Theory of Yarn Structure
Prof. Dipayan Das
Department of Textile Technology
Indian Institute of Technology – Delhi

Lecture - 06
Relations among Yarn Count, Twist, and Diameter

Welcome to you all to this MOOC's online video course theory of yarn structure. Today, we will start module 3, relations among yarn count, twist and diameter. As you all know, the relations among yarn count, twist and diameter are related to specific geometrical and mechanical properties of yarns. The basic quantity which is underlying this relationship is called packing density.

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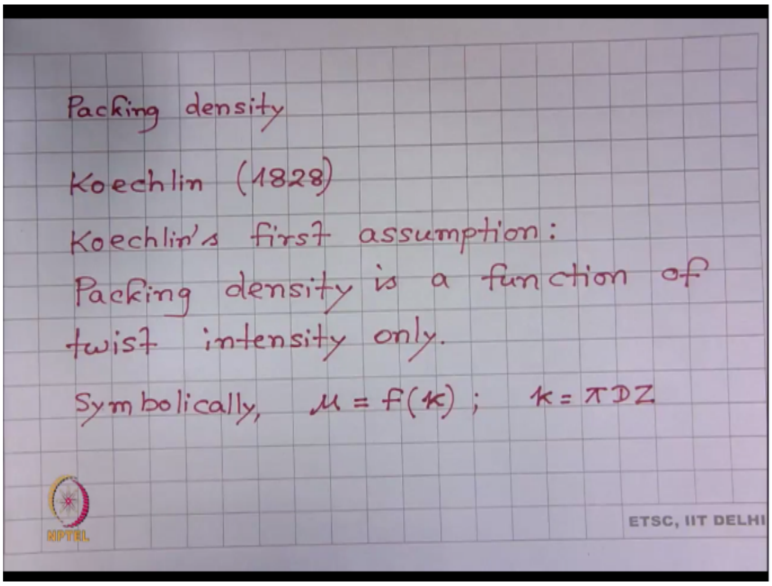

The fibre packing density in yarn basically decide this relationship among yarn count, yarn twist and yarn diameter. The first traditional model regarding this relationship was established by Koechlin in the year 1828. So you can imagine this concept is quite old. Now in the year of 1828, the specific regulations in yarn structure especially the mechanics of fibrous assembly inside the yarn was not well known.

The specific tensors, stress tensor in yarn were also not well known. So Koechlin introduced a few basically two important assumptions. As those relationships were unknown in the year of 1828, Koechlin introduced two assumptions. The first assumption packing density is a function of twist intensity only. So this was the first assumption of Koechlin. Packing density is a function of twist intensity only.

So symbolically mu packing density is a function of yarn twist intensity kappa where kappa is=pi times D times Z. So this was the first assumption of Koechlin. Now let us analyze the consequence of this assumption.

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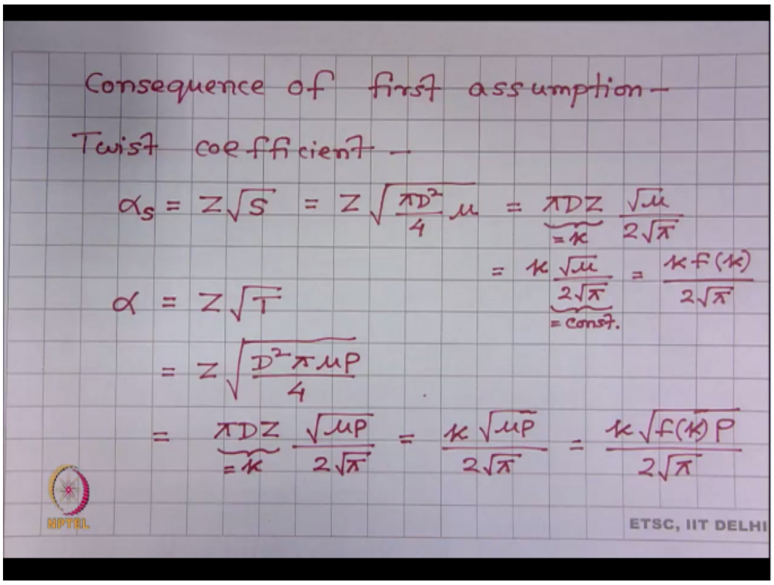

Consequence of first assumption, we will discuss with twist coefficients or twist multiplier and diameter multiplier. So the question is what are the consequence of Koechlin's first assumption on twist coefficient or twist multiplier and diameter multiplier. So we will answer to this question now. First, let us start with twist coefficient. First, twist coefficient as we know that alpha S Z root over S and alpha is=Z root over T.

So these two relationships we learned in module 2 right. Now what is S? S is the substance cross-sectional area of yarn. So that is pi D square/4*mu. This relation also we learned in module 2. So let us write this expression pi right. Now this pi DZ, this quantity is equal to kappa twist intensity. So we can write this expression further as kappa times mu 2 root pi. Now look at this carefully.

Twist intensity kappa 2 root over pi is a constant. This is a constant right and root over mu. Now we assume packing density is a function of twist intensity only, so we can write kappa function of kappa and 2 root pi, this function f is right now unknown okay. So this was related to twist coefficient. Second twist coefficient, let us see Z, now T we know that T is D square pi mu rho/4 right.

So we can further write this as pi D times Z*pi right. Now what is pi DZ? Pi DZ is kappa. So we can write kappa/2 root over pi okay. Now mu is a function of twist intensity only. So we can write kappa function of kappa rho/2*pi okay. So these were the consequences of Koechlin's first assumption on twist coefficient. Now we will analyze the consequence of first assumption on diameter multiplier.

But before that what we see here is alpha subscript is a function of kappa only right and alpha is also a function of kappa only because fibre density is assumed to be constant because Koechlin studied yarns produced from same fibrous material using same technology for analogical end users. So that is why rho same fibrous material was used so rho is constant here.

So that means what we see is that alpha S as well as alpha both twist coefficients are functions of kappa only. Now let us proceed to the consequence of first assumption on diameter multiplier.

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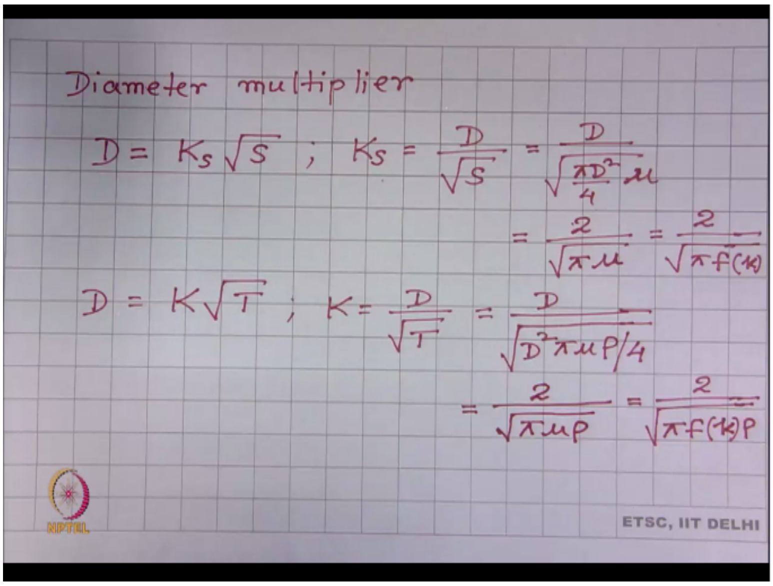

Now diameter multiplier, two relations we learned in module 2 regarding diameter multiplier D is=K times root S. Second, what we learned is D is=K times root T. So these two relations we already learned in module 2 right. So what we see here KS D/root S. What is S? Let us substitute pi D square/4*mu right. So this DD will cancel out and what we will see is 2 root over pi*mu right.

Now mu is a function of twist intensity only. So we write 2/pi this function, look 2 is a constant, pi is a constant, function of twist intensity that means KS is a function of twist intensity only. What happens to K? So K is D/root T, is not it? Now what is T? Let us write 4 so what we see here is 2 this DD will cancel out and as a result you will get this expression, is not it? Now 2/root over pi function of twist intensity only*rho.

So what we see here, 2 constant, pi is another constant, rho same fibrous material so rho is constant. So diameter multiplier D is also a function of twist intensity only.

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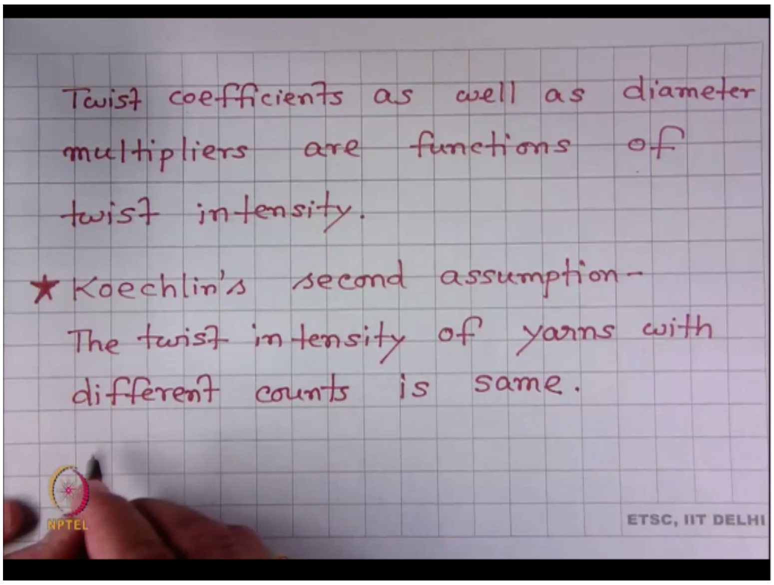

So if we summarize these two results then what we obtain is twist coefficients as well as diameter multipliers are functions of twist intensity. So this statement is the consequence of Koechlin's first assumption alright. Now we introduce Koechlin's second assumption and the very interesting assumption was introduced by Koechlin, what was that? The twist intensity of yarns with different counts is same.

Look at this second assumption of Koechlin. The twist intensity of yarns with different counts is same.

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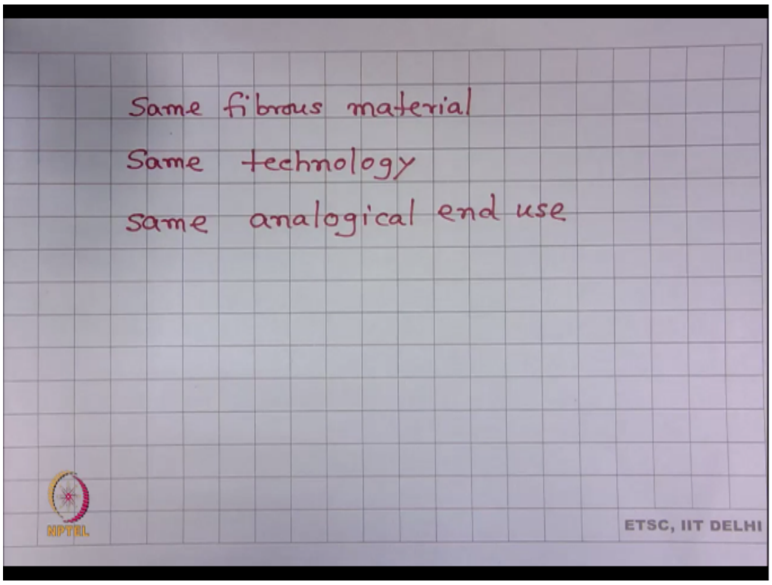

That means and as I told you earlier Koechlin used same fibrous material. He used same technology for yarn production and those yarns were of same analogical end use, apparel end use right and Koechlin's second assumption was the twist intensity of yarns with different counts is same. So that means twist intensity is same. Then, what happens to our consequences let us see now.

So this was the consequence of first assumption and now we will analyze the consequence of both assumptions. Alpha S was a function of twist intensity only and twist intensity of yarns with different fineness is same, so alpha S is same for all yarns. Similarly, first assumption told alpha is a function of twist intensity only and second assumption told twist intensity is same for all yarns, so alpha is same for all yarns right.

Then, what happens to diameter multiplier? KS is a function of twist intensity only and twist intensity is same for all yarns. So KS is same for all yarns. Similarly, K is a function of twist intensity only and twist intensity is same for all yarns, so K is same for all yarns.

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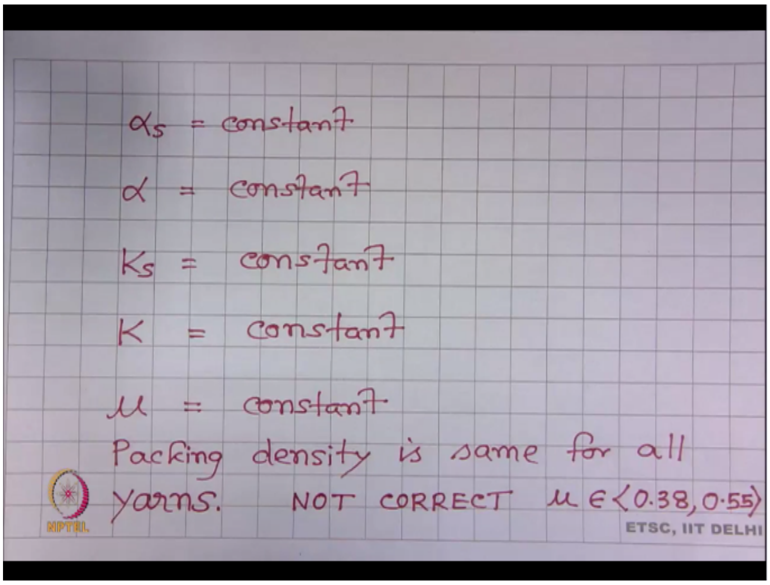

That means what we see is that alpha S is a constant, alpha is a constant right, KS is a constant, K is a constant. They are same for all yarns. That means finally if they are same for all yarns, then mu packing density for all yarn is same. Packing density is same for all yarns. This is not correct. Why? Because we know packing density ranges from 0.38 to roughly 0.55, so packing density is not same for all yarns.

Experimentally, we have analyzed it and we observed that packing density ranges from 0.38 carded yarn to 0.55 for combed yarn. Why is it so? What went wrong in Koechlin's assumption? Let us now analyze these two assumptions once again. Koechlin's second assumption twist intensity of yarns with different count is same.

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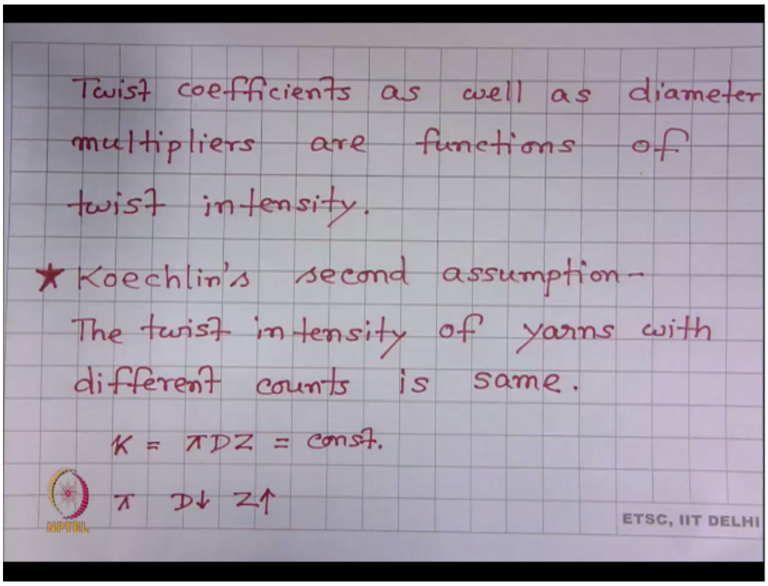

That means kappa is pi times D times Z is constant right. Now as for our experience on yarn manufacturing is concerned, this assumption is probably not too far from reality. When we insert higher twist, yarn becomes more compact and diameter reduces. So Z is increasing, D is decreasing, pi is constant. So probably kappa is constant is not a too bad assumption. Then, what went wrong?

Let us then come back to first assumption. Koechlin's first assumption packing density is a function of twist intensity only. Twist intensity is not a function of packing density only. It depends on yarn count too. So probably this assumption Koechlin's first assumption was not too real. So that is why empirical corrections to Koechlin's equation were necessary and many scientists, researchers worked on yarn structure proposed different corrections to Koechlin's expression. Let us see those empirical corrections to Koechlin's expression.

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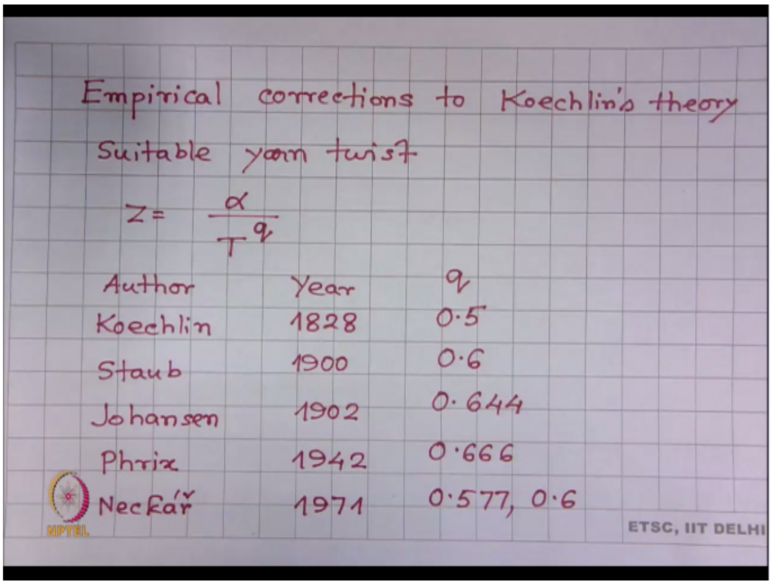

So now we would like to see empirical corrections to Koechlin's theory. Now if we generalize those corrections, then we will see that suitably yarn twist as per those scientists who proposed different corrections, Z alpha/T to the power q. So this must twist is required to give yarn. Now alpha you know twist coefficient or twist multiplayer T to the power q. Now the scientists who proposed different corrections empirical corrections to Koechlin's theory, they proposed different values of q.

Say let us summarize those. Say author Koechlin himself first and probably in the year if you look at 1828, he proposed the value of q 0.5 right. Then, there was another researcher who worked on yarn structure Staub; in 1900 he proposed a value 0.6. Remember these values came from experimental results. They are empirical. Then, there was another scientist Johansen in 1902 he proposed a value 0.644.

Then, there were many important Phrix in 1942 he proposed 2/3 this. Then, Neckar in 1971 he published research article where he mentioned the value of q ranging from 0.577 0.6 like that, so there were many, we could list 5 here. So what we see is that there are two trends, the first trend is different researchers proposed empirical corrections to Koechlin's theory. So the value of q is different according to different researchers.

Second which is interesting all the values of q, if you look at them, you will see that all the values are >=0.5. Not a single value till date proposed by any researcher falls below 0.5. So they are higher than 0.5 or else Koechlin proposed it is equal to 0.5. It is a very interesting trend. As a student of theory of yarn structure, you should basically look at those results and then start thinking why is this happening, why all the values are >0.5, why not a single value falls below 0.5.

So this gives you inspiration for future research. So you have to think critically and analytically alright. So these were the empirical corrections to Koechlin's theory. Now problem with empirical corrections is that if we use different yarns say yarns from different fibres or say yarns with different technologies, ring spinning technology, rotor spinning technology, air jet yarn manufacturing technology.

So if we use different technologies or different fibrous materials, the value of q will be different but this value of q is required for yarn production, is not it? Then, how will you predict before producing the yarn what should be the value of q? Accordingly, I will set the yarn manufacturing technology process parameters. So it is very much necessary to predict the value of q in this case before production of the yarn.

But all relations are empirical, so that was probably the necessity of a theoretical model which can propose a theoretical scientific relation among yarn count, yarn twist and yarn diameter. Such a model exists, so but it is a theoretical model. As typically happens, every theoretical model is based on certain assumptions, certain hypothesis. Then, the equations are derived and then finally final results are compared with experimental ones.

So here we will describe one theoretical model which accounts for a relationship among yarn count, yarn twist and yarn diameter. So we will start with certain assumptions. We will develop mathematical equations until we achieve the final one and we will show you the comparison of the final theoretical expressions with experimental results. So it will take quite a long time.

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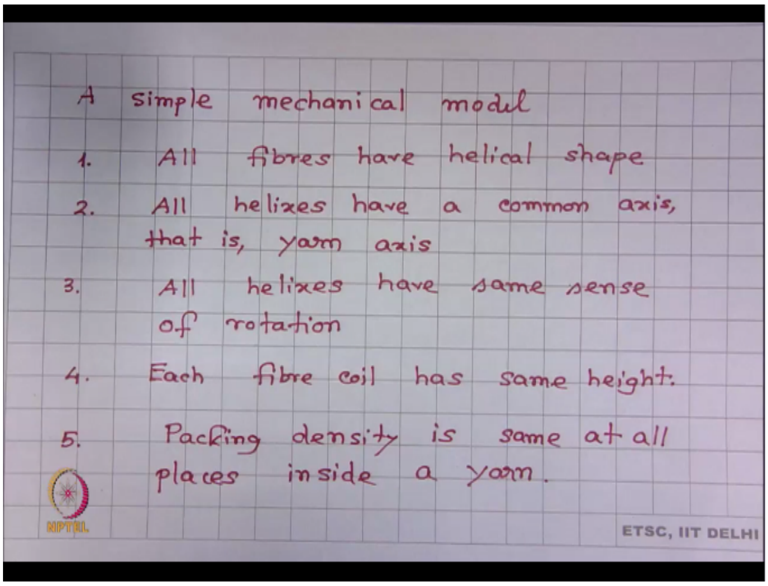

Let us name this model as a simple mechanical model. This model is based on a few assumptions. The primary assumptions, the 4 primary assumptions I will write down, also while deriving relationships we will introduce a few more assumptions. First assumption, all fibres have helical shape. So yarn consists of many fibres and inside the yarn, the fibres follow helical pattern, helical shape, all fibres follow helical pattern.

Second assumption, all helixes have a common axis, that is yarn axis. So all helixes have a common axis that is yarn axis. Third, all helixes have same sense of rotation, same direction of rotation. So they are not randomly follow rotation. They have a same sense, same direction of rotation. Fourth, basically it is a direction of twist. Fourth as a result of helical path fibre coil will be developed.

So each fibre coil has same height right. These are all primary assumptions. The fifth assumption is a very important one. This assumption is packing density is same at all places inside a yarn. Look at this fifth assumption, what we say? We say that in one yarn packing density is same at all places. Mind it we do not say that packing density of all yarns is same, we are not saying that.

What we are saying is that in one particular yarn, packing density will be same at all places inside the yarn right. So these 5 are the initial assumptions. Now we will start developing this model.

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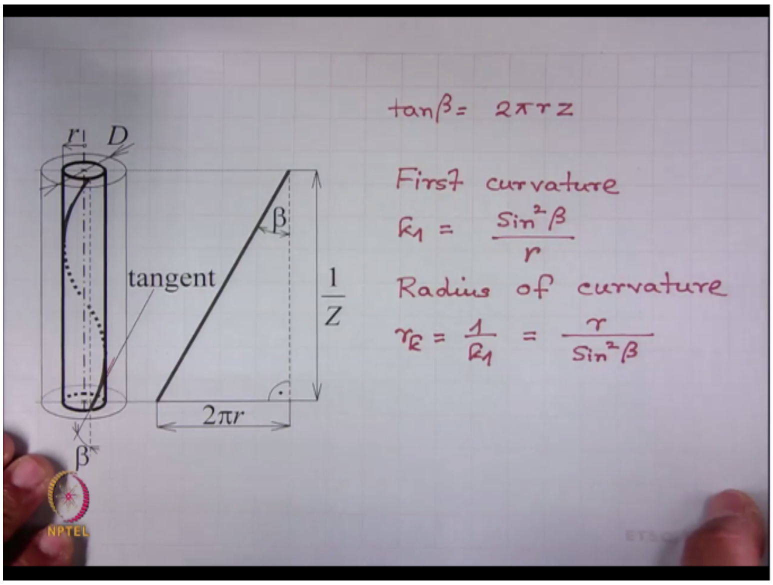

Look at this image. A cylindrical yarn, one fibre coil is shown here, this on the surface so you see it, inside dotted you do not see and then again it comes on the surface you see it right. D is the diameter of the yarn and r is the radius at which the fibre is present and if we draw a tangent along this line, this tangent makes an angle beta from yarn axis. So beta is twist angle of a general fibre right.

Now if you unroll this cylinder, the thick cylinder then what you will see is that this distance will be 2 pi r, this will be 1/Z because Z is the number of coils per unit length. So the length will be 1/Z and this angle is beta, so you can understand tangent of beta is=2 pi r times Z okay. Now this fibre trajectory is a basically curve. So what will be the first curvature? We know from simple mechanics, the first curvature k1 is=sin square this angle beta/radius r right.

Then, what will be radius of curvature? Radius of curvature is 1/curvature right. So r/sin square beta right. So this expression you will recover later on, radius of curvature is=r/sin square beta. What is the beta? Beta is the twist angle at radius r and r is radius okay. Now let us because of the twist a lot of forces are generating. Some forces are acting along the fibre;

some forces are centripetal in nature. So we would like to study the centripetal force per unit volume of fibre.

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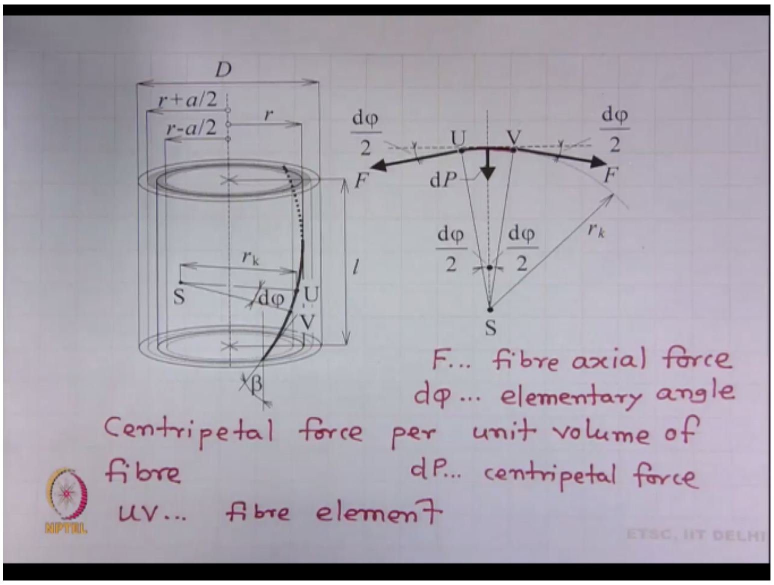

So what is the amount of centripetal force which is generated because of twist or twist insertion per unit volume of fibre? So we would like to find out this amount. So what do you do for that we take one small element of fibre UV. This is U, this point is V. So we take a small element UV. The same element is shown here, U and V this element, we are talking about this element okay.

Now so this is fibre element, so there is one force which is acting along the fibre, that force is F. So UV is basically fibre element, a small element UV. We would like to derive an expression for centripetal force per unit volume of this UV and what is F? F is fibre axial force okay and what is d phi? d phi is the elementary angle. So here you can see d phi is the elementary angle and rk is the radius of curvature right.

So d phi is elementary angle okay. What more you see in this image? Yarn diameter D, now one interesting you can see that there is something gray color. So this gray color cylinder basically a cylinder where we will see later on some significant compression of fibrous assembly happens because of twist. So this cylinder is situated at a distance from r+a/2 and r-a/2, these two ready right okay.

Beta you already know twist angle alright. Now yeah one more symbol is not known to you that is called dP, dP look at this direction of the force dP is acting here towards S. So dP is the centripetal force, dP is we need to find out centripetal force per unit volume of fibre. So first we have to find out dP, then we have to find out the volume of UV, then we will divide, we will find out dP by that volume ratio right. So we will achieve our target.

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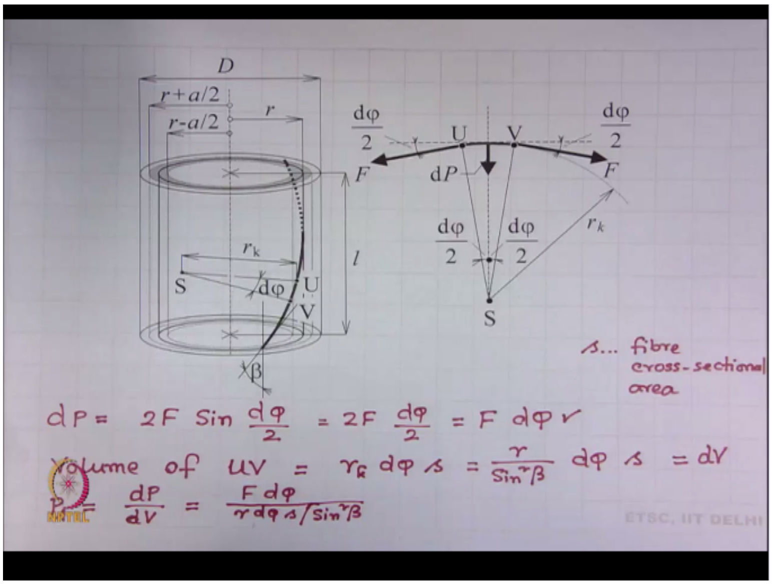

Now what is dP? If you look at this one, then what you see is that dP is=2 times F*sin d phi/2. If you analyze this simple mechanical analysis, you will find out dP is=2 times F*sin d phi/2. Now is that elemental angle, d phi is an elemental angle? It is very small, so when theta is small, sin theta is=theta. So when d phi is small, sin d phi/2 is=d phi/2. So if we write that 2F d phi/2 finally we find out F*d phi.

So centripetal force is equal to fibre axial force multiplied by elementary angle okay good. Now find out the volume because what is our target, our target is to find out an expression for centripetal force per unit volume. So if we find out volume and if we divide this expression by that volume, we will obtain our result. So volume of UV we need to find out. How will you find out volume?

Volume is simple. Radius of curvature*d phi*s, s is fibre cross-sectional area, is not it? So radius of curvature we have already known from our earlier one that is r/sin square beta. So if we write r/sin square beta*d phi and s is fibre cross-sectional area right. So let us write s here okay. Remember s small s is fibre cross-sectional area, we learnt it in module 1 right. So now centripetal force per unit volume let us write P1 centripetal force per unit volume, so P1 is defined by dP/this volume we can write it as dv small volume dv right.

So what is dP? dP is F d phi here and what is dv? dv is here, r d phi*s/sin square beta right. So if we rearrange it, then what we see is that let us do it once again.

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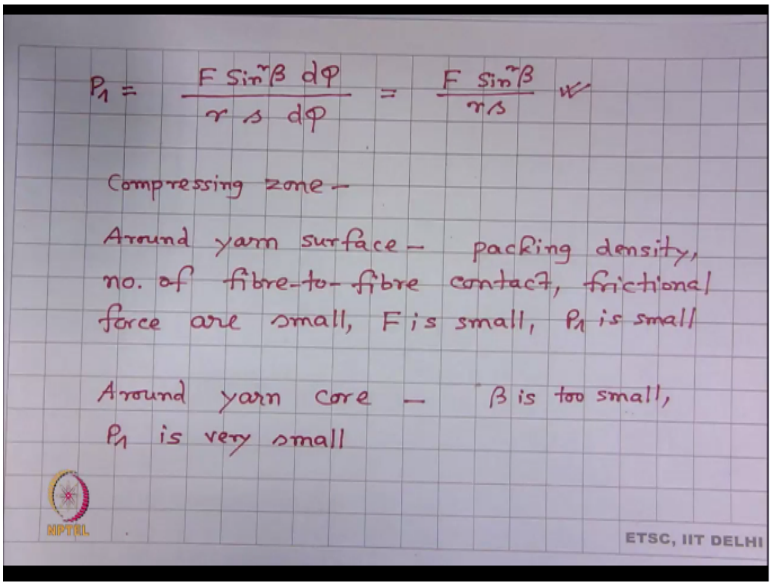

P1 is=F sin square beta*d phi/r*s*d phi right. So d phi d phi will cancel out and we obtain F sin square beta/r*s right. So we wanted to find out an expression for centripetal force per unit volume of fibre and we obtained it. So this is our desired expression P1 centripetal force per unit volume of fibre is=fibre axial force sin square beta. What is beta? Beta is twist angle, r is the radius of yarn and s is fibre cross-sectional area okay right.

Now this model what we are going to study is based on idea of compressing zone. When we insert twist fibres in certain region inside the yarn are significantly compressed. Why certain region? Because same amount of compression is not happening throughout inside the yarn. Why? Think about. Around yarn surface, packing density is very small; number of fibre to fibre contact is very small, frictional forces are very small.

So F will be very small, so P1 will be very small. So what we say is that there will be one significantly compressed zone inside the yarn structure. Where is that? Where is that located at? That is our question now. Around yarn surface it cannot be. This cannot be present around yarn surface why? Because packing density, number of fibre to fibre contact, frictional forces are small.

So F is small, so P1 is small, so this compressing zone cannot be present around the yarn surface. Can it be present then around yarn core? Around yarn core, it also cannot be present.

Why? Around yarn core, fibres are more or less straight. So beta, angle beta is very small. As a result curvature is also very small. So P1 is small. That means around yarn core, beta is too small, so P1 is very small.

So this significantly compressed zone can neither be present around yarn core nor be presented around yarn surface, so where it can be located at?

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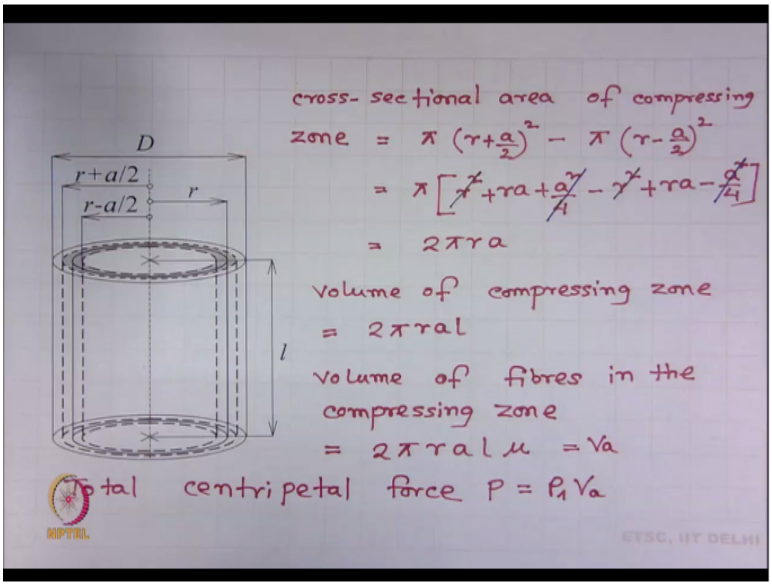

Let us assume that this significantly compressing zone what we are talking about is present somewhere in between yarn core and yarn surface. So let us then say that this gray color region is the region of cylinder where significantly compressing zone is present. It is not at core nor at surface it is in between. So we assume that because of twist fibres will be significantly compressed and this significantly compressing zone is present at a distance from yarn.

As a distance in between r+a/2 and r-a/2, these two ready. Length of the cylinder is still remains l, so what is the cross-sectional area of this compressing zone? Look at this gray color region, this is equal to pi r+a/2 square-pi r-a/2 square, is not it? Okay so this pi is common, pi now expand it r square+r a+a square/4-r square+r a-a square/4. So what we see is that this r square and r square cancel out.

Then, this a square/4 and a square/4 cancel out. So we find a and r a. So that is equal to 2 times pi r a right, very simple expression. So what is the cross sectionality of the compressing zone is 2 times pi times r*a, a is the thickness of the compressing zone okay. Then, what is the volume of this compressing zone? Simple, what is volume? Cross-sectional area*length, length is l, cross-sectional are is 2 pi r a.

So volume is 2 times pi times r a l. Now this is the volume of compressing zone. What is the volume of fibre in this compressing zone? Volume of fibres in the compressing zone/volume of the compressing zone is equal to packing density right. So volume of fibres in the compressing zone will be 2 pi r a l*mu, mu is the fibre packing density in yarn right. So what will be total centripetal force?

The total centripetal force will be say P, let us use the symbol P is=P1 earlier*this volume V. So let us write this volume as Va, so Va okay. So this will be total centripetal force.

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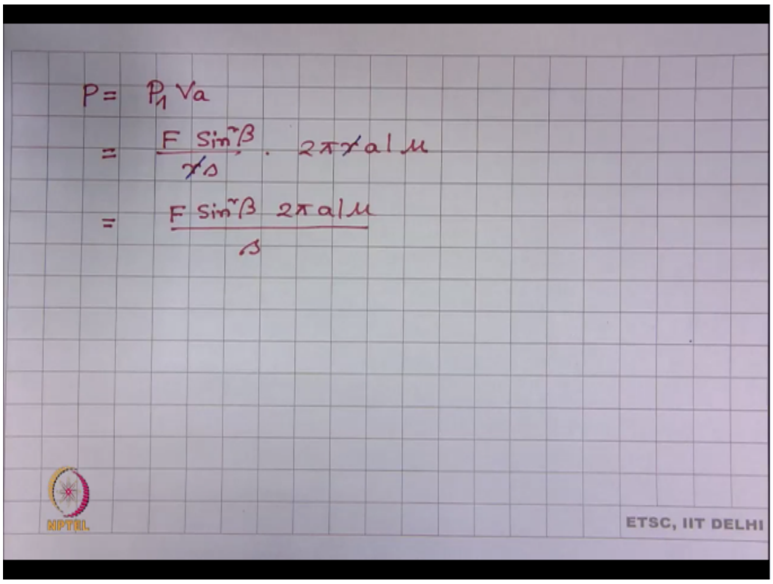

So P is=P1*Va. What is P1? P1 we have probably derived earlier right, F sin square beta/r*s that is P1 and what is Va? Va is 2 pi r a l*mu. So this is the total centripetal force right. If we write it as F this r r will be cancel out, so what we obtain, F*sin square beta 2 pi a l mu/s okay. Now we need to find out the, so we have found out the total force. Now we stop here. In the next class, we will start from here and try to find out the pressure developed in this compressing zone okay. Thank you for your attention.