

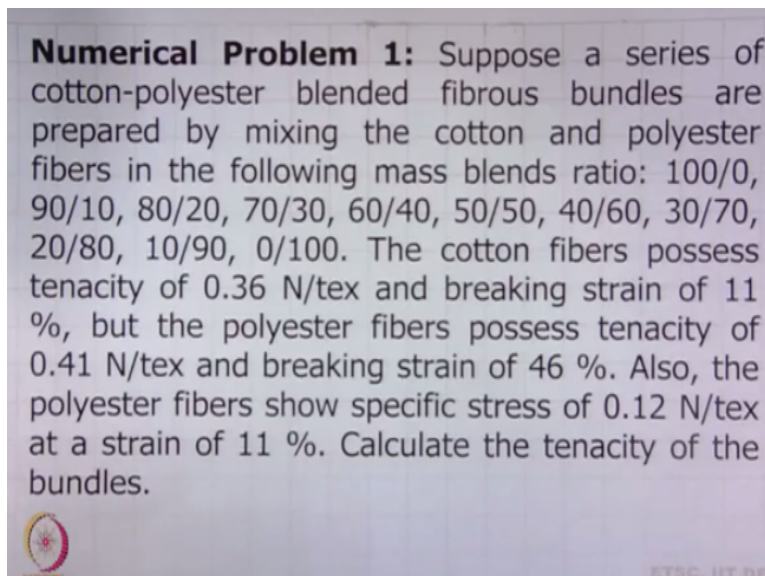
Theory of Yarn Structure
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Lecture – 21
Tensile Mechanics of Yarns (contd.,)

Welcome to you all to this MOOCs online video course, theory of yarn structure, in the last class we started module 8; tensile mechanics of yarn, we discussed about 2 situations of fibrous bundle; one was very trivial case where all fibers have identical stress strain behaviour then what will be the bundle characteristics, tensile characteristics we discussed. Second case; we talked about hamburgers theory.

That theory predicts the tensile behaviour of a parallel fiber bundle consists of 2 different types of fibers and then we talked about how hamburgers theory can be applied to yarns, so a simple note also we discussed. Now, we will start with hamburgers theory numerical problems, the first numerical problems reads as follows.

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Numerical Problem 1: Suppose a series of cotton-polyester blended fibrous bundles are prepared by mixing the cotton and polyester fibers in the following mass blends ratio: 100/0, 90/10, 80/20, 70/30, 60/40, 50/50, 40/60, 30/70, 20/80, 10/90, 0/100. The cotton fibers possess tenacity of 0.36 N/tex and breaking strain of 11 %, but the polyester fibers possess tenacity of 0.41 N/tex and breaking strain of 46 %. Also, the polyester fibers show specific stress of 0.12 N/tex at a strain of 11 %. Calculate the tenacity of the bundles.

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Suppose a series of cotton polyester blended fibrous bundles are prepared by mixing the cotton and polyester fibers in the following mass blends ratio; 100/0, so 100% polyester cotton fibers, 0% polyester fibers, 90/10, 80/20, 70/30, 60/40, 50/50, 40/60, 30/70, 20/80, 10/90, 0/100, 0/100 means 100% polyester fiber, 0% cotton fibers. The cotton fibers possess tenacity of 0.36 Newton per tex and breaking strain of 11%.

But the polyester fibers possess tenacity of 0.41 Newton per tex and breaking strain of 46% also, the polyester fiber show specific stress of 0.12 Newton per tex at a strain of 11%, calculate the tenacity of the bundles.

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$$\frac{P_1}{t_1} = 0.36 \text{ N/tex}; \quad \frac{P_2}{t_2} = 0.41 \text{ N/tex}, \quad \frac{S_2(a_1)}{t_2} = 0.12 \text{ N/tex}$$

$$a_1 = 0.11; \quad a_2 = 0.46$$

$$\text{Bundle tenacity } \frac{P_\Sigma}{T} = \max \left[g_1 \frac{P_1}{t_1} + g_2 \frac{S_2(a_1)}{t_2}, g_2 \frac{P_2}{t_2} \right]$$

$$\xi \quad g_1 = 1; g_2 = 0; \quad \frac{P_\Sigma}{T} = \max \left[(1 \times 0.36) + (0 \times 0.12), (0 \times 0.41) \right]$$

$$= \max (0.36, 0) = 0.36 \text{ N/tex}$$

$$\xi \quad g_1 = 0.9; g_2 = 0.1; \quad \frac{P_\Sigma}{T} = \max \left[(0.9 \times 0.36) + (0.1 \times 0.12), (0.1 \times 0.41) \right]$$

$$= \max (0.336, 0.041) = 0.336 \text{ N/tex}$$

$$\xi \quad g_1 = 0.8; g_2 = 0.2; \quad \frac{P_\Sigma}{T} = \max \left[(0.8 \times 0.36) + (0.2 \times 0.12), (0.2 \times 0.41) \right]$$

That means, what are given to us is $P_1/t_1 = 0.36$ Newton per tex is given also, $P_2/t_2 = 0.41$ Newton per tex given, $a_1 = 11\%$, $a_2 = 46\%$ and one more important value is given that = 0.12 newton per tex, so these items of information are given to us, we have to calculate bundle tenacity at different blend ratios. What is bundle tenacity? From hamburgers theory, we learn that bundle tenacity is maximum of $g_1, p_1, t_1 + g_2/t_2; g_2 p_2 t_2$.

So, we need to calculate this function for different blend ratios, so let us first start, when g_1 is 1, g_2 ; 0, 100% cotton fiber, so we will substitute all these quantities here, this will be maximum of g_1 is $1 * p_1/$ this 0.36 + g_2 is $0 * 0.12$, g_2 is $0 * 0.41$, so if we calculate, this will be 0.36 and this will be 0, so out of these 2, what is maximum; 0.36, so bundle tenacity will be 0.36 Newton per tex.

Second; g_1 is given say 90% cotton fibers, g_2 10% polyester fibers, what will be the bundle tenacity; we use the same formula, maximum of g_1 is $0.9 * p_1/t_1$ is 0.36 + $0.1 * 0.12$, the second value $0.1 * 0.41$, what will be the maximum value? Maximum of this quantity you will find it out this should be equal to 0.336 and 0.041, so what is the maximum of these 2; 0.366 and 0.41 of course, 336 Newton per tex, right.

Then we proceed to third; g1 cotton fiber percentage is 80 and g2, 20, what will be the bundle tenacity? We use the same formula here, maximum of g1 is $0.80 * 0.36 + 0.2 * 0.12$, $0.20 * 0.41$.

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$$\frac{P_{\Sigma}}{T} = \max(0.312, 0.082) = 0.312 \text{ N/tex}$$

$$\xi \quad g_1 = 0.70; \quad g_2 = 0.30; \quad \frac{P_{\Sigma}}{T} = \max[(0.70 \times 0.36) + (0.30 \times 0.12), (0.30 \times 0.41)]$$

$$= \max(0.288, 0.123)$$

$$= 0.288 \text{ N/tex}$$

$$\xi \quad g_1 = 0.60; \quad g_2 = 0.40; \quad \frac{P_{\Sigma}}{T} = \max[(0.60 \times 0.36) + (0.40 \times 0.12), (0.40 \times 0.41)]$$

$$= \max(0.264, 0.164)$$

$$= 0.264 \text{ N/tex}$$

$$\xi \quad g_1 = 0.50; \quad g_2 = 0.50; \quad \frac{P_{\Sigma}}{T} = \max[(0.50 \times 0.36) + (0.50 \times 0.12), (0.50 \times 0.41)]$$

$$= \max(0.240, 0.205) = 0.240 \text{ N/tex}$$

What will be the maximum; 0.312, 0.082, so what is the maximum; 0.312 Newton per tex, so at 80/20 blend ratio, the bundle tenacity will be 0.312 Newton per tex, so gradually it is reducing, so there will be 1 blend ratio at which the bundle tenacity will be minimum then again it will increase, we have to see what is that blend ratio. Fourth; let us consider g1 is 0.70, g2 is 0.30, what will be bundle tenacity?

Max $0.70 * 0.36 + 0.30 * 0.12$, $0.30 * 0.41$; 0.288, 0.123; 0.288 newton per tex, so at 70/30 blend ratio, the bundle tenacity will be = 0.288 newton per tex, then we configure the next one; 0.60, g2 is 0.40, so the bundle tenacity will be $0.60 * 0.36 + 0.40 * 0.12$, $0.40 * 0.41$, so what will be these two values; these two values will be 0.264, 0.164, what will be the maximum in between these 2, Newton per tex.

So at 60/40 blend ratio, bundle tenacity will be 0.264 newton per tex, now we come to the next one, 50/50, what will be the bundle density at 50/50 blend ratio; $0.50 * 0.36 + 0.50 * 0.12$, $0.50 * 0.41$, what will be these 2 values? These 2 values will be 0.240, 0.205, maximum is 0.240 Newton per tex. So, when cotton and polyester fibers are mixed in equal proportions in terms of weight, then the bundle tenacity will be 0.24 newton per tex.

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$$\begin{aligned} \xi \quad g_1 = 0.40; \quad g_2 = 0.60; \quad \frac{P_\Sigma}{T} &= \max \left[\frac{(0.40 \times 0.36) + (0.60 \times 0.12)}{(0.60 \times 0.41)} \right] \\ &= \max (0.144, 0.246) \\ &= 0.246 \text{ N/tex} \\ \xi \quad g_1 = 0.30; \quad g_2 = 0.70; \quad \frac{P_\Sigma}{T} &= \max \left[\frac{(0.30 \times 0.36) + (0.70 \times 0.12)}{(0.70 \times 0.41)} \right] \\ &= \max (0.192, 0.287) \\ &= 0.287 \text{ N/tex} \end{aligned}$$

We continue for the other blends; $g_1 = 0.40$, $g_2 = 0.60$, at this blend ratio, what will be the bundle tenacity; maximum of $0.40 \times 0.36 + 0.60 \times 0.12$, these 2, so what will be these 2 value; max of 0.144, 0.246, look at these values now, this value is higher than this value, 0.246 Newton per Tex, so the minimum bundle tenacity will be obtained when the polyester fiber percentages in between 50 to 60%, right.

Now, we proceed further to obtain the other blend ratios, g_1 is 0.30, g_2 is 0.70, what is the bundle tenacity; max of $0.30 \times 0.36 + 0.70 \times 0.12$, 0.70×0.41 , what will be these 2 values; these 2 values will be maximum of 0.192, 0.287, which will be 0.287 Newton per tex, see again that increasing strain started at 30/70 blend ratio, the bundle tenacity value will be 0.287 newton per tex.

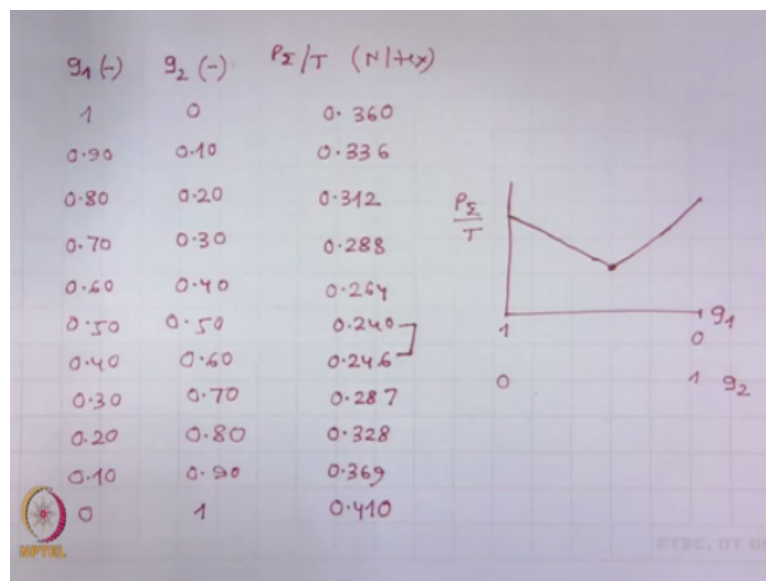
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$$\begin{aligned} \xi \quad g_1 = 0.20; \quad g_2 = 0.80; \\ \frac{P_\Sigma}{T} &= \max \left[\frac{(0.20 \times 0.36) + (0.80 \times 0.12)}{(0.80 \times 0.41)} \right] \\ &= \max (0.168, 0.328) = 0.328 \text{ N/tex} \\ \xi \quad g_1 = 0.10; \quad g_2 = 0.90; \\ \frac{P_\Sigma}{T} &= \max \left[\frac{(0.10 \times 0.36) + (0.90 \times 0.12)}{(0.90 \times 0.41)} \right] \\ &= \max (0.144, 0.369) = 0.369 \text{ N/tex} \\ \xi \quad g_1 = 0; \quad g_2 = 1; \\ \frac{P_\Sigma}{T} &= \frac{P_2}{t_2} = 0.41 \text{ N/tex} \end{aligned}$$

Next blend ratio; g_1 is 0.20 and g_2 is 0.80, so at this blend ratio, bundle tenacity will be maximum of these 2; $0.20 * 0.36 + 0.80 * 0.12$, $0.80 * 0.41$, these 2 maximum will be 0.168, 0.328; 0.328 Newton per Tex. Next blend ratio; g_1 is 0.10, g_2 is 0.90, at this blend ratio, will be maximum of these 2; $0.10 * 0.36 + 0.90 * 0.12$, $0.90 * 0.41$, so this 2 will be 0.144, 0.369; 0.369 Newton per Tex.

The last one 100% polyester fibers, when the bundle is prepared from 100% polyester fiber will be equal to very simple that is basically, P_2/t_2 that = 0.41 Newton per tex.

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If we summarize the results what we see is that g_1 , g_2 and bundle tenacity in Newton per tex, dimensionless, dimensionless, so 1, 0, 9, 0.1, 0.8, 0.2, 0.7, 0.3, 0.6, 0.4, 0.5, 0.4, 0.3, 0.2, 0. this, 0, 1, so this value is 0.41, this value will be 0.369, this value will be 0.328, then this value will be 0.287, this value will be 0.246, this value 40, 60, afterwards, this value will be 0.240, then this value will be 0.264, then this value 0.288, this value will be 0.312, then 312, then this value will be 0.366, then this value will be 3. this.

So, these values we calculated now, what we see is that it is decreasing, somewhere here we see it is again started increasing, so this value continuously decreasing, after this it is again started increasing, so the critical blend ratio lies in between 50 to 60, so the behaviour schematically will be g_1 say 1, 0 here and g_2 0, 1 here and this will be the bundle tenacity, so it starts decreasing somewhere here again, it starts increasing, this is the critical blend ratio.

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Numerical Problem 2: Suppose a series of cotton-polypropylene blended fibrous bundles are prepared by mixing the cotton and polypropylene fibers in the following mass blends ratio: 100/0, 90/10, 80/20, 70/30, 60/40, 50/50, 40/60, 30/70, 20/80, 10/90, 0/100. The cotton fibers possess tenacity of 0.36 N/tex and breaking strain of 11 %, but the polypropylene fibers possess tenacity of 0.53 N/tex and breaking strain of 25 %. Also, the polypropylene fibers show specific stress of 0.30 N/tex at a strain of 11 %. Calculate the tenacity of the bundles.

This we obtain from this numerical example now, let us consider a second numerical example, very similar manner suppose, a series of cotton polyester blended fibrous bundle are prepared by mixing cotton and polypropylene fibers in the following mass blend ratio; 100/0, 90/10, 80/20, 70/30 and so on, the cotton fiber possess a tenacity of 0.36 newton per tex, breaking strain of 11%.

But the polypropylene fiber process tenacity of 0.53 Newton per tex and breaking strain of 25% also, the polypropylene also specific stress of 0.30 newton per tex at 11%, calculate the tenacity of the bundles?

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Bundle tenacity

$$\frac{P_{\Sigma}}{T} = \max \left[\underbrace{g_1 \frac{P_1}{t_1} + g_2 \frac{S_2(a_1)}}_A ; \underbrace{g_2 \frac{P_2}{t_2}}_B \right]$$

$$\frac{P_1}{t_1} = 0.36 \text{ N/tex}, \quad \frac{P_2}{t_2} = 0.53 \text{ N/tex}, \quad \frac{S_2(a_1)}{t_2} = 0.30 \text{ N/tex}$$

$$a_1 = 0.11 ; \quad a_2 = 0.25$$

g_1 [%]	A [N/tex]	B [N/tex]	$\max(A, B) = \frac{P_{\Sigma}}{T}$ [N/tex]
1	0.36	0	0.36
0.9	0.354	0.053	0.354
0.8	0.348	0.106	0.348
0.7	0.342	0.159	0.342
0.6	0.336	0.212	0.336
0.5	0.330	0.265	0.330
0.4	0.324	0.318	0.324
0.3	0.318	0.371	0.318
0.2	0.312	0.424	0.312
0.1	0.306	0.477	0.306
0	0.3	0.53	0.3

So, how do we solve this problem; this problem we would solve in a similar manner, bundle tenacity = maximum of these 2, t_2 right, here this p_1/t_1 is given as 0.36 Newton per tex, p_2/t_2

is given as 0.53 Newton per tex and your s2 all right, a1 is given as 11%, a2 is given as 25%, so s2 a1/ t2 is given as 0.3 Newton per tex now, what we will see, let us consider this as capital A, this as capital B, we will show you the results of calculations.

Now, g1 then we will show you the value of Newton per tex B, Newton per tex max of A and B that is equal to this Newton per Tex, right so, 1, 0.9, 0.8, 0.7, 0.6, 0.5, 4, 3, 2, 1, 0, so this value will be 360, what is the maximum; 36, 0.9; it will be 0.354 and 0.053, what is maximum; 354, 0.8; it will be 0.348 and this will be 0.106, what is the maximum; 348, at 7, it will be 0.342, this will be 0.159, what will be maximum; 342.

At 0.6, it will be 0.336, this will be 0.212, what is maximum; 336, at 0.5, it will be 0.330, this will be 0.265, what is maximum; 330, at 0.4, this value will be g2 4 and this value will be 0.318, maximum is 324, at 0.3, this value will be 0.318 but this value will be 0.371 and what is maximum in between these 2; 371, at 0.2, this value will be 312, this value will be 424, at 0.1, this value will be 306, this value will be 477, this value will be 477.

And at 0, this value will be 0.3, this value will be 0.53, so this value will 0.53, what we see here is; it is continuously decreasing but from here 0.324 to 0.71 then again it is increasing, so the very similar behaviour initially decreasing, then it is increasing so, this was problem number 2, we stop here, in the next class we will continue with this module.