

**Theory of Yarn Structure**  
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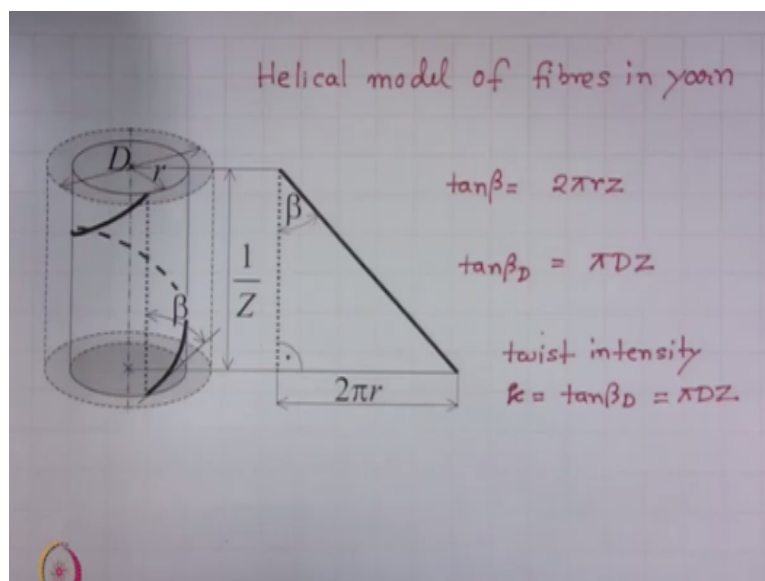
**Lecture – 19**  
**Yarn Shrinkage Due to Washing**

Welcome you all to this MOOCs online video course, theory of yarn structure, so far we have completed 6 modules today, we are going to start module 7, this module talks about yarn shrinkage due to washing. Almost all of us experienced this problem when you wash a cotton cloth, it shrinks in dimension generally, we have a tendency to reduce the effect of shrinkage by chemical processing so called stenting technique we do that.

However, the actual reason of this shrinkage of fabric remains unknown in order to understand this shrinkage behaviour of fabric say woven fabric or knitted fabric, it is necessary for us to understand the behaviour of yarn shrinkage, why does a yarn shrink, which parameters of yarn are responsible for yarn shrinkage, is it yarn fineness or is it yarn twist or it is some other factor which is dominant in deciding yarn shrinkage due to washing.

In this module, we will try to understand this behaviour and we will try to see the influence of yarn parameters on yarn shrinkage due to washing, we consider the helical model of fibres in yarn, it is the well-known concept often used in yarn modelling, we have discussed it in detail in one of the modules in this course.

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Helical model of fibres in yarn, we recapture at this model, a general fibre is situated at a radius  $r$  in the yarn, it makes an angle  $\beta$  from the yarn axis, if we cut the yarn along its axis and if we unravel then, we obtain this channel, the base of the triangle is  $2\pi r$  and the height of the triangle is  $1/z$ , where  $z$  denotes yarn twist and  $\beta$  is this angle, so if we apply trigonometry we can find out tangent of  $\beta$  is  $2\pi r$  times  $z$ .

Now, of course there will be a fibre which is situated at diameter  $D$  as well in that case for that fibre, we will write tangent of  $\beta$   $D$  is  $\pi$  times  $D$  times  $z$ , in theory of yarn this quantity has a name which is known as twist intensity generally, we denote the symbol  $K$  for this which is equal to  $\pi$  times  $D$  times  $z$ , so this is a very well-known expression as we know from helical model of fibres in yarn and we consider helical model of fibres in yarn in this module 2.

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Yarn shrinkage -

Quantify ?

$\zeta$  = original length of yarn

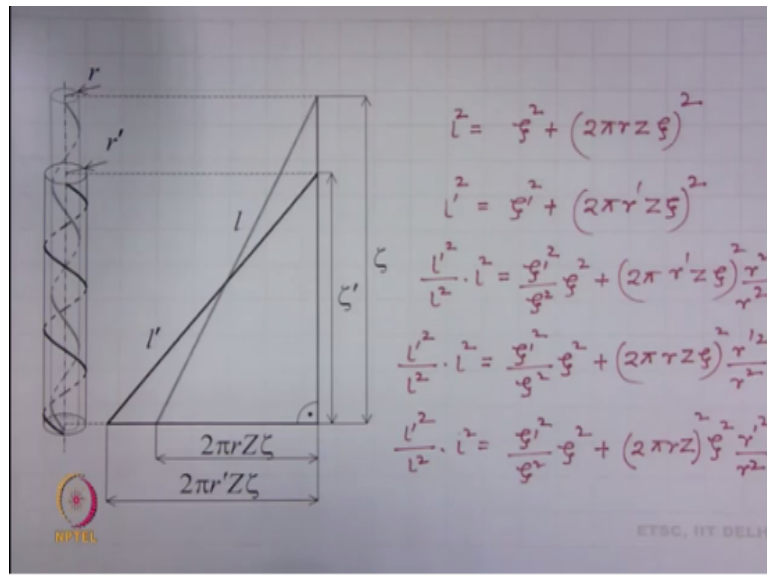
$\zeta'$  = length of yarn after washing and drying

$\phi = \frac{\zeta - \zeta'}{\zeta} = 1 - \frac{\zeta'}{\zeta}$       $\phi$  ... yarn shrinkage

Now, what is yarn shrinkage? Let us first define; the cotton yarn contracts its length due to washing, this phenomenon is called as yarn shrinkage, how we quantify this effect; the effect of shrinkage, let us consider  $\zeta$  is the original length of yarn, if we wash this yarn into hot soap solution then its length will shrink suppose, this is the length of yarn after washing and of course, drying so, what is the change of length?

$\zeta - \zeta'$  is the change of length divided by original length, this is how we quantify shrinkage,  $\phi$  is yarn shrinkage, so yarn shrinkage is defined by the change in length due to washing divided by original length, we can write this further as this okay, so this is how we define yarn shrinkage. Now, we consider a helical fibre inside the yarn before shrinkage and before washing and after washing.

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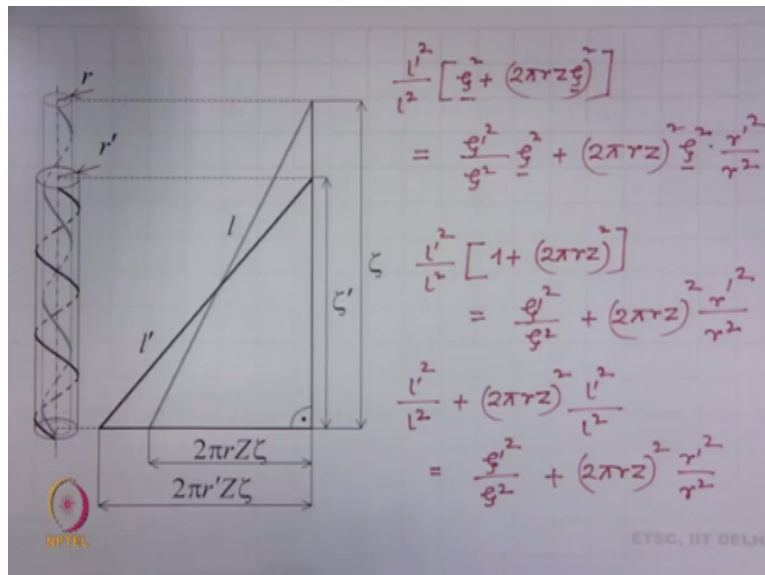
So, how it will look like; so what you see is 2 cylinders; one is having length bigger however, its radius is small; small arc, second cylinder what you see is that shortened in length but looking bigger in diameter, its radius is r prime, so r is the initial radius and r prime is the final radius after washing and drying that is shrunk yarn and l; now if we cut along the axis of these 2 cylinders and unravel it then we will find out 2 triangles.

In the one triangle this is a fibre length, original length of fibre small l, what will be the base; base will be 2 pi r times Zeta \* z; why z \* Zeta because that is basically number of coils; initial number of coils so that is why 2 pi r z \* Zeta and this length will be zeta, after washing and drying the same fibre will have a different length, there are; let us consider that length is l prime and of course, radius will change.

So, this base will be now 2 times pi times r prime z \* Zeta and this length is however z prime, we consider number of coils z times zeta does not change because of washing and drying, so if we apply now Pythagorean theorem to the initial fibre length l square will be = zeta square + 2 pi r z zeta square, right. Similarly, if we apply Pythagorean theorem to the triangle after shrinking, then we will obtain l prime square + 2 times pi r prime z zeta square.

This equation; next equation, second one can be written in this form, so 2 pi r z zeta square r squared by this okay, so what we can see is that we can further write as prime square / l square \* l square + 2 pi r z zeta square \* square prime/ r square, right. Now, this l square we will substitute from this equation.

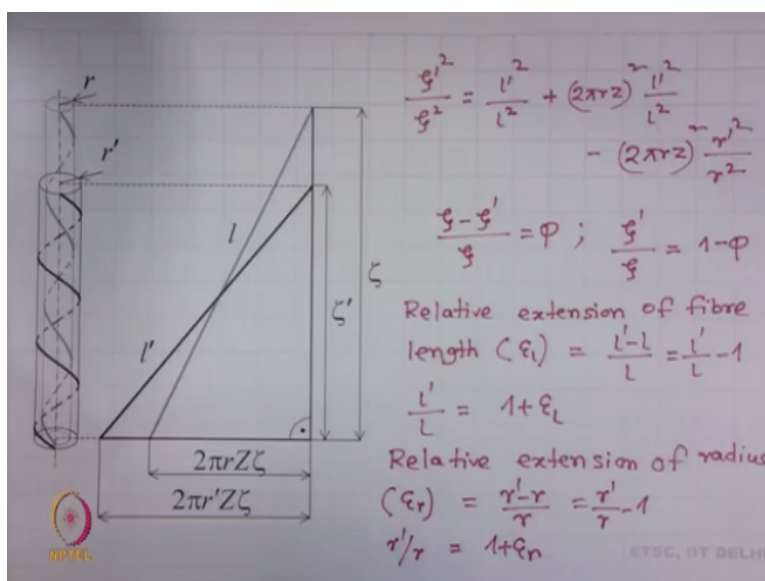
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So, what we will get is  $l' \text{ square} / l \text{ square} \zeta \text{ square} + 2 \pi r z \zeta \text{ square}$ , this =  $\zeta \text{ square} + 2 \pi r z \text{ square} \zeta r^2 \text{ square} / r \text{ square}$  now, in all these quantities this is common, so it can be removed, so what we get is;  $1 + 2 \pi r z \text{ square} = \zeta \text{ prime square} \zeta \text{ square} + 2 \pi r z \text{ square} / r \text{ square}$ , right. So, further we can write this expression this +  $2 \pi r z \text{ square} * \text{this by this} = + 2 \pi r z \text{ square} / r \text{ square}$ , right.

Now, this equation we can further write as; so what we do in the earlier expression now, this expression is somehow related to shrinkage, so we will keep it in the left hand side rest all in the right hand side.

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If we do so then we obtained is equal to – this, all right, how we define shrinkage; change in length by original length is equal to this, so this by this = 1 – this, is not it, we now introduce 2 quantities; one is relative extension of fibre length, we symbolize it by epsilon subscript l, how we define it; change in length divided by original length, so  $L' / L - 1$ , so  $L' / L = 1 + \epsilon_L$ .

We also introduce another quantity here that is relative extension of radius, epsilon r change in radius by original radius, so we can write, plus this, now these 2 expressions we substitute, these 3 expressions we substitute here, so this becomes  $1 - \phi^2$ .

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$$(1-\phi)^2 = (1+\epsilon_L)^2 + \underbrace{(2\pi r z)^2}_{= \pi D z \frac{2r}{D}} [(1+\epsilon_L)^2 - (1+\epsilon_r)^2]$$

$$(1-\phi)^2 = (1+\epsilon_L)^2 + (\pi D z)^2 \left(\frac{2r}{D}\right)^2 [(1+\epsilon_L)^2 - (1+\epsilon_r)^2]$$

$$\phi = 1 - \sqrt{(1+\epsilon_L)^2 + (\pi D z)^2 \left(\frac{2r}{D}\right)^2 [(1+\epsilon_L)^2 - (1+\epsilon_r)^2]}$$

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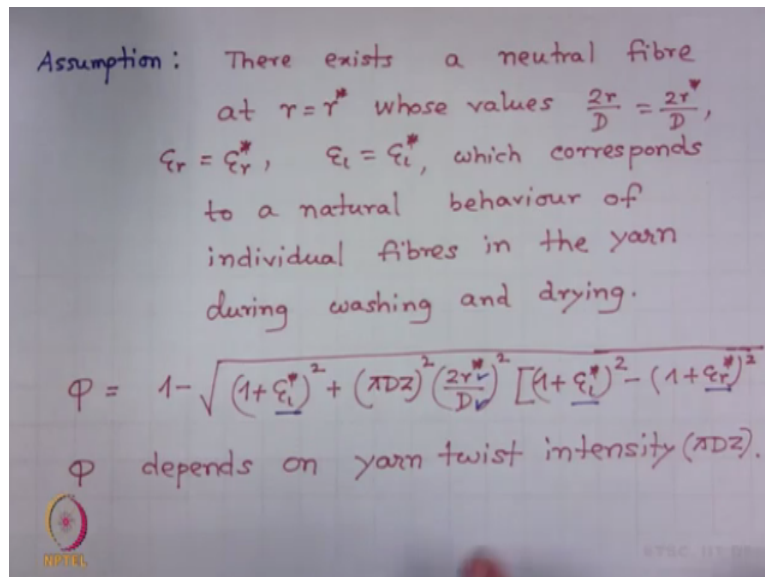
Yarn shrinkage is different at different radii. NOT POSSIBLE !!!

So, finally we obtain =  $L' / L$  is  $1 + \epsilon_L^2 + 2 \pi r z$  square, common  $L' / L - 1 + \epsilon_r^2$  now, this  $2 \pi r z$  can also be written as  $\pi D z * 2r / D$ , right, so what we obtained is  $1 - \phi^2 + \pi D z^2 * 2r / D^2 * 1 - \epsilon_L^2 + 1 + \epsilon_r^2$  square, if we take the square root and put phi at one side, so we will obtain  $\phi = 1 - \sqrt{1 + \epsilon_L^2 + \pi D z^2 * 2r / D^2 * 1 + \epsilon_r^2}$  square.

Look at this expression, this is the expression for yarn shrinkage due to washing, right now, what this equations tells to us; it tells us a very interesting story, these parameters epsilon L twist intensity kappa, epsilon r, this diameter D, they are constant for a given yarn, so for a given yarn, epsilon L, Kappa, capital D, epsilon r are constant, so for that yarn, shrinkage is a function of radius.

That means this equation tells us there is shrinkage of yarn different at different radii, so yarn shrinkage is different at different radii, this is not possible, how a yarn can show different values of shrinkage at different radii, this is not possible that means, there is something interesting, so we introduced one assumption here. The assumption is that there exists a neutral fibre at radius  $r = r^*$  that fibre represents the whole yarn in terms of shrinkage.

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So, the assumption is something like this, they exist a neutral fibre at radius  $r = r^*$  whose values say  $2r / D = 2r^* / D$   $\epsilon_r = \epsilon_r^*$ ,  $\epsilon_l = \epsilon_l^*$ , these are the values they correspond; which corresponds to a natural behaviour of individual fibres in the yarn during washing and drying, so we imagine if fibre which is a neutral fibre which is situated at this particular radius whose values are  $2r / D = 2r^* / D$   $\epsilon_r = \epsilon_r^*$ ,  $\epsilon_l = \epsilon_l^*$  which corresponds to a natural behaviour of individual fibres in the yarn during washing and drying.

Then we are able to write  $\phi = 1 - \sqrt{(1 + \epsilon_l^*)^2 + (\pi D z)^2 \left(\frac{2r^*}{D}\right)^2} \left[ (1 + \epsilon_l^*)^2 - (1 + \epsilon_r^*)^2 \right]$ , so then this factor this quantity becomes a constant,  $r^*$  becomes a constant,  $D$  is a constant, this is constant, this is constant so, in that case yarn shrinkage depends on yarn twist intensity;  $\pi D z$ . So, what we learn is; yarn shrinkage depends on yarn twist intensity only, it is not only yarn twist, it is not only yarn fineness, it is not yarn twist, it is not yarn fineness, it is yarn twist intensity that dictates yarn shrinkage due to washing.

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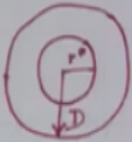
parameter  $\epsilon_l^* = 1.1\% - 1.2\%$  (literature)

parameter  $\epsilon_r^* = 9.5\% - 28\%$  (literature)

parameter  $\frac{2r^*}{D} = ?$

$\pi r^{*2} = \frac{1}{2} \frac{\pi D^2}{4}$

$\frac{2r^*}{D} = \frac{1}{\sqrt{2}} = 0.7$



Now, we would like to talk about these quantities epsilon L star, epsilon r star and 2r star/ D about these quantities we would like to tell you first we start with parameter epsilon L star, what is this parameter; this parameter characterizes relative extension of fibre due to washing and drying so, generally speaking in different literature, you will see this value is reported as 1.1% to 1.2%.

In the book of physical properties of fibres by a Morton and Hearle, you will see that a very similar values are reported for this quantity, then comes another parameter epsilon r star, what is epsilon r star; it characterizes relative extension of radius, so this is generally put it as 9.5% to 28% in literature, 9.5 % to 28% generally reported in literature, third parameter is to 2r star/ D, what is the value of this quantity?

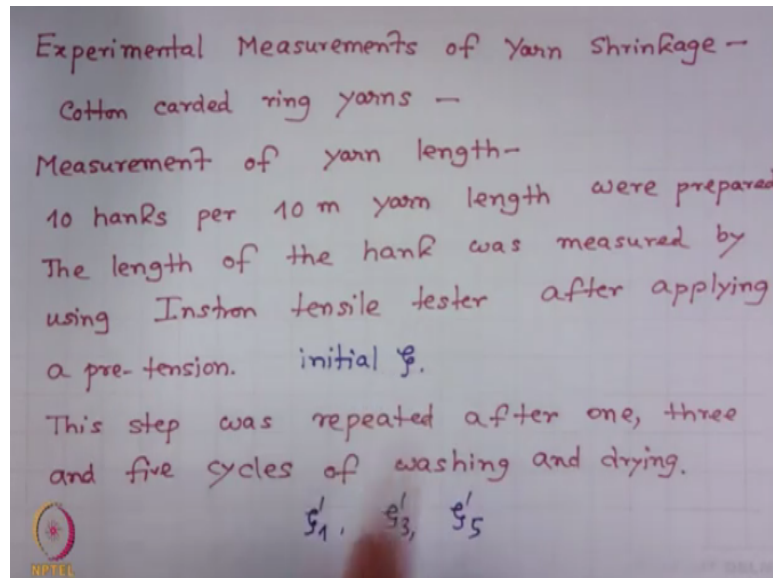
This is the relative position of the neutral fibre inside the yarn, right based on helical model, constant value of packing density at each radii, we can estimate this position by dividing the area into 2 halves that means, this the radius is r star and this is a yarn whose radius is D that means what that means  $\pi r^{*2} = 1/2$  of  $\pi D^2 / 4$ , okay, so what will be the value of this quantity?

This pi and this pi cancel out so, root 2, so  $1/\sqrt{2}$ , so this becomes  $1/\sqrt{2}$ , so  $2r^*/D = 1/\sqrt{2}$  that means, this will be roughly = 0.7, so based on helical model constant packing density if we assume that at the; where this radial where this neutral fibre is existing, it makes the yarn cross sectional area into 2 equal portions, so  $2r^*/D = 0.7$  now, of course this quantity might be different for different technologically spun yarns.



They may be for ring yarn, rotor yarn, air jet yarn, friction yarn probably this value might be different right, so this was about the discussion with 3 parameters, so these parameters remains constant, this parameter however is a function of number of cycles of washing and drying, so the beginning this value will be less as you go on washing and drying, this value will increase probably after certain number of washing and drying, this value will not practically change.

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Now, we would like to go for experimental measurement of shrinkage, so experimental measurements of yarn shrinkage, in this particular study cotton carded ring yarns were taken, so how the yarn length was measured, what was done is; 10 Hanks per 10 meter yarn lengths were prepared, the length of the hank was measured by using Instron tensile tester after applying a pre tension, right.

Then this yarn underwent one cycle of washing and drying then this step was repeated after one cycle of drying; washing and drying, this step was repeated after 1, 3 and 5 cycles of washing and drying, so this gives you initial length, these 3 measurements gave you length of yarn after first cycle, length of yarn after third cycle, length of yarn after fifth cycle, so from these differences we obtain shrinkage values at different cycles; first, three and fifth.

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Washing - 10 minutes in water solution of soap (3g/l) and soda (2g/l) at 90°C. This was followed by rinsing using distilled water.

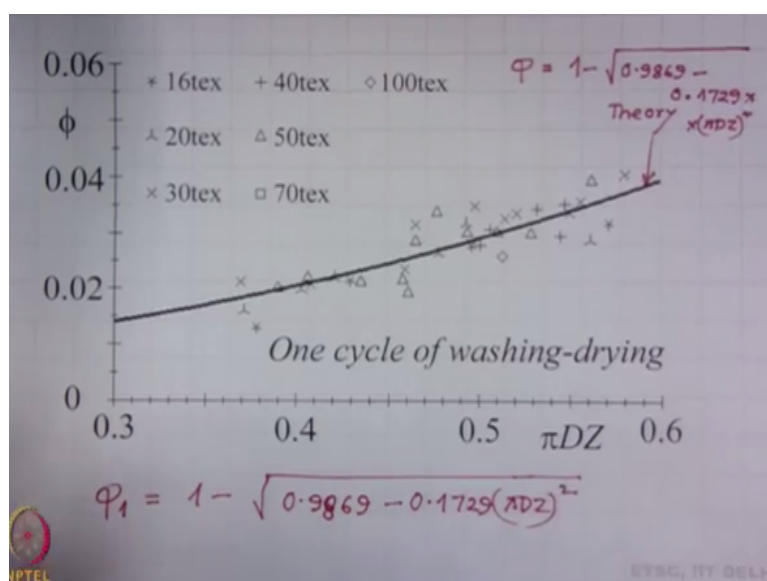
Drying - Free drying on air for 6h. Then, 4 hours at 60°C, Afterwards, conditioning at 20°C and 60% R.H. for 24 hours.

Diameter measured  $R = \pi DZ$   
 Twist measured

How did you was this washing was carried out 10 minutes in water solution of soap, 3 gram per litre and soda 2 gram per litre at 90 degree Celsius temperature, this was followed by rinsing using distilled water, so this is how we carried out washing and how we carried out drying; drying initially was carried out on air for 6 hours then, 4 hours at 60 degrees Celsius, after that conditioning at 20 degree Celsius and 60% RH for 25 hours; 24 hours.

So, this is how the experiment was carried out and we measured the shrinkage and we obtained the results, I will show you now the results, these are the experimental results of shrinkage after one cycle of washing and drying we used many yarns, cotton carded yarns, all were cotton yarns but of different counts.

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16 tex a star, so wherever you will see star that correspond to 16 tex, wherever you will see +40 tex, rhombus 100 tex, 20 tex, 50 tex, 30 tex, 70 tex, so different fineness of yarns we prepared and we obtained the value of pi times D times z, so how we measured diameter, so yarn diameter was also measured, twist measured from there we obtained pi Dz, yarn twist intensity.

So, this yarn twist intensity we plot along the x-axis and these are the results correspond to first cycle of washing and drying shrinkage and this line is coming from theory, what is the expression of this line;  $1 - \sqrt{0.9869 - 0.1729 * \pi D z \text{ square}}$  now, let me tell you one thing here.

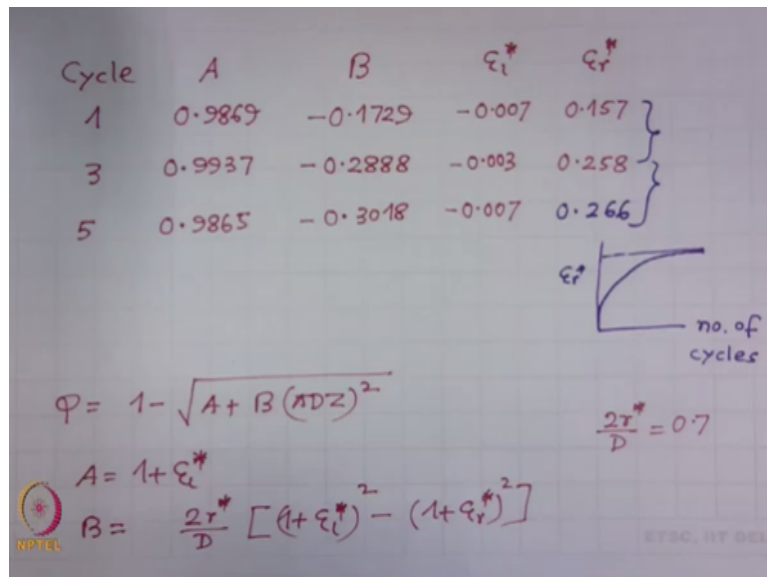
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$$\begin{aligned} \Phi &= 1 - \sqrt{(1 + \epsilon_L^*)^2 + (\pi D z)^2 \left(\frac{2r^*}{D}\right)^2 [(1 + \epsilon_L^*)^2 - (1 + \epsilon_r^*)^2]} \\ &= 1 - \sqrt{(1 + \epsilon_L^*)^2 + \frac{2r^*}{D} [(1 + \epsilon_L^*)^2 - (1 + \epsilon_r^*)^2] (\pi D z)^2} \\ 1 + \epsilon_L^* &= A \\ \frac{2r^*}{D} [(1 + \epsilon_L^*)^2 - (1 + \epsilon_r^*)^2] &= B \\ \Phi &= 1 - \sqrt{A + B (\pi D z)^2} \end{aligned}$$

As we have seen this expression,  $1 + \text{this} - \pi D z \text{ square} * 2r \text{ star} / D \text{ square} 1 + \text{epsilon L star} - \text{this}$ , right now,  $1 -$ ; see how we are writing this equation,  $\text{this} + 2r \text{ star} / D 1 + \text{squared} - * \pi \text{ times } D \text{ times } z$ , then we consider  $1 + \text{epsilon L star}$  as  $A$  and  $2r \text{ star} / D * 1 + \text{epsilon L star} - 1 + \text{epsilon r star}$ , this we consider as  $B$  then, we will be able to write yarn shrinkage;  $1 - \text{root over } A + B * \pi D z$ .

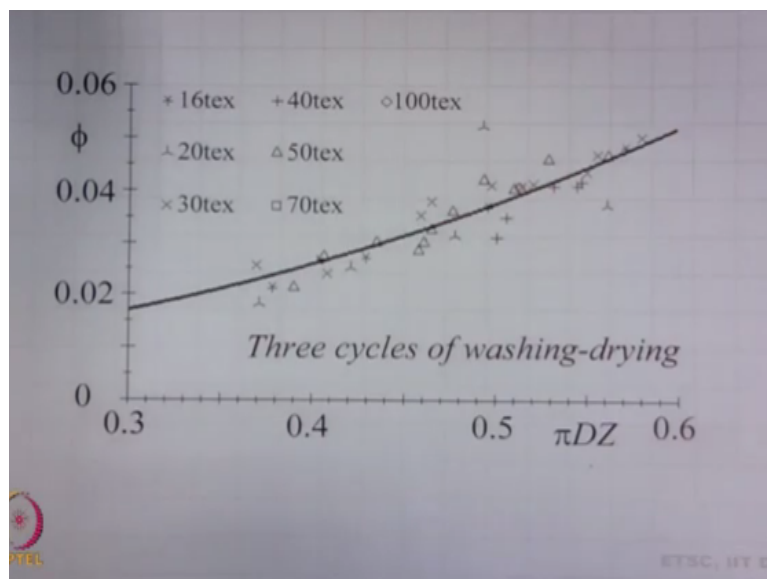
So, this line was obtained by using statistical regression technique and the equation what we use for regression is  $\phi = 1 - \text{root over } A + B * \pi D z \text{ square}$ , by using statistical regression technique, we obtained the values of  $A$  and  $B$ , these values are written here. So, for first cycle we obtain  $1 - 0.9869 - 1729 * \pi D z \text{ squared}$ .

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So, for cycle 1, what is the value of A; the value of A is 9869, what is the value of B; the value of B is 0. This, right for cycle 1, now if we see here  $\phi = 1 - \sqrt{A + B \cdot \pi DZ^2}$ , A; what is A;  $A = 1 + \epsilon_i^*$ , so we will be able to obtain this value and what is B;  $B = \frac{2r^*}{D} [1 + \epsilon_i^*^2 - (1 + \epsilon_r^*)^2]$ ,  $\frac{2r^*}{D}$  was considered to be 0.7, so this is 0.7, the moment we know this value, we will be able to find out this as well, right.

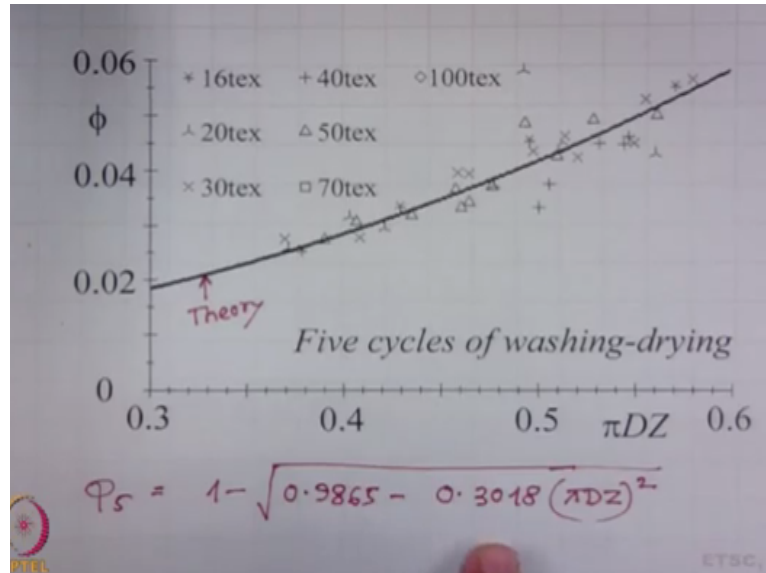
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So, this was the result after first cycle of washing and drying then we carried out for third cycle, so this is the result for third cycle of washing and drying, what we see as compared to the previous one, it is little increasing, so for this what we found similarly, this thick line comes from theory, so what is the value of shrinkage after third cycle,  $1 - \sqrt{0.9937 - 0.2888 \pi DZ^2}$ .

So, this line follows this equation, this you obtained from statistical regression technique, so what is the value of A for third cycle; the value of A 0.9937 and the value of B 0.2888 similarly, we measured yarn shrinkage after 5 cycles of washing and drying.

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And after 5 cycles of washing and drying, we obtained this result similarly, this comes from theory, after 5 cycles we obtained  $1 - \sqrt{0.9865 - 0.3018 \pi DZ^2}$ , so the value of A is this the value of B is this after 5 cycles, so 5 cycle the value of 0.9865 and -0.3018 is the value of this after this. So, now if we use A from this table, we will obtain the value of epsilon star, this value was obtained as this similarly, if we substitute A is here then we will obtain the value of epsilon star for third cycle, it was this similarly, if we substitute A = this in this equation.

So you will find out the value of epsilon L star, this value will be 0.07, you know B, all ready for first cycle, third cycle, fifth cycle, you know epsilon L for first cycle, third cycle, fifth cycle. You know  $2r^* / D = 0.7$  we consider it by using this equation, you will be able to find out this value epsilon r star, relative extension of radius, this value was for the first cycle 0.157, for a third cycle 0.258 and for the fifth cycle interestingly, it was 0.266.

What we see that this difference is much higher than these difference so probably, number of cycles of washing and drying, epsilon r is probably after a few cycles there is a play to that comes in. Now, why does it happen? Because yarn structure initially because of washing, it swells and it shrinks, so it opens up, once it opens up then further swelling is probably not possible that is why this play to comes, so after 5 cycles of washing, we did not find much significant difference in results.

So, what we see is that our equation of this form is able to explain the experimental results quite satisfactorily. So, probably the way it was imagined helical model and then simple extension of helical model, the shrinkage of yarn was explained satisfactorily now, we would like to solve one numerical problem on this module, the numerical problem is like this.

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19.43 m<sup>-1</sup> twist, considering that the fiber shows 0.7% decrease in length, the yarn exhibits 16% increase in its diameter due to washing.

29.5 tex ; z = 719.43 m<sup>-1</sup>

$\epsilon_f = -0.007$  ;  $\epsilon_r = 0.16$

Eq 4  $\Rightarrow D [mm] = \sqrt{\frac{4T [tex]}{\pi \mu [kg/m^3] \rho [kg/m^3]}}$

$= \sqrt{\frac{4 \times 29.5}{3.14 \times 0.46 \times 1520}} = 0.2318$

1/2)

Predict the shrinkage of a cotton carded ring yarn of 29.5 tex count and 719.43 meter inverse twist, so this yarn has a count 29.5 tex, zeta is the twist, 719.43 meter inverse considering that the cotton fibre shows 0.7% decrease in length, -0.007 and yarn exhibits 16% increase in its diameter due to washing, so diameter increase and radius increase will be proportional, 16, so you have to predict the shrinkage, okay.

So, shrinkage you have to find out yarn diameter, diameter is not given, twist given, count given, diameter is not given, so if we go back to module 4, we can find out diameter, how you know; we will use this expression, only thing is that we need to know about packing density because for cotton fibre rho can be taken as 1 phi 20 kg per meter cube, capital D is given, 29.5 tex, what is not given is mu, so we have to use module 4, some formulas to find out mu, which formulas?

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$$\frac{\mu^{2.5}}{\left\{1 - \left(\frac{\mu}{0.8}\right)^3\right\}^3} = Q z^2 T^{1/2}$$

$$= (9.61 \times 10^{-8}) \times (719.43)^2 \times (29.5)^{1/2}$$

$$= 0.2702$$

$$\mu = 0.46$$

$$D = 0.2318 \text{ mm}; \quad R = \pi D z = 3.14 \times 0.2318 \times \frac{719.43}{1000}$$

$$= 0.5236$$


If you remember this equation,  $z^2 T^{1/2}$  this equation we derived in module 4, the value of  $Q$  for cotton carded ring yarn,  $9.6 \times 10^{-8}$   $Q$ , right and  $z$  was  $719.43 \text{ square} \times T$  to the power  $1/2$ ;  $29.5$  to the power  $1/2$  this value you will see will come as  $0.2702$ , if you solve this by using a suitable numerical technique or by a table prepare before, please refer to module 4, then you will find out  $\mu = 0.46$  for this yarn.

If you now know  $\mu = 0.46$ , then you substitute here,  $4 \times T$ ;  $T$  is  $29.53$ ,  $3.14 \times 0.46 \times 1520$ , this value you will get as  $0.2318$ , so you now know  $D$ , the moment you know  $D$ ,  $2318$  millimeter, what will be the value of  $Kappa$ ;  $\pi$  times  $D$  times  $z$ , so  $3.14 \times 0.2318 \times z$ ,  $719.43$  meter inverse, so this you will get  $0.5236$ , this will be the value of  $Kappa$ .

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$$\epsilon_l^* = -0.007$$


$$\epsilon_r^* = 0.16$$

$$R = \pi D z = 0.5236$$

$$\frac{2r^*}{D} = 0.7 \text{ (consider)}$$

$$\Phi = 1 - \sqrt{(1 + \epsilon_l^*)^2 + R^2 \left(\frac{2r^*}{D}\right)^2 [(1 + \epsilon_l^*)^2 - (1 + \epsilon_r^*)^2]}$$

$$= 1 - \sqrt{(1 - 0.007)^2 + (0.5236)^2 (0.7)^2 [(1 - 0.007)^2 - (1 + 0.16)^2]}$$

$$= 0.0761 \text{ Am.}$$


So now, you know all values you know this value, 0.007, you know this value, you know kappa, you will consider this, this we consider, then shrinkage =  $1 - \frac{1 + \epsilon L^2 + \kappa^2 - 2r^*/D^2}{1 + \epsilon L^2 - \epsilon r^2}$ , you substitute all values what are available to you,  $1 - \frac{0.007^2 + 0.5236^2}{1 - 0.007^2 - 1 + 0.16}$ , all values are given if you do you will get this answer 0.0761.

So, there will be approximately 7.6% shrinkage of this yarn, this ends the solution of this problem and also this ends these module 7, thank you very much, thank you for your attention.