Theory of Yarn Structure

Prof. Dipayan Das
Department of Textile Technology
Indian Institute of Technology – Delhi

Lecture - 08
Helical Model of Fiber in Yarn

Welcome to you all to this MOOC online video course Theory of Yarn Structure. In the last class we started module 3 relations among yarn count, yarn twist and yarn diameter. If you remember, we first started with Copeland's theoretical concepts on how to twist a staple fiber yarn. Then we started with a simple mechanical model and we established a very important relation on packing density, yarn twist and yarn count so today we will start from there.

When we measure twist angle surface fiber twist angle that is angle beta D then it is probably not correct to consider diameter D. It is better to consider the diameter at which the axis of the surface fibers lie.

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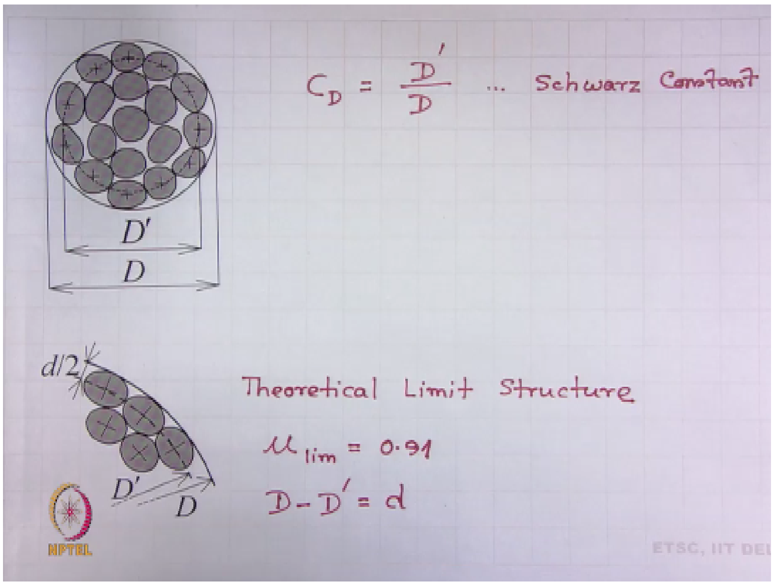

So in this image what do you see is that yarn diameter is capital D however when we make a twist angle then we look at this surface fibers at that time it is better to consider diameter D prime. D prime is a diameter constituting the axis of the surface fibers. Then we talk about one ratio C subscript D which we define by D prime/D this C subscript D is well known as Schwarz constant right. So C subscript D= D prime/capital D is known as Schwarz constant.

We need to find out a value for Schwartz constant. This ratio is not sufficient to determine

Schwarz constant because it is very difficult to measure D prime. We need to have an expression for C subscript D Schwarz constant so that we can determine practically and easily. So we need to first find out an expression for that so that is our first aim of this lecture. For that purpose we consider theoretical limit structure.

Theoretical limit structure we talked in module 2 hexagonal packing with closest packing. So distance between fiber H=0 and then we derived the packing density of this limit structure 0.91. This we have done in module 2. So in this limit structure what we see that the distance between the yarn diameter and D prime=D so that is basically the distance in case of theoretical limit structure then how we can find out now C subscript D.

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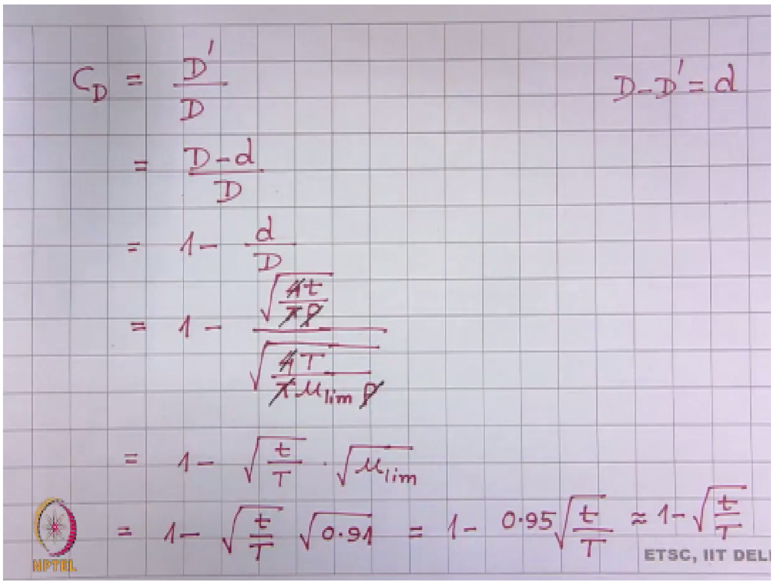

So C subscript D D prime/D right and we know that for theoretical limit structure D-D prime is small d. So we can write down D prime=capital D-small d/capital D see all are measurable quantities. Capital D yarn diameter possible to measure small d is fiber diameter also possible to measure so then we can write this=1-small d/capital D right. Now this small d we would like to write as 4 t/ pi * rho this we learnt in module 1 fiber diameter= square root of 4 times fiber fineness/pi time rho fiber density and what is capital D.

Capital D also we learned in module 2 4 times capital T/pi mu limit because it is theoretical limit structure*rho right. So what we see that this 4 and 4 cancels out this pi and this pi cancel out and this rho and this rho cancel out. And then as a result we find 1-root over this* mu limit right. Now if we substitute the value of mu limit then we obtain 1-/T 0.91 this ratio you will see as 1-0.95*/small t.

Now you see Schwarz constant for theoretical limit structure=1-0.95 very close to 1 square root small t/capital T small t is fiber fineness capital T is yarn count in tex. So both are measurable quantities small t and capital T so we can determine it. However, this 0.95 is very close to 1 so we can write this expression as (()) (09:16) right. So this is the expression for Schwartz constant in case of theoretical limit structure.

What happens to practical yarn structure? In practical yarn structure capital D is higher than that of theoretical limit structure. Also D prime in practical yarn structure is proportionately higher than the theoretical limit structure as a result of this Schwarz constant remains practically same as it is in case of theoretical limit structure. So the value of Schwarz constant does not deviate too much in case of practical yarn structure as compared to that in case of theoretical limit structure right then we try to find out modified angle.

So angle at D prime the surface for the twist angle at yarn diameter D or beta subscript. Now what is the angle at diameter D prime.

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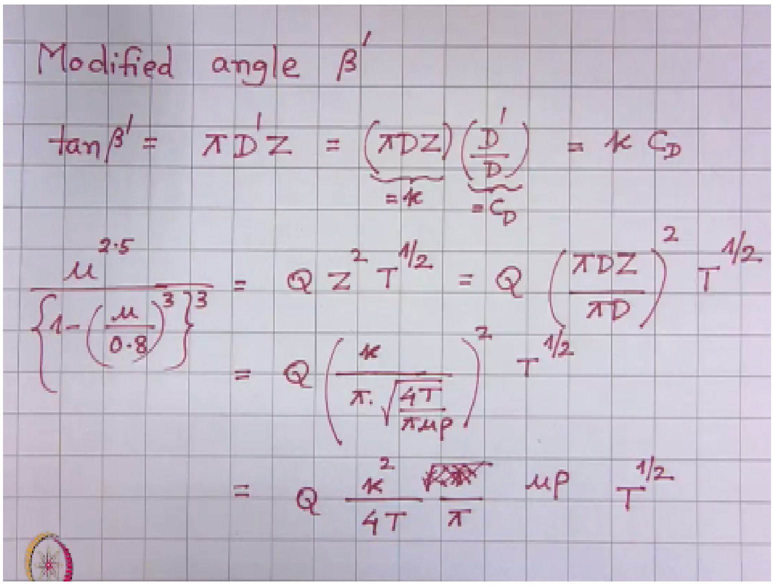

So this angle let us say modified angle we name it as beta prime. So tangent of beta prime is= pi times D prime Z okay. Let us write this in simple different little different form pi DZ D prime/D. Now you see what is pi DZ pi DZ is twist intensity that is=kappa so this is= kappa and what is your D prime/capital D that is=Schwarz constant C subscript capital D. So this expression tangent of beta prime is= kappa multiplied by Schwarz constant right.

Then what happens to our last class expression mu to the power 2.5 1-mu/0.8=Q Z square T to the power 1/2. Remember this expression from previous class. Now if we need to change this expression we need to work with Z why because this Z is involved in tangent of beta prime is not it so let us do that. So let us write this expression Q instead of Z we would like to write pi DZ/ pi D that is basically Z square and T to the power 1/2 we do not want to change this yarn fineness okay.

So what we can write in the next step is Q what is pi DZ kappa and what is pi D pi times D. D is your square root of 4 times T pi mu rho that is your D now this square T to the power ½ okay. So Q kappa square 4 T will come here and this will go * mu rho*T to the power ½. No we did a mistake here this will come here yes this will come here right.

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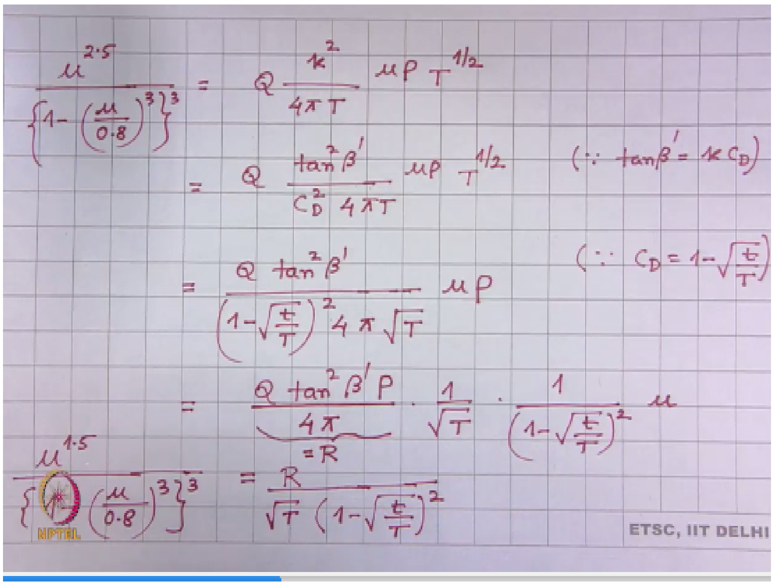

Then we will further work on this expression mu to the power 2.5 1- mu to the power 0.8=Q then your kappa square/4 pi T mu rho T to the power 1/2. Now what is kappa we know that tangent of beta prime=kappa times Schwarz constant then we can substitute Q tan square beta prime/Schwarz constant square 4 T mu rho T to the power 1/2 right. Now what is CD? CD is 1-we have derived it earlier 1-this is not it that is your CD.

So we substitute this Q beta prime 1-t/T square 4 pi. So this T and this square root will cancel out we will find out capital T *rho okay. So we write it little different from Q tan square beta prime*rho/4 pi let us club them together *1/ capital root T *1/whole square and what is remaining is mu okay. So this mu should come to the left hand side then we will be obtaining an expression for mu.

So if it comes to the left hand side then what we obtain is mu to the power 1.5 right not 2.5 because this 2.5-1 so 1.5. 1-mu so this is the left hand side. Right hand side let us write this as a parameter capital R we will discuss about this little later. So Q tan square beta prime*5/density/4 pi it is a parameter we call this parameter R so this R okay/root over T*1-this square right. So let us write it in a nice manner.

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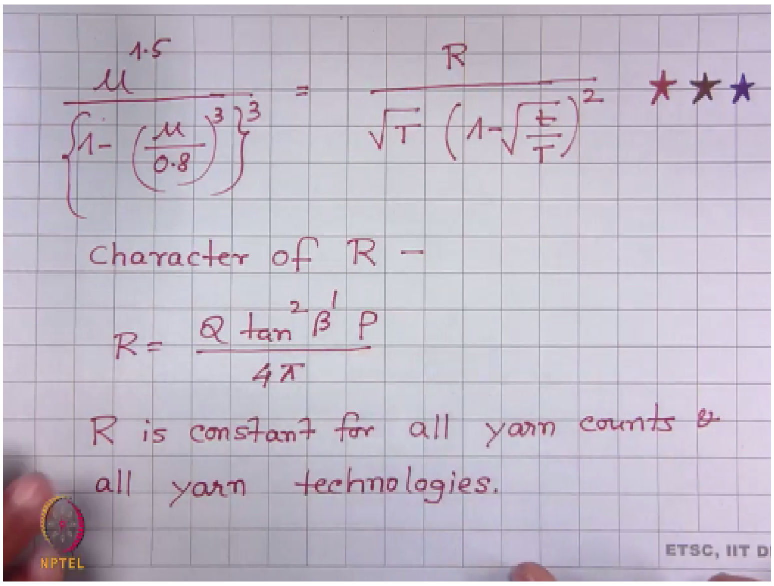

So what do we obtain is mu to the power 1.5/1-mu cube to the whole cube R/root over T *1-small t/T square right. So this is another important expression. In the last class we obtain one important expression in today's class we obtain another important expression. They have tremendous application in practice okay. Now before going into the application part let us discuss a little on the character of R.

What is R? R is Q tan square beta prime* phi by density /4 pi. This 4 pi is constant and Q is a parameter rho is constant for a given fiber and tan square beta prime. Now it is generally seen our practical experience say that R is typically constant for all yarn counts and all yarn technologies and there is an R varies for different fibers because of the involvement of rho and also other parameters. So it is necessary for us to know the value of R for different fiber.

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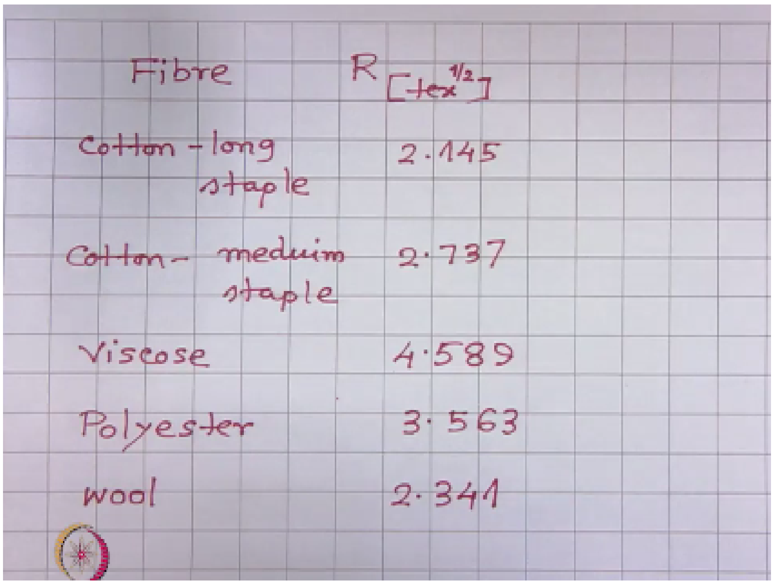

So fiber and R by the way what is the unit of R unit of R will be this cotton long staple. We generally find the value of R for cotton long staple as 2.145 cotton fiber medium staple the value of R we found little higher 737. Similarly, for viscose this value we found much higher 4.589 what is the value of R for polyester 3.563 and also a value of R is reported for Wool fiber what is the value 2.341. So these are the values of R reported in literature so now finally we will come to application point of view. Now we have 2 important equations.

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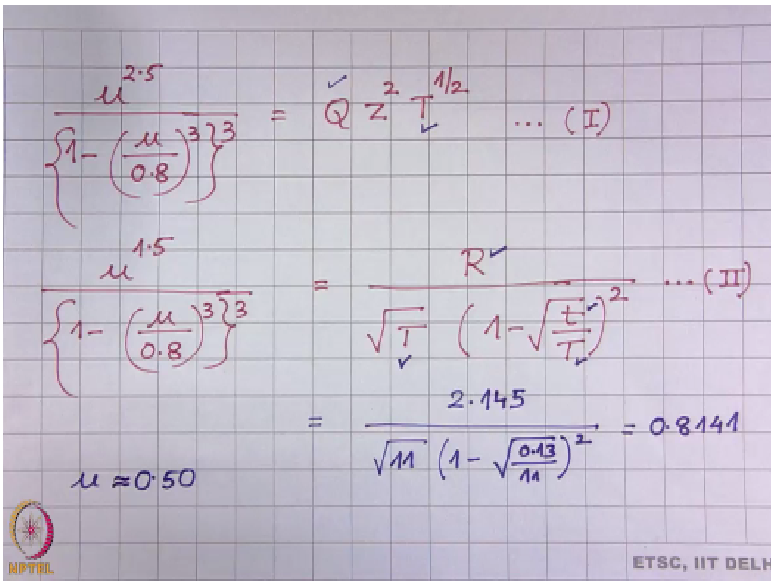

First equation what we derived in the last class Q right so this was let us write our first expression and what is the second expression second expression we derive today this is our second expression all right how we can use these 2 equations for practical application so this is our question. Now when practical application comes that time we generally know the desired yarn count in a spinning industry we have to produce yarn.

So generally what is the count of yarn that we have to produce this information is generally known. So capital T is generally known before production of yarn. Yes, which fiber I have to use whether it is cotton or it is polyester, or it is viscose if it is cotton then it is long staple cotton or short staple cotton and what is its fineness. This information is also known prior to yarn manufacturing that means small T is also known.

The moment you know about fiber whether it is cotton fiber or polyester fiber or viscose fiber that means you are aware about R how because you know about this value so you know about R that means the right hand side of second expression you will be able to calculate the value. So you will be able to calculate the right hand side of equation 2 then you will be able to solve this resulting second expression you will able to find mu.

You can use a suitable numerical method interval splitting method or you can already prepare table like in the last class we demonstrated you for solving one numerical problem so you will be able to find out mu. So the moment you find out about mu that mu that value of mu you will substitute in left hand side of equation one. So the value of mu you obtain from expression 2 you will substitute that value to the left hand side of expression 1 right.

Then which yarn technology we have to use this information is also well known before yarn production it is ring spin technology or rotor spinning technology. So that means the value of Q is known to you before yarn production and value of T is also known to you. So left hand side is known Q is known capital T is known what is unknown only one variable that is yarn 2 is DZ so if you solve this equation you will find out Z yarn twist.

So that twist you have to insert to yarn is that clear. Let me repeat it again so we need to learn how we can use these 2 equations for practical application. When the question of yarn manufacturing comes into picture we know about fiber fineness, we know about yarn count and also we know about which type of fiber we have to use R. So first we need to find out this expression right hand side of equation 2 then we have to solve for mu.

The value of mu what we obtain we will substitute in left hand side of equation 1 before yarn production Q is known capital T is also known what is unknown only Z. So one expression one unknown you will find out the value of Z so that much of Z that is yarn twist you have to insert to the yarn. So that is what is called suitable yarn twist how much twist you need to insert to yarn. You remember when you started module 3 such type of question came.

So by using these 2 equations we can find out how much twist is required to be inserted to yarn. Let us now solve 2 numerical problems this part then we will be very clear. So we will solve first this problem.

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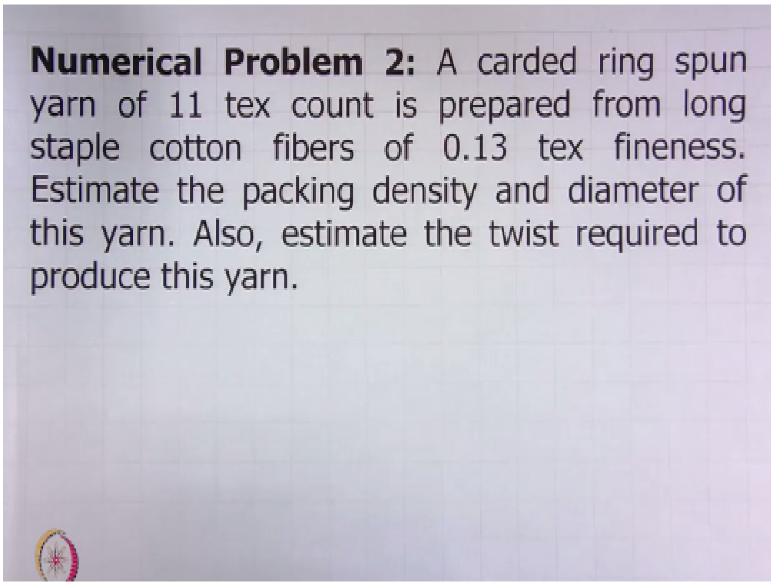

In the last class we solved one numerical problem numerical problem 1. Now we will solve we will solve 2 more numerical problem number problem 2. A carded ring spun yarn of 11 tex counts is prepared from long staple cotton fibers of 0.13 tex fineness typical practical problem. We have to produce a carded ring spun yarn count of that yarn is given 11 tex which fiber we have to use long staple cotton fiber what is its fineness 0.13 tex.

Estimate the packing density and diameter of this yarn also estimate the twist required to produce this yarn right. So we have to solve this problem. Now we will use these 2 equations for solving this problem. First we find out this so we start from here. It is long staple cotton so what is the value of R for long staple cotton 2.145 how I find this value from this table 2.145 this value.

Now what is yarn count 11 tex where from I find this already given in this problem 11 tex. What is fiber fineness 0.13 tex already given so 1-0.13/11 square. So you can find our this the right hand side of expression 2. What will be the value? This value will be 0.8141 okay. So this expression is=this what is the value of mu how you can find out as I told you 2 ways.

One you can adopt a suitable numerical method maybe interval splitting method or you can already prepare a table before hand and from that table you can find out the value of mu.

So we will start with the second case let us prepare a table.

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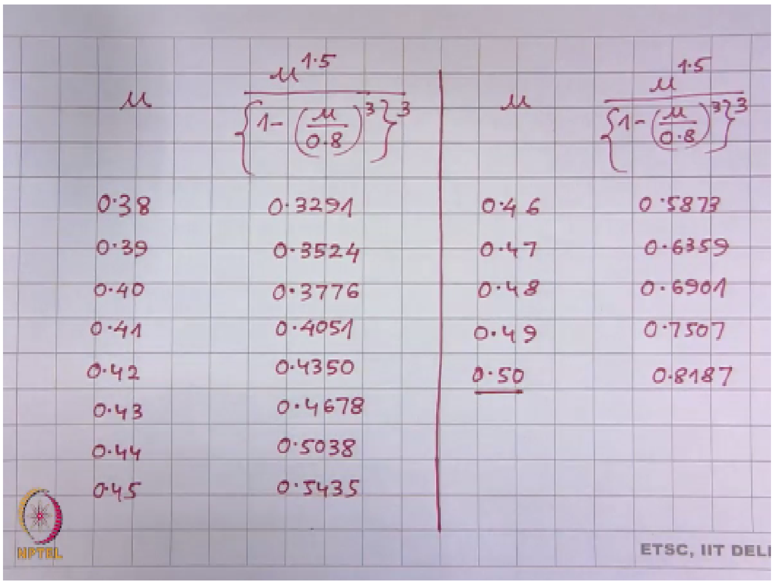

μ	$\frac{\mu^{1.5}}{\left\{1-\left(\frac{\mu}{0.8}\right)^3\right\}^3}$	μ	$\frac{\mu^{1.5}}{\left\{1-\left(\frac{\mu}{0.8}\right)^3\right\}^3}$
0.38	0.3291	0.46	0.5873
0.39	0.3524	0.47	0.6359
0.40	0.3776	0.48	0.6901
0.41	0.4051	0.49	0.7507
0.42	0.4350	0.50	0.8187
0.43	0.4678		
0.44	0.5038		
0.45	0.5435		

Table mu and mu to the power 1.5 like previous class 1-mu/0.8 cube to the power cube. So the value of mu is 0.38 you substitute mu 0.38 here and also 0.38 here then use a simple calculator and find out this value you will find out this value will be 3291 right. In earlier case what was the value the value was 0.8141 and this is 0.3291 so mu is not=0.38. Then you find out 0.39 you substitute 0.39 here you substitute 0.39 here what will be the value you obtain 0.3524 is that the value no.

Then 0.40 what will be the value of mu 0.3776 similarly 0.4051, 0.42, 0.4350, 0.43 0.4678 0.44, 0.5038, 0.45, 0.5435, 0.46 this value will be 5873 0.47 this value will be 0.6359, 0.48 this value will be 0.6901 0.49 this value will be 0.7507, 0150, this value will be 0.8187. And what was earlier 0.8141. So mu is approximately=0.50. If you use a suitable numerical method, you will find out the exact value right.

So come back to this problem if you solve then you will find out mu is 0.50 okay because this expression=0.8141 and when mu=0.50 this expression becomes 0.8187 right. So mu is roughly= 0.50 okay. This value of mu now you have to substitute here so if you substitute mu 0.5 here and also 0.5 here what you will obtain. You will obtain a value of this expression what will be this expression value now in the last class we solved one table.

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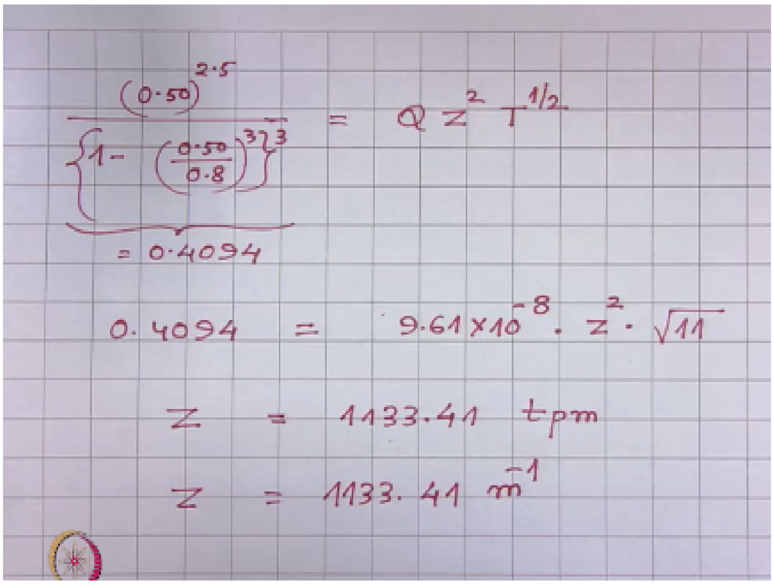

So now if we substitute 0.50 here to the power 2.5 1- 50.8 Q Z square T to the power1/2. So this value will be=0.4094 okay so 0.4094=Q what is the value of Q which yarn is this. Carded ring yarn for carded ring yarn what was the value of Q in the last class we discussed these values. Carded ring yarn cotton Q is 9.61*10 to the power -8 Z we need to find out T what is yarn count yarn count is given 11 tex right.

So 11 tex now this you will be able to solve and you will find out the value of Z so in this manner you will be able to solve this problem. So Z you will find out 1133.41 twist per meter. So basically Z is your 1133.41 meter inverse right. So in this manner you will be able to find out suitable yarn twist. Would you like to do one more exercise let us do it so it will be fully clear then.

Numerical problem 3 the last numerical problem of this module numerical problem 3 very similar problem, very relevant for practical application right.

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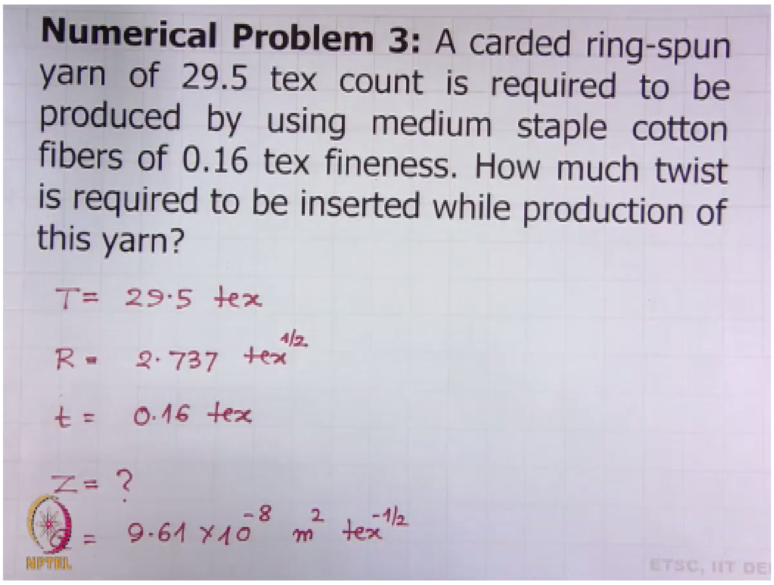

A carded ring spun yarn of 29.5 tex count so what is capital T 29.5 tex to be produced by using medium staple cotton fiber medium staple cotton fiber so the value of R is 2.737 of 0.16 tex fineness. So fiber fineness how much twist is required to be inserted while production of this yarn very typical industrial problem. You need to find out Z okay and the value of Q is also given is not it the value of Q is= carded ring yarn so 9.61*10 to the power -8 meter square right.

So this was the value of Q is also given you need to find out Z. So how we can find out Z.

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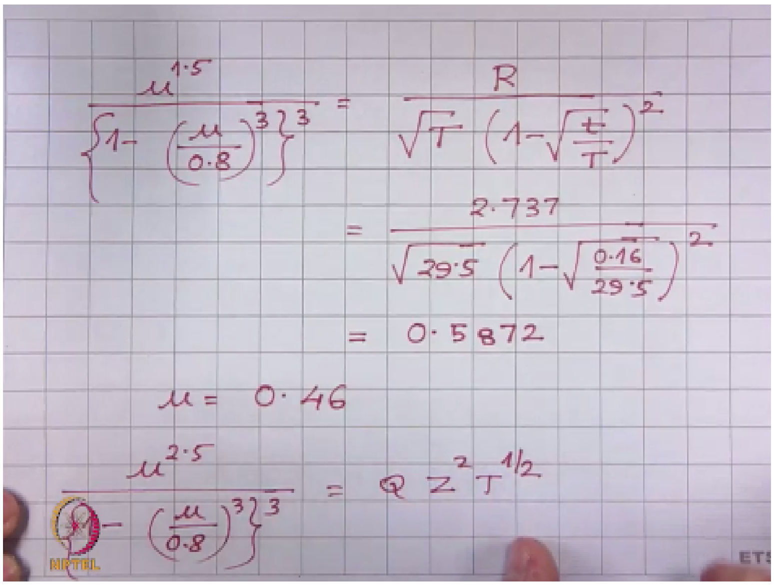

First we have to work with this expression okay. So what are the values given the value of R is given 2.737 value of capital T and also value of small t are given what is the value of capital T capital T is 29.5 tex is not it and what is your small t, small t is 0.16 okay. So what will be this value this value will be= 0.5872 then you find out the table again already prepared this table where is the value lying 0.5872, 0.5873 is very nearby right.

So the value of mu is 0.46 so then we write the value of mu 0.46 all right then we solve our first equation 2.5 1-0.8 cube to the power 3. Q Z square T to the power ½. If we now substitute mu 0.46 here.

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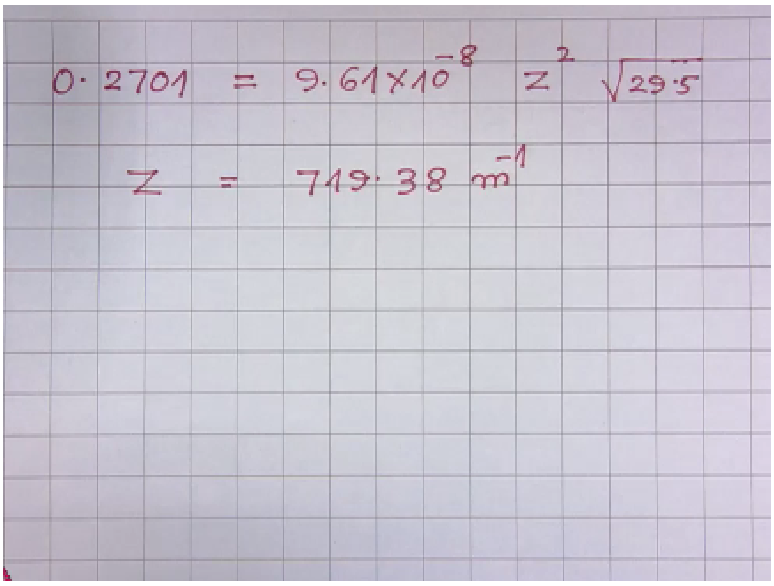

Then we will find out this value=0.2701=Q what is the value of Q here in this problem the value of Q is 9.61*10 to the power -8 Z square and what is the value of T, T is 29.5 okay. So if you solve you will find out Z=719.38 meter inverse. So roughly 720 turns to be inserted per meter of this yarn so if we summarize module 3 the target or the aim of this module was to find out a relation among yarn twist, yarn count and yarn diameter.

As I told you fiber packing density in yarn basically dictates this relationship. We started with Copeland's theoretical model which we did not find too much precise then we discuss certain empirical corrections to Copeland's model however they are empirical. Thereafter we started with a simple mechanical model and we derived 2 important relations.

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$$\frac{\mu^{2.5}}{\left\{1-\left(\frac{\mu}{0.8}\right)^{3}\right\}^{3}} = Q Z^{2} T^{1/2} \quad \ldots (\mathrm{I})$$

$$\frac{\mu^{1.5}}{\left\{1-\left(\frac{\mu}{0.8}\right)^{3}\right\}^{3}} = \frac{R}{\sqrt{T}\left(1-\sqrt{\frac{t}{T}}\right)^{2}} \quad \ldots (\mathrm{II})$$

One relation was mu this relation where mu is packing density Q is a parameter for different fiber, for different technologies the values of Q will be different and we have already told about those value Z is yarn twist capital T is yarn fineness then we talked about one more equation where mu is again packing density R is a parameter which depends on fibers for different fibers the values of R will be different.

We have already told about those values capital T is yarn count small t is fiber fineness then we have discussed how these 2 equations can be used to solve practical application. For example, typically in practice we need to know how much twist needs to be inserted so that is the purpose then first the right hand side of this expression 2 we need to find out generally capital T is given small t is given R is also given so we will be able to find out this value.

We will solve this and we will find out mu that value of mu we will substitute here we will find out this expression Q is generally known from the table T is already given so we will find out Z so we will find out suitable yarn twist. So that is how these 2 expressions can be used in practice and also we have shown how this expression falls in front of experimental results.

And we have found out that this expression can explain the yarn twist, yarn count and yarn diameter satisfactorily so this completes module 3. Thank you for your attention.