Theory of Yarn Structure
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Lecture - 07
Relations among Yarn Count, Twist, and Diameter (contd.,)

Welcome to you all. So in the last class, we started module 3, relations among yarn count, twist and diameter. So as I told you in the last class, the relations among yarn count, twist and diameter are related to specific geometrical and mechanical behavior of yarns. The underlying variable which governs this relation is packing density, fibre packing density in yarn.

Now the very first traditional model on the relations among yarn count and twist was found by Koechlin in the year 1828 he based on two important assumptions proposed the relationship; however, experimentally we observed that Koechlin's model was not too precise. Then, many researchers in the area of yarn structure proposed many relations related to yarn count, twist but all relations were empirical only.

The question that remained is, is it possible to theoretically derive a relation among yarn count, twist and diameter so that we can predict beforehand and then accordingly we can set our yarn manufacturing process parameters. So with that aim we started a model, simple mechanical model. In the last class, you learned that this model was based on 4 primary assumptions and one important assumption.

Then, we derived an expression for centripetal force per unit volume of the fibre. Then, we were analyzing where the compressing zone is situated inside the yarn. That we have analyzed in the last class and then we also analyzed the total centripetal force in the compressing zone. So that was the final results in the last class.

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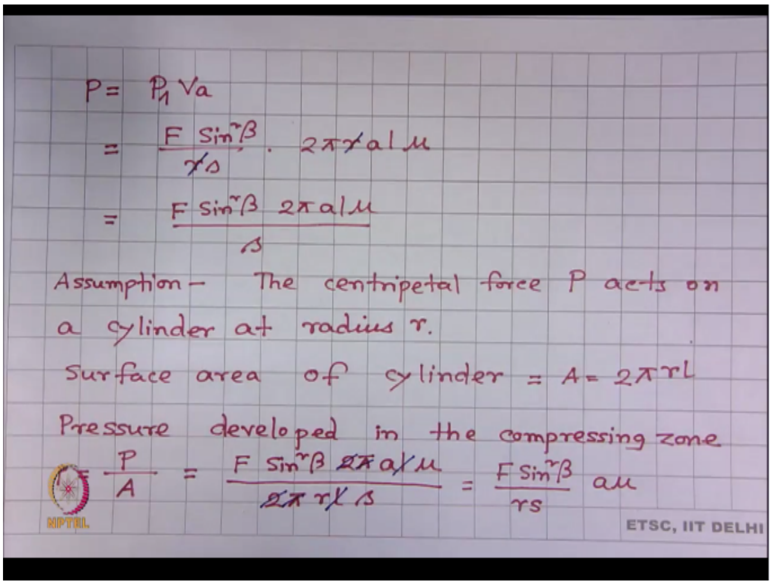

Now we assume that this centripetal force P acts on a cylinder which is situated at radius r. That means what we do, all the partial centripetal forces we centralized at one particular cylinder that is situated at radius r where the centripetal force P acts. What is the surface area of that cylinder? Surface area A=2 pi r*l.

So what is the pressure developed in the compressing zone? This pressure P is=the total force capital P by this area, so this force is this, area is this. So let us write it F sin square beta 2 pi a times l*mu/2 pi r l s right. So this 2 2 cancel out, pi also, l also. So what you obtain is that F sin square beta a*mu/r*s. So F sin square beta/r*s a*mu okay. So this is the expression for the pressure developed in the compressing zone.

Now what is difficult to obtain is the F. That is the axial force on the fibre. So we have to find out a suitable expression for the axial force acting on the fibre capital F. So because of the insertion of the twist, fibers are inclined.

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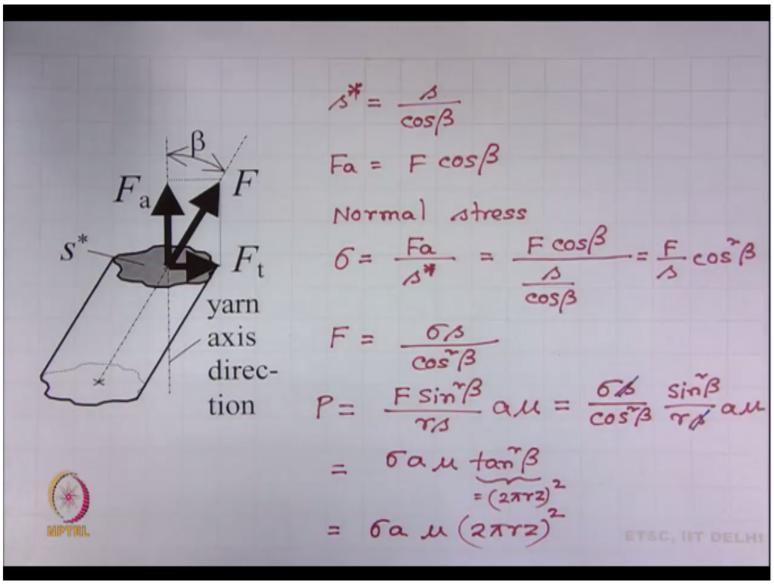

So if this is the yarn axis, let us think a fibre which is inclined at an angle beta. So along the axis of the fibre force F is acting, along the axis of the yarn a force F subscript a is acting and this is the transverse force F subscript t right and what is this area? This is not cross-sectional area. This is sectional area. Sectional area we denote by s superscript star right. Now what is your relation between this star and s, cos beta.

This we have already derived in module 2 okay and what is the relation between Fa and F? Fa is F times cos beta. So what is the normal stress? (()) (08:39) sigma, sigma is Fa/s star right. So what is Fa? F cos beta/s/cos beta, so F/s cos square beta right. So what is F? F is sigma times s/cos square beta okay. Then, we come back to our original expression of pressure. So this pressure was F sin square beta/r*s a*mu.

We substitute F from here, so what we obtained is sigma s/cos square beta sin square beta/r*s a*mu right. So only this s is probably getting canceled out. So as a result what we obtain is sigma a mu tan square beta right. Now what is tan beta? Tan beta is 2 pi r Z. So what is this tan beta? 2 pi r Z square right. So if we substitute, we obtain sigma a mu 2 pi r Z square right. Now we have to work on this expression.

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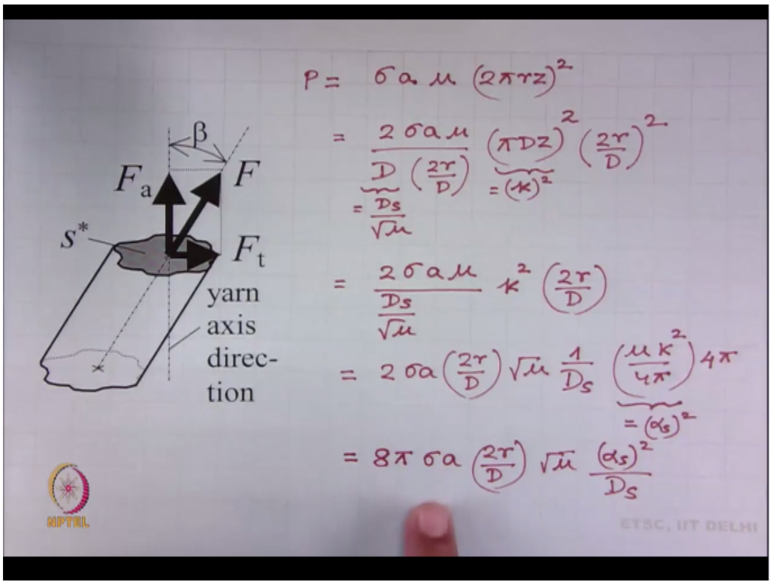

P is sigma a mu 2 pi r Z square okay. Let us write it in this manner, 2 sigma a u/D*2r/D that means this cancel out into pi DZ this into 2r/S square right, 2 sigma a u*D/2r/D pi DZ square 2r/D square. So what do we see here? So this is the expression for P. Now what we see, this D is=Ds/square root of mu, 2r/D let us remain this is equal to kappa okay. So 2 sigma a times mu/Ds/mu.

Then, your kappa square and 2r/D, is not it? Okay 2 sigma au/Ds/mu and kappa square 2r/this. Further we write it as 2 sigma a 2r/D within bracket root over mu then 1/Ds right and then kappa mu kappa square/4 pi*4 pi. Why do we write in this manner? Because this is equal to alpha S square so that is why we write in this manner. So finally what we see, 8 times pi sigma a 2r/D root over mu alpha S square/Ds is not it? Right by Ds so now this a so let us write this expression in an another form.

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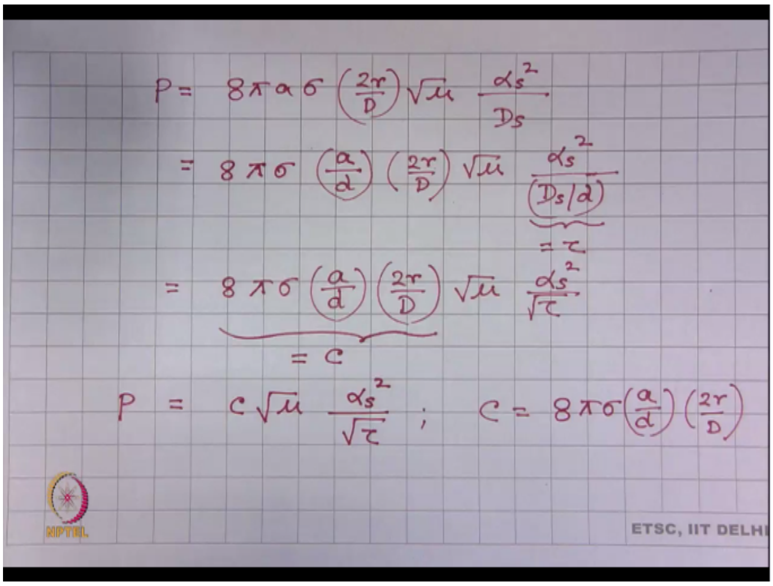

So what we obtain? P=8 pi a sigma 2r/D root over mu alpha S square/Ds right Ds, we obtain this now, 8 times pi*sigma. Then, this a/small d, so introduce small d. Then, 2r/D root mu now this alpha S square Ds/small d okay. Now what is Ds/small d? This is equal to tau, relative fineness tau. So finally we obtained 8 pi*sigma a/d 2r/D root mu squared/tau right, 8 sigma this a/d 2r/D root mu alpha S square Ds/this tau.

So we obtained root over tau okay. Now let us consider this ratio is equal to capital C. So we can then write pressure capital C root mu alpha S square/root over tau, where C is 8 times pi sigma a/d 2r/d okay clear. Now let us learn about this ratio C.

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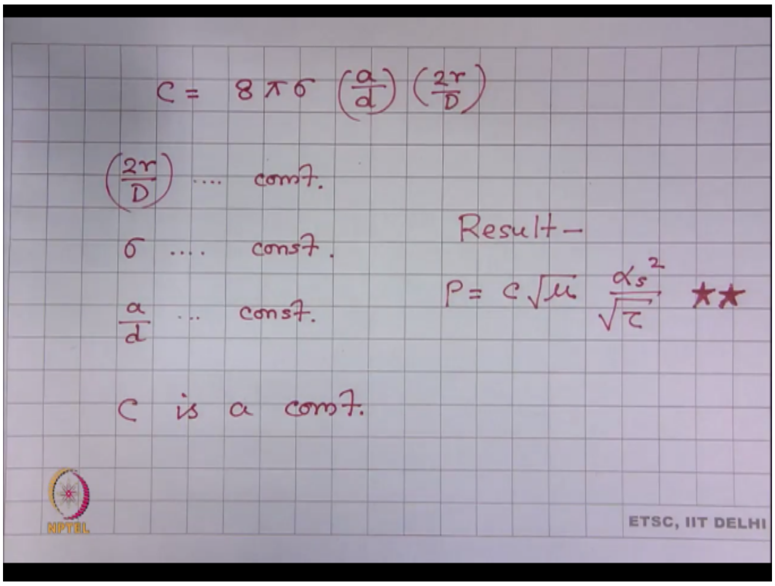

C is 8 times pi rho a/d 2r/D. What does it indicate? What is the physical meaning of C? That is what we need to learn okay. Now what is this 2r/D? 2r/D we assume it is constant because if D is increasing let us assume r is also proportionally increasing. If D is decreasing, then R is also proportionally decreasing, so that this 2r/D remains constant okay. Sigma is the axial stress in yarn cross section.

Axial stress in yarn cross section can also be a constant because the centrifugal force in spinning is perhaps the same. The centrifugal force in spinning is perhaps the same, so sigma can also be considered to be a constant. Then, you have a/d, what is this a/d? a/d is the relative thickness of the compressing zone, relative thickness of the compression zone right. Let us assume this is also a constant.

In that case, your C will be a constant too right. So C is a constant. It is very difficult, probably impossible to measure experimentally the value of C right. So what is our result? Our final result is P is=C root mu square/tau right. So important relation P is the pressure in the compressing zone. How much pressure is developed in the compressing zone because of twist which is related to packing density alpha S twist coefficient or twist multiplier right.

And tau relative yarn fineness so yarn tex/fibre tex. Now if that means what, if you try to increase twist multiplier then your pressure will be increasing and vice versa right. So this is a very important relation. We need to work on this relation later on okay. Now we think about a very similar situation which is often talked about in the book of mechanics.

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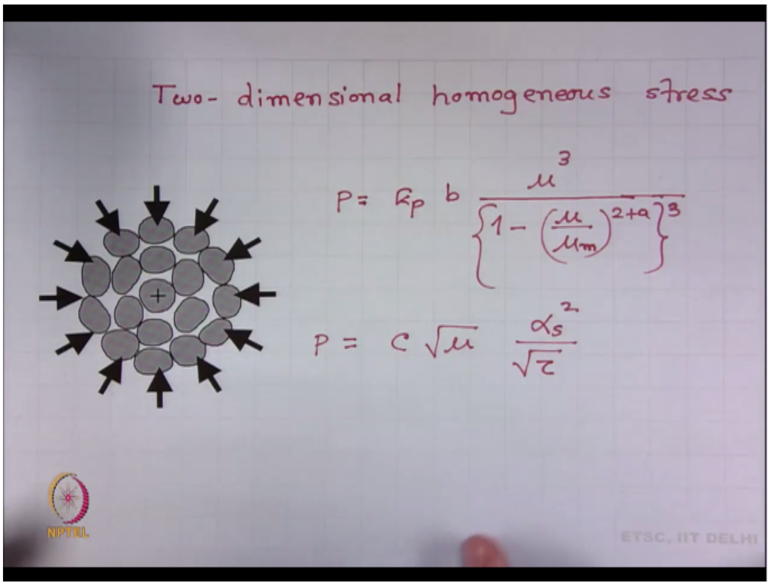

Two-dimensional homogenous stress, so these fibres in the cross section of yarn and the pressure is acting on all directions. So the homogenous stress, then it is possible to derive that this pressure P is a coefficient kP times b mu cube/1-mu/mu maximum to the power 2+a and then these 3. So it is possible to derive this relation. We are not going into the detail of this derivation.

It is possible to derive this expression. So P is k times P, it is basically a coefficient, b is another coefficient, mu is packing density, mu m is the maximum packing density. Typically, in yarn mu m is considered to be 0.8. Practically, we have seen this. So mu m is considered to be 0.8 and a is found to be 1, so this relationship we can later on think about that, mu m is 0.8 and a is 1.

Now we will compare this equation with our earlier derived expression. Our earlier derived expression was C times mu alpha S square/root over tau right. Now what we will do? We will make these two equations equal.

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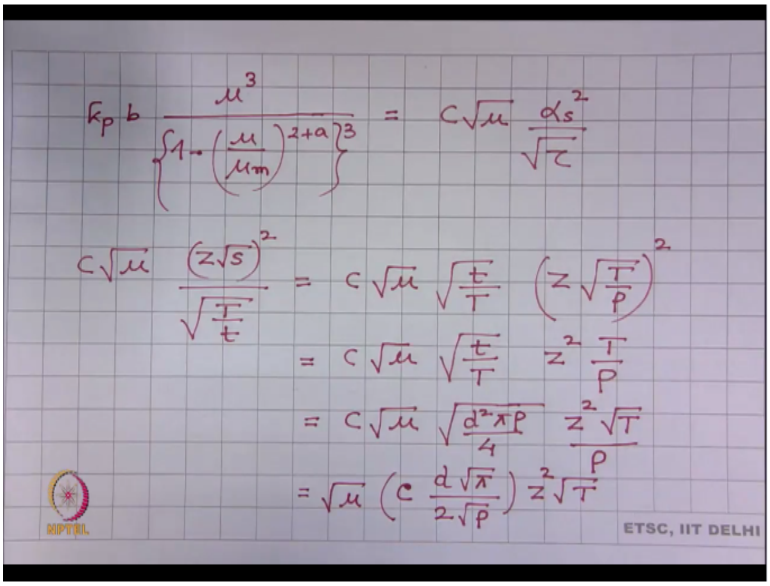

So kP times b mu cube/1-mu/mu m 2+a cube is=C times mu alpha S square/tau right. These two are equal to pressure, so they are equal. Now what is your alpha S? C times mu alpha S is=Z root S so Z root S squared right that is your alpha S and what is tau? Tau is your root over yarn fineness/fibre fineness. These two expressions we learned in module 2 okay. So this is equal to C mu.

Now Z root S so okay let us write it small t/capital T. Now Z root S? What is S? S is root over T/rho. This also we learned in module 2, square okay. Now C root mu t/capital T Z square

T/rho okay. So what we see is that C root over mu small t, small t is fiber fineness. Now that is equal to d squared then pi rho/4 that is your fibre fineness and Z square root T/rho okay.

So what we see is that root over mu then your C right, then this d will come out, let this d come out of the square root. Then, root pi will remain there and 4 square root that will be 2 and 1 root rho will remain here into Z square root T okay. Now we will substitute this expression here and make these two equal.

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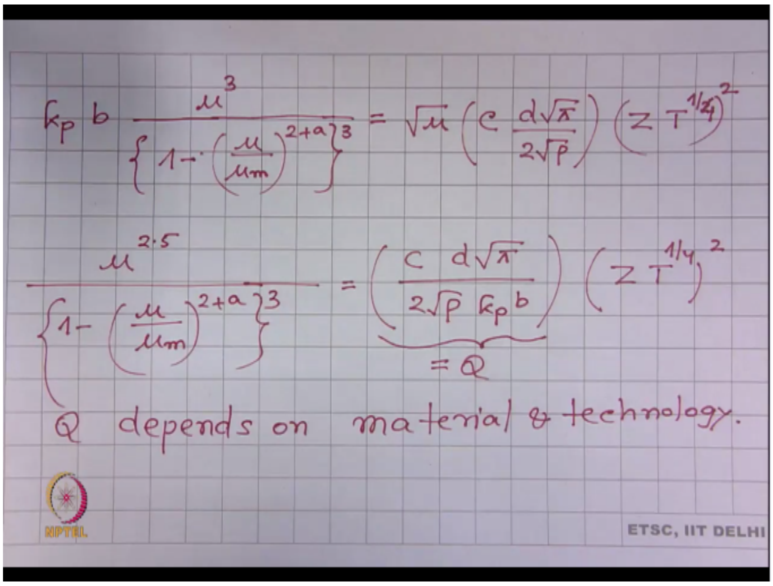

So how will you do that, kP*b mu cube/1-mu/mu m 2+a 3 is=root mu*C d root pi/2 root rho Z T 1/2 to the power 4 squared okay. So what we did, we have derived this right hand side equal to this and then we substituted this in place of here and we obtained this expression. This is T to the power 1/4 okay. Now if we little make it in a better manner say mu to the power this square root 2.5/1-mu/mu m to the power 2+a cube is=C d to the power pi/2 root rho and this kP and b will come, kP and b, this is one Z T to the power 1/4 squared okay.

Now let us consider this is equal to Q. Now what is Q? Q is a parameter. Parameter depends on C, d fibre diameter, rho fibre density, kP coefficient of compression and b another coefficient. So Q depends on material and technology. Why material because D is fibre material, rho fibre density and why technology, kP will be influence of technology, b will be influence of technology, capital C will be influence of technology.

So basically Q depends on material and technology. As of now, it is impossible to find out the value of Q experimentally because lot of parameters are unknown, kP is unknown, b is unknown, C is also not well known. So it is not possible to determine experimentally the value of Q; however, it is possible to find out value of Q by some other means. That we will discuss today. Well then we will write the final expression here.

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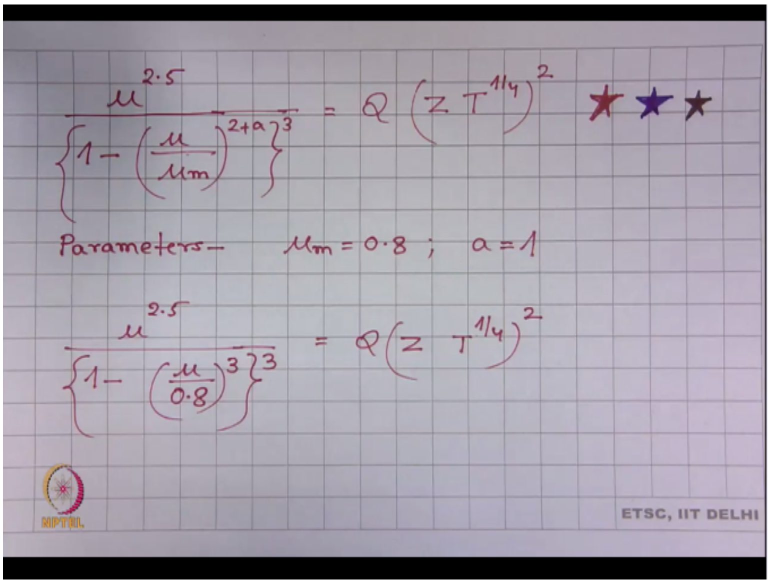

Mu to the power 2.5/1-mu/mu m 2+a 3 is=Q Z T to the power 1/4. This is the very important one of the two final expressions, one of the two final expressions right. Now let us talk about these parameters mu m and a. Typically, maximum packing density practically in a yarn is=0.8. We are not talking about theoretically maximum packing density, that is 1 but practically maximum packing density in a yarn in one of the region is 0.8.

And this a is typically 1, so then this relationship will become this okay. Now Q is a parameter for different technology, the value of Q will be different, for different fibres, fibre materials, cotton fibre, polyester fibre, viscose fibre within same technology say carded ring yarn Q will be different. So we will inform you about the value of Q for different fibres, different technology.

Suppose now if you know the value of Q, you know which count you have to produce, capital T and you know your targeted packing density mu. Then, if you solve this equation, you will be able to know how much twist is required capital Z right. Say this equation can also be read in a different way. The value of Q will be given to you, Z suppose how much twist has been inserted to the yarn you know, what is the count of the yarn you know.

So if you find out this value and if you solve this equation, the root of the equation you will find out mu, so packing density will be known to you. So this equation can be practically applied for different purposes but before going to that let me tell you the value of Q. As far as the experimental results were concerned, the value of Q were obtained.

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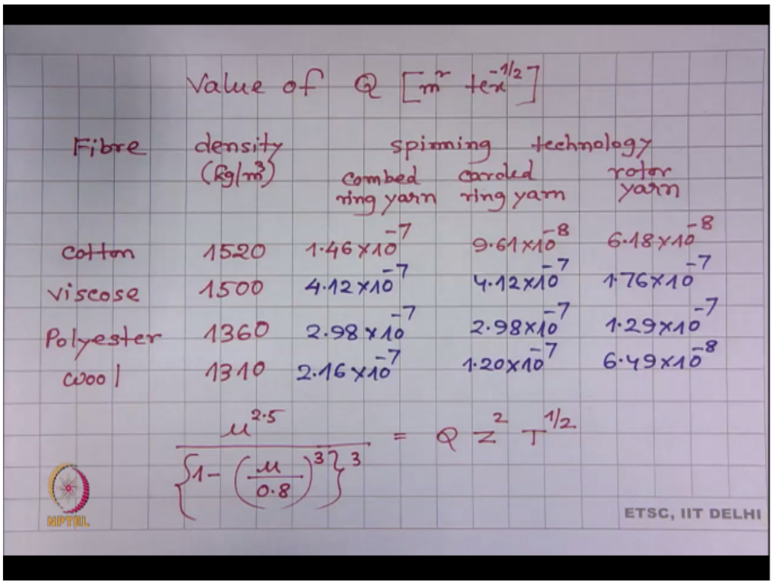

Value of Q, what is the unit of Q? This is the unit of Q okay. Now say fibre and then we will talk about the density of fibre, density of fibre we will talk about in this unit and then spinning technology. Spinning technology, we will talk about combed, we will talk about carded and we will talk about rotor yawn okay. Fibre say cotton, typically the density of cotton fiber we considered 1520 kg per meter cube.

Then, viscose, viscose fibre density we considered 1500. Polyester, polyester 1360 kg (()) (39:29) and then wool 1310. This is the value of Q for combed ring yarn cotton, for carded -8 and for rotor 8 right. So this is the value for viscose. Now let us start with viscose, the value for viscose is viscose is same basically because we do not comb viscose, so 1.76*10 to the power -7 for viscose right.

For polyester, when you consider 1360 kg per meter cube density. Then, polyester also will be same value for carded and combed for obvious reason. For rotor yarn this but for wool -7, 1.20*10 to the power -7, 6.49*10 to the power -8 right. Now so this table suppose you have now you know the value of Q for different fibre material and for different spinning technologies.

What is the value of Q when we produce rotor yarn using polyester fibre, this value 1.29*10 to the power -7. What will be the value of Q when you produce carded cotton ring yarn? 9.61*10 to the power -8 okay. So if you know the value of Q, you will be able to find out the expression mu to the power 2.5/1-mu/0.8 3 power cube is=Q Z square T to the power 1/2 okay.

Now so as I told you before starting before at the beginning of this module that in any theoretical work the final expressions are often compared with the experimental results. You have obtained this final result. Obviously, this final result is based on certain assumptions. When you compare this expression with experimentally obtained results, if they are matching probably our assumptions were close to reality.

If they are totally different, then probably one or more of the assumptions was were not close to reality. So we have to change those assumptions, we have to modify them and we have to again rebuild the model, we have to find out the final expressions once again and we compare with the experimental results. So that is how the theoretical, this is a typical character of any theoretical work.

So now we obtained the final results, now we would like to compare with the experimental results. This is the comparison between theory versus experiment.

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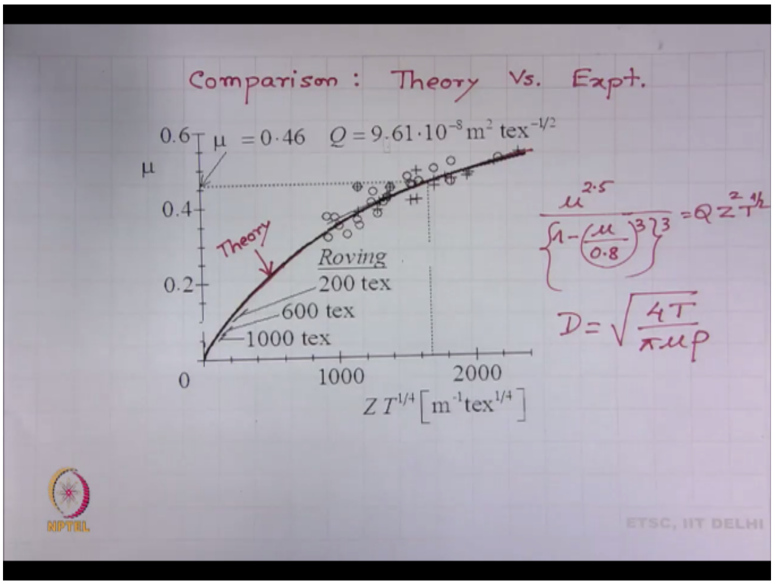

So a lot of yarns also rovings were prepared by using different by yarns are produced. Those yarns were carded ring yarn, carded cotton ring yarn showed the value of Q 9.61*10 to the power -8 meter square tex to the power -0.5. Now those yarns twists were given, so value of the twist were known, fineness of the yarns were measured and also those yarns the cross-section of those yarns were analyzed to find out packing density mu, average packing density mu okay.

Then, those experimental results were plotted in a graph. This graph is shown here. Along the x axis, Z T to the power 1/4 is plotted, along the y axis mu is plotted. So for different experimentally obtained results you see hollow circle, positive sign right. So all these are experimental results for yarns were produced. Then, using this value of Q, using the value of Z experimentally measured and also the experimentally measured value of T, the right-hand side was calculated for different yarns.

Then, this equation was solved and we found out different values of mu. This solid line, thick black one obtained from theory. So this line was obtained from theory and all these are experimental points. So how we obtained this line? We solved this equation, Q we considered 9.61*10 to the power -8, Z we measured the yarn count, we measured the yarn twist. So for one yarn, this value was known.

We solved this equation, we found out mu for different values of Z, for different values of T, we found out many values of Q, mu. We plotted all those values and we obtained this thick black line curve that is from theory and they are the experimental values. What we see is that this curve matches quite well with the experimental results. So that means this equation what we derived what it talks about relation between yarn twist, yarn count and packing density is probably a very well expression.

We can use it in practice right. Then, how do you find out diameter. Now diameter is known by this form rho. If we obtain mu, if we substitute mu here, T is already known, fibre density is known, we can find out D. So how we can use these equations in practice, now this we have to learn because we have to finally apply this equation. Suppose we have to produce a cotton carded yarn of given fineness.

So capital T is known, cotton carded yarn and if it is cotton fibre we will know and it is cotton fibre so we know value of Q, some of the packing density is targeted 0.46. So the left hand side will come to know, right hand side Q is known, capital T is known, we will find out

Z, so that much of twist we will insert okay. Also for research purpose you can use it in other way how is this cotton carded ring spun yarn.

You have produced by giving say certain amount of twist and yarn fineness is also known, so capital Q is known to you, capital Z is known to you, capital T is known to you, so right hand side you will be able to find it. You solve this equation, you find out mu, that mu is substitute here, you find out capital D diameter. So the relation among yarn twist, yarn count and yarn diameter you can find out.

So as I told you at the beginning of this module, packing density is the variable which governs this relation. You see in both relations packing density is significantly present. So basically packing density is the variable which governs these two relations, relation among yarn count, yarn twist and yarn diameter. Now let us solve a numerical problem to understand it in a better manner.

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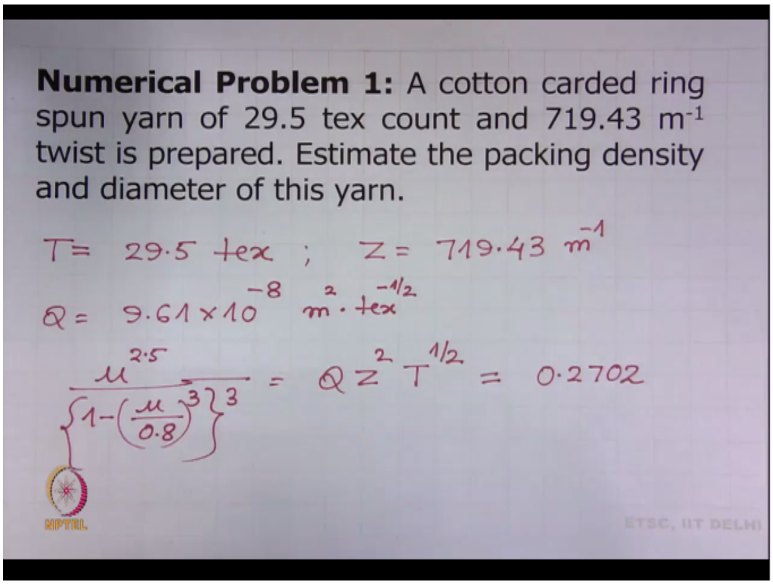

This numerical problem reads as follows. A cotton carded ring spun yarn of 29.5 tex count. So capital T is given 29.5 tex and twist is also given meter inverse is prepared. Estimate the packing density and diameter of this yarn? So we will now demonstrate you how we can use this the previous equation to solve this problem okay. Now it is cotton carded ring yarn right, so cotton carded ring yarn using this table we can find out cotton carded ring yarn the value is this okay.

So Q we will consider 9.61*10 to the power -8 meter square tex to the power 1/2 okay. So mu to the power 2.5/1-minus mu/0.8 cube whole cube is equal to Q Z square T to the power 1/2. Q you will substitute this value, Z you will substitute 719.43 meter inverse and T you will substitute 29.5 tex. So as a result what you obtain is 0.2702. So you will obtain this value, 0.2702. So this expression is equal to 0.2702.

How will you find out mu? Mu can be found out by two ways, one numerical method. You can find out the value of mu from this equation by using a suitable numerical method. Otherwise, you can also find out mu by already prepared table. What does that mean? Suppose beforehand you will prepare a table where mu and this will be known to you.

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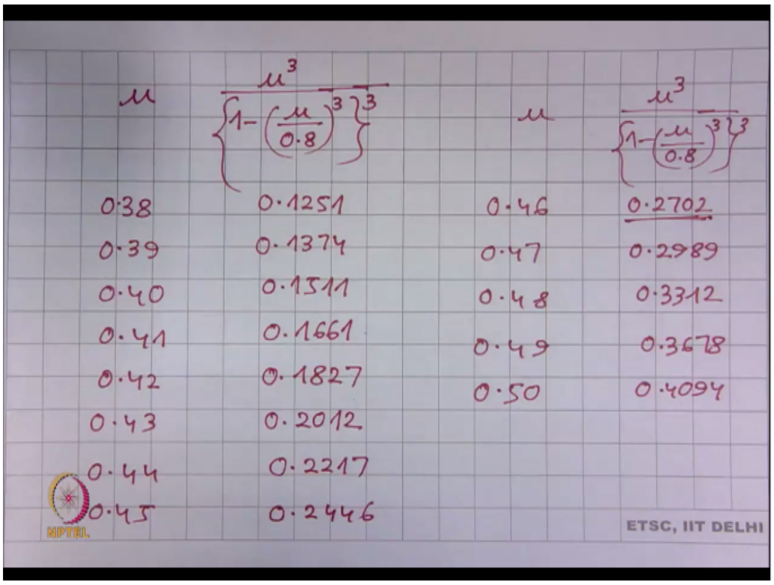

Suppose mu 0.38 okay, you substitute mu 0.38, 0.38 here you will find out the value 0.1251. Similarly, 0.39 you will find out 0.1374, 0.40 you will find out 0.1511, 0.41 you will find out 0.1661, 0.42 you will find out 0.1827, 0.43 you will find out 0.2012, 0.44 you will find out 0.2217, 0.45 you will find out 0.2446, 0.46 the value you will find out 0.2702, 0.47 you will find out this value 0.2989, 0.48 you will find out this value 0.3312, 0.49 you will find out this value 0.3678, 0.50 you will find out this value 0.4094.

Suppose you produce this table beforehand, now what was the value, 0.2702, so this is the value 0.2702. You will find out where this 0.2702 comes here you see, 0.2702 has come. What is the corresponding mu? Corresponding mu is 0.46.

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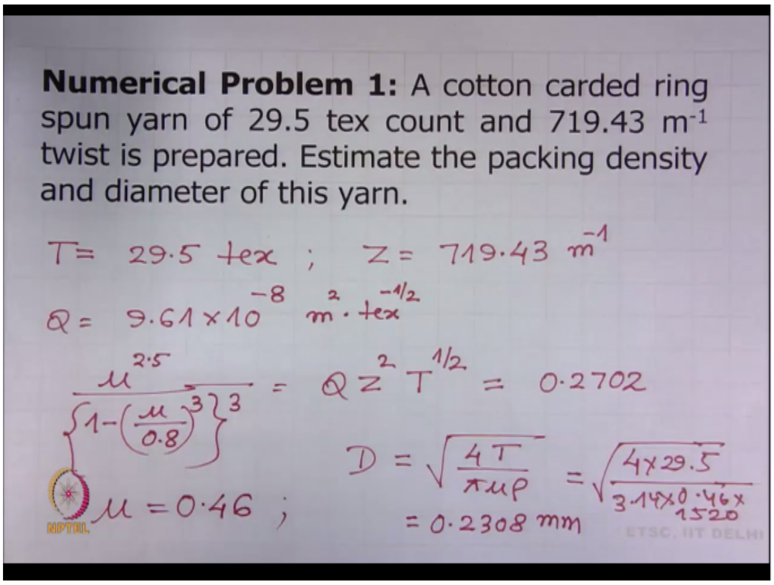

So you come back to this chart and then write mu is 0.46, so you find out the packing density right clear. Now how will you find out diameter? Diameter is root over 4T/pi mu rho right. Now what is your 4*29.5 tex/3.14*0.46*1520 right. So you will find out this value as 0.2308 millimeter clear. You will find out 0.2308. So in this way this problem can be solved. I hope it is clear to you.

In the next class, we will continue with this module and solve one more numerical problem. Thank you very much for your attention.