Theory of Yarn Structure
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Lecture – 20
Tensile Mechanics of Yarns

Welcome to you all to these MOOCS online video course, theory of yarn structure, today we will start module 8, tensile mechanics of yarns. Mechanics deals with the study of force stress strain relation.

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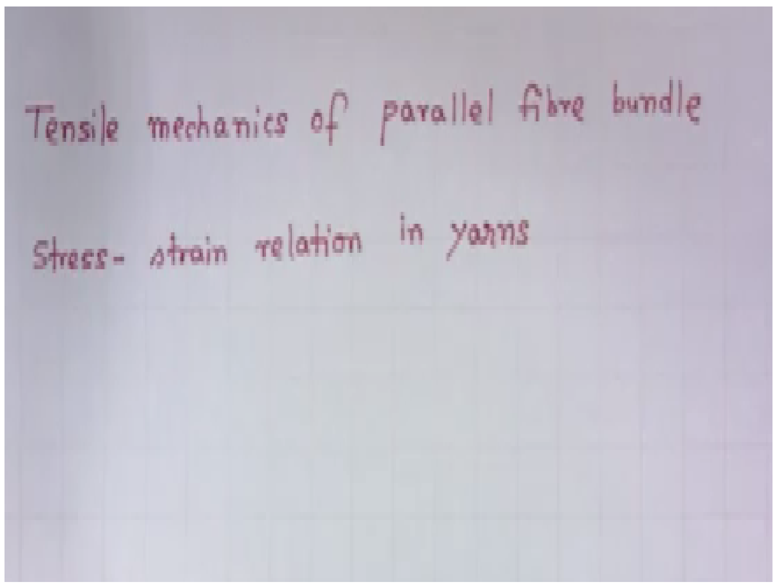

In this module, first we would like to learn about the tensile mechanics of parallel fibre bundle; afterwards we would like to learn about stress strain relations in yarns, so we will cover these 2 aspects under this theme, tensile mechanics of yarns.

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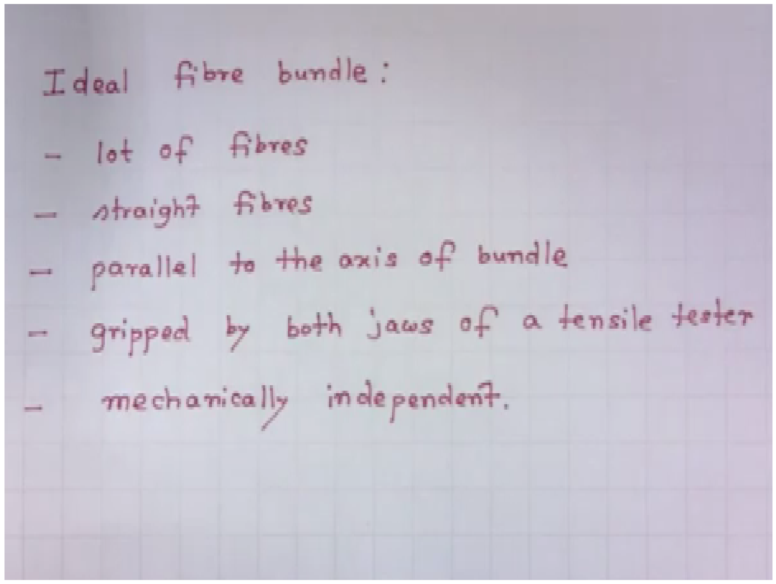

Now, what is an ideal fibre bundle; an ideal fibre bundle consists of lot of fibres, where the fibres are straight, they are all parallel and also they are parallel to the axis of the bundle, when we apply force to such a bundle, we assumed that all fibres are gripped by both jaws of a tensile tester and we will also assume that the mechanical behaviour of a given fibre is independent to the mechanical behaviour of all other fibres.

So, by an ideal fibre bundle, we assume the bundle consists of numerous fibres where all the fibres are straight, they are parallel to the axis of the bundle and when you stress such a bundle then we assume that the all fibres are gripped by both jaws of a tensile tester and the fibres are mechanically independent that means, when you apply force to a fibre, so the behaviour of this fibre is independent to the behaviour of other fibres, they are mechanically independent.

Now, we talk about the stress of one fibre, then we will talk about the stress of a; of an ideal fibre bundle, so let us first talk about one fibre.

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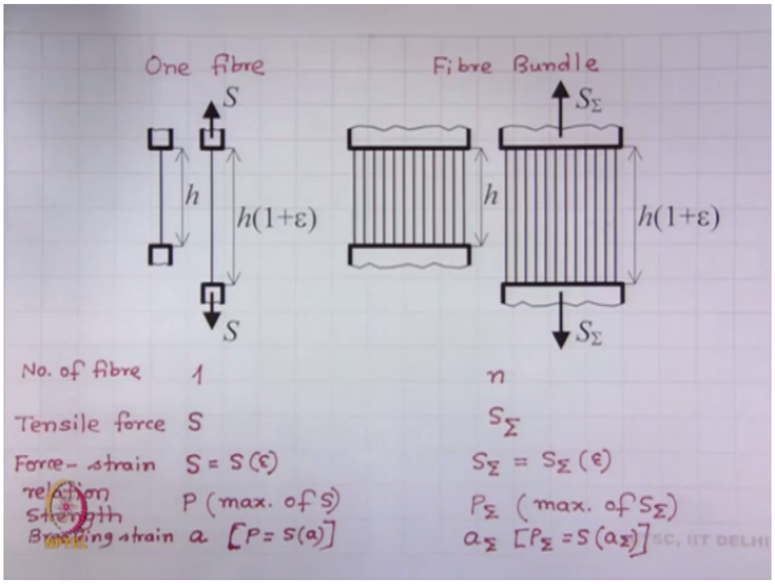

Here is one fibre and there is a fibre bundle, so what you see here, these are the jaws of a tensile tester, a fibre is gripped by both jaws of the tester and here the distance between the jaws is h, when you apply a force, capital S as a result, it elongates, so the length becomes h * 1 + epsilon, where epsilon is the elongation similarly, we consider a bundle, an ideal fibre bundle where the initial distance between the jaws is h.

Because of the application of a force S subscript summation, this bundle extends, the fibres extends and the distance between the jaws become h * 1 + epsilon, so let us now analyse this situation in respect of the mechanical quantities, number of fibres, first. How many fibres here we consider? One, so here it is 1, how many fibres we consider here, say it is small n now, tensile force, what is the tensile force we consider here; capital S.

And what is the tensile force we consider here; capital S subscript summation, what is the force strength relation here; force strength relation here is a function of strength and here it is a summation force = is a function of strength. What is the strength of this fibre? As you know, strength is the maximum force at which fibre breaks, so here you consider P, which is basically, maximum of S.

Similarly, here the strength is P star; P subscript summation which is also maximum of this and the last quantity is breaking strain or strain at break; breaking strain, suppose, breaking strain is here, small a, what does that small a means; P will be obtained when the breaking strain is a, similarly here it is a subscript summation, this will be obtained under this situation, so these are the symbols we are going to use.

Now, we will consider 2 cases, in the first case is very trivial where we will assume all fibres exhibit identical characteristics of stress strain, so we will analyse what will be the mechanical tensile stress, tensile strength in such a bundle.

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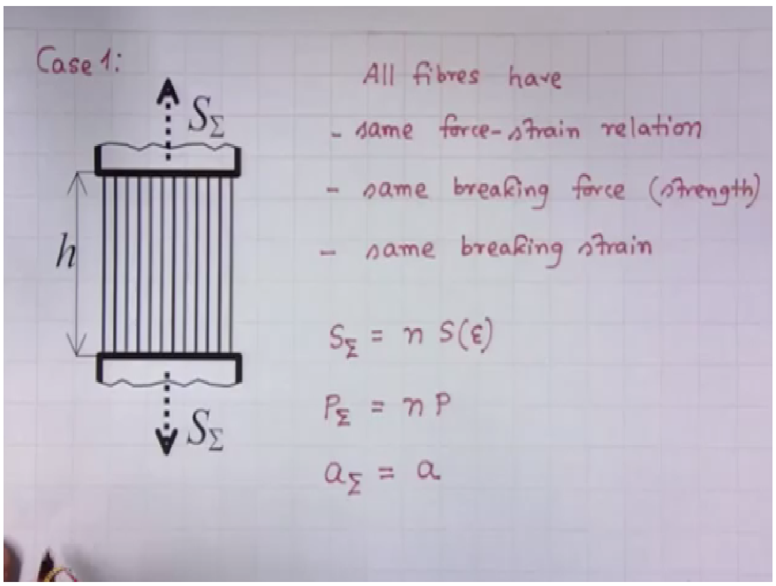

So, this is a case 1, very simple case, we assume all fibres have same force strain relation, all fibres have same force that is strain, they have same breaking strain. What does that mean? That means all fibres are identical so far their tensile characteristics are concerned, in such a situation the following relations are valid, we will be able to multiply them similarly, strength will be equal to strength of the bundle will be = number of fibres, n * strength of one fibre.

Similarly, breaking elongation; breaking strain of the bundle will be = breaking strain of the fibre, this is a very idealised situation now, we will make it little complicated, we will come to case 2.

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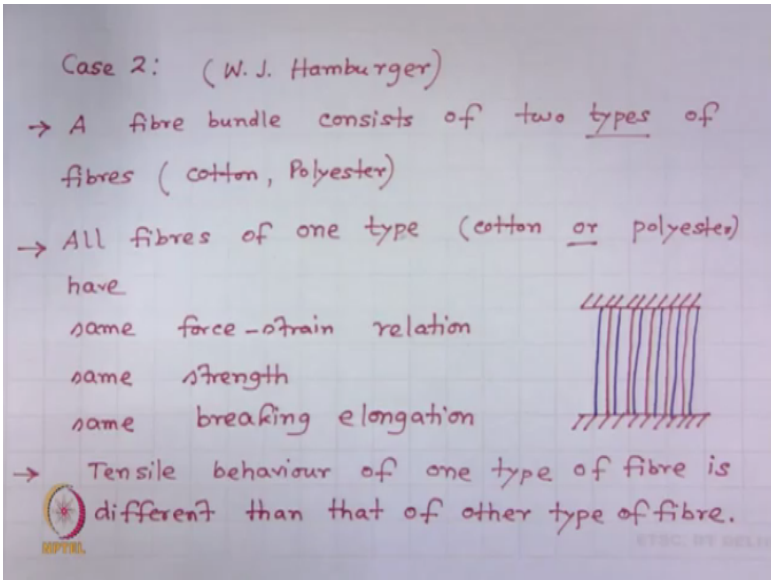

In case 2, what we will consider, a bundle consists of 2 types of fibres say, cotton, polyester, right now, then all fibres of one type say cotton or polyester, have same stress strain, force strain relation, same strength, same breaking elongation or breaking strain, so if you consider cotton fibres, all cotton fibres have same force strain relation, all cotton fibres have same strength, all cotton fibres have same breaking elongation.

If you consider polyester fibres, then all polyester fibres have same force strain relation, all polyester fibres have same strength, all polyester fibres have same breaking strain however, the tensile behaviours of cotton fibres is different than the tensile behaviour of polyester fibres of one type of fibre is different than the tensile behaviour of the other type of fibre, so it means tensile behaviour of all cotton fibres, though they are same but they are different than the tensile behaviours of all polyester fibres.

Though all polyester fibres exhibit same tensile behaviour, so now we will consider such a bundle of fibres, so it will be something like this, suppose this is the upper jaw of tensile tester, this is the bottom jaw of the same tensile tester, now if I have one fibre say, red colour, all straight parallel to the axis of the bundle then there will be other type, we will indicate it by blue colour such is the bundle.

What will be the tensile behaviour of this bundle, this problem was theoretically solved many years ago by one researcher, his name was W.J. Hamburger, so the theoretical concept what we are going to discuss now for this kind of bundle are also known as hamburger's model now, before going to the detail of the model, let us define the symbols that we are going to use in Hamburger's model.

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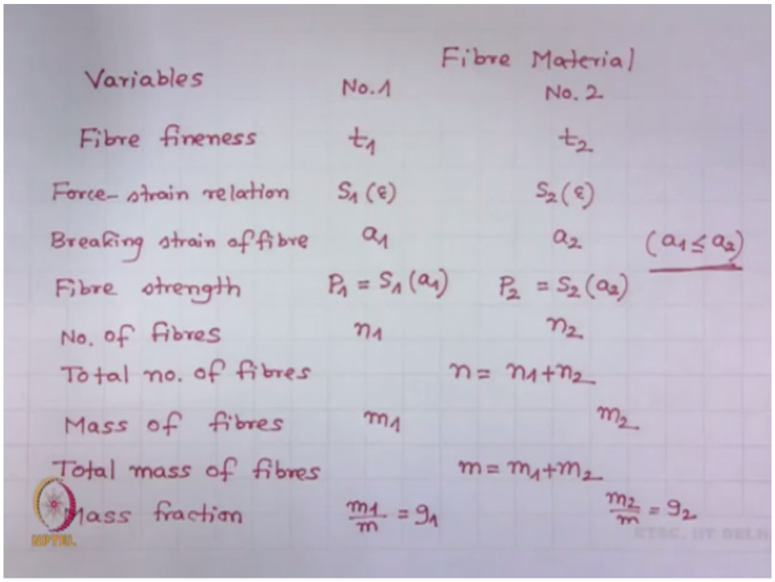

So, variables enables fibre material or fibre, one type number 1, second type number 2, first about fibre fineness for type 1, we will denote it by t subscript 1, for type 2, we will denote t2, as an example, if a fibre bundle consist of cotton and polyester fibres then t1 may denote the fineness of cotton fibre, t2 may denote the fineness of polyester fibre, then second is force strain relation.

For type 1, S subscript 1 is a function of strain, S subscript 2, function of strain, breaking strain of fibre or strain at break of fibres, suppose this is a subscript 1 and this is a subscript 2, these 2 will be different, so a subscript 1 and be < = a subscript 2, so this is a very important consideration here then, we come about fibre strength, fibre strength of type 1, P1 which is = S subscript a1 because this is the breaking elongation so, function at breaking elongation will be equal to strength.

Similarly, P2, function is S2 and breaking elongation, a2 now, number of fibres, suppose this is n1, this is n2, so total numbers of fibres in the bundle n; n1 + n2, mass of fibres; suppose, this is your m1, this is your m2, what is the total mass of fibres; m1 + m2. Now, mass fraction of type 1, m1/m, let us say this is g1 and here it is m2/m which is g2. What will be summation of g1 and g2; g1 + g2 = 1, right.

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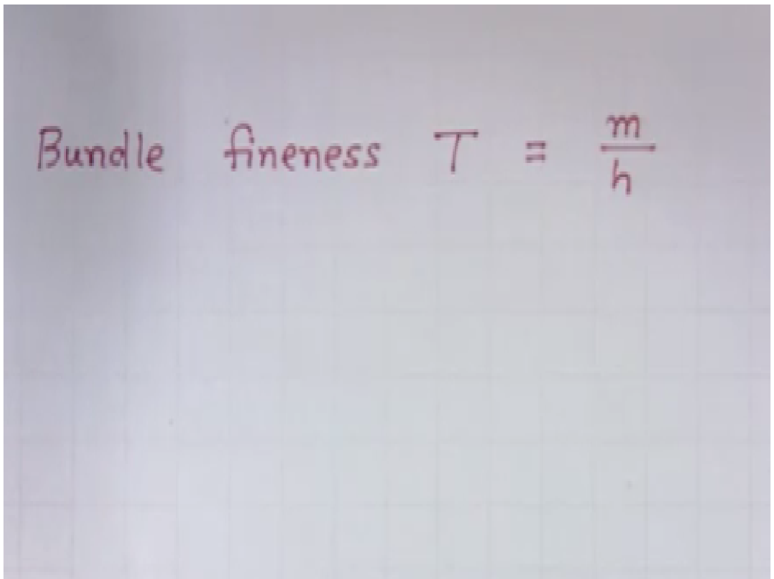

So, these are the characteristics of 2 types of fibres; 1 and 2, now we will consider the fineness of the bundle say, bundle fineness, we will denote by the symbol T which = mass per unit length, what is the mass of bundle; small m, what is the length; small h, so m/h, okay, so all characteristics if you see subscript 1, they denote to type 1, say cotton, there will be some characteristics which will have subscript 2, all those characteristics are related to type 2 fibre that is polyester.

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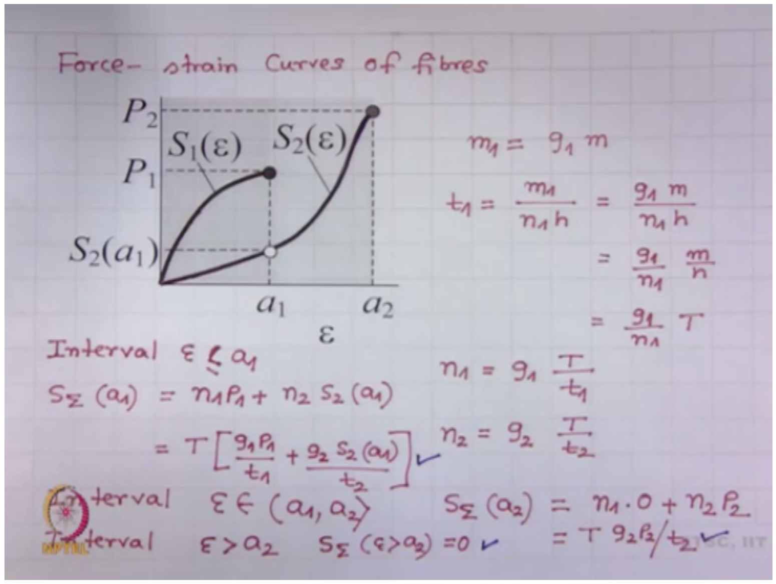

And there will be some subscript, there will be some variables without subscript, they are common to both now, we consider this kind of stress strain force strain curve of fibre, as I said you 2 types of fibres, one and two, suppose this is the force strain curve of type 1 fibre and this is the force strain curve of type 2 fibre, so type 1 and this will be type 2, this twisting behaviours of these fibres are very different.

However, all type I fibres have same stress strain behaviour, all type 2 fibres have same stress strain behaviour but type 1 fibres have different stress strain behaviour than type 2 fibres, right, now if we look carefully to this graph, a1 is a breaking strain of type 1 fibre however that a1 type 2 fibres do not break, type 2 fibres have a force S2 subscript a1, at higher elongation a2, all type 2 fibres will break, so they are breaking force is P2.

P1 is breaking force of type 1, P2 is breaking force of type 2, so this is what we see now, what do we have to now consider; we have to consider if this is the fibres tester behaviour then what will be the behaviour of the bundle, so first we need to know how many type 1 fibres are present, how many type 2 fibres are present, so in order to do that let us say, m1, what is m1; m1 is mass of type 1 fibres, so that is = g1 * m; m is mass of all fibre, g1 is mass fraction of type 1 fibre.

What is the fineness of type 1 fibre, single fibre fineness, so what is the fineness of type 1 fibre; it is the mass of all type 1 fibres divided by length of all type 1 fibres, so mass of all type 1 fibres is m1 divided by length of all type I fibres, what is the length of one fibre; small h, how many fibres are present, n1, so what is the total length; n1 * h, right, if you substitute m1/g1 m n1 * h, you can write it further, g1/n1, m/h is bundle fineness * T; T is bundle fineness m/h.

So, n1 = g1 capital T/ small t1, so how many type 1 fibres are present is possible to calculate using this formula now, imagine in a textile company, we blend fibres in terms of their weight, their mass, so generally mass fractions are generally known, so g1 and g2 are known quantity, they are readily available quantities, capital T; what is the count of the sliver, what is the count of the sliver, what is the count of the yarn, these all quantities are generally known.

What is the fineness of fibre; is also known, so capital G1, so small g1, capital T small t1, these are all known quantities, so by using all known quantities, you will be able to calculate how many type 1 fibres are present in the bundle similarly, we can write n2 = g2 capital T/t2, so by using this relation, one will be able to calculate how many type 1 fibre, how many type 2 fibres are present in the bundle, right.

So, now we come back to this, we have to analyse this situation, in order to analyse this situation, what we do; we divide this region * 3 regions, first we consider what is happening at when the breaking; when the elongation is < = a1, then we will consider what is happening when the elongation is in between a1 and a2, third region; we will consider when the elongation is > a2, so these 3 situations we would like to analyse.

First is this interval, (()) (31:32) epsilon is < = a1, so this situation, so when they are equal to; epsilon = a1, the force exhibited by one type of fibre will be n1 P1, so what will be the force at elongation a1; n1 times P1 + type 2 fibres are also stressed, what is the force; coming from type 2 fibres, at this strain this is a force and how many type 2 fibres; n2, so n2 this, if we substitute n1 from here and n2 from here, what we will see is g1 P1/t1 + g2 S2 a1/ t2.

So, this is how it can be calculated now, we consider the interval a1, a2 excluding a1, so now we consider interval epsilon excluding a1 but including a2, open interval, close interval, what will be the force at a2; at a2, there is no type 1 fibre, so n1 * 0 + n2 * P2, so if we substitute n2 here, then it will be T g2 P2/ t2, right, then the last interval; interval epsilon > a2, there is no fibre, there is no bundle, so what will be the force at that strain, 0.

So, we obtain 3 expression for force, one is this interval, second is this interval, and the third is this interval, so what will be the force; what will be the strength of the bundle; strength of the bundle will be the maximum of this 3 forces, this we can neglect is 0, so strength of the bundle will be maximum of these 2 forces.

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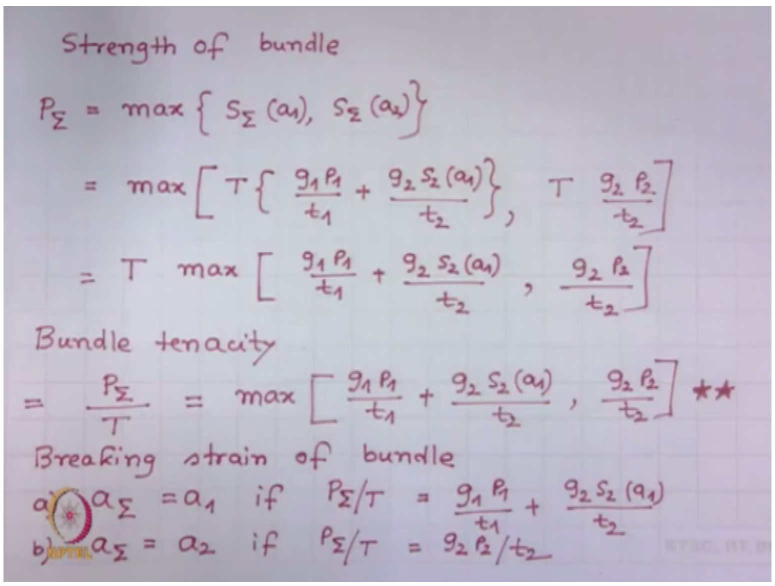

So, we will write strength of the bundle will be; this is the symbol we used earlier, maximum of a1, a2, right so, what is that? Maximum of capital T g1 P1/ t1 + g2 S2 of a1/t2, this is for this function and for this will be T * g2 P2/t2, whatever will give maximum value that will be = strength of the bundle, now what will be alright, so here this T is common, constant, it can out of this maximum operator, right.

So, what will be bundle tenacity; bundle tenacity is this which is equal to maximum of this 2, this will be expression for bundle tenacity, so this is about the strength of the bundle, what is about the breaking strain of the bundle. Now, we will discuss that breaking strain of bundle, 2 situations can arise, breaking strain is denoted by this quantity, this can be = a1, if this T = t2, so if this is quantity is maximum, then the breaking strain of fibre type 1 will be = the bundle strain of the breaking strain of the bundle.

Otherwise, breaking strain of the bundle can be = a2, breaking strain of the type 2 fibre, if this is = g2 P2/t2, right, now we would like to know the graphical representation of this expression, how this expression looks like graphically?

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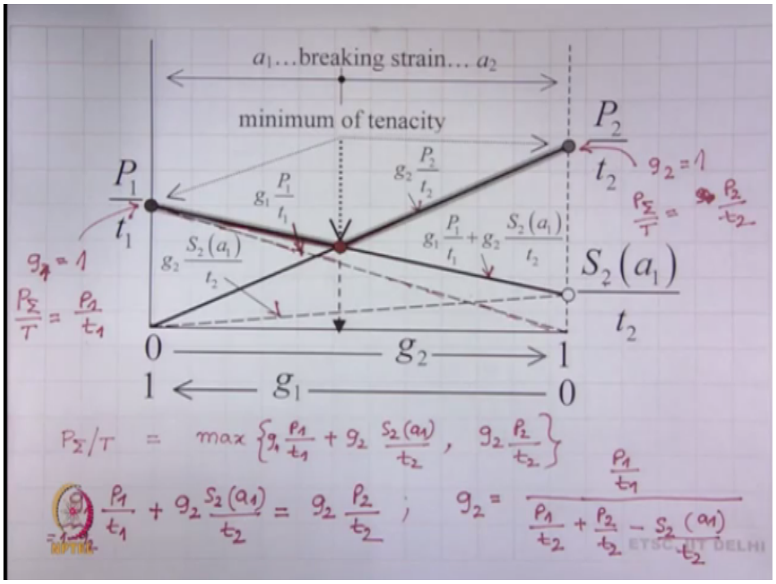

So, this is the graphical representation of this expression, this, let me explain you what is what in this diagram, x axis mass fractions are plotted, g1 denotes mass fraction of type 1 fibre, g2 denote mass fraction of type 2 fibre, when g1 = 1 at this point, mass fraction of type 2 fibre is 0, similarly at the end when g2 = 1, mass fraction of type 1 fibre is 0, it has 2 y axis, this y axis, we plot this point is basically P1/t1.

And this point is higher, P2/t2, this point is S2 a1/t2, so this is basically specific stress of fibre type 2 at breaking elongation of fibre type 1, so if we join this line; this dotted line, this line denotes g1 P1/t1, this dotted line similarly, the line joining from 0 to P2/t2 will give you g2 P2/ t2 and this is this line g2 P2 / t2 this line right, similarly the line joining between these 2 points gives you g2 S2 a1/ t2, so this dotted line gives you g2 S2 a1/t2.

And what is this 2 joining lines; this line gives you g1 p1/t1, then + g2 S2 a1/t1, okay, now what is of concern; our concern is of minimum tenacity, we should not makes type 1 and type 2 fibre in such a manner that we should not arrive at minimum tenacity that is basically is a concern while mixing. If we choose wrong blend ratio at which the bundle tenacity will be minimum, then we spoil the material.

So, minimum tenacity is always is our concern that means, this point, so this will be the behaviour of these bundle and then increase, this point is the host point at any case, we must avoid this point, right so what we see; we see 3 important points, first is this point, second is this point, third is this point, this point is very crucial now, this first; this point, this point can be obtain when g1 = 1.

So in that case bundle tenacity will be P1/t1, at this point g2 = 1, so what will be the bundle tenacity at this point; P2/t2, these 2 points are easy but the host point is this, how will you find out the bundle tenacity at this point, so the behaviour of the bundle; tensile behaviour of the bundle is this and then it increases at any cost, we must avoid this blend ratio, so these point can be obtained is basically the intersection of 2 lines; this line and this line.

So, this point gives is the intersection of 2 lines that means, how will you find out this point, so g1 is this line, so g1; this expression will be = this expression, so this line and this line, so at this point g1 P1/t1 + g2 S2 a1/ t2 = g2 P2/ t2, right now, let us write g1as 1 – g2, to so if you write g1 = 1 – g2, then if you club all g2 together in one side, remaining on the other side, then we will be able to find out g2 = P1/t1 * P1/t1 + P2/t2 – S2 a1/ t2, right.

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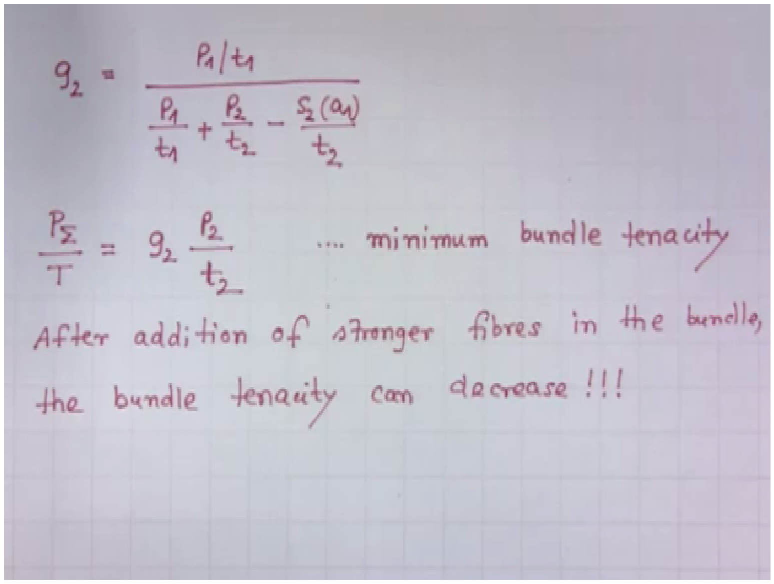

So, how will you obtain such a blend ratio; g2 = P1/t1 divided by P1/t1 + P2/t2 – S2 a1/t2 and at that blend ratio, bundle tenacity will be g2 P2/t2, so this is the minimum bundle tenacity which we must avoid in practice, so in textile industry we should not mix 2 fibres in such a manner that at that particular blend ratio, yarn tenacity is minimum or sliver tenacity is minimum or roving tenacity is minimum, we must avoid such a blend ratio under all circumstances.

And this theory; Hamburger's theory predicts what will be the blend ratio of 2 different types of fibres at which we obtain minimum bundle tenacity. What is interesting here to see that come back to this graph, look at this here g2 = 0 and g2 = 1, so and g2 is stronger than g1, type 2 fibre is stronger than type 1 fibre, you have already seen this; this is type 2 fibre, type 2 fibre is stronger than type 1 fibre, P2 is > P1.

So, we go an adding stronger fibres in the bundle, the bundle tenacity initially is reducing, it is lowest here, then it is increasing, so what we observe this figure is that very interesting after addition of stronger fibres in the bundle; bundle tenacity can decrease, we are adding stronger fibres however, bundle tenacity is decreasing, after certain point it again start increasing and finally it goes on increasing.

So, the interesting part of this theory is that after addition of stronger fibres in the bundle, the bundle tenacity can decrease very much surprising however, it is true, Hamburger's theory tells us about this now, 2 other situations can also happen however, they are not of our concern.

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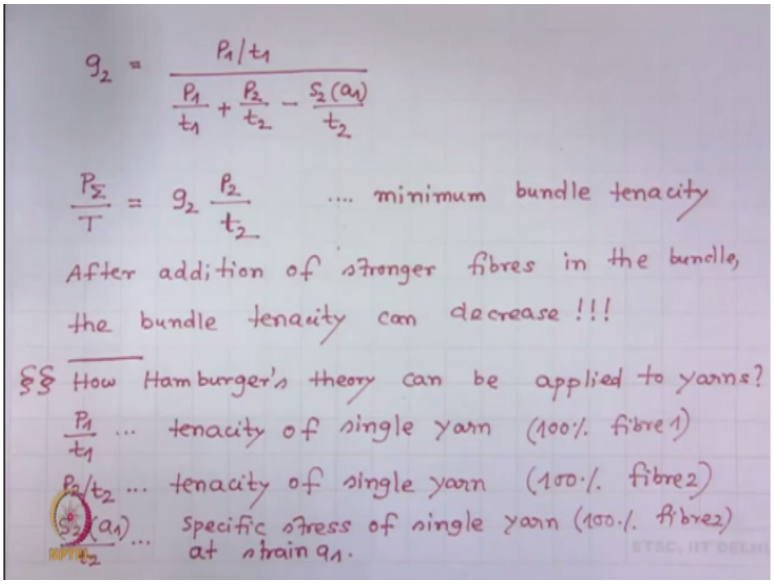

What kind of situations can happen, this 2 are also the possibilities, the curve can be something like that here, highest here, lowest here and it is continuously decreasing, in this case lowest here, highest here continuously increasing, they are of not our concern, problem is only when initially decreases then it is increases, so this is possible only when the stress strain diagram of the fibres is something like this?

So, what we learn is that the stress strain curves of the fibres ultimately decide the tensile behaviour of the bundle now, how this theory can be applied to yarn, so far we have discussed about parallel fibre bundle however, yarn is a twisted fibre bundle, so the question remains how this theory can be applied to yarn. If one wishes to apply this theory exactly the way it is to yarn, then there will be a huge mismatch between the actual results and the predicting results.

One has to modify, how do you modify, let us talk about that. How this theory can be applied to yarn? What we will do; in this case, the meaning of this symbol was little different, P1/t1 in a hamburger's theory it was the tenacity of fibre 1 but in this particular case, we will consider this as tenacity of single yarn 100% fibre 1, so if the blend is a carbon and polyester, this means tenacity of 100% cotton yarn.

Similarly, P2/t2 will be tenacity of single yarn but 100% fibre type 2 so, P2/t2 will be the tenacity of 100% polyester fibre yarn and then what will be by t2, this is the specific stress of single yarn, which yarn; 100% fibre 2 at strain a1, so if you consider these 3 meanings in this manner then we use this theory, you will be able to obtain the tensile behaviour of the yarn, so this modification or this consideration you have to follow.

If you wish to apply Hamburger's theory in case of yarns, so now we will stop here, we would like to discuss in the next lecture, new medical problems relate to this module, thank you very much for your attention.