

Theory of Yarn Structure
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Lecture – 24
Tensile Mechanics of Yarns (contd.,)

Welcome to this MOOCs online video course, theory of yarn structure. In the last few classes we discussed about module 8 related to tensile mechanics of yarns. In the last class we solved the problem of yarn stress and fiber stress and also yarn strain and fiber strain. Also we have started solving numerical problem 5 and we solved for different twist angles beta.

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Numerical Problem 5: Show how the ratio of fiber strain to yarn strain changes from the center (core) to the surface (sheath) of a yarn for different values of angle of twist ($10^\circ, 20^\circ, 30^\circ, 40^\circ$) and for different values of contraction ratio (0, 0.25, 0.50, 0.75).

$$\frac{\epsilon_l}{\epsilon_a} = \cos^2 \beta - \eta \sin^2 \beta$$

$\beta [^\circ]$	$\eta = 0$	$\eta = 0.25$	$\eta = 0.50$	$\eta = 0.75$
10	0.9698 ✓	0.9623	0.9547	0.947
20	0.8830 ✓	0.8538	0.8245	0.795
30	0.7500 ✓	0.6875	0.6250	0.56
40	0.5868 ✓	0.4835	0.3802	0.27

If we assume there is no radial contraction of yarn what will be the ratio of fiber strain to yarn strain. So we solved this column. Today we will continue with solving for the rest of the columns. Let us start with contraction ratio 0.25. So if in this expression $\eta = 0.25$, when $\beta = 10$ degree, so the value of this ratio ϵ_l / ϵ_a will be 0.9623. Similarly, when $\beta = 20$ degree.

ϵ_l / ϵ_a will be 0.8538, similarly for $\beta = 30$ this value will be further less 0.6875, when $\beta = 40$ then this value is remarkably less 0.4835. In the similar manner we can solve what is this ratio when contraction ratio is 0.5 and there are 4 different angles of β . This answers will be 0.9547 then for 20 degree β , the answer will be 0.8245, for 30 degree the answer will be 0.6250 and for 40 degree the answer will be 0.3802.

In a similar manner we can solve when contraction ratio is 0.75, what is the ratio of fiber strength to yarn strength when beta = 10. This value will be 0.947, for 20 this value will be 0.795, for 30 this value will be 0.56 and for 40 this value will be approximately 0.27. Now if we carefully observe this table, if we go along this direction for a given contraction ratio when the surface twist angle, when the twist angle increases this ratio fiber strength to yarn strength decreases.

The same trend is observed here, here also, and here also. Now if we go along this way, for a given twist angle, when contraction increases the ratio of fiber strain to yarn strain decreases; however, when beta is less, this effect is less. When beta is little high as high as 20 degree then we see as the contraction ratio increases, same trend is here; however, the effect is little higher. This effect you see when beta is 40, if we increase contraction ratio the values are changing from say 0.59 to 0.48 to 0.32 to 0.27, remarkable change.

So this can also be observed from this data. So we have solved numerical problem 5. Now we are going to solve numerical problem 6 of this module.

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Numerical Problem 6: A cotton carded ring spun yarn of 29.5 tex count and 719.43 m⁻¹ twist is prepared. This yarn is tested for its stress-strain behavior. Calculate tensile force utilization coefficient of this yarn, assuming yarn contraction is equal to 0, 0.25, 0.50, and 0.75, respectively.

$$T = 29.5 \text{ tex} ; Z = 719.43 \text{ m}^{-1}$$

$$\varphi = (1 + \eta) \cos^2 \beta_D + \eta \frac{\ln |\cos \beta_D|}{\tan^2 \beta_D}$$

$$\tan \beta_D = \pi D Z$$

$$= \pi \sqrt{\frac{4T}{\pi \mu \rho}} \quad Z = 3.14 \times \sqrt{\frac{4 \times 29.5}{3.14 \times 0.46 \times 1.520}} \times 719.43$$

$$= 0.5237$$


This problem reads as follows, a cotton carded ring spun yarn of 29.5 tex count and 719.43 meter inverse twist. So you have given T = 29.5 tex and Z is also given 719.43 meter inverse. This yarn is tested for its stress-strain behavior. Calculate tensile force utilization coefficient of this yarn assuming yarn contraction is = 0, 0.25, 0.50, 0.75 respectively. So we have to basically use this expression phi tensile force utilization coefficient of yarn 1 + eta cos square beta D + eta, tan square beta D.

Eta given 0, 0.25, 0.50, 0.75, what is not given is beta D. Now tangent of beta D = pi times D times Z. Z is given, D is not given, what is D? D as we know root over 4 times T/pi mu rho * Z. T is given, rho is the cotton fiber, so rho we can consider 1.52 gram per centimeter cube, mu is not given. How to find out mu? mu we can find out in a manner we learnt earlier. Let me show you how.

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$$\frac{\mu^{2.5}}{\left\{1 - \left(\frac{\mu}{0.8}\right)^3\right\}^3} = Q Z^2 T^{1/2} \quad (\text{Module 4})$$

$$\frac{\mu^{2.5}}{\left\{1 - \left(\frac{\mu}{0.8}\right)^3\right\}^3} = (9.61 \times 10^{-8}) (719.43)^2 (29.5)^{1/2}$$

$$\mu = 0.46$$

$$\tan \beta_D = 0.5237$$

$$\beta_D = 27.65 \text{ degree}$$

mu to the power 2.5/1 – mu to the power 0.8 cube, Q Z square T to the power 1/2. Probably we learn this in module 4. So it is cotton coordinating spun yarn 9.61 * 10 to the power -8 is the value of Q. What is the value of Z, Z is given 719.43 and what is the value of T? 29.5 to the power 1/2. So then you can solve for this expression, you will find out mu will be = 0.46.

The moment you find out 0.46 mu then you will come back to this page and you will see what is given, 3.14, 4 times T, what is T? 29.5, 3.14 mu is 0.46 and rho is 1520 kg per meter cube * Z 43. So this tangent beta you will get as 0.5237 of course it is dimensionless. So tangent of beta D = 0.5237 then beta D is changed for this value will be coming approximately = 27.65 degree.


So in this manner you obtain beta D. Now you come back to this expression, beta D you obtain 27.65 degree, eta for different values you need to calculate phi.

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$$\varphi = (1 + \eta) \cos^2 \beta_D + \eta \frac{|\ln |\cos \beta_D||}{\tan^2 \beta_D}$$

$$\beta_D = 27.65^\circ$$

η	φ
0	0.7864
0.25	0.7597
0.50	0.7278
0.75	0.7098



So φ is $= 1 + \eta \cos^2 \beta_D + \eta \frac{|\ln |\cos \beta_D||}{\tan^2 \beta_D}$. β_D you obtain as 27.65 degree. So you need to find out this. For η 0, second is 0.25, third is 0.50 and fourth is 0.75. So you need to find out 4 values of φ , tensile force utilization coefficient in yarn. So when η is 0, so these vanishes, this 0, so $\cos^2 \beta_D$, $\cos^2 27.65$ degree, the value will be 0.7864.

So $\cos^2 27.65$ degree $=$ roughly 0.7864, when contraction ratio $=$ 0.25, $\eta =$ 0.25, so you substitute 0.25 here, 0.25 here and β_D 27.65 degree you calculate, you will get this value will be approximately $=$ 7597. Now third one, η contraction ratio $=$ 0.50, so you substitute $\eta =$ 0.50 and consider $\beta_D =$ 27.65 degree, $\beta_D =$ 27.65 degree. You solve for it, you will find out these value will be roughly $=$ 0.7278.

The last one when the contraction ratio is 0.75 and β_D 27.65 degree. So you substitute contraction ratio as 0.75 here and you substitute β_D as 27.65 degree, you will find out this value will be $=$ 0.7098. What do we see is that for a given yarn where β_D is fixed constant 27.65 degree, for contraction ratio increases tensile force utilization coefficient decreases. This we see from this numerical problem. So we solved numerical problem 6 also. So module 8 ends here. Thank you very much for your attention.