Theory of Yarn Structure
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Lecture - 18
Radial Migration of Fibres in Yarns (contd.,)

Welcome to you all to the MOOC's online video course theory of yarn structure. In the last lecture, we started with module 6 radial migration of fibres in yarns. We have covered about general fibre path in yarn, how to characterize this. Then, we talked about fibre element, then we talked about some angles to define fibre path in yarn. Then, we discussed about Treloar's ideal fibre migration model.

And at the end we observed that Treloar's ideal fibre migration model is not able to explain experimental results of radial migration of fibres in yarn. So we start from there in this lecture. So the obvious question is that why Treloar's ideal migration model does not explain the experimental results of migration. We observed totally different trends.

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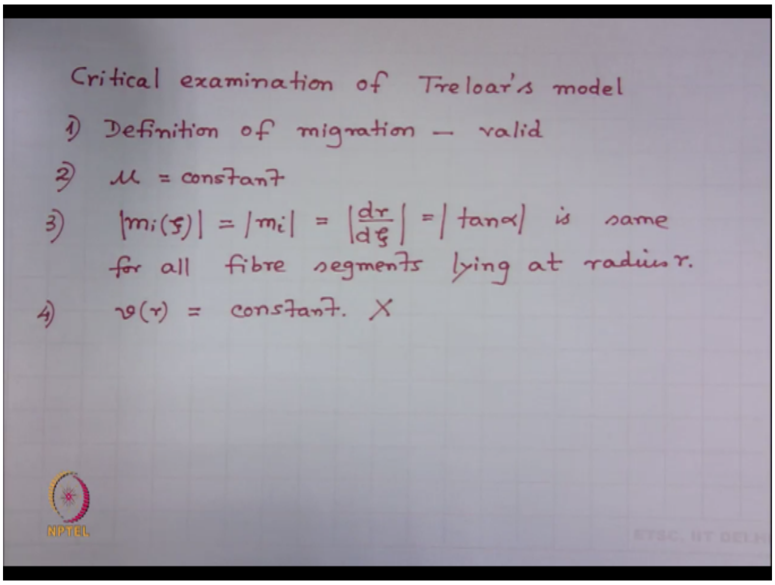

So we wish to critically examine the assumptions of Treloar's ideal migration model. If you remember, the first assumption was related to definition of radial migration which is no doubt valid. Second assumption of Treloar's model was packing density constant at all places inside the yarn. This assumption although we know it is not experimentally fully correct; however, this is an idealized situation, so we can to some extent agree to this.

Third assumption was the absolute value of this is same for all fibres present at radius r. This assumption is idealized and is acceptable. The fourth assumption was nu r is constant, nu r if you remember, it is the number of fibre segments intersecting cylinder at radius r per fibre per unit length of yarn. This assumption is probably not correct. Why do we feel so?

We feel so because if we imagine the inside of a cylindrical yarn, then we see that at the core the volume of yarn is very less because radius is very less, so fibre to fibre contact is very less there. At the same time, if you think about the surface of the yarn where packing density is very less. If the packing density is very less, then the fibre interaction is also very less. Therefore, we feel that this number of fibre segments intersecting a cylinder at any radius r per fibre per unit length of yarn is probably not constant, it varies.

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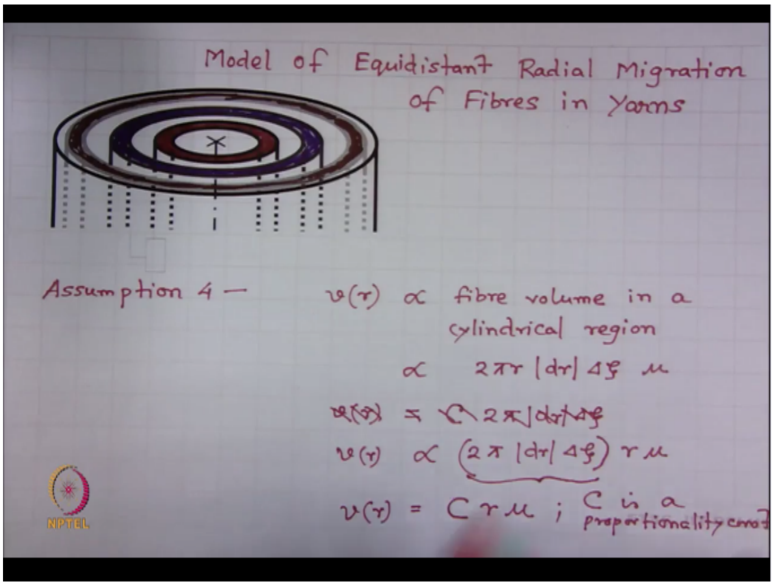

Here you see a cylindrical yarn. Near to this region near to the core in this region, the volume of the cylinder is very less because it is very close to center of the yarn. So its radius is very small, so fibre intersections must be small here. If you think about a cylindrical region near to the surface, say this region near to this region as we know packing density of actually yarn is very less at the surface.

So the packing density is less, so fibre volume is also very less, number of fibre is very less, as a result interaction among fibres must be less too. Somewhere inside the yarn in between these two cylindrical surfaces, packing density is significant. So number of fibre intersections must be very high somewhere near to this cylindrical region because number of fibres is too high here.

So fibre to fibre interactions, fibre to fibre contact, packing density all are very high in this region. So therefore it is felt that the packing density in this region is high, fibre to fibre interaction is very high, as a result it is thought that this nu r number of fibre intersections is not same at all places inside the yarn, wherever fibre volume is too high it is very high, so assumption 4 is modified.

So in this model, assumption 4 is modified. What is this assumption 4? Assumption 4 is modified in this way. Number of intersections, number of fibre to fibre interactions must be proportional to the fibre volume in a cylindrical region. So what is fibre volume? What is the volume of the cylinder? 2 pi r*dr*zeta. This is the volume of the cylinder and what is the fibre volume, multiplied by packing density.

Because as you know, packing density is defined by the ratio of fibre volume to yarn volume. When the yarn volume is 2 pi r dr times delta zeta*packing density will give you the fibre volume. So nu r is=Capital C 2 pi dr delta zeta is proportional to 2 pi dr delta zeta*r*mu. If you look carefully, this quantity within the parenthesis is a constant. This is same for all radii. So we can write nu r is a constant*r*mu where C is a proportionality constant.

So this way we modify the fourth assumption of Treloar's model. So we are now discussing a new theory which is known as model of equidistant radial migration of fibres in yarns. So in this model assumption 4 is modified in this manner.

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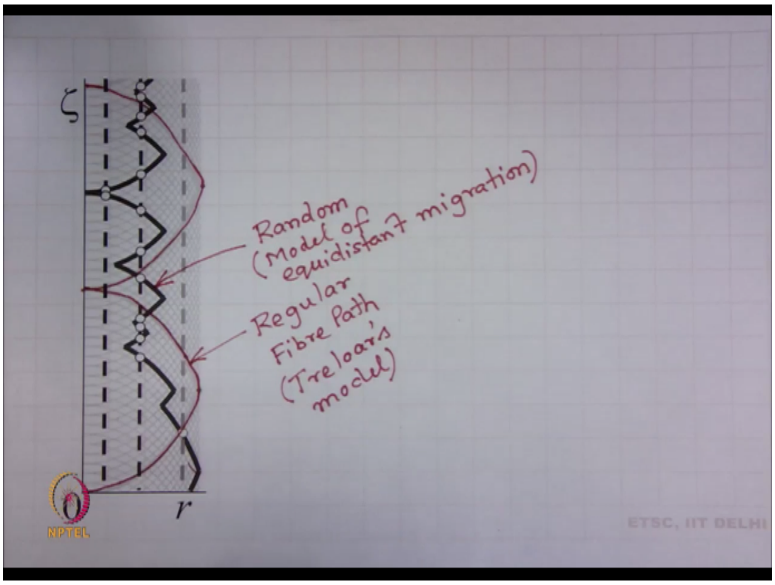

So it is evident that the model path of fibres if we imagine then in this model, model of equidistant fibre migration model, it starts somewhere here, then it has very less intersections near to the surface, maximum intersection fibre to fibre intersections happening over here at the core also is very less; however, if you remember Treloar's model this fibre intersections was same was considered to be same at all places.

So in Treloar's model, the path was assumed to be like this regular fibre path but in this model fibre path is random. So this is the basic difference between Treloar's model and model of equidistant radial migration.

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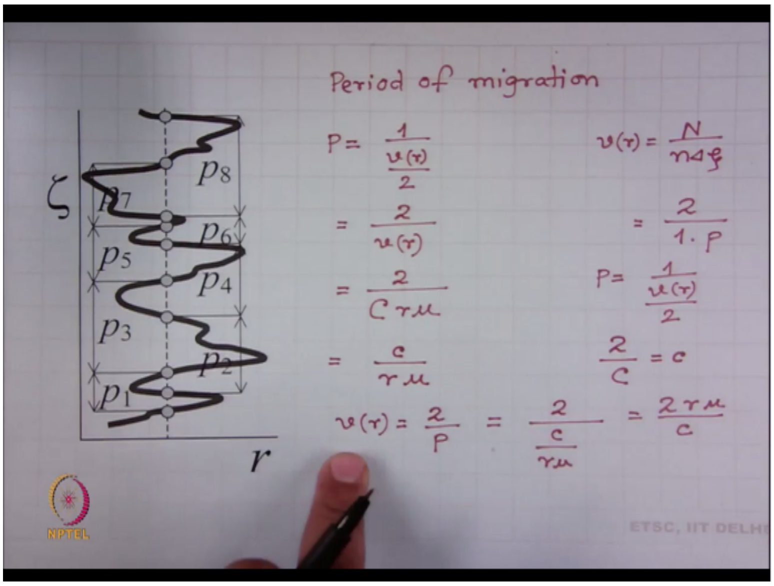

Now if you look at what will be the value of period of migration, it is definitely varying in this model. This period of migration can be written by 2 because you remember nu r was defined by this so when small n is=1, how many fibres or fibre segments will intersect the axis in order to get a period p? 2 so is nu r/2 right. So you can write period is here, now varying, now what was nu r, nu r was capital C r*mu right.

We consider this 2/C is a constant because capital C is a constant, we can write as a small c. So we write small c*r*mu right. So nu r is=2/p and p is=small c/r*mu. So 2 r mu/small c. So this is the expression for nu r. You see here if r varies nu r will vary because c is a constant and mu is also assumed to be constant in this model too. So this is how this model is different from Treloar's model.

Now we will go back to the fundamental equation of radial fibre migration. We will substitute nu r there and we would like to see what is the expression comes out and what does that expression tells to us.

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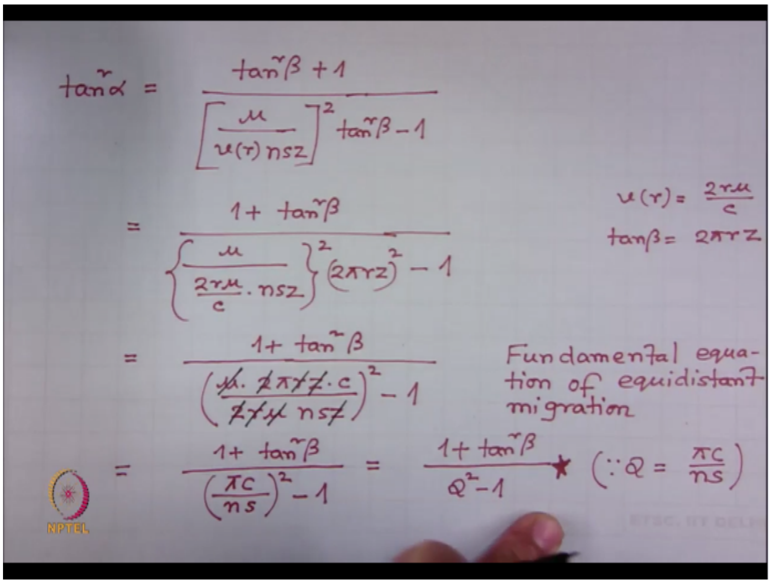

So if you remember the fundamental equation of radial fibre migration, mu/nu r-1, this was the fundamental equation of radial fibre migration. Now we substitute nu r this and tangent beta 2 pi r z. We will make these two substitutions into this expression and we would like to see what it comes out. So tan square beta let us write as tan square beta here. We will substitute in the denominator mu/2 r nu/small c*n s z square tangent beta 2 pi r z square-1.

So what it becomes is 1+tan square beta/mu*2 pi r z*c/2 r mu n s z square-1 right. So we would like to see what it comes out. Numerator we do not change; denominator you would like to see what is coming out. These two and these two cancel out. This mu this mu cancel out. This z this z cancel out. This r this r cancel out. So we are left with a simple form pi*c/n*s square-1.

Pi is a constant, small c is a constant, n number of fibres in the cross-section of the yarn is constant, small s is fibre cross-sectional area that is also constant. So pi c/ns is a constant. Let us assume this is equal to capital Q which is a constant. Then, we write as Q squared-1. This is a very important expression. This expression is known as fundamental equation of equidistant migration.

Last expression is known as fundamental equation of equidistant migration. If you like to know how to find out Q, then we can do one thing.

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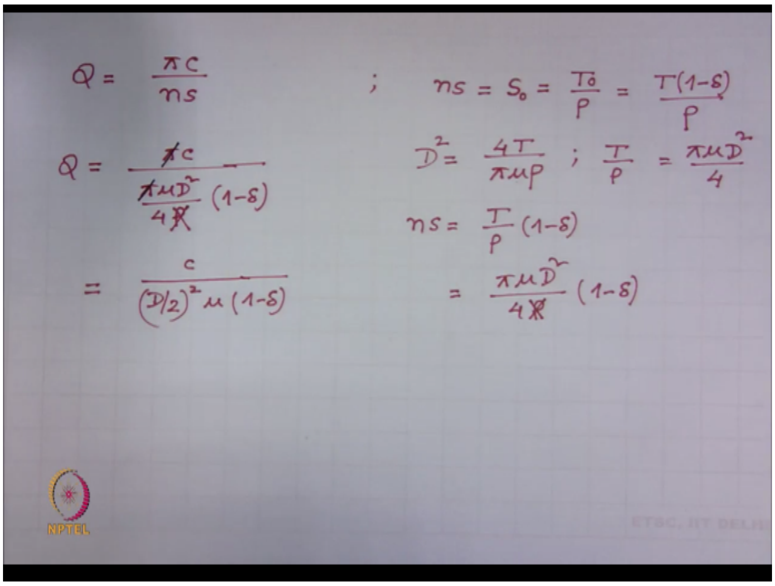

So Q is=pi times small c/n*s. Now what is ns? Number of fibres in the cross-section of yarn*cross-sectional area of one fibre, so this is the cross-sectional area of all fibres, substance cross-sectional area S0. What is S0? S0 is capital T0/rho is the starting fineness of parallel fibre bundle which is equal to fineness of the yarn*1-retraction. We discussed this in helical model of fibres in yarn.

Now we know yarn diameter square take this form, so T/rho is=pi mu capital D square/4. If we substitute this here, then pi mu D square/4*rho 1-retraction right. Then, if you come back here in Q, so pi times small c/pi mu D square/4*rho 1-retraction. What we see that this pi and pi will cancel out, so sorry this rho will not be here. So we can write it further small c/D/2 square mu*1-retraction.

So this is about capital Q okay. Now we would like to derive the expression for fibre path at different radius, so zeta versus small r. So we go back to our fundamental equation of radial migration.

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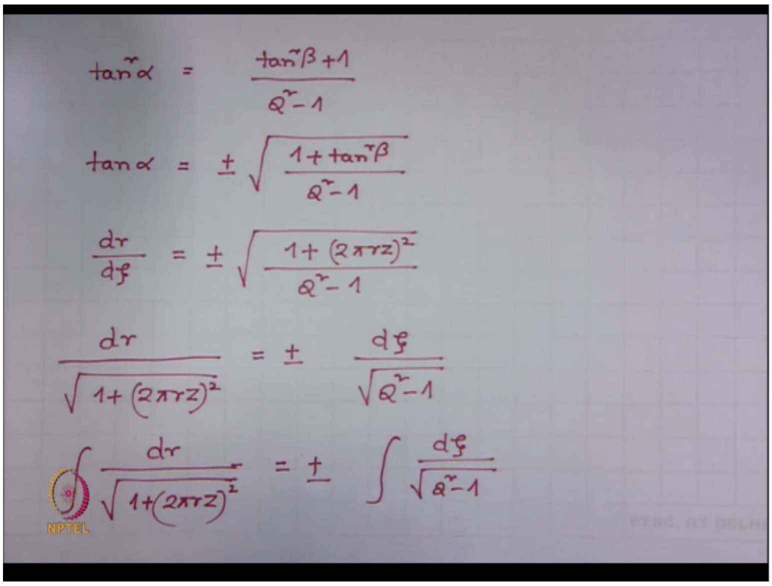

Tan square alpha is=tan square beta+1/Q square-1, so tan alpha will be equal to +-1+tan square beta/Q square-1. So what is tangent of alpha? dr/d zeta from definition, dr/d zeta=+-root over 1+tangent of beta is=2 pi r times z square/Q square-1 right. So we can write further dr/root over 1+2 pi r z square is=+-d times zeta/root over Q square-1 right. So if we integrate this +-this. So we need to do this integration. Let us think about this part.

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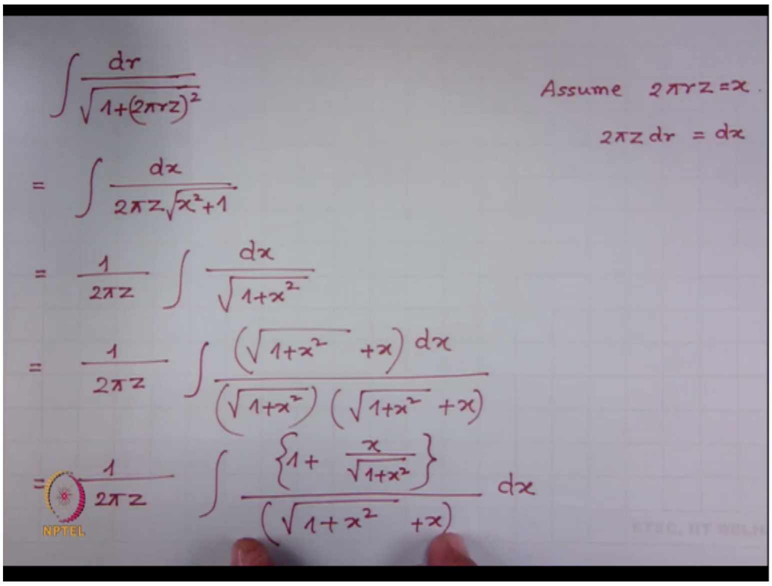

So integration dr/root over 1+2 pi r*z square. How to integrate this? Let us assume 2 pi r*z is=x so integration by substitution. So 2 pi z dr is=dx so integration dr is dx/2 pi z and this is your x no s square+1 so 1/2pi z is a constant, it can now come out of the integral. This we write in a little different manner. Let us multiply by this in the numerator; it must be the same in the denominator also right.

Now we can further divide this by this quantity. So what we will get is 1+x by into dx. Now we will consider this quantity as Y.

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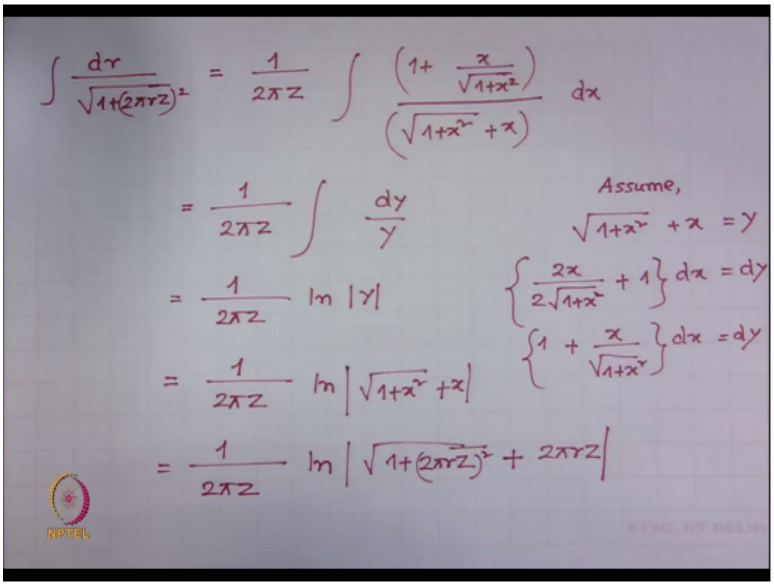

So we rewrite the last step for our convenience. This is=1/2 pi z integration 1+ divided by 1+x square+x*dx. Assume root over 1+x square+x is=Y. So this is 2x/this+1 dx=dy. So 1+x/root over 1+x square*dx is=dy, so then we can write 1/2pi z, this is your dy and this is your Y. So integration dy/Y is logarithmic Y, 2 pi z ln*Y, what is your Y? Y is root over 1+x square-x, what is your x? x was 2 pi r z 1+2 pi r z square-2 pi r z right.

So 1/2 pi z logarithmic of root over 1+2 pi r z square+ sorry this must be +2 pi r z. So this was one integration. What was the second one?

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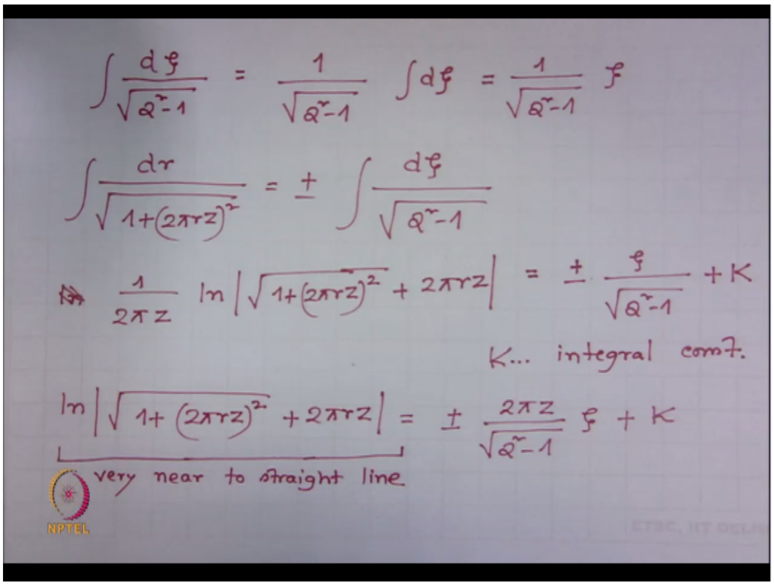

Second one was simple one, d zeta/this. Now here this is constant d zeta, so root over Q square-1*zeta okay. So what was our original form of integration? So our original form of integration was this dr/root over 1+2 pi r z square is=+-integration d zeta this. So this integration we found as 1/2 pi z ln root over 1+2 pi r z square+2 pi r z is=+- zeta/root over Q square-1+say K, K is integral constant right.

So we can rewrite it further 1+2 pi r z square+this is= +-2 pi z/root over Q square-1*zeta+K. So this expression will give you the fibre path inside the yarn. Now if you put different values of r, then you will realize that this gives you an almost straight line. So we can say that the infinitesimally small curve of a fibre follows almost a linear path inside the yarn. So imaginatively inside the yarn it follows this kind of straight line.

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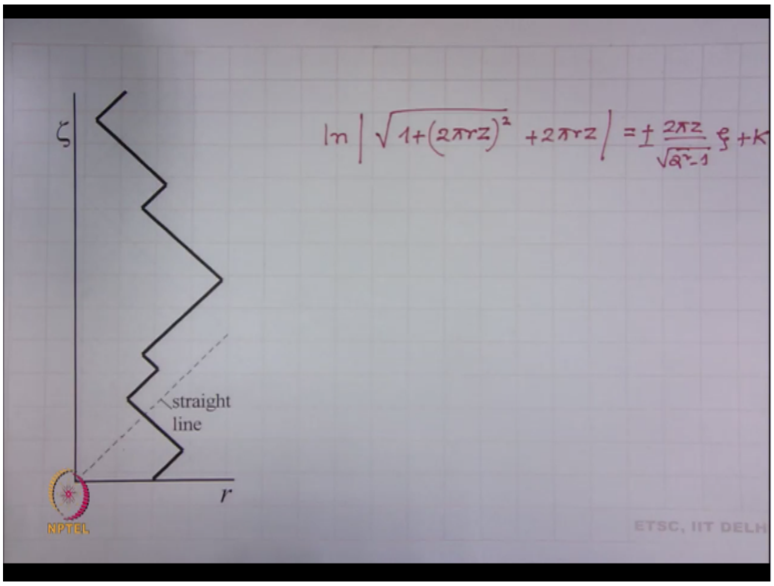

So in the last expression if you put different values of r then you can find out different values of zeta. If you plot them r versus zeta you will obtain a curve which will be resembling very similar to this straight line. So this infers that the path of infinitesimally small fibre segments inside a yarn is very near to a straight line. So 1+2 pi r z square+2 pi r z is=2 pi z root over zeta+K.

So if you take different values of small r, you will get different values of zeta. If you plot, you will get such behaviour. Now we said this model as model of equidistant migration of fibres in yarn why we said so? So why this model is called as equidistant migration of fibres in yarn? So we would like to talk about it now.

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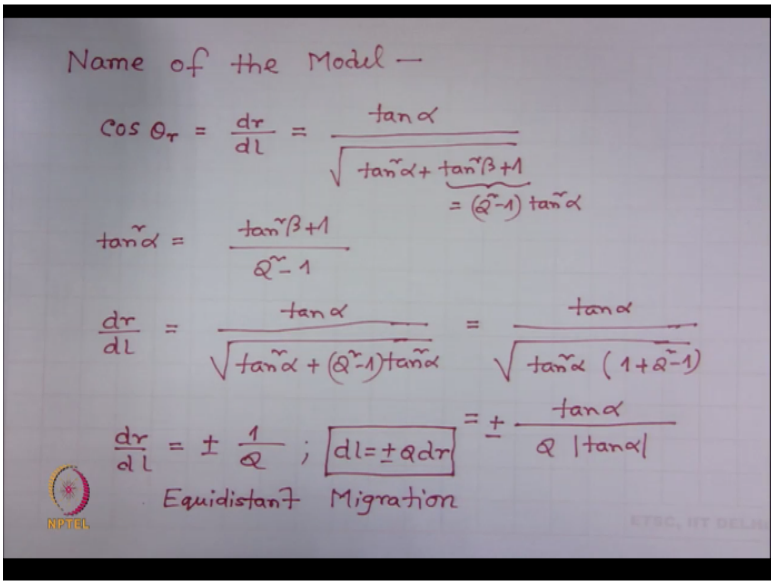

So name of the model we will discuss now. If you remember, one point of time we derived this expression. In fact, this expression we derived at the beginning of this module, dr/dl is=tan alpha/square root of tan square alpha+tan square beta+1. Now what this model gives? This model gives us tan square alpha is=tan square beta+1/Q square-1 right. So tan square beta+1 can be substituted as Q square-1 tan square alpha.

So what we will get, dr/dl is=tan alpha square root of tan square alpha+Q square-1 tan square alpha. So in the denominator tan square alpha if we take then what we see 1+Q square-1, so what you will get, we will get tan alpha/Q tan alpha right. So dr/dl is=+-1/Q right. What does it mean? This means dl is +-Q times dr. Look at this expression, dl is +-Q times dr, dl is the change in length of fibre segment, dr change in radius, Q is a constant.

So the fibre length increases equidistantly with the steps of radius. That is why this model is called as equidistant migration model. That is why this model is called as equidistant migration. Now we will consider some approximations.

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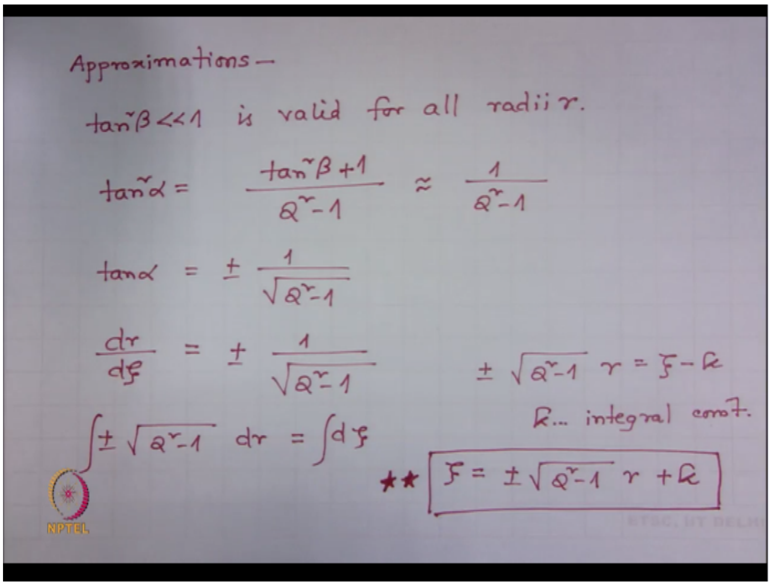

If you remember in Treloar's model also we considered two approximations, one was slow migration, period of migration was very high, so slow migration, so the factor K capital K there was very high. Second assumption was tan square beta is much <1. In this case, we will consider the same that is tan square beta is much <1 is valid for all radii r. So what will become tan square alpha tan square beta+1 Q square-1 right.

Now if tan square beta is very small, then 1+ a very small quantity is approximately 1. So we can write it as 1/Q square-1. So tangent of alpha will be +-1/root over Q square-1. What is tangent of alpha? dr/d zeta 1/root over Q square-1. So root over Q square-1 dr will be=d zeta. So if we integrate this expression what we will get? We will get we will write here, +- root over Q square-1*r is=zeta-small k, small k is integral constant right.

So zeta is=+-root over Q square-1*r+k, look at this expression, this is an equation of a cone. That means the fibre path approximately follows conical inside yarn. So how does it look like?

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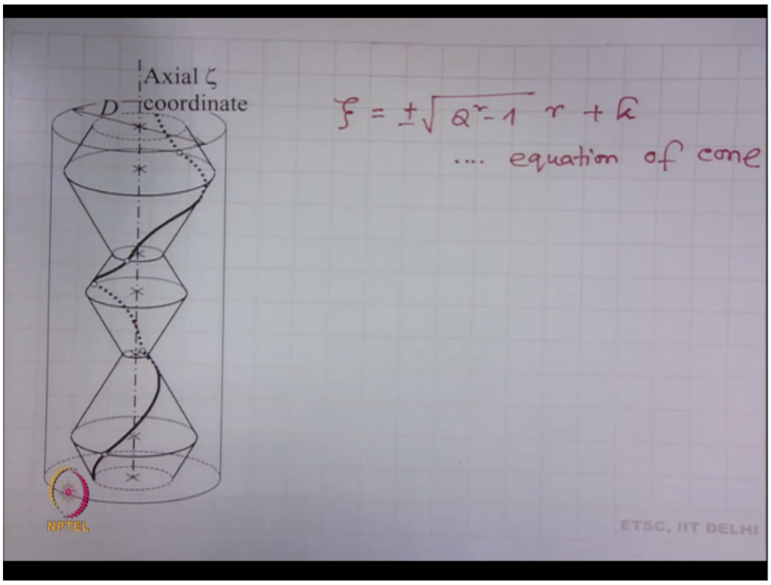

Imaginatively, this is how the fibre path will look like. So it starts from somewhere here, it touches the surface. So this is how it touches, then again it goes inside, again it comes out touches the surface, goes inside. So this path resembles path of a cone. So in equidistant radial migration, if we consider the approximation then fibre path follows cone. Otherwise, we can say the fibre path inside a yarn approximately follows a cone.

So this is all about equidistant migration of fibres in yarn. Now the very important question arises whether this model is able to explain the experimental results correctly, so this is how is the comparison between theory and experimental results.

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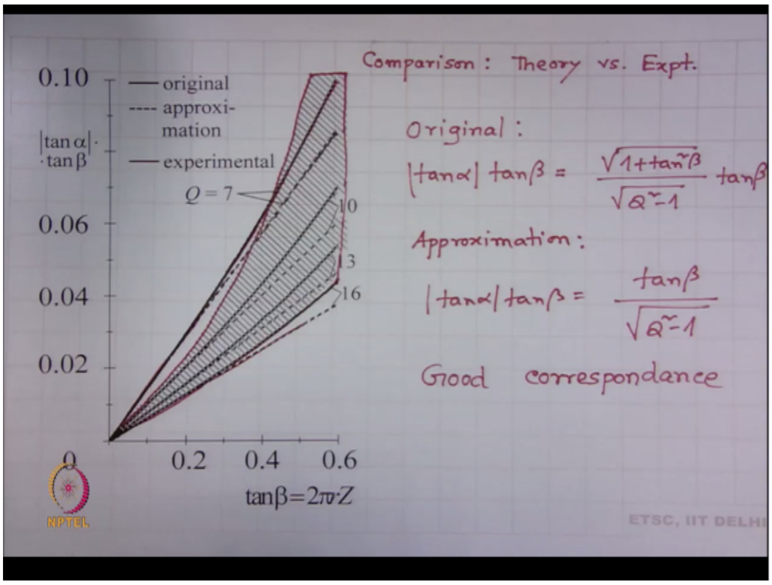

Comparison, theory versus experiment, what you see there in this graph along the x axis, tangent beta 2 pi r times z is plotted, along the y axis absolute value of tangent alpha*tangent beta. So this region is same in the comparison of Treloar's model, so this is basically the experimental region. All experimental data fall in this region right. So this is your experimental region.

Now two types of curves are there, one is original, second we see approximation. The original we have derived tan alpha*tangent beta is=root over 1+tan square beta/root over Q square-1*tangent beta. So this was original fundamental equation. So this continuous thick lines black color for different values of Q we obtained. For example, Q is=7 here, so this line. So this line is obtained from this expression right.

And also you see one approximation line, approximation line is dotted line is this line for Q is=7. How we obtained this approximation line? The approximation line we obtained by tangent alpha*tangent beta is=tangent beta root over Q square-1. So if you substitute Q is=7 for different values of tangent beta, you will obtain this dotted curve. For Q is=10, you will obtain this dotted curve.

For Q is=3, you obtain this dotted curve 13, for Q is=13 you obtain this dotted curve, for Q is=16 you obtain the last one dotted curve. So what is interesting to see here that the equidistant migration model shows a very good correspondence between theory and experiment. That means by changing one assumption in Treloar's model that was assumption 4, nu r is proportional to radius r.

We can see that the derived model explains the experimental results satisfactorily. So this was about the model of equidistant migration of radial fibres in yarn. Now we will have one numerical exercise now. So the numerical problem is set in this manner.

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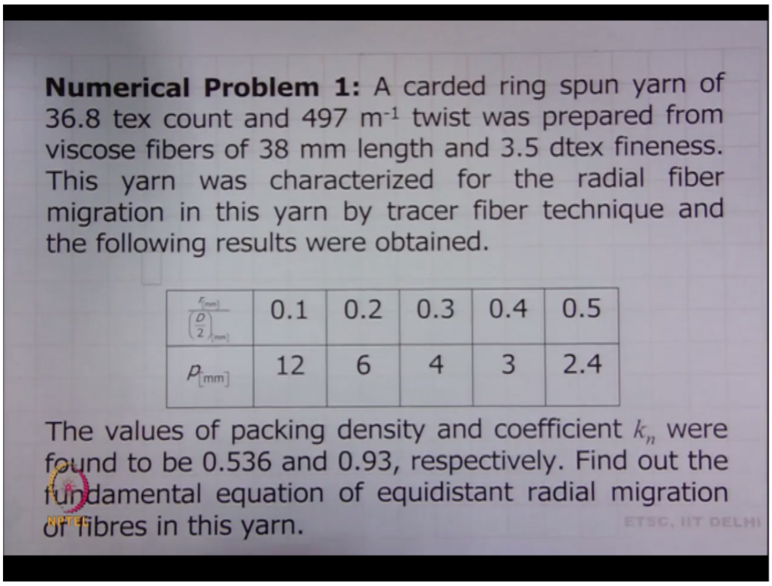

A carded ring spun yarn of 36.8 tex count and 497 meter invers twist was prepared from viscose fibres of 38 millimeter length and 3.5 decitex fineness. This yarn was characterized for the radial fibre migration in this yarn by tracer fibre technique and we obtained the following results. For different value of small r/1/2 of yarn diameter 0.1, 0.2, 0.3, 0.4, 0.5 we measured different values of period of migration in millimeter 12, 6, 4, 3, 2.4.

The values of packing density and coefficient kn were found to be 0.536 and 0.93 respectively. Find out the fundamental equation of equidistant radial migration of fibres in this yarn. So this is how is this problem. So let us solve this problem.

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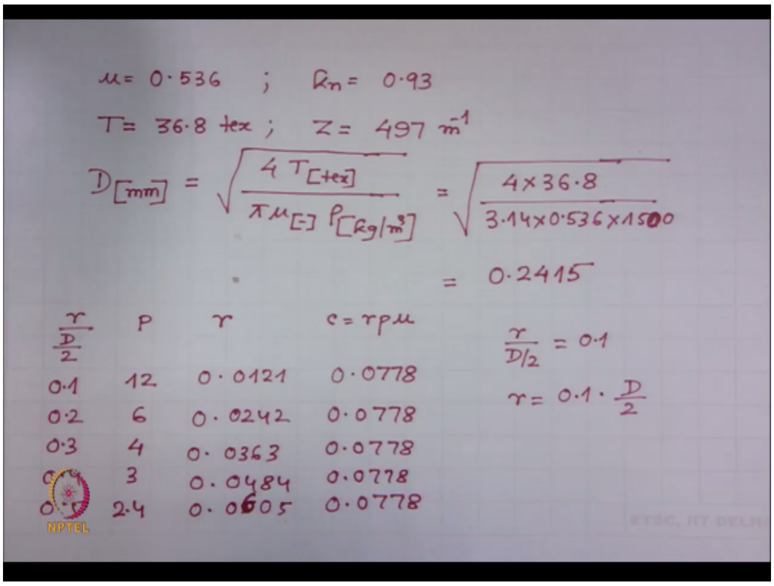

What is given is mu 0.536 and coefficient kn is given 0.93, also given yarn count capital T 36.8 tex, twist is given 497 turns per meter. So you have to first find out yarn diameter. As we know from yarn diameter module 2, so if you substitute the values here 4*36.8/3.14*0.536 viscose fibre*1510 kg per meter cube is density right. So this value you will obtain 0.2415 in millimeter.

Let us consider it as 1500. Viscose fibre density 1.5 gram per cc. Then what is given here? In the table, this value is given 0.1, 0.2, 0.3, 0.4, 0.5; p is given 12, 6, 4, 3, 2.4. From there we can find out r because we know now D, so r is=so r/D/2 is=say 0.1, so r is=0.1*D/2, D is given here. So if we do this then we will find out 0.0121. 0.0242, 0.0363, 0.0484, 0.0605. Roughly, we will find these values.

Now what was small c, r times p times mu in equidistant radial migration we know r, we know p, we know mu, so if we multiply these three then we will get this value 0.0778. We will see that this value will be same in all other cases right. So we obtain now this table. We have to now find out fundamental equation of equidistant migration.

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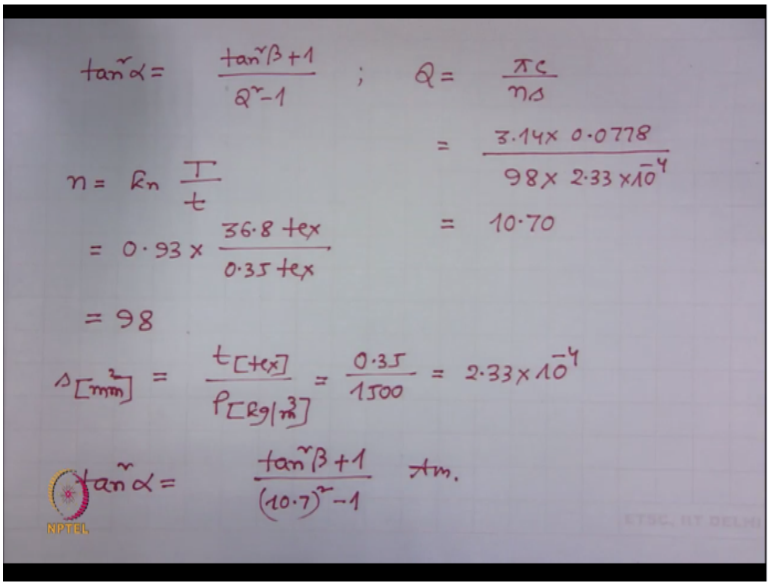

As we know, the fundamental equation is tan square beta+1*Q square-1, where Q is pi times c/n*s. The value of c we obtained as 0.0778. We have to obtain values of small n number of fibres in yarn cross-section and small s that is fibre cross-sectional area. How to find out small n? Small n is=coefficient kn*yarn fineness/fibre fineness. The value of coefficient kn is given here 0.93*36.8 tex/0.35 tex.

So this value will be coming to 98, so this is your 98*small s. How to find out small s? Cross-sectional area we know this relation from module 1, so this was 0.35/1500 so this value will be equal to this. So if you substitute 2.33*10 to the power -4, this will be coming is equal to 10.70. So the fundamental equation becomes tan square beta+1/10.7 square-1. So this is the fundamental equation of radial fibre migration in this yarn.

So for different value of beta, we can obtain different value of tan alpha from this example also. So this completes the numerical problem in this module. Thank you very much for your attention.