if you take the first derivative, substitute, solve this integration, you will see that this value you will get as 2/p. You can try this. What we see is that if period of migration is high, J is less, slow migration. When p is very high, migration is happening very slow rate, intensity also indicates the rate of migration.

So the rate of migration is slow. You remember when we used approximations, two assumptions we considered, one is tan square beta much <1 and the second was about p is very high, then we talked about slow migration because of this. When p is very high, the migration is very slow. So these 3 characteristics can be obtained from Treloar's model of migration. Now the very important question.

How is the correspondence between Treloar's migration model and reality experimental results? Is it so that Treloar's model explains the experimental results very well or is it so that Treloar's model fails to explain the experimental results? In order to find out this answer, lot of experiments were carried out and the comparison between theory and experiment is shown here.

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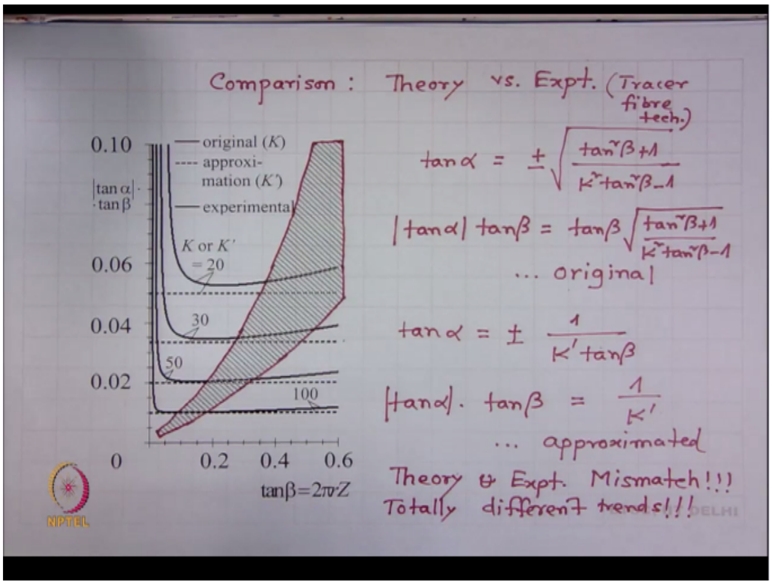

This region is the experimental region. Lot of yarns were tested by using tracer fibre technique, migration characteristics were measured and in this region all experimental results fall right and what you see here, tangent beta is plotted here 2 pi r z and this is tangent alpha*tangent beta. Two curves you see, one is the thick continuous curve, one is the dotted curve. Thick continuous curve is original expression, fundamental expression.

What was that? Tan alpha was=+-square root of tan square beta+1/K square tan square beta-1, is not it? That was your original expression. Now if we take the absolute value*tangent beta is equal to what, tangent beta*this, so this is basically the original. So the thick line denote this expression original for different value of K. K is=20 this curve, K is=30 this curve and K is=50 this curve, K is=100 this curve.

So it is decreasing then increasing, decreasing increasing, decreasing increasing, decreasing slowly increasing this original and approximated equation is shown by the dotted line. What was the approximated relation? You remember tangent of alpha+-1/K prime tangent of beta. So tangent alpha*tangent beta and if we take the absolute value then it becomes this. So this was the approximated one K prime.

So obviously this is constant because K prime is constant, so this expression is constant. That is why you see line parallel to x axis. This is a line parallel to x axis for K prime is=30, K prime is=50, K prime is=100. So this is about this curve and the experimental results were obtained using tracer fibre technique, Morton's tracer fibre technique. What we see that? There is a complete mismatch between the theoretical results and the experimental results.

So for the experimental results are concerned when tangent of beta is increasing, this function is increasing; however, the theoretical results original one decreasing then increasing and approximated is constant. So there is a complete mismatch between theory and experiment, totally different trends. So conclusion is Treloar's model could not explain the experimental results. They are totally different.

Why is it so? Is there any possibility to correct Treloar's model and check with experiments? Is it possible to obtain a more precise model to explain radial migration of fibres in yarn? This we will discuss in the next class. Thank you very much for your attention.