Theory of Yarn Structure
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Lecture – 02
Fibre - The Building Block of Yarns (contd.,)

Welcome to you all to this MOOCS online video course, Theory of Yarn Structure. Today, we will continue discussion about module 1. It is on fibre, the building block of yarns. If you remember earlier in this module we talked about the characteristics of individual single fibre. And also we solved certain numerical problems on individual characteristics of fibres. Today, we will discuss about the characteristics of fibre blends.

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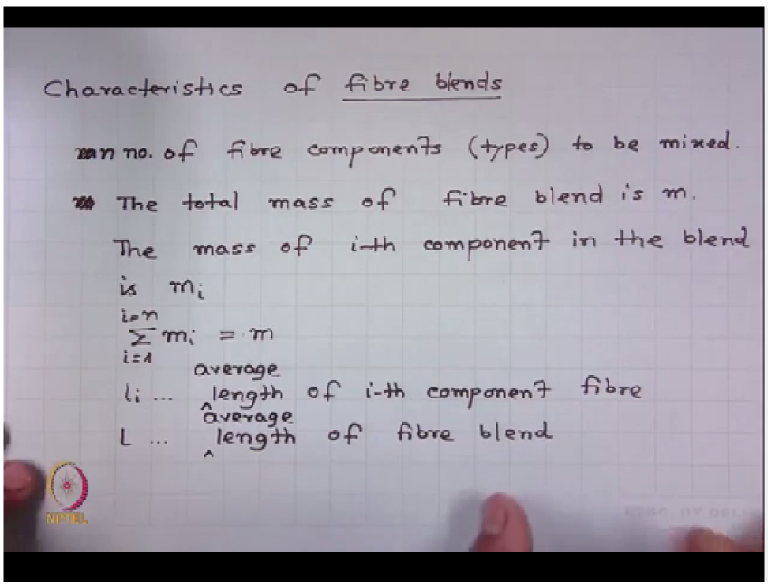

What is fibre blends? Often we mix different fibres, say cotton and polyester fibres or polyester and viscose fibres to produce yarns. When we mix 2 different types of fibres, the mix, the fibre mix is often called as fibre blends. This fibre blends are important to achieve functional properties, aesthetic properties, performance of the materials and also it is done to reduce the cost of materials.

So blending of fibres is a very important component in yarn manufacturing process. How to characteristics such fibre blends is basically the learning objective of this module. Let us consider that there are m number of fibre components or types to be mixed. m subscript, let us consider this as n, n number of fibre components to be mixed is a very general situation. Now the total mass of fibre blend is m.

The mass of ith component in the blend is m subscript i. It is therefore evident that if we sum all these ith component where i from 1 to i=n, individual mass of blend m, right. Now we will consider certain characteristics of individual components and also we will try to derive the expressions for those characteristics of the fibre blend. Whenever we will talk about certain components, we will use the subscript.

In the symbols where you do not find any subscript, those indicate the characteristics of the blend. So suppose L subscript i, this is the length of ith component fibre. When we will use L without subscript, that will be denoting length of fibre blend. Now here this L subscript i actually denote the average length of ith component fibre. To be very precisely speaking, this is the average length of ith component fibre. Similarly, precisely l denotes the average length of fibre blend.

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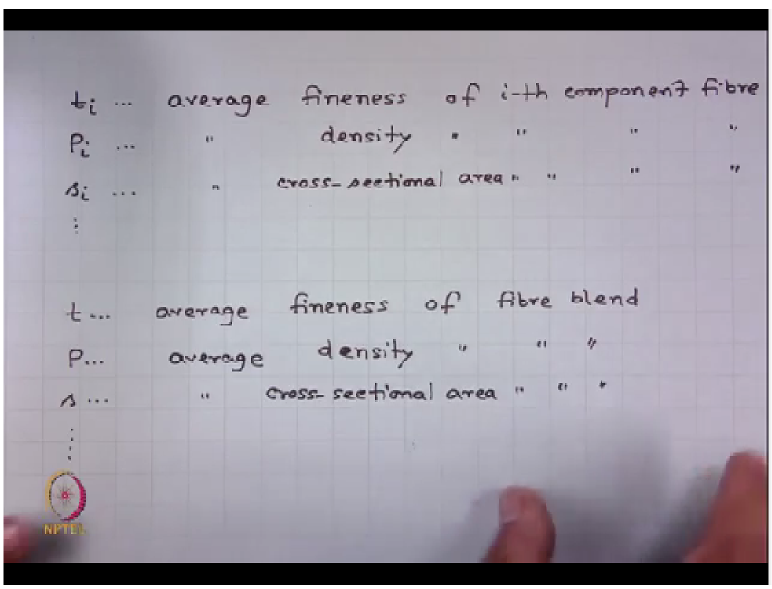

Similarly, t subscript i will denote the average fineness of ith component fibre and t will denote average fineness of fibre blend or fibre mix. Similarly, rho subscript i denotes average density of ith component fibre. Whereas rho denotes average density of fibre blend. Similarly, s subscript i denotes average cross-sectional area of the ith component of fibre. Similarly, s denotes average cross-sectional area of fibre blend. This is how we will use the symbols. First we talk about mass fraction of fibres in the blend.

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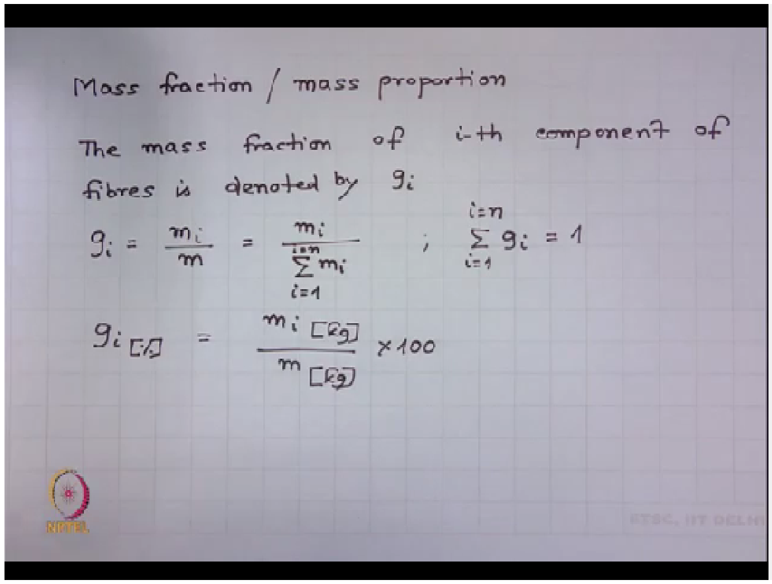

Sometimes we call it as mass proportion. The mass fraction of ith component of fibres is denoted by g subscript i. How do we define g subscript i? g subscript i is defined by the mass of ith component fibre/total mass of the blend, right. And the total mass of the blend can be written as this. So what will be summation of all mass fractions? Will be equal to 1. Often in textile industries, we mix fibres in terms of mass fraction or mass proportion.

Also it can be expressed as percentage say for example g subscript i when we wish to express it in terms of percentage, then suppose this is your kg, this is your also kg, multiplied by 100. So in textile industry, we sometimes hear 67% cotton fibres and 33% polyester fibres are mixed. That means here g for cotton fibre is 67% and g for polyester fibre is 33%. As mass or weight is easy to measure, we often in industry go by mass fraction or mass proportion.

However, there could be possibility of volume fraction, length fraction, number fraction. We will discuss them in this module.

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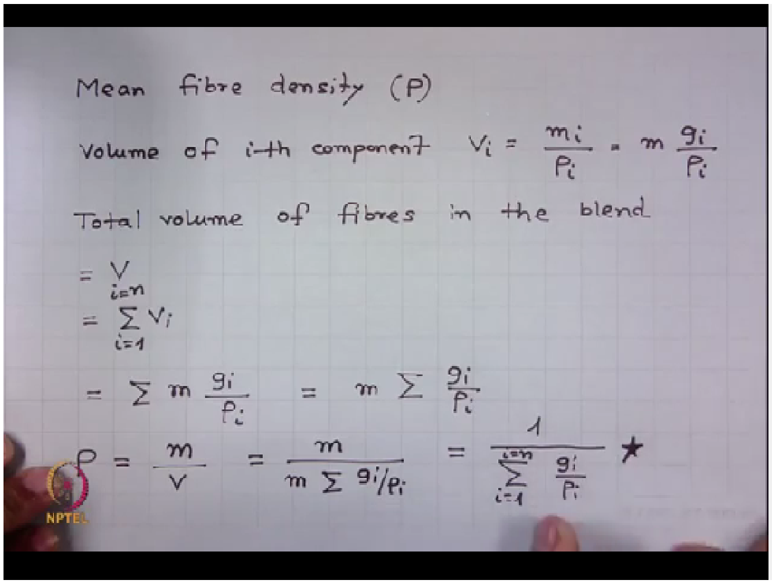

Now we will discuss one important characteristics of blend that is known as mean fibre density, that is rho of blend. It is known that volume of ith component fibre V subscript i=mass of ith component fibre/density of ith component fibre. What is m subscript i? m subscript i is m*gi. So the total volume of fibres in the blend=V which is equal to summation of V subscript i when i=1 to n that is equal to m*gi/rho i.

m is a constant. It can come out of the summation. So we obtain m subscript gi/rho i. Please remember that this subscript starts, this summation starts from i=1 to i=n. So we can now the mean fibre density we can write of the blend is total mass of fibres in the blend/total volume of fibres occupied in the blend. So V=m*the summation. So what we obtain? We obtain 1/summation gi/rho i where I varies from 1 to n.

What we see is that the density, the mean fibre density of the blend is not an arithmetic mean, not a geometric mean. However, it is the harmonic mean of the individual component fibre densities weighted by mass fraction of the fibres in the blend. Now we talk about volume fraction.

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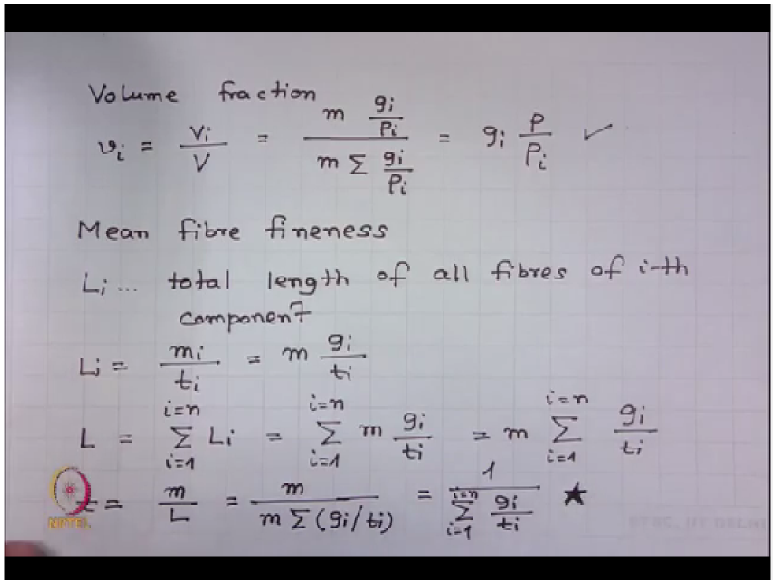

We know about mass fraction. Now we talk about volume fraction. Volume fraction is defined by the volume of ith component of fibres/the total volume. Now what is the volume of ith component fibre? We have just now derived this expression. What is the volume of all fibres? That also we have just now derived, m*summation gi/rho i. So what we obtain? We obtain gi*rho/rho i.

So if we write in terms of percentage, we can also find out similar expressions. So this is the expression for volume fraction of fibres in the blend. So by now we know about 2 fractions or 2 proportions. One is the mass fraction. Second is the volume fraction. Now we come back to another important characteristics of the blend is the mean fibre fineness. How to find out the mean fibre fineness of the blend?

Suppose the total length of fibres of ith component is Li. So Li denotes total length of all fibres of ith component. So what is this total length, Li? Total mass/fineness of individual component fibre. So the total mass of ith component is m subscript i and the fineness of ith component is this. Now what is m subscript mi? m*gi. So m*gi/ti. What is the total length of all fibres in the blend? L without subscript that is equal to summation L subscript i, where i from 1 to n.

So summation m*gi/ti, i=1 to n. This m is a constant. So we write gi/ti. Now what is the mean fibre fineness, t? What is fineness? Mass per unit length? What is the total mass? m. What is the total length? L. So t=m/L. What is your L? L is m, gi/ti. So gi/ti, i=1 to n. So here again we see that the mean fibre fineness of the blend is the harmonic mean of the fineness of individual components weighted by mass fraction of the blend, fibres in the blend. Now we go for another characteristics that is mean fibre cross-sectional area.

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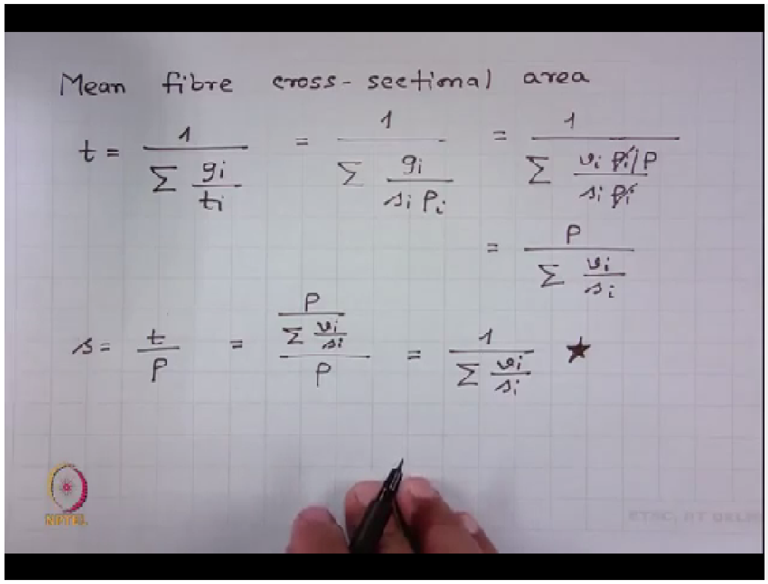

Mean fibre cross-sectional area. Now we have just now obtained t=gi/ti. If you remember from this module 1, fineness is related to cross-sectional area and density of fibre. How is the relation? The relation is fineness=cross-sectional area*density. So we can write this, this we can write in terms of volume fraction, *rho i/rho. So this rho is going there and then this gets cancelled. So what we obtain volume fraction/cross sectional area of ith component.

So what is mean fibre cross-sectional area? Mean fibre cross-sectional area=t/rho. What is t? t is mean fibre fineness of the blend. What is rho? Mean fibre density of the blend. Now if we substitute rho/volume fraction/cross-sectional area and what as rho? Keep it as rho. So this rho and this rho will cancel out. What we will have? We will have 1/, si. So here again the mean fibre cross-sectional area=the harmonic mean of the individual components cross-sectional area weighted by the volume fraction, not by the mass fraction.

Now we come to mean equivalent fibre diameter. We will use this expression.

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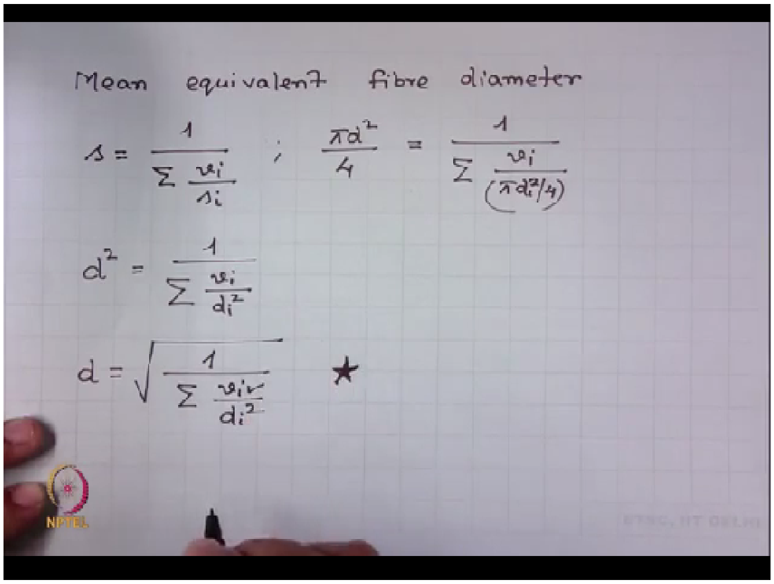

We have just now derived that the mean cross-sectional area of fibres in the blend is 1/volume fraction by the individual cross-sectional area. What is this s? s we can write as pi d square/4, where d is the mean equivalent fibre diameter in the blend which is equal to 1/volume fraction and si, pi di square/4. So we can write down that d square=1/summation volume fraction/di square.

So square of mean equivalent fibre diameter=1/summation of volume fraction of ith component/square of fibre diameter of ith component. If we take the square root, we obtain mean equivalent fibre diameter, 1/di square. So this is the expression for mean equivalent fibre diameter in the blend. Now we come to length fraction of fibres in the blend.

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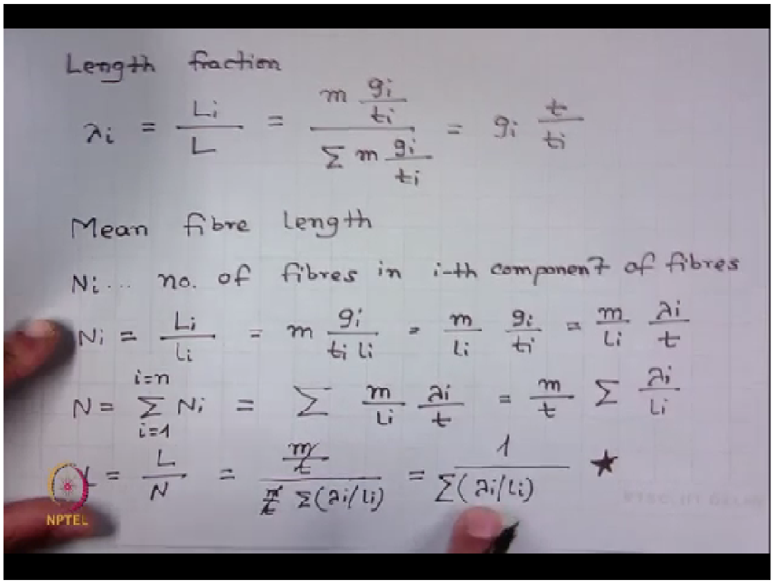

Earlier we came to know about 2 fractions. One is mass fraction, second is volume fraction. This is the third fraction, length fraction. If length can be characterized not only by mass fraction which is typically done in industry, but also by volume fraction, length fraction, number fraction and so on. What is length fraction? Length fraction we denote by this symbol which is equal to total length of all fibres of ith component/length of fibres in the blend.

What is L subscript i? mgi/ti and what is L, mgi*ti, right. So this is your t. So we write down gi t/ti. So if we know mass fraction of ith component, if we know fibre fineness of ith component, if we know the fineness of fibres of the blend, we will be able to know about the length fraction. So this is how we can also obtain about the length fraction. Now we discussed about another important characteristics of the blend that is mean fibre length.

What is the mean fibre length of fibres in the blend? Mean fibre length, to derive mean fibre length, we have to know about number of fibres of ith component. Let us configure n subscript i denote number of fibres in ith component of fibres, right. How we can find out n subscript i? This is equal to total length of all fibres of ith component/length of 1 fibre of ith component. So total length of all fibres of ith component/length of 1 ith component fibre, Li/li.

What is Li? We have derived m*gi/ti and this li will be there. So we can further write m/li*gi/ti. What is gi/ti? Look at this expression. gi/ti=lambda i/t. So m/l subscript i, gi/ti is lambda i/t. So what is total number of fibres available in the blend? N=Ni. Remember this summation is from i=1 to n. So what is n subscript i? m/li lambda i/t. This m and t are related to the fibre blend. So this can be coming out of the summation.

Lambda i/li, okay. Then what is mean fibre length? Mean fibre length is total length of all fibres in the blend/number of fibres in the blend. What is total length of all fibres in the blend? Is L. What is total number of fibres in the blend? Is N. So now we substitute L. L is m/t, m is mass of all fibres in the blend. t is average fineness of fibres in the blend; /N. We have to write this expression. So m/t lambda i/li. So this m/t cancels out.

What we have is 1/lambda i/li. So if we know mass fraction of ith component fibres, if we know fineness of ith component fibre, if we know the average fineness of fibres in the blend, we can find out length fraction. If we know length fraction, if we know individual length of ith component fibres, we will be able to find out mean fibre length, right. Then we come to another important fraction which is number fraction.

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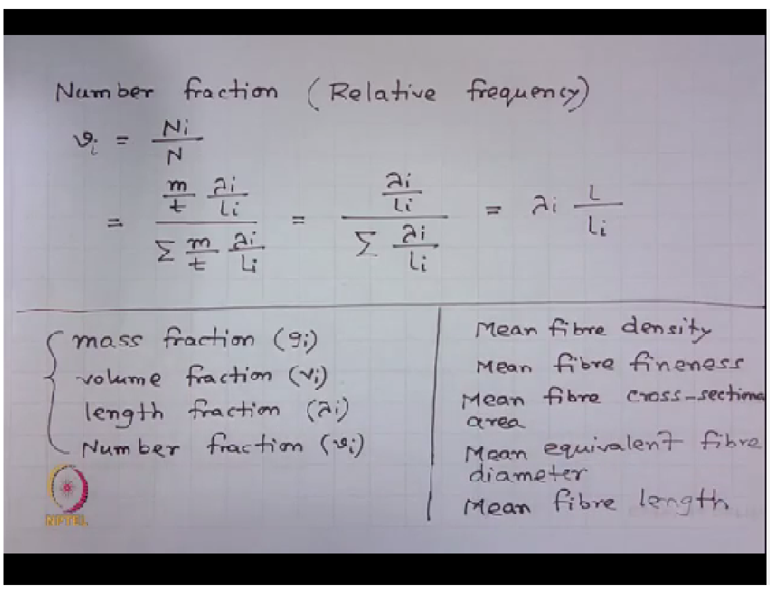

This number fraction is often termed as relative frequency. It can be asked like this, what is the relative frequency of cotton fibres in the blend of polyester and cotton fibres? That means you are required to find out the number fraction of cotton fibres in the blend of cotton polyester fibres. This number fraction we can use a symbol nu i=N subscript i/N. Number of fibres of ith component/total number of fibres in the blend, that is number fraction by definition.

Now what is Ni, N subscript i we have derived here. m/t lambda i/li, we write it here, m/t lambda i/li. And what is N? N is summation Ni. So summation m/t lambda i/li. This m/t can be cancelled. So lambda i/li. L if we substitute lambda i L/li. Well, so in this way we obtain the number fraction, relative frequency of fibres in the blend. So what we have used here? We used the expression from the last page, L=mean length fibre, Length=total length of all fibres in the blend/total number of all fibres in the blend.

This L=mass of all fibres in the blend/fineness of a fibre in the blend. So m/t. This N we derived from here. So this m/t and m/t cancel out, we obtain, this summation should be there. So lambda i/li. So this we substitute here to obtain this expression. So if we summarize, by now we obtain expressions for 4 fractions. One is mass fraction, gi. Second is volume fraction, Vi. Third is length fraction. Fourth is number fraction, nu i.

Although typically in textile industries we talk only about mass fraction. Because simply mass or weight is easy to measure. you require simple a weighing balance. However, theoretically it is possible to calculate volume fraction, length fraction, number fraction as well. We have just now seen these fractions are important to find out mean characteristics of fibres in the blend. What are the main characteristics of fibres in the blend we have obtained? Starting from mean fibre density we obtained. So this was the expression for mean fibre density.

So this was related to fibre blend. So this is a characteristics of fibre blend, mean fibre density. So we obtained an expression for mean fibre density. Then we obtained expression for mean fibre fineness. This was the expression for mean fibre fineness. Mean fibre fineness. Then we obtained expression for mean fibre density. This was the expression, mean fibre density is already obtained. Mean fibre fineness is obtained. Then we obtained expression for mean fibre cross-sectional area. All are the harmonic means of individual component.

The difference is here. It is volume fraction, not mass fraction. So mean fibre cross-sectional area was the third characteristics of the blend we obtained. Afterwards, we obtained another characteristics of fibre blend, mean equivalent fibre diameter. So this was the expression for mean equivalent fibre diameter. Here also it is the volume fraction.

Mean equivalent fibre diameter and the last one we obtained is mean fibre length. So mean fibre length is this expression. But here this is length fraction. So you see all the fractions are somehow important to derive the characteristics for fibre blend. Mean fibre length. so we obtained 4 fractions and we obtained 5 characteristics of blend.

So a blend can be characterized comprehensively in terms of these 5 characteristics, mean fibre density, mean fibre fineness, mean fibre cross-sectional area, mean equivalent fibre diameter and mean fibre length as well as in terms of these 4 fractions, mass fraction, volume fraction, length fraction, number fraction. Now what we will do? We will now apply whatever we learnt by now with the help of one numerical problem. In that way, we will characterize comprehensively a fibre blend.

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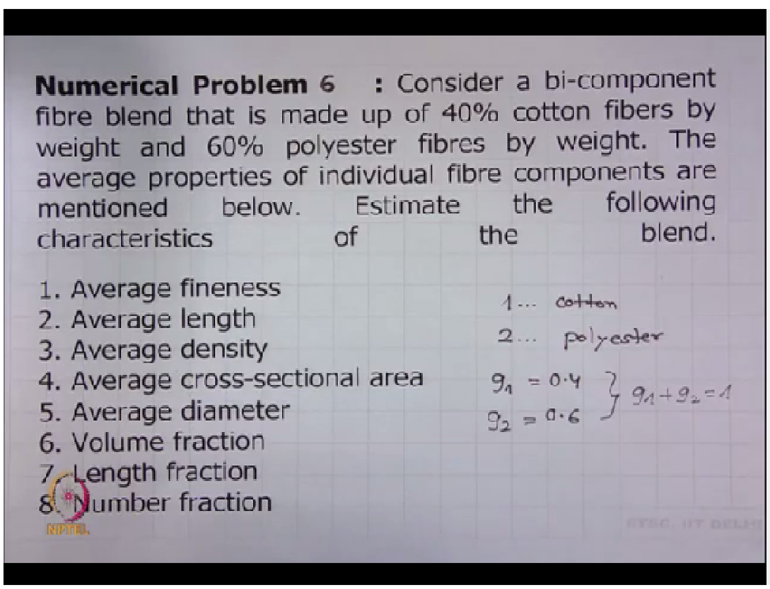

So we have our sixth numerical problem. Consider a bi-component fibre blend. Bi-component fibre means it consists of 2 components. Tri-component means 3 component. Mono-component means 1. So it is a bi-component fibre blend which consist of 40% cotton fibres by weight. So g1 if we consider 1 stands for cotton, 2 stands for polyester, then g1 is 0.4, g2 is 0.6. You see g1+g2, it must be equal to 1 and it is 1. The average properties of individual fibre components are mentioned below. Let me tell you the average characteristics.

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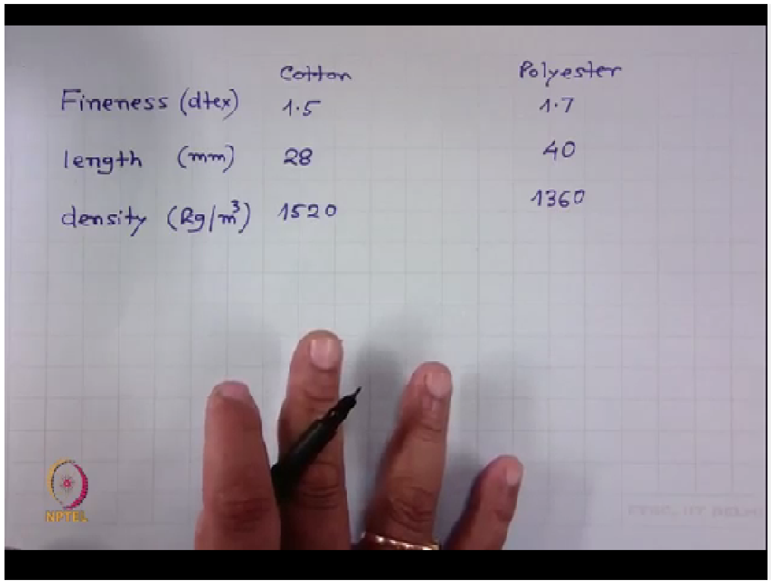

Say fineness cotton and polyester. Cotton is fineness in terms of decitex, 1.5 and polyester 1.7. Then we talk about length. Length in terms of millimeter. Cotton, the length is 28 mm, polyester you consider 40 mm. Density in terms of kg per meter cube. Cotton you consider 1520, polyester you consider 1360. So these 3 characteristics let us consider, these 3 are the basic characteristics which are often known in practice.

When we mix 2 fibres, say cotton polyester, we generally know their length, we generally know their fineness. Their densities are also well known. Based on this and also given is mass fraction. It is dimensionless. It is given here 0.4 and this is 0.6. These 4 data are given. What we have to find out? You have to find out these characteristics of the blend. Average fineness of the blend, average length of the blend, average density of the blend, average cross-sectional area of the blend, average diameter of the blend.

Five characteristics I told you earlier and mass fraction is given. So remaining 3 characteristics of fractions, volume fraction, length fraction, number fraction. So let us solve this problem. Whatever formulas we learnt, we will use them directly.

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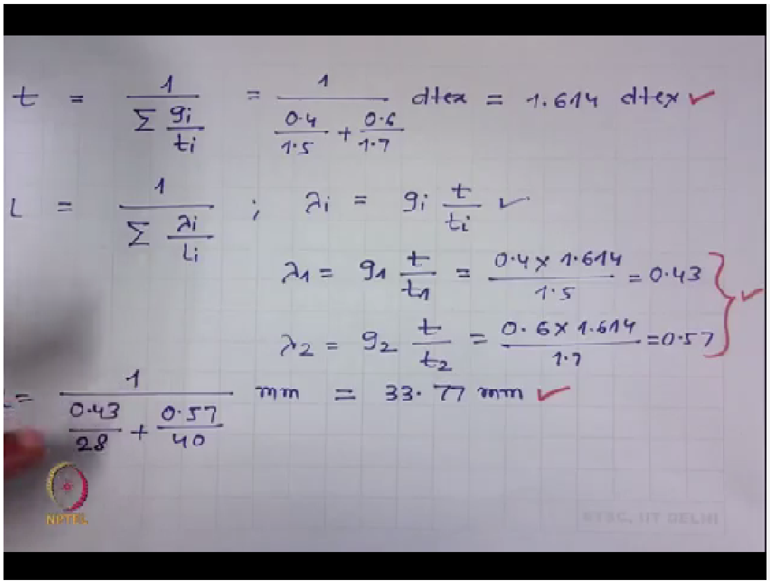

So we start with fineness of blend, t. We have learnt that t is related to individual component/this expression. Now all values are given in this table. g1 is 0.4, g2 0.6, t1 1.5, t2 1.7 decitex. So 1/gi is 0.4 and t1 is 1.5+g2 is 0.6 and t2 is 1.7. So this value will come in decitex. What will be the value? This can be calculated as decitex. So we obtained our first answer. The mean fineness of fibres in the blend is 1.614 decitex.

Now we calculate mean length, l. What is the formula we have learnt? Mean length of fibres in the blend is 1/length fraction/individual component length fraction. Now what is lambda i? Lambda i is gi t/ti, right. So lambda 1 for cotton, g1t/t1. What is g1 given? g1 is 0.4*t is 1.614 and t1 is 1.5, dimensionless. What will be its value? Its value will be 0.43. Lambda 2 will be of course 1-0.43, so 0.57.

It can also be calculated in another way, lambda 2 g2 t/t2. What is g2? 0.6*t is 1.614 and t2 is 1.7. You will see it will also come 0.57. So this is your lambda 1, lambda 2. Now we will use this formula directly. 1/, lambda 1 0.43 and what is the length of fibre given? Cotton l1 28, polyester l2 40 mm. So 28+0.57/40. This unit will be in mm. So this value will come approximately 33.77 mm. Now we will calculate mean fibre density, rho.

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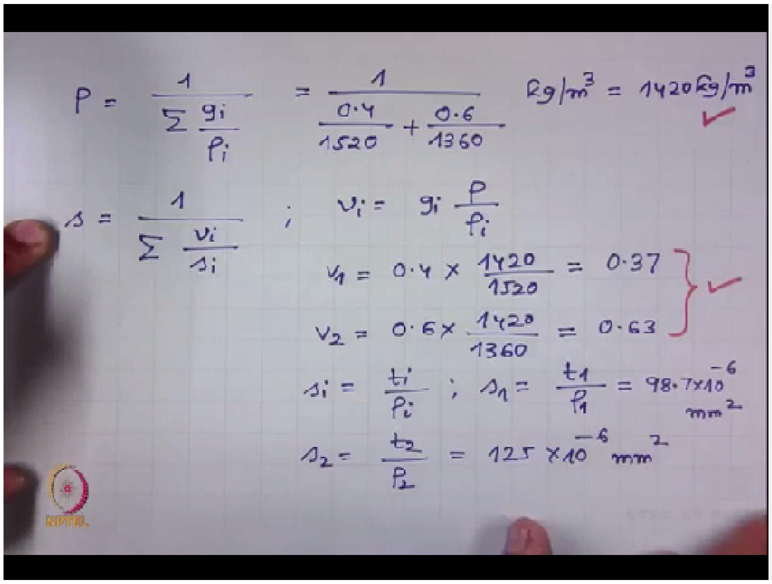

What is Rho? Rho is 1/summation gi/rho i, we have derived this expression, 1/. What is g1? 0.4. 1520, cotton fibre density, polyester 6. What is this value given? It is given 1360 kg per meter cube. Sure. We substitute 1360 kg per meter cube. What will be the unit? Kg per meter cube. If you calculate, you will obtain 1420 kg per meter cube. So this is our third one. Now we calculate the fourth one, mean fibre cross-sectional area, s.

What is s? s=1/summation volume fraction/individual this. Now what is volume fraction? Volume fraction=mass fraction, mean fibre density, individual fibre density. Now, so let us calculate v1 first. g1 is 0.4 cotton, this is 1420 and cotton fibre individual density is 1520. So what is the volume fraction? Volume fraction is 0.37. This is the volume, v1, right. Now v2=0.6*1420/1360.

So this will be coming roughly equal to 0.63. So we have already now calculated this volume fraction, right. We need to calculate now s subscript i. What is s subscript i? s subscript i is ti/rho i. So what is s1? s1 is t1/rho 1. Now t1 is given. t1 is given as 1.5 decitex, t1. t2 is 1.7 decitex and rho 1 is given 1520 kg per meter cube, rho 2 is given 1360 kg per meter cube. So if we use this expression, then and if we make the unit balance, then you will see the value will come somewhere 98.7*10 to the power -6 mm square.

Similarly, for s2, t2/rho 2. So if we use t2 1.7 decitex, rho 2 1360 kg per meter cube and make the unit balance, then this value you will get as 125*10 to the power -6 mm square. Now we know all 4 quantities. We will be able to calculate s. Let us do that.

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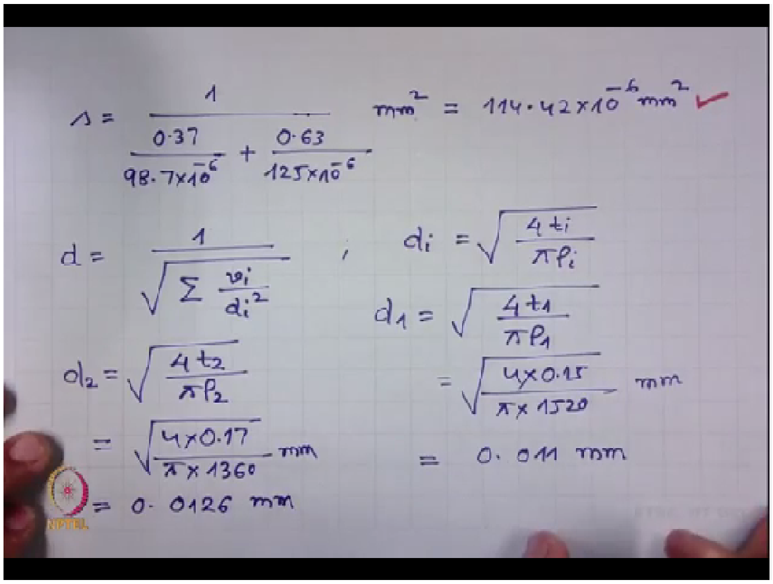

So s=1/v1 0.37, /area 98.7*10 to the power -6+v2 0.63 and 125*10 to the power -6. What will be the unit? Unit will be mm square. So if you calculate, you will say this value will come mm square. So this is the fourth important characteristics of the blend. We are remaining with the last one, mean equivalent diameter. So mean equivalent diameter is 1/square root of vi/di square. So this is known but di is not known.

What is di? di is root over 4ti/pi*rho i. Suppose d1=4t1/pi rho 1. So let us calculate this, 4. What is t1? t1 is 1.5 decitex. So 0.15 tex and rho 1 1520 kg per meter cube. So this unit will come in millimeter. So what will be the value of d1? 0.011 millimeter. So 11 micrometer. So this is d1. What is about d2? d2, similarly, 4t2/pi rho 2. So 4*1.7 decitex is t2. So 0.17 decitex pi*1360 millimeter. So this value you will see as 0.0126 millimeter. So 12.6 micrometer. Now you know all 4 quantities, v1, v2, d1, d2. You will be able to calculate d.

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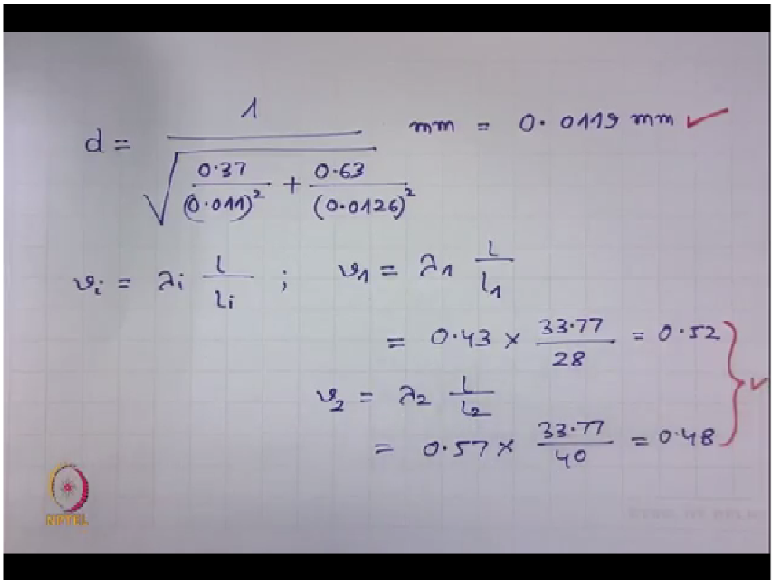

d will be = 1/square root of v1. v1 was 0.37, d1 square. d1 square is 0.011 square+v2, 63, d2 square 0.0126 square come in millimeter. So this value will come 0.0119 millimeter, that is equal to, roughly equal to 11.9 micrometer. So this was our fifth characteristics. We obtained about the fractions, mass fraction and volume fraction. Also we obtained about the length fraction. Length fraction was here.

So we are remaining with the only 1 fraction, that is your number fraction. Number fraction, we use the formula nu. We know length fraction*L/individual length. So nu1=lambda 1L/l1. You remember lambda 1 was 0.43. Here, lambda 1 0.43, lambda 2 0.57, we use these 2 values, 0.43*mean length. Mean length was 33.77 millimeter and l1, cotton fibre length was given as 28. So this value will come 52.

Similarly, nu2 lambda 2L/l2. Lambda 2 was given as 0.57, was obtained as 0.57. This is 33.77 and length of polyester fibre was given 40. So this value will come 0.48. This was our last one. So mass fraction was given, we obtained length fraction, volume fraction and number fraction. And we obtained mean fibre diameter, mean fibre cross-sectional area, mean fibre density, mean fibre fineness and mean fibre length.

All characteristics we obtained. So in this way we can characterize a fibre blend comprehensively. So this completes module 1. Thank you. Thank you very much for your attention.