Theory of Yarn Structure
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Lecture – 22
Tensile Mechanics of Yarns (contd.,)

Welcome to this MOOCs online video course, theory of yarn structure. In the last 2 classes we discussed about module 8, tensile mechanics of yarns. We started with discussion on mechanics of parallel fiber bundle, there we discussed Hamburgers theory and also we solved 2 numerical problems. Today we are going to start with the third numerical problem on module 8.

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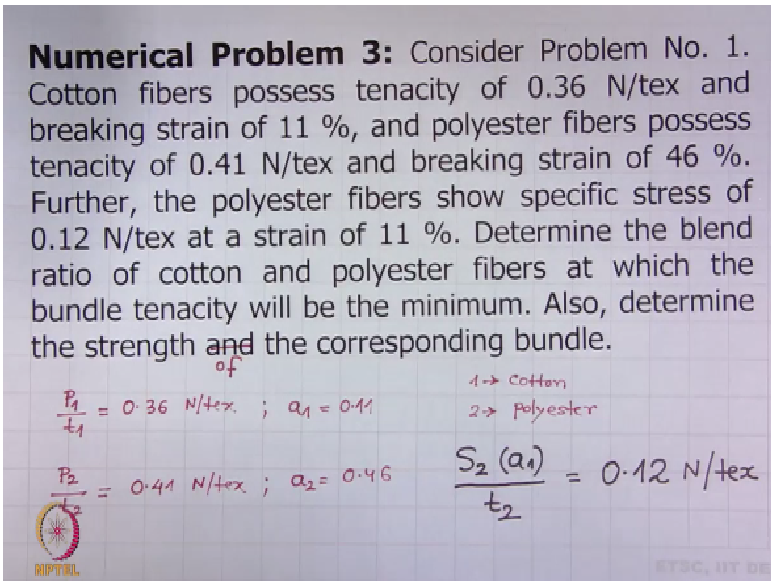

So this problem is basically an extension of problem number 1 of this module. You remember in problem number 1, this information was given, cotton fibers possess a tenacity of 0.36 Newton per tex, breaking strain 11%, polyester fibers possess tenacity of 0.41 Newton per tex and breaking strain 46%. Further the polyester fiber source specific stress of 0.12 Newton per tex at a strain of 11%.

Determine the blend ratio of cotton and polyester fibers at which the bundles tenacity will be the minimum. Also, determine the strength of the corresponding bundle. So what information are given let us summarize. P1/t1 = 0.36 Newton per tex is given. We consider 1 denotes cotton and 2 denotes polyester okay. Then another information is given P2/t2 = 0.41 Newton per tex, breaking elongation is also given here. Breaking elongation is also given here.

Another information which is given as S2 a1/t2 is given as 0.12 Newton per tex. So these all information are given, we have to determine the blend ratio at which the bundle tenacity will be minimum. This we have already derived in the last class. We will straightaway use the expression here.

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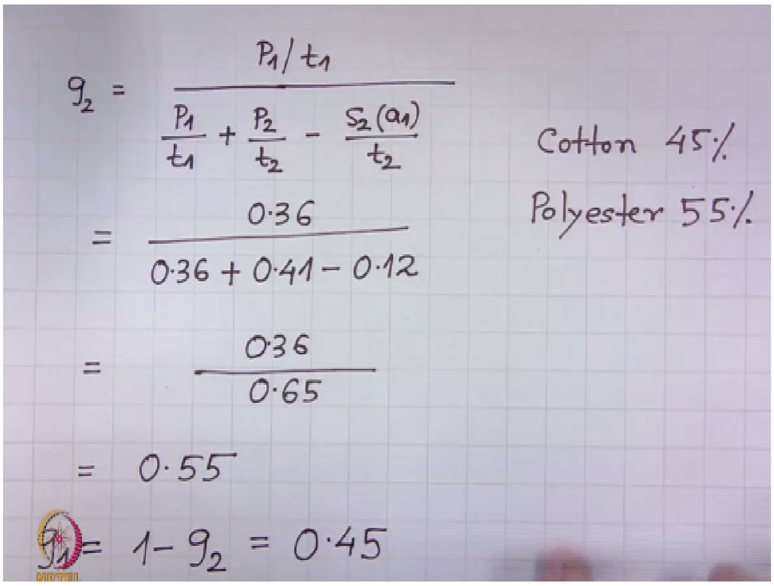

So the bundle tenacity will be minimum when g2 = P1/t1 / P1/t1 + P2/t2 – S2 a1/t2. So these data are given here. What is P1/t1? Is given as 0.36/0.36 + P2/t2 is given as 0.41 – S2 a1/t2 is given as 0.12. So this value will come as the ratio of 0.36 to 0.65 which is = 0.55 that means when, so g1 is 1-g2 0.45. When cotton fiber mass percentage is 45 and polyester mass percentage is 55, the resulting bundle gives the minimum strength.

So this was the first part of the problem. Now the second part is what will be tenacity of this bundle consists of 45% cotton fibers and 55% polyester fibers. So let us solve that part of the problem.

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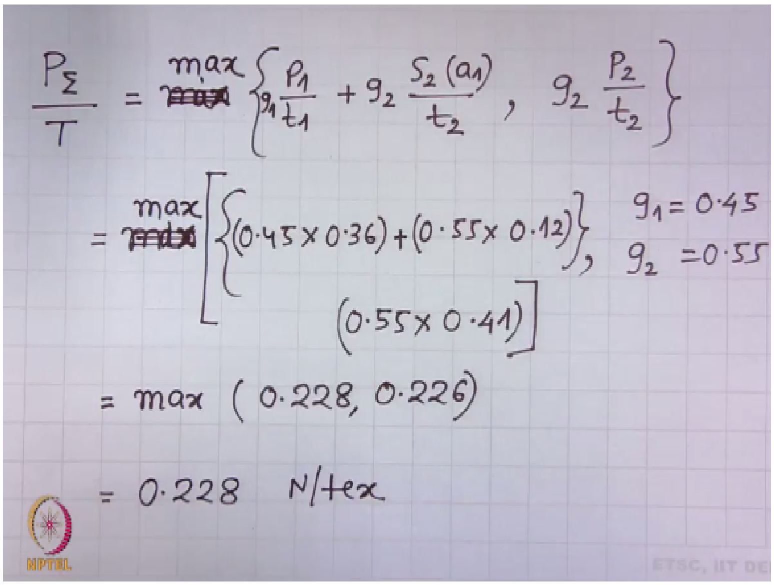

As we know bundle tenacity is the minimum of 2 values right. So here g1 is given as 0.45 and g2 is given 0.55, so minimum of 2 values, 0.45 * 0.36 + 0.55 * 0.12, this value, 0.55 * 0.41. So these, sorry it is maximum. Maximum of these 2 values show, these 2 values will come 0.228 and 0.226, which will be = 0.228 Newton per tex. So the bundle consisting of 45% cotton fibers and 55% polyester fibers will show minimum strength.

And what is the minimum strength? Minimum strength is 0.228 Newton per tex. So this is the answer of the second part of the problem. Now we will discuss another numerical problem, the fourth one, which is basically an extension of problem number 2.

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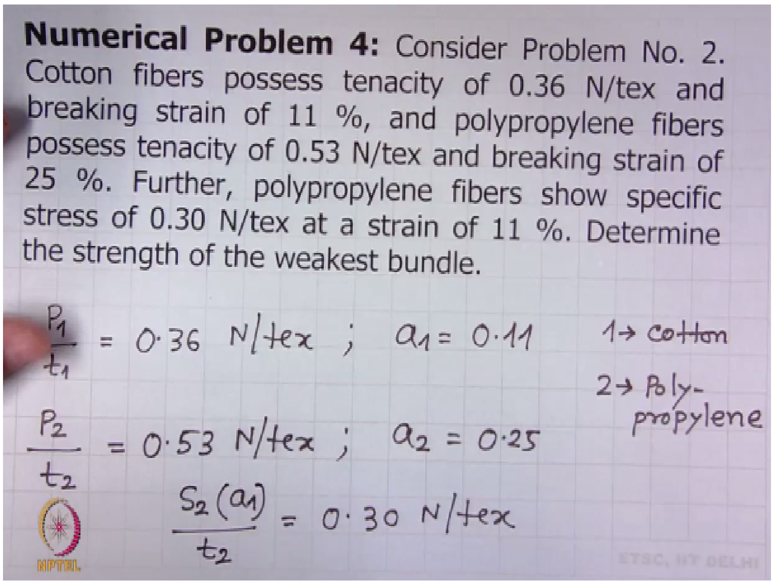

If you remember in problem number 2 in this module we calculated bundle tenacity, when cotton and polypropylene fibers of given data were blended at different proportions. In this particular problem what is asked is the minimum with the determine the strength of the weakest bundle. So in problem number 2 also these information were given. P1/t1 = 0.36 Newton per tex, then a1 0.11, same were given in problem number 2.

P2/t2 0.53 Newton per tex, a2 = 0.25, another information is given, S2 a1/t2 0.30 Newton per tex. Now we have to first find out what is the blend ratio at which bundle tenacity will be minimum. Then we have to determine the strength of the bundle at that particular blend ratio. So let us first determine what is the blend ratio at which bundle tenacity will be minimum. So in this particular problem subscript 1 denotes cotton and subscript 2 denotes polypropylene.

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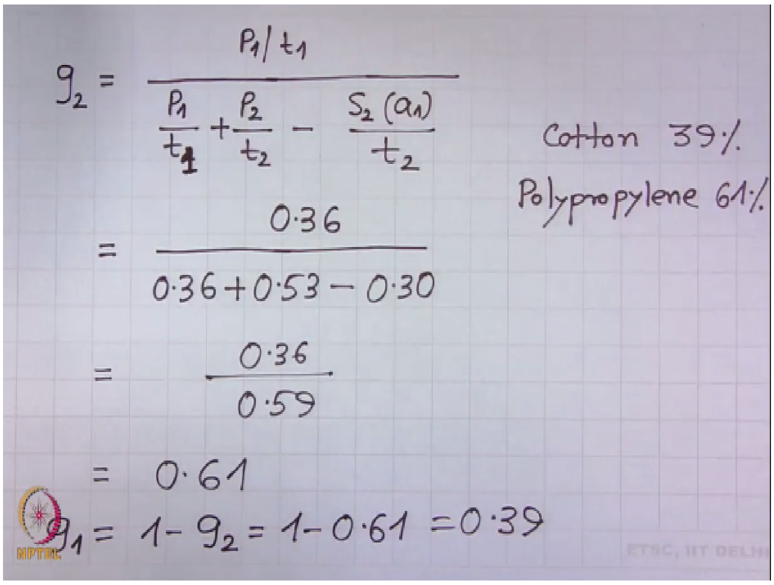

So the blend ratio at which bundle tenacity will be minimum is (()) (11:22) P1/t1 + P2/t2 – St a1/t1 is this. Now in this particular problem P1/t1 is given as 0.36, P2/t2 is given as 0.53 – S2 a1/t2 is given as 0.30 right. So what will be this value 0.36 / 0.59, so this will be = 0.61. Then what is g1? g1 is 1- g2. So 1 – 0.61. So 0.39, what does that mean? The blend ratio cotton 39%, polypropylene 61%.

So a bundle which consists of 39% cotton fibers and 61% polyester fibers gives the minimum tenacity. Then what is the minimum tenacity? So now we will solve that part of the problem.

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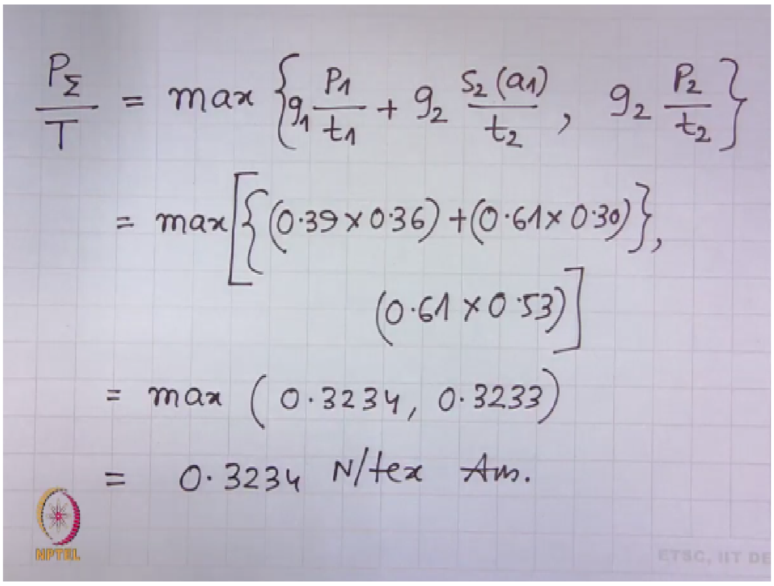

So minimum tenacity will be obtained in this manner right. So here what is your g1? 0.39. What is P1/t1? 0.36 + what is g2? 0.61. What is S2 a1/t2? 0.30, so this is one value. The next value is 0.53. So there will be 2 values 0.3234 and another will be Newton per tex is the answer. So the bundle which gives you minimum tenacity where the minimum tenacity is 0.3234 Newton per tex and that bundle consist of 39% cotton fibers and 61% polypropylene fibers.

So far we have solved 4 numerical problems. Now we will go to discuss the next part of this module that is related to stress-strain relation in yarn. So we will now start that part, stress-strain relation in yarn.

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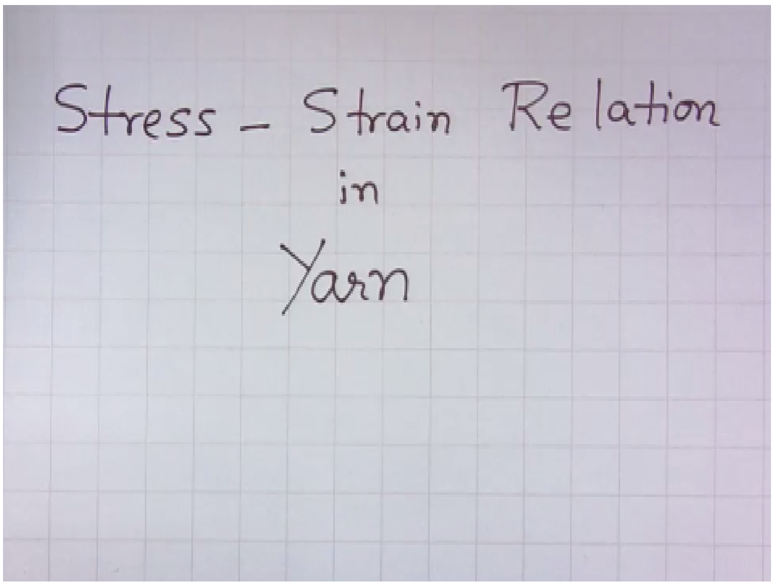

If you remember on the first class of this module we told that in this module we will be going to discuss 2 things, one is the tensile mechanics of parallel fiber bundle, second is the stress-strain relation in yarn. So far we have discussed the tensile mechanics of parallel fiber bundles, now we are going to discuss about stress-strain relation in yarn. So this part we will start with a diagram.

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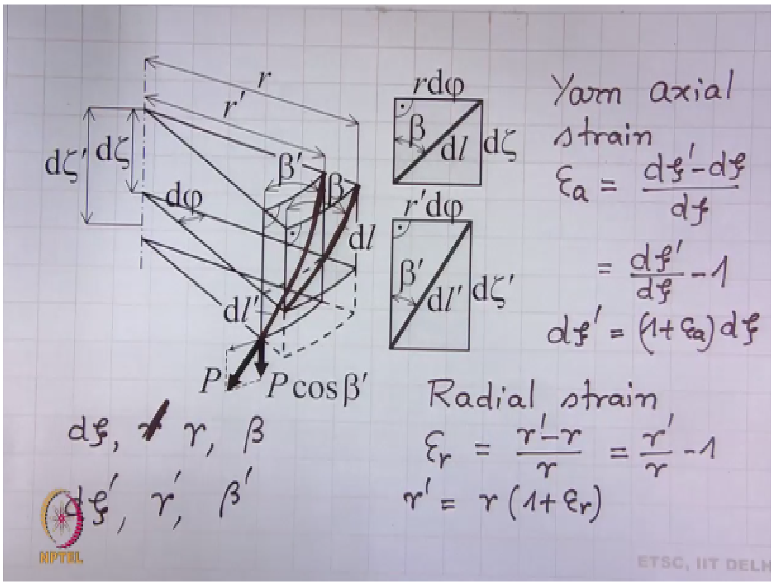

This diagram is very much related to the diagram that we discussed in the module of radial migration of fibers in yarn. What we see here is there is a general fiber element dl is its length, this element. So as we know from the module of radial fiber migration in yarn where we talked about general fiber path, fiber element, different angles. So this fiber element initial fiber element of length dl is characterized by 3 quantities.

One is d zeta, second is r, and the third is the angle beta. Now we are applying load force to the yarn as a result yarn elongates. So dl prime is this length, this part you are not seeing, dl prime is the length of the fiber element after elongation of the yarn. So the axial position is changing. So this is the increment along the axis of the yarn its radial position is changing and also its angle is changing.

So these 3 quantities determine the position of the fiber element after elongation of the yarn. Now if we see this top picture, this is the length dl, length of the fiber element, angle of the fiber element to the axis of yarn is beta. This width becomes r times d phi and this is d zeta. After elongation this fiber element will change its position. This change is shown here, the length becomes dl prime, angle becomes beta prime.

As a result the width becomes r prime d phi and the length axial distance becomes d zeta prime. So this is about this image, now we will define 3 quantities. One is yarn axial strain. Yarn axial strain, we use this symbol epsilon subscript a. This is the change in the axial distance / the original distance. So what is the change in the axial distance? d zeta prime – d zeta is the change in the axial distance / original distance.

Original distance is d zeta. So d zeta prime - d zeta / d zeta. So this is = -1. So we obtain 1 + epsilon a d zeta, right. Second is the radial strain. How do we define radial strain? Radial strain is the change in radius / original radius. We denote this by epsilon r, what is the change in radius, r prime - r is the change in radius / original radius. So this becomes r prime / r -1. So r prime is = r * 1 + epsilon r.

Then we define contraction ratio similar to Poisson ratio. What is Poisson zero? is the ratio of radial strain by axial strain.

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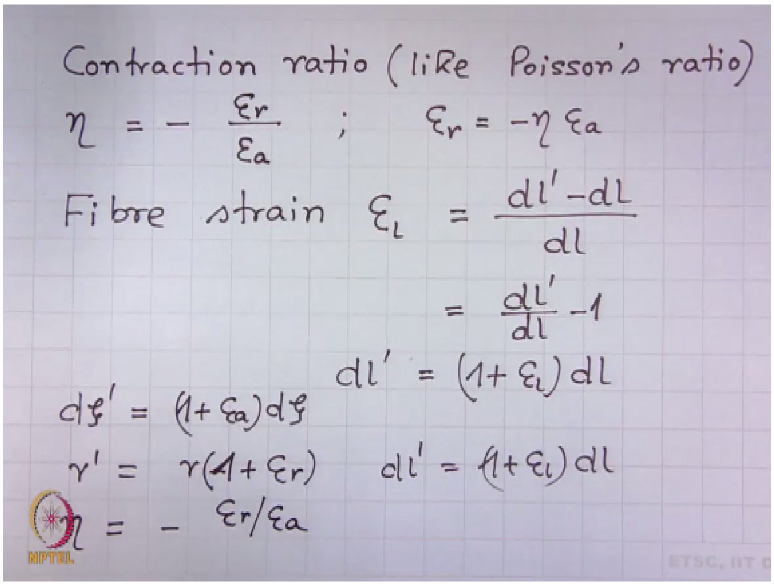

So we define contraction ratio is like Poisson's ratio, we use the symbol eta for that is = - radial strain / axial strain, this gives you – eta times this. The last quantity is fiber strain because of the application of force the fiber is also stressed, it also exhibits strain, so what is fiber strain? Fiber strain let us use this symbol epsilon L again it is defined by the ratio of change in fiber length / original fiber length.

What is change in fiber length? dl prime – dl is the change in fiber length / original length. So dl prime / dl -1. So dl prime is 1 + epsilon * dl. So if we summarize these 4 quantities, first one was d zeta prime is = this, second one was new radius 1 + epsilon r, third was contraction ratio and the fourth is this one right. Our next step if we come back to the original drawing our next step will be applying Pythagorean theorem to this triangle and also to this triangle.

So if we apply Pythagorean theorem let us see what happens, dl square is = r d phi square + d zeta square. So let us write down.

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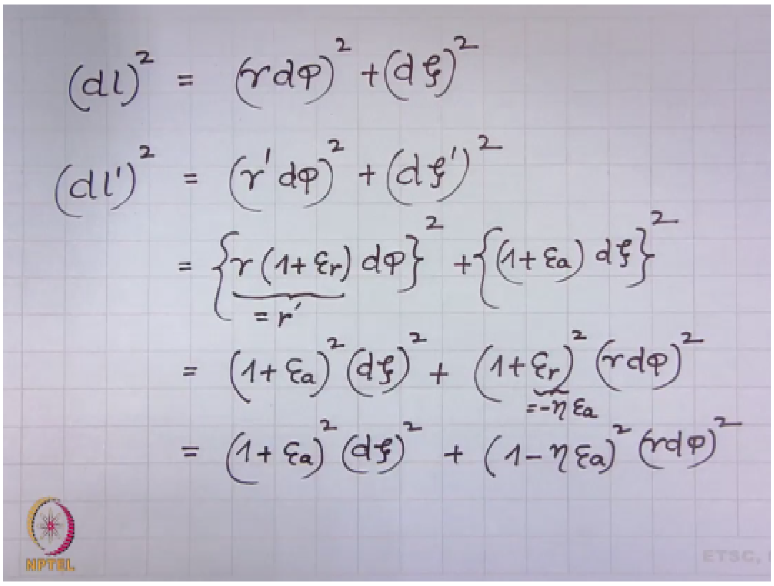

dl square is = r d phi square + d zeta square, similarly if we apply Pythagorean theorem to this triangle then we will see dl prime square = r prime d phi square + d zeta prime square. So let us write down square r prime d phi square + plus this. You see one more time, this square = this square + this square okay. Now we use our 4 basic relations, r prime was = r times 1 + epsilon r d phi square.

This was = r prime right and what is your d zeta prime, d zeta prime we have derived square. So if we rearrange it then it becomes 1 + epsilon a square, square + 1 + epsilon r square d phi square right. Further this epsilon r = - eta * epsilon a, definition of contraction ratio, so what we obtain is square square + 1 - square r d phi square right. We continue with derivation.

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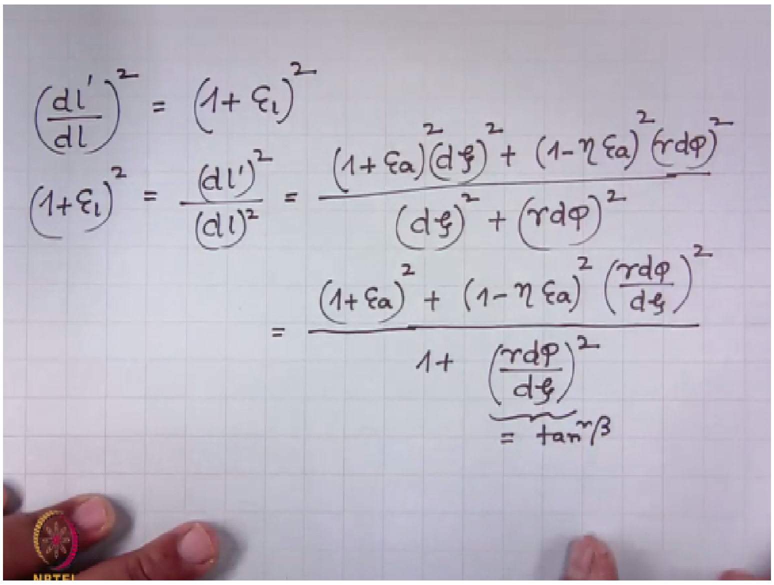

What is your dl prime/ dl square? dl prime/dl square was defined by 1 + this. So 1 + epsilon l square is this / this. We have just now derived dl prime square is this. So we write this in the numerator, 1 + epsilon a squared square + 1 – eta epsilon a square r d phi. In the denominator it is dL square. So by using Pythagorean theorem we obtain this relation, we will use this in the denominator. So we write this as square + r time d phi square.

Now we divide numerator as well as denominator by this quantity d zeta square. Let us see what we obtain, 1 + epsilon a square 1 – eta square r d phi/d zeta square 1 + r d phi / d zeta square. What is this quantity r d phi /d zeta? we come back to our diagram. Tangent beta, tangent of this = r times d phi/d zeta. So r times d phi / d zeta = tangent of beta. So this quantity is = tan square beta right. So we substitute this then what we obtain is.

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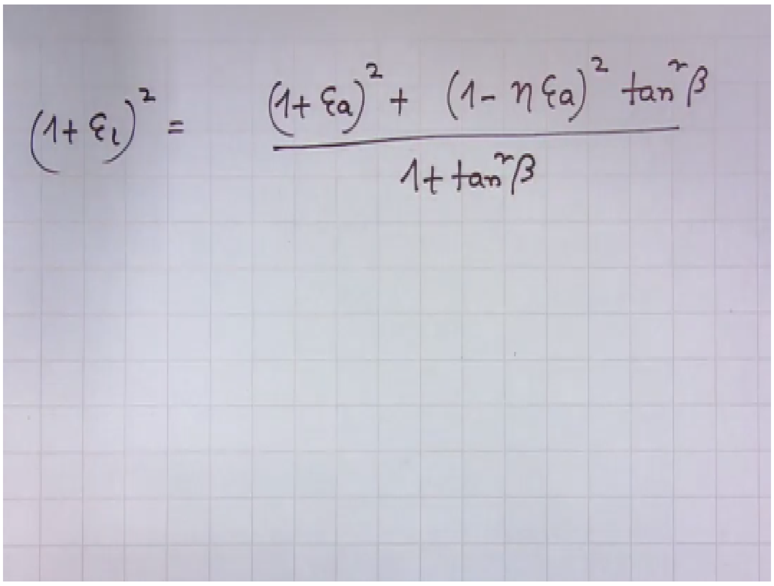

Square + 1 -. We stop here today. In the next class, we will continue from this expression. Thank you. Thank you very much for your attention.