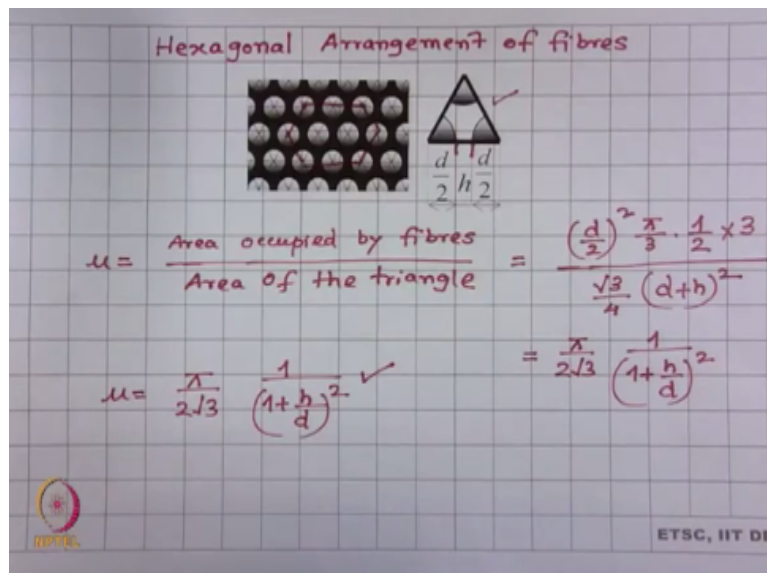


**Theory of Yarn Structure**  
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**Lecture – 04**  
**Basic Characteristics of Yarns (contd.,)**

Welcome to you all today, we will continue with our last class on module 2, basic characteristics of yarns, we will start with packing density. In the last class, we started with the definition of packing density, if you remember well packing density is defined by the ratio of volume of fibers occupied in the yarn to the volume of the yarn itself.

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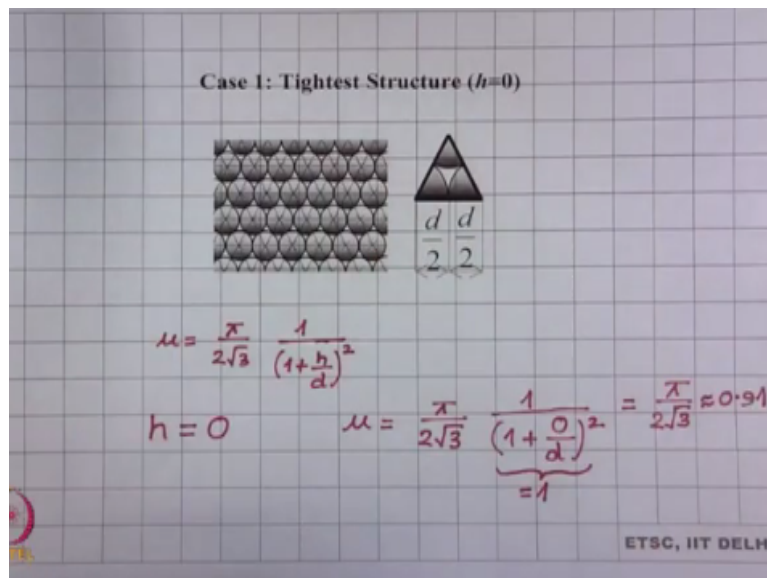
Now, it is possible also to interpret packing density in Area terms that is the ratio of area occupied by fibers to the area of the yarn now, packing density is a dimensionless quantity theoretically, it varies from 0 to 1, suppose if we say packing density of this air is 0.5, what do we interpret; we interpret that 50% of the volume of this yarn is occupied by fiber and the rest 50% is occupied by the air, this is correct.

Can we infer more information about the yarn structure? The fiber packing arrangement inside the yarn structure is characterized by packing density that means, packing density gives certain ideas about the packing arrangement fiber; packing arrangement inside the yarn structure, so how it gives certain arrangement about the fibers in the yarn structure that we will discuss today. Let us take the model which is often used in case of yarn, hexagonal packing model.

So, in this model fibers are packed in an hexagonal manner, you can see this hexagon now, we need to find out packing density of this model structure now, this model structure is a repetition of one unit cell that unit cell is shown here, it is basically a triangle, in this triangle, free spaces there and also some areas are occupied by fibers. Now, how much is the area occupied by the fibers?

So, packing density is defined by area occupied by fibers to area of the triangle, then in yesterday's class itself, we derived this expression;  $\mu$  is the packing density  $1$  divided by  $1 + h/d$  whole square, where  $h$  is the distance between 2 fibers and  $d$  stands for fiber diameter. Now, we will study 4 different variants of this model, we will use this equation, is a generalized equation to find out packing density of those 4 different variants.

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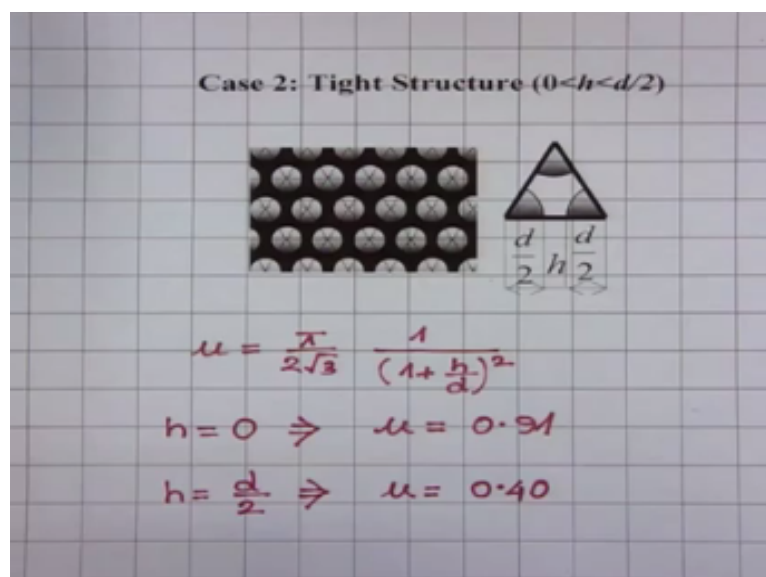
Let us start with the first variant so, first case; tightest structure, how we define tightest structure? The structure where distance between 5 bars = 0 that means, fibers are touching each other. So here you see in this image, the fibers are touching each other, very little white spaces you can see that is basically air, occupied by air. Now, we need to find out packing density of this structure.

So, what is the packing density of the structure; very similar unit cell is shown here, there is the fibers are touching each other, so  $h = 0$  here then, what is the packing density of this structure, the packing density of this unit cell? In general, for hexagonal packing arrangement, this relationship is valid, let us put  $h = 0$  here, so what we will obtain; we will obtain  $\mu = \pi/2 \text{ root } 3$   $1$  divided by  $1 + 0/d$  whole square.

So, this term is 0 and  $1 + 0$  is 1, 1 divided by 1 is 1 that means this whole term becomes equal to 1, so this becomes  $\frac{\pi}{2\sqrt{3}}$ , this is roughly = 0.91, so what we infer is that in case of tightest structure according to hexagonal fiber packing arrangement, the packing density = 0.91 that means 91% of the volume or 91% of the area is occupied by fibers, rest 9% is occupied by air only.

Now, what will be the behaviour of this structure; no doubt this structure will be mechanically very strong but this structure is less porous that means, fluid transmission properties air permeability for example, will be very less for this kind of structure, so this kind of structure is mechanically enough stable. However, it is not very soft, it is not very porous and its fluid transmission properties will be less.

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So, this was the case 1, now we will proceed to another case that is case 2 that this is your case 2, we call it as tight structure or compact structure. How we define this structure? In this structure, the distance between fibers is small, it is  $\neq 0$  however, it is small, how much small; it varies from 0 to  $1/2$  of the diameter of fiber so, 0 to  $d/2$ , so scheme of this structure is shown here, you can see little gap is there between fibers, unit cell of this structure is shown here where there is a value of  $h$ , which is very small.

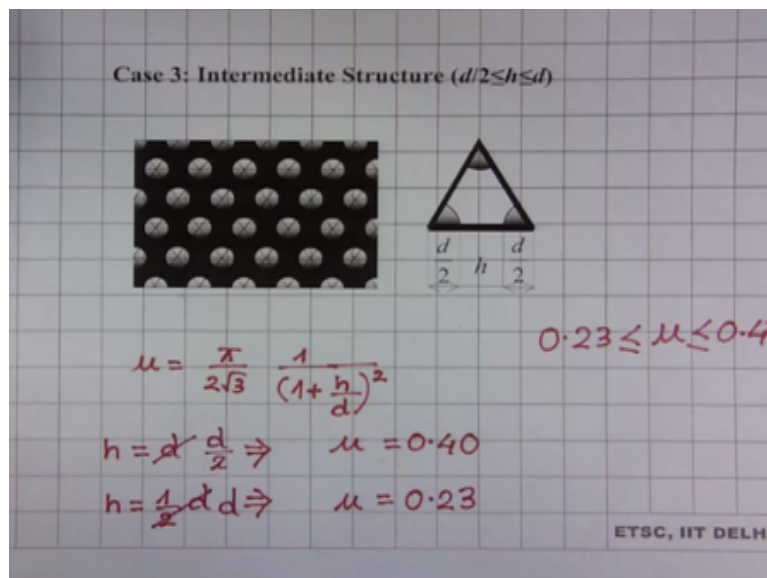
Now, what is the packing density of this structure?  $\mu$  for general hexagonal structure, this relationship we have already derived if we put  $h = 0$  here, we obtain  $\mu = 0.91$ , just now we have seen. If we put  $h = d/2$  here, what will be the value of  $\mu$ ; you can try this value will be

roughly = 0.4 that means, the packing density of this structure will be somewhere ranging from 0.4 to 0.91.

What will be the behaviour of the structure, this structure is a tight or compact structure, then the little space available in between fibers, so this structure will be mechanically strong at the same time, it will be porous, it will be soft so, fluid transmission it will have quite okay fluid transmission properties, so this will be the behaviour of tight structure. Now, to give you a note you remember what was the packing density of; typical packing density of yarn?

It is; it varies from say 0.38 to 0.55 that means our yarn is very similar to this kind of structure; tight structure, it is mechanically strong at the same time, it is quite soft white porous and exhibit quite okay fluid transmission properties, so that is the beauty of yarn structure so, now we will proceed to the third case that is intermediate structure case 3.

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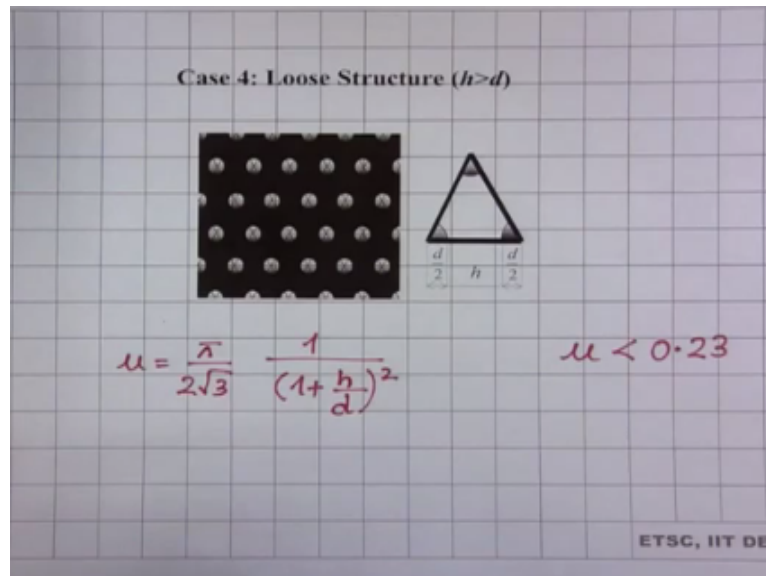
How we define intermediate structure? In intermediate structure, the distance between fibers is higher, then that in case of tight structure so, you can see here fibers are more spaced as compared to earlier tight structure, so the unit cell is shown here first, what is the packing density of the structure? The packing density of this structure can again be found out from our known relation if you put  $h = d$ , we will obtain  $\mu$  is equal to roughly = 0.40.

If you put  $h = d/2$ , half of diameter then you will find out  $\mu = 0.23$  that means, for this structure, packing density will vary in this range 0.23 to 0.4 well, there is one small mistake is here, in place of  $d$ , it will be  $d/2$  and here in place of  $d/2$ , it will be  $d$ , so when  $h = d/2$ , you will

get  $\mu = 0.4$ , when  $h = d$ , you will get 0.23, so in case of intermediate structure, packing density ranges from 0.23 to 0.4, how will be the behaviour of the structure?

This structure will be quite soft quite, white porous, it will have good fluid transmission properties but this structure will not be mechanically as strong as earlier structures, so this is intermediate structure.

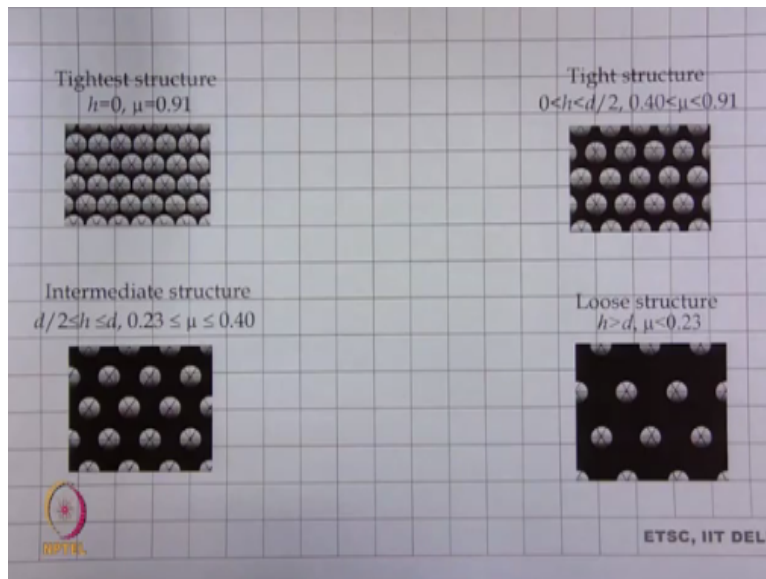
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Now, we will proceed to the last variant that is loose structure, scheme of the loose structure is shown here. Here, the distance between fibers is too large; you can visualize it, what is the packing density of this structure? This is the general formula for packing density of hexagonal fiber packing arrangement, if you put this condition then you will find out packing density of the structure will be  $< 0.23$ .

So that means  $< 23\%$  of the area is occupied by fibers, more than 77% of the area is occupied by air that means, this structure will exhibit very good fluid transmission properties, air permeability for example and it is very soft, very much porous structure but from mechanical point of view, it is not as strong as our earlier structures, just to tell you that in textiles, non-woven materials generally exhibit packing density in this range.

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So that is why non-woven materials are known to be very porous, very soft kind of materials, so if we now summarize these 4 structures what we see is that the first one is the tightest structure, distance between fiber is 0, packing density will be roughly = 0.91, tight structure where distance will be vary from 0 to 1/2 of the diameter accordingly packing density will vary from 0.40 to 0.91.

Most of our yarns generally fall in this packing density range, intermediate structure; the distance between fiber ranges from 1/2 of the diameter to diameter accordingly packing density ranges from 0.23 to 0.40, for loose structure distance between fiber is much greater than fiber diameter itself accordingly, packing density will be very low  $< 0.23$ , so this completes our discussion on packing density.

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Yarn diameter

$$D = \sqrt{\frac{4S}{\pi\mu}} = \sqrt{\frac{4T}{\pi\mu\rho}}$$

$$D = \sqrt{\frac{4S}{\pi\mu}} = \frac{D_s}{\sqrt{\mu}}$$

$$D = \sqrt{\frac{4T}{\pi\mu\rho}} = \sqrt{\frac{4}{\pi\mu\rho}} \sqrt{T} = \frac{2}{\sqrt{\pi\mu\rho}} \sqrt{T} = \frac{2}{K} \sqrt{T}$$

$$D = K \sqrt{T}$$

$K$  ... coefficient of yarn diameter

Logos for NPTEL and ETSC, IIT DELHI are visible at the bottom.

Now, we will proceed to yarn diameter, a very important characteristics of yarn; yarn diameter. How we find out yarn diameter, no; in the last class we have already derived yarn diameter =  $4S / \pi \text{ times } \mu$ , now what is S; S is; D is yarn diameter, S is substance cross sectional area of yarn,  $\mu$  is packing density of fibers in yarn and also in the last class, we derived this  $S = \text{capital } T / \rho$ .

So, if we substitute that for  $4T / \pi \mu \rho$ , so T is a yarn fineness in direct count,  $\rho$  is a fiber density, so yarn diameter =  $\sqrt{4 * T / \pi * \mu * \rho}$ , you can see that capital T, yarn fineness is easy to measure, fiber density is also possible to measure,  $\mu$ ; packing density is also possible to measure that means, all 3 quantities on the right hand side are possible to measure experimentally accordingly, D can be determined, right.

Now, so again if we write this into this, so then we can write this equal to  $D_s / \mu$ , what is  $D_s$ ; in the last class if you remember  $D_s$  stands for substance diameter, so we talked about 2 diameter; one is yarn diameter, second is substance yarn diameter, how they are related? Yarn diameter = substance diameter root over packing density, so if we know packing density, if we know yarn diameter we can find out substance diameter.

Now, let us rewrite our expression of yarn diameter, T, this let us say K, then we can write  $D = K \sqrt{T}$ , what is K? K is called coefficient of yarn diameter, now let us point out one interesting fact.

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$D = K\sqrt{T} ; K = \sqrt{\frac{4}{\pi \mu \rho}}$   
 K depends on  $\mu$  and  $\rho$ .  
 $D_{[inch]} = \frac{1}{28\sqrt{T_{[Ne]}}}$   
 $D_s = \sqrt{\frac{4S}{\pi}} = \underbrace{\sqrt{\frac{4}{\pi}}}_{=K_s} \sqrt{S} = K_s \sqrt{S}$   
 $K_s \dots$  coefficient of substance yarn diameter

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So what we learnt,  $D = K \cdot \sqrt{T}$ , what is K; K is coefficient of yarn diameter which is  $\sqrt{\frac{4}{\pi \mu \rho}}$ , K depends on  $\mu$  and  $\rho$  suppose, you produce yarn, ring spun cotton yarn of different say, counts, do you think packing density will be same; packing density will not be same that means, a group of that means all ring spun cotton yarns do not have same value of K then you remember this expression which is very popularly used in our branch, cannot be applied for all yarns even if they are prepared by using same fibers.

Because packing density is not constant as packing density is variable, this relationship is not true for all yarns, even if they are prepared by using same fibers, this is a very popular error that we commit right, now we go back to our relationship of substantial diameter  $D_s$  is  $\sqrt{\frac{4S}{\pi}}$ , right \* S, okay so this is a constant let us say  $K_s$ , then we can write this expression is  $K_s \sqrt{S}$ , this  $K_s$ , okay then what is  $K_s$ ?

$K_s$ , we will call coefficient of substance yarn diameter, right if you read this equation  $D_s$  substantial diameter =  $K_s \sqrt{S}$ ; capital S is substance cross sectional area of yarn and what is  $K_s$ ,  $K_s$  is coefficient of substance yarn diameter. What do you observe? Generally, in practice we work with K not with  $K_s$ , right but what is interesting in this expression is that  $K_s$  is dimensionless why; simple.

Because if S is a millimeter square area,  $D_s$  diameter is millimetre, this is square root, so this millimeter and millimeter will cancel, so  $K_s$  will become dimensionless generally, speaking these dimensionless quantities are very, very important they give lot of internal information about yarn structure, so for theoretical research work, scientists prefer to work with  $K_s$  because it is dimensionless quantity, it gives you lot of information.

There are many dimensionless quantities probably, we have learnt already and we are going to learn many in the subsequent modules, one dimensionless quantity is packing density, right, it is a ratio of volume to volume and by now, you know the importance of packing density, it talks a lot about the internal structure of yarn, right. So, we have learned about yarn diameter, substance yarn diameter and also the coefficient of yarn diameter, capital K and coefficient of substance yarn diameter capital K subscript s, okay.

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Yarn Twist

$$Z = \frac{N_c}{L}$$

inch<sup>-1</sup>, m<sup>-1</sup> ... units of Z

Twist Intensity

$$\kappa = \pi D Z$$

The; yes, now we will proceed to yarn twist, another important characteristics of yarn, yarn twist as known, the traditional staple yarns are strengthened by means of twisting although too much twist is not good from mechanical point of view of yarn now, this twist is characterized by number of coils inserted in a given length of yarn divided by length of the yarn, so Z we denote yarn twist,  $N_c$ ; number of coils inserted, L; length of the yarn.

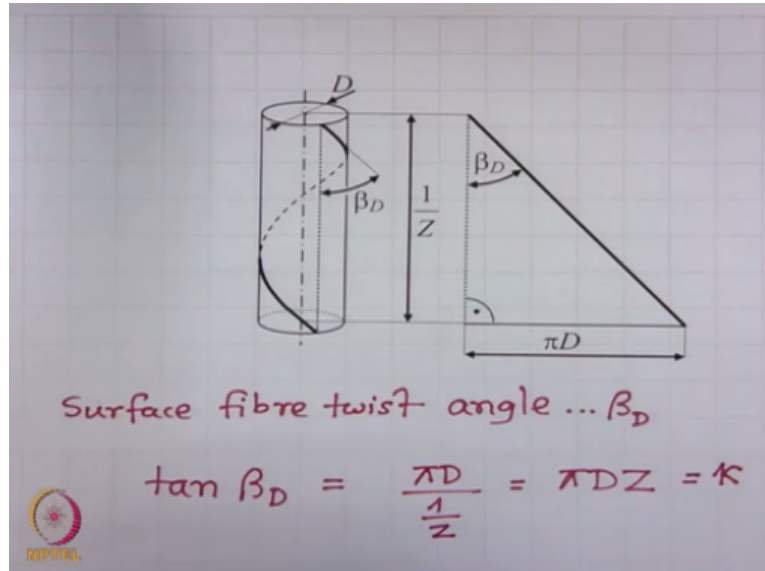
So, how many coils are inserted in a given length of yarn that is basically untwist,  $N_c$  is a number, dimensionless, L the length quantity, it has the length dimension that is why this yarn twist will have inverse length dimension, you will see sometimes inch universe or meter inverse like that this will be the units of Z, right. Now, we will talk about another interesting character of a yarn, twist intensity.

Look at this quantity, capital D multiplied by capital Z, what is capital D; yarn diameter, what is capital Z, yarn twist. What is interesting about this quantity? This quantity is dimensionless, right if we multiply this quantity by pi, we obtained twist intensity, so twist intensity is characterized by symbol Kappa = pi times D times Z, so twist intensity = pi DZ. Now, there is a very interesting physical meaning of this parameter, twist intensity we will come to that.

But before that look at the dimension as I told you probably, in the first class that in the course of theory of yarn structure, you should always look at the dimensions of the quantities; if you find out any dimension any structure, any dimensionless quantity thinks about it, what does it say? Generally speaking, dimensionless quantities are very important to develop to think about yarn more deeply.

Here again we come across with another dimensionless quantity that is twist intensity now, what is the meaning of twist intensity? Let us learn about the meaning of twist intensity.

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What you see in this image is that a cylindrical yarn of diameter  $D$ , capital  $D$  then you see a fiber this on the surface, so you see it then it goes inside, you do not see it by dotted line and then again it comes to the surface you see it so the thick bold line you see, dotted line you do not see again, thick line you see, fiber starts from surface goes inside then it comes back to the surface, so this is one coil.

So, what is the length? Length is  $1/Z$ ,  $Z$  is yarn twist right, all right, there is one more information that is this surface fiber makes an angle  $\beta$  times  $D$  from the yarn axis, so surface fiber twist angle is  $\beta D$ , please note that  $\beta D$  is not the twist angle of all fibers, it is the twist angle of one surface fiber right. Now, imaginatively let us unroll this cylinder along its axis.

So, you unroll and you obtain this image; a triangle, this angle is  $\beta D$ , this length is  $\pi$  times  $D$ , let us find out what is this angle and this height is  $1/Z$ , so tangent of  $\beta D$ , tangent of this angle, this divided by this, so  $\pi D$  divided by  $1/Z$ , so  $\pi D Z$ , what is  $\pi D Z$ ; twist intensity just now we learnt,  $\kappa$  so, what we see that twist intensity = tangent of the surface fiber twist angle, so this is the physical meaning of twist intensity.

Twist intensity means tangent of the twist angle that a peripheral fiber makes from yarn axis later on you will see that this variable  $\pi DZ$  is very important from physical behaviour point of view of yarn, right.

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Twist multiplier  
or  
Twist coefficient

$$\alpha = Z T^q$$

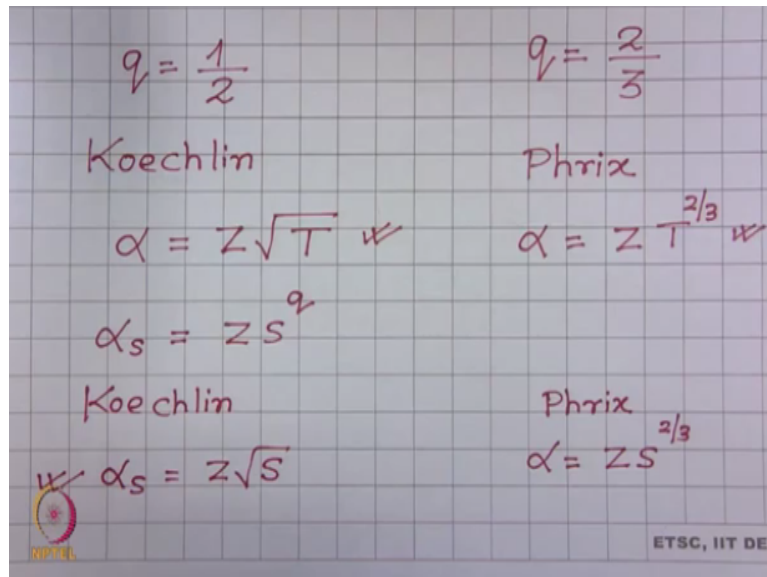
$\alpha$ ... twist multiplier/twist coefficient  
 $Z$ ... yarn twist  
 $T$ ... yarn fineness  
 $q$ ... exponent

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So, now we will come to our last characteristic of yarn that is twist multiplier or twist coefficient sometimes, we use this word, this phrase twist multiplier or sometimes we use this phrase twist coefficient, they mean same now, what is twist multiplier or twist coefficient? Alpha is  $Z T$  to the power  $q$ , later on we will speak in some of the models, we will speak about this relation.

This alpha is twist multiplier or twist coefficient,  $Z$  is yarn twist,  $T$  is yarn fineness in direct count, we have defined throughout this model yarn fineness in direct count, so even if I tell or I do not tell capital  $T$  denotes mass per unit length of yarn,  $q$  is important, is a coefficient is an exponent right now, this exponent has different numerical values.

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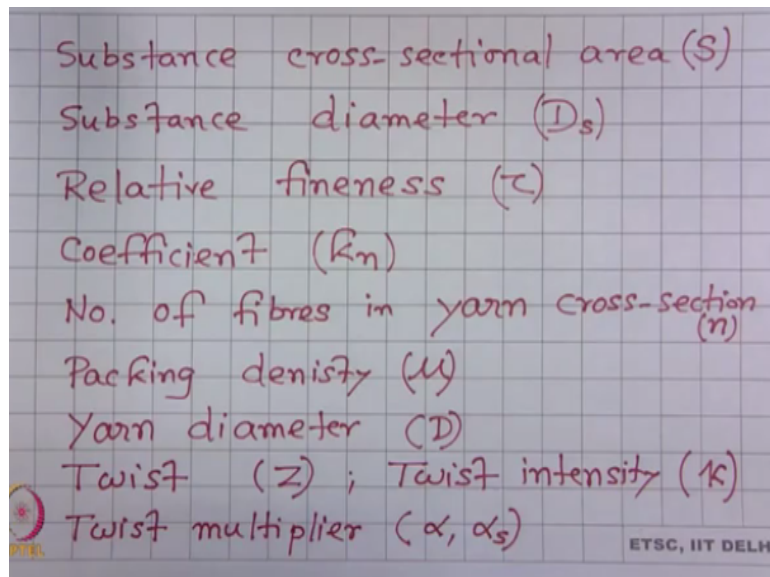
In India and probably in major parts of the world, we use this exponent  $q = 1/2$  for the first time there was one scientist whose name was Koechlin, he for probably for the first time thought  $q = 1/2$  why he thought about this that we will discuss in module 3, next module, so that is why when we write  $\alpha = Z\sqrt{T}$ , we talked about Koechlin's twist coefficient, right but in some parts of the world especially, Eastern European countries,  $q$  is not taken as  $1/2$ ,  $q$  is taken as  $2/3$ .

So that is often called as Phrix coefficient of twist multiplier in that case we obtain  $ZT$  to the power  $2/3$ , in India all industries, all universities this relationship is generally taught however, if you visit Eastern European countries, if you visit some of the spinning companies, you will see they use some different twist multiplier, do not get surprised, there are many different values of this exponent  $q$  are available in the world, in third module we will speak more about them.

So, it is not only  $1/2$  it is not only  $2/3$ , there are many more so similarly, if we use substance cross sectional area, we can find  $\alpha_s$  will be  $= Z * S$  to the power  $q$ , right and if we use Koechlin then we will use  $\alpha_s = Z\sqrt{S}$ , when we consider Phrix coefficient then we will write  $\alpha = ZS$  to the power  $2/3$ , as I told you sometimes in theoretical work, we often work with these expressions.

But probably for practical purposes in industrial practice, we use  $\alpha$  but for theoretical work many times we prefer to use  $\alpha_s$ , so that is why it is probably better for you to know about  $\alpha$  as well as  $\alpha_s$ , right. So, I think all the characteristics of; all the basic characteristics of yarn are now known to you.

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We summarize, we talked about substance cross sectional area capital  $S$  in the last class then, we talked about substance diameter of yarn, we defined it in the last class and also we continued in today's class then, in the last class we talked about third important characteristics of yarn relative fineness, we use this symbol to denote relative fineness, then we talked about very important coefficient; coefficient  $k_n$  which generally talks about orientation of fibers inside the yarn structure.

Then we spoke about number of fibers in yarn cross section then today, we discussed about packing density  $\mu$ , here we used  $n$  then today, we talked about yarn diameter and also partly talked about substance diameter again then, we talked about yarn twist we use the symbol capital  $Z$  for that then, we talked about twist intensity  $\pi$  times  $T$  times  $Z$ , we use symbol  $K$  for that.

Then at last, we talked about twist multiplier or twist coefficient, we use  $\alpha$ ,  $\alpha_s$ , right so these are the basic characteristics of yarn that we have discussed so far now, what we will do; we will solve a few numerical problems on this module, so that you will have further better understanding of this module. So, we will start with our first numerical problem.

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# Numerical Problems on Module 2

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Numerical Problem 1: State the expressions along with the physical dimensions (units) for the yarn characteristics given in Column A in terms of the fiber and yarn characteristics given in Column B.

Column A	Column B
Count of yarn	Mass of yarn and length of yarn
Count of yarn	Density of fiber and cross-sectional area of yarn
Diameter of yarn	Count of yarn, Density of fiber, and packing density in yarn
No. of fibers in cross-section of yarn	Coefficient $\lambda$ , and relative yarn count
Diameter of yarn	Diameter multiplier and count of yarn
Twist intensity of yarn	Diameter of yarn and twist of yarn
Twist of yarn	Twist multiplier and count of yarn

$$T [\text{tex}] = \frac{m [\text{g}]}{L [\text{m}]} \times 1000$$

$$T [\text{tex}] = P [\text{kg/m}^3] S [\text{m}^2] \times 10^6$$

So, this numerical problem we will start first, so how it reads; state the expressions along with the physical dimensions for the yarn characteristics given in column A in terms of the fiber and yarn characteristics given in column B a very similar problem we solved in module 1 that means, in this column, yarn characteristics are mentioned, in this column fiber as well as some the yarn characteristics are mentioned.

You need to express the variables in column A in terms of the variables in column B for example, count of yarn you need to express count of yarn in terms of mass of yarn and length of yarn along with unit, right so, let us start, count of yarn say, capital D tex = mass per unit length mass you will express in say gram, L; length of yarn if you wish to express in meter, you have to multiply by 1000, right.

Because tex 1 gram; 1 tex is 1 gram power 1 kilometer, so if you express here L in terms of millimeter you have to change this accordingly okay, first is done, second; count of yarn density of fiber and substance cross sectional area of yarn; substance cross section area of yarn, so capital D tex = rho \* into S, if we express rho in kg per meter cube and if we express S, substance cross sectional area of yarn say, in meter square then, you need to multiply by 10 to the power 6 in order to balance both side, so this was the second.

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$$D [mm] = \sqrt{\frac{4T [tex]}{\pi \mu [ ] \rho [kg/m^3]}}$$

$$n [ ] = z [ ] k_n [ ]$$

$$D [mm] = K [mm \cdot tex^{-0.5}] \sqrt{T [tex]}$$

$$K [ ] = \pi D [mm] z [m^{-1}] / 1000$$

$$z [m^{-1}] = \frac{\alpha [tex^{0.5} \cdot m^{-1}]}{\sqrt{T [tex]}}$$

Third; diameter of yarn in terms of count of yarn, density of fiber, packing density in yarn, so let us write next page, diameter of yarn D, suppose in millimeter in terms of count of yarn divide by pi times Mu packing density is dimensionless into rho kg per meter cube, you can verify this is a balanced expression, so diameter in terms in unit millimeter = square root of 4 times T, T is yarn fineness in tex pi Mu, dimensionless packing density rho fiber density in kg per meter cube.

So, this is over, we will go to the next one; number of fibers in cross section of yarn in terms of coefficient kn and relative yarn count, relatively easy, number of fibers in yarn cross section dimensionless and relative yarn fineness also dimensionless, capital D / small d and coefficient kn is also dimensionless, very easy expression. So, you go to the next one; diameter of yarn diameter multiply yarn and count of yarn.

So, this you need to be little careful, diameter of yarn in millimeter K times root T; T is tex, so what will be the unit; millimeter \* tex to the power - 0.5, right, so diameter; yarn diameter in

millimetres, coefficient of yarn diameter  $K$  unit is millimeter \* tex to the power  $-1/2$  \* root over  $T$ ;  $T$  is yarn count in tex, okay. Then we go to the next one; twist intensity of yarn in terms of diameter of yarn and twist of yarn.

So, twist intensity of yarn,  $K$  dimensionless  $\pi$  times diameter \*  $Z$ ;  $Z$  let us write in terms of meter inverse, so you have to divide by 1000, okay then the last expression twist of yarn, in terms of twist multiplier and count of yarn, so twist of yarn  $Z$  let us assume its unit is this alpha, alpha unit will be into meter inverse divided by root over  $T$  tex, so twist of yarn in terms of twist multiplier alpha and count of yarn; count of yarn  $T$  tex and this is unit of alpha, twist multiplier has unit.

So this completes our numerical problem 1, thank you very much for your kind attention.