Theory of Yarn Structure

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Lecture - 09
Helical Model of Fibers in Yarns (Contd.,)

Welcome to this MOOC online video course Theory of Yarn Structure. Today, we are going to start module 4 Helical Models of Fibers in Yarns. As you are aware of Helical Model of Fibers in Yarns is a very popular model and often quoted in research articles. It is relatively old concept in Theory of Yarn Structure started from Cook Lyn 1828, then Muller, then Marchief, Giggof, Swargch, Brassler, Budnikov, Neskerds, Trailor many scientists worked on Helical Models of Fibers in Yarns.

Today this model is well established. The importance of Helical Model of Fibers in Yarn lies in 3 distinct cases. This model helps us to calculate number of fibers present in cross-section of yarn. This is a very big importance of this model second another very important application of this model is it explains yarn retraction. What is yarn retraction? When we twist a yarn the length of the yarn shortened this phenomenon is called yarn retraction.

In some books you will find out yarn contraction that basically denotes the same phenomenon however little difference exist between them. Why it was important to understand yarn retraction because what governs yarn retraction. If we do not know about this, we will not be able to control the shortening of yarn so it was necessary to understand the phenomenon of yarn retraction.

This model Helical Models of Fibers in Yarn explains yarn retraction phenomenon scientifically. Third important application of this model lies in yarn twisting. What happens if we go on increasing the twist in yarn after the so called saturated twist if we still increase the twist then the coils do not go inside the yarn and we obtain a very undesired structure of yarn. So we should not go to that level of twisting that means in practically also there is a limit of twisting what is that limit of twisting we must know beforehand.

We should not reach to that twist then we will spoil yarn structure. So basically these 3 phenomena are well explained by Helical Model of Fibers in Yarns. In this module we will study these 3 phenomena under Helical Models of Fibers in Yarns. So first let us clarify this concept what is Helical Model of Fibers in Yarn. The Helical Model of Fibers in Yarn is based on 4 important assumptions.

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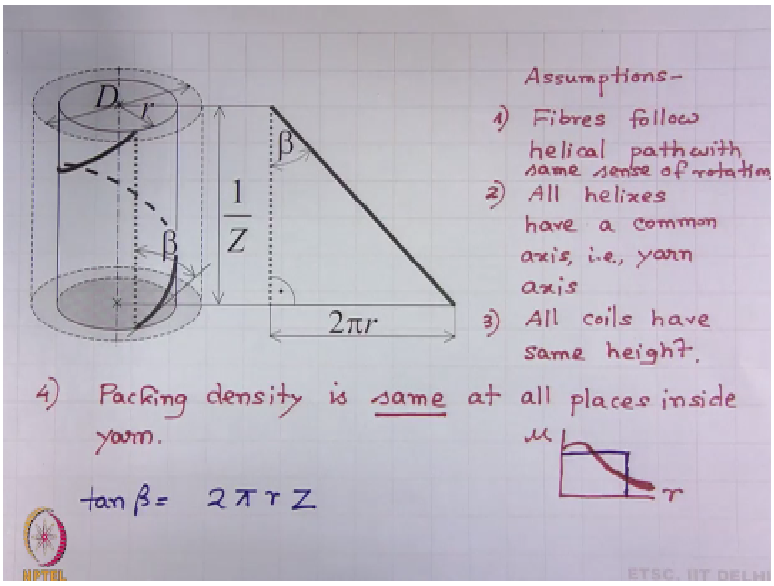

First 3 are very general assumptions and the fourth one very important is a very special assumption. The first assumption fibers follow helical path in yarn. So you see it is a yarn cylindrical yarn and this fiber follows helical paths here you see thick portion of the fiber which comes on the surface then the fiber goes inside dotted line you do not see them then again it goes to surface so you see them thick black line. So the fibers follow helical path second all the helixes have a common axis of rotation that is yarn axis okay.

Third all coils it seems to be a coil so all coils have same height. So little elaborate the first assumption fibers follow helical path with same sense of rotation let us write in this manner. So as I told you helical model is based on 4 assumptions the first 3 are very general assumptions. First assumptions fibers follow helical path inside the unstructured with a same sense of rotation so the direction of rotation is same.

Second all helixes have a common axis that is your yarn axis. Third all helixes have a common height; coil height is same. Fourth the very, very special assumption packing density is same at all places inside yarn. Packing density is same at all place inside yarn. So actually the first 3 are quite general assumption, but the fourth one is a very special assumption why? In practice you know mu packing density is not constant however it is a function of R.

So this is the practically we see this kind of behavior the real behavior. This probably goes a little low something like that, but here the assumption is same that means what we think is that you probably think in this manner this is the difference right. So the red is indicating the real behavior however the blue indicate this assumption. Now this helical model is based on this 4 assumptions right.

Now this image you see it is basically a yarn cylinder, it has a diameter capital D and the inside this cylinder or diameter D there is one small cylinder of radius r where a fiber helix is shown. This helix makes an angle beta from yarn axis if you unroll this cylinder make it flatten then you will obtain this triangle where the base of the triangle=2 times pi times small r.

R is yarn radius D is yarn diameter and this beta is the twist angle. The height of this triangle is 1/Z, Z is number of turns per unit length of the yarn so this length is 1/Z and this angle is beta. So what do you obtain is tangent beta is 2 pi R Z right we obtain this 1 tangent beta is 2 pi RZ so this is the concept of helical model. Now we will find out number of fibers present in yarn cross-section.

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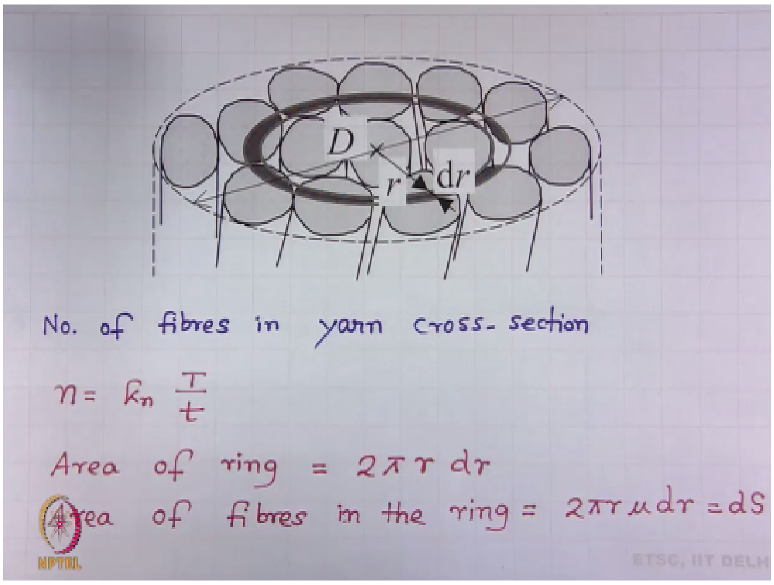

So our aim is to determine a relation which will predict number of fibers in yarn cross-section. You will ask me what is the need to predict we know that n is this, this is true. Capital T is very easy to find out small t is also very easy to find out, but kn coefficient kn is relatively difficult to determine experimentally why because you require microtome apparatus, you require suitable utensils for cutting cross-sections of yarn, you require a light microscope.

So many common textile laboratories may not have this facilities that is why it become very difficult to find out number of fibers experimentally, but often we need to predict it for theoretical research or for some empirical research we need to find out what is the value of n how we find out that is why it is necessary to find out a relation theoretical relation that predicts number of fibers in yarn cross-section right.

So our aim is to find out a relation that predicts number of fibers in yarn cross-section and we will use helical model for that purpose. What do we see here in this image it is this is the cross-section of the yarn lot of fibers are present say n numbers of fibers are present? This is yarn diameter and we see one thick annular ring which is situated at a distance r from yarn center and the thickness of this annular ring is d times r right.

That means first we will find out how many fibers are present in this annular ring then if we integrate that expression from r=0 to r=d/2 we will find out number of fibers in yarn cross-section right. So let us follow the same strategy. Now what is a area of this ring 2 pi r dr so this is the area of ring what is the area of fibers in the ring this is the area of the ring what is the area of fibers in the ring if we multiply this by packing density we will find out because packing density one of the interpretation is area of fibers/area of yarns right.

So 2 pi r mu dr mu is packing density which is constant at all places packing density here is mu, here is mu, here is mu, here is mu everywhere packing density is same that was our fourth assumption of helical model right. Let us think that this area is=this. You remember the symbol capital S substance cross-section area of yarn which basically consists of sectional areas of fibers so ds okay. Now you will continue with this image with one more additional image here it is shown.

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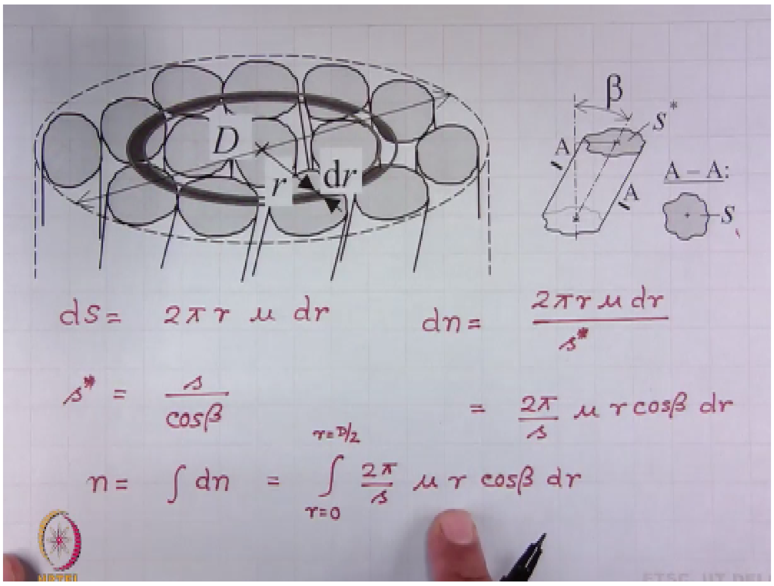

So what we found ds is 2 pi r mu dr. This image is basically the image of a fiber inclined fiber because of the twist fiber is not straight they are not parallel along yarn axis they are inclined at an angle. This angle is beta so this area is a sectional area S star and if we go along this axis at cross-section we will find out cross sectional area this is your this S okay. Now what is the relation between these 2 areas.

This we have already found out in module 2 s star= s/ cos beta okay. Now you tell me how many fibers or fiber sections precisely are present in this annular ring that is small in numbers. How many fibers or how many fibers sections are present in this small ring how will you find out. Suppose that number is dn total area/mean sectional area is not it. So total area is your 2 pi r mu dr/sectional area s star is not it.

So if we substitute dr if we substitute s star then we find out this expression right. So what was our target our target was to find out an expression for n how will you find out n, if you integrate one dn so let us indicate r=0 D/2 2 pi/s mu r cos beta dr. Now beta is a function of r is not it how they are related. Okay before going to that this 2 pi is constant small s is fiber cross sectional area also constant mu we assumed constant because of fourth assumption. So then r is a variable and beta is also a function of r.

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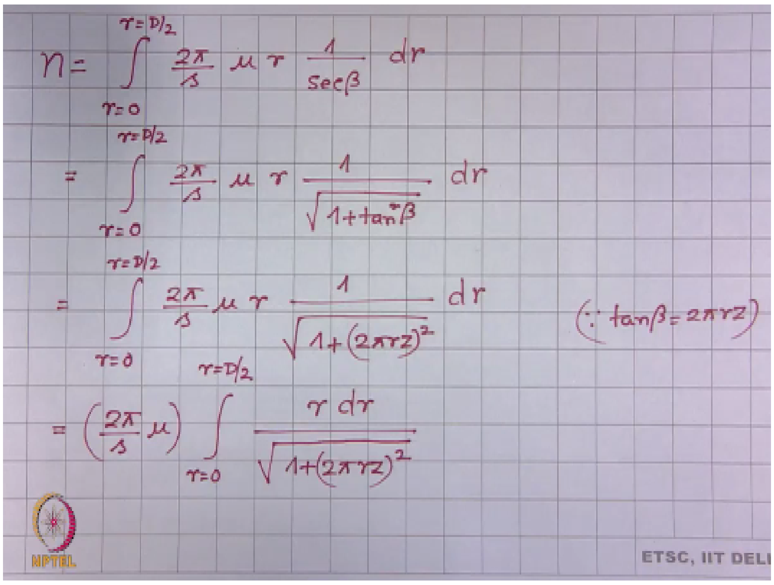

So what we will do now n is= integration r=0 r=D/2 2 pi/s mu r cos beta let us write cos beta as 1/sec beta dr 0 D/2. Now basically what we want to convert this as a function of r what is sec beta square root of 1+tan square beta dr what is tan beta we have already derived it dr. Now we will be able to write it in a better manner 2 pi/s * mu this we assume to be a constant integration r dr square this integration we have to solve. We will follow method of substitution for this.

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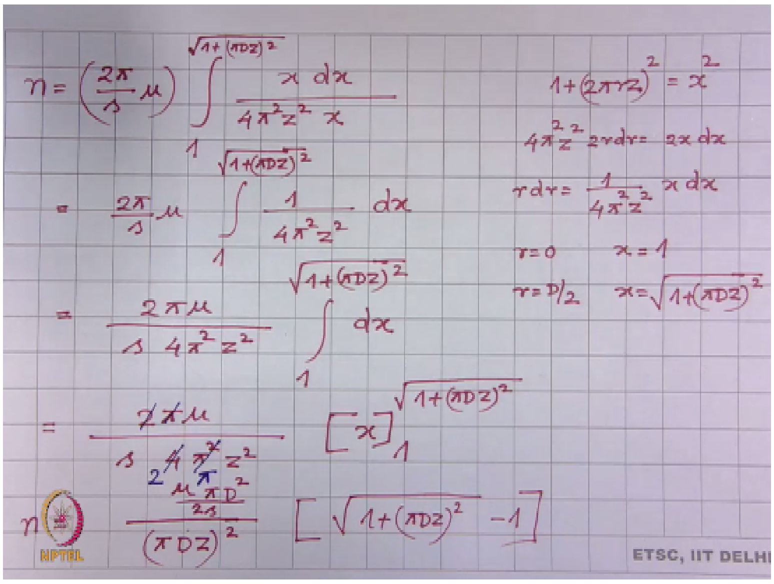

So what we will consider 1+2 pi rz square= you will consider x square. Okay so r dr= 1/4 pi square Z square * x dx this substitution we will consider okay. So then what we will write n= 2 pi/s * mu this limit now we will change. When r= if r=0 then what will be x x=1 when r=D/2 this 2, 2 will cancel pi DZ x will be root over 1+pi DZ square okay clear about this limit okay now we substitute so x will be 1 and this will be 1+pi DZ square upper limit.

And r dr so if it is r dr so then we will write x dx/4 pi square Z square and in the denominator also the denominator was x so we write x. So then become simple 2 pi/s mu integration 1 to 1+pi DZ square 1/4 pi square Z square dx. So this 4 pi square Z square is also constant it can come out of this integration. So 2 pi mu s 4 pi square Z square 1 1+pi DZ square* dx integration of dx=x 2 pi mu s 4 pi square Z square integration x lower limit 1 upper limit 1+pi DZ square right.

Now let us make it little simple so now what do we see here is that this 2 and this 2 this pi and this pi square pi. So what we do n is= let us write it here as pi DZ square if we write in the denominator pi DZ square then what will be in the numerator. In the numerator it will be mu pi D square right because one pi was already there. So pi square D square Z square pi Z square all right and then you will have this 2 times s okay.

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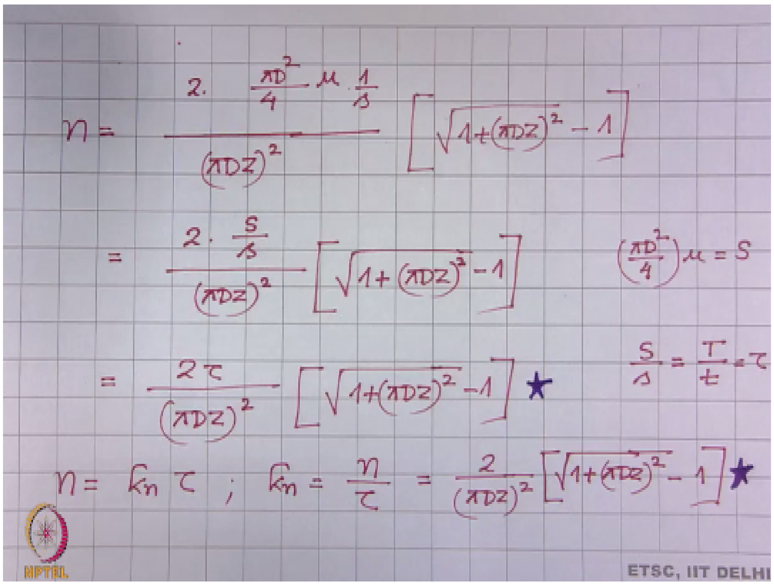

Now n is= this in the numerator what we will do we will write 2* pi D square/4 *mu* 1/s root over 1+pi DZ square-1 right pi D square/2 s* mu clear. Now what is pi D square/4 *mu that is= capital S so we can write 2* capital S/small s pi DZ square root over 1+pi DZ square-1 okay. What is capital S substance cross sectional of yarn what is small s cross sectional level of fiber So this ratio capital S/small s we have already derived that this=capital T/small t right so that is= relative fineness tau.

You remember in module 2 this relationship we have already derived in module 2 relative yarn fineness tau= capital T/small t yarn count in tex/fiber fineness that is= substance cross-section area capital S/fiber cross-section area small s. So we can now write down 2* tau pi DZ square root over 1+pi DZ square-1. This relationship is what we were looking for. So we have now got a relation for n tau relative fineness capital T/small t very easy to determine practically.

Pi DZ also possible to determine because Z is already known to you or using module 3 we will be able to determine Z and capital D also root over 4t/pi mu rho so we will be able to determine capital D. So n is now possible to calculate using this expression right. Okay let us now proceed a little further this is your n and what we know n is kn* tau this is true always whether it is helical model or not does not matter this expression is true always.

Then what is your kn? kn is n/tau this expression we have already derived in module 2 n/tau so if we substitute n from here what we see is 2/pi DZ square root over 1+pi DZ square-1 right. So we can also find out the value of kn we can calculate the value of kn also we can find out the value of small n number of fibers in yarn cross-section. Now this is also another important expression so let us put a star mark here okay. It is also possible to express n and coefficient kn in a different manner so let us do that.

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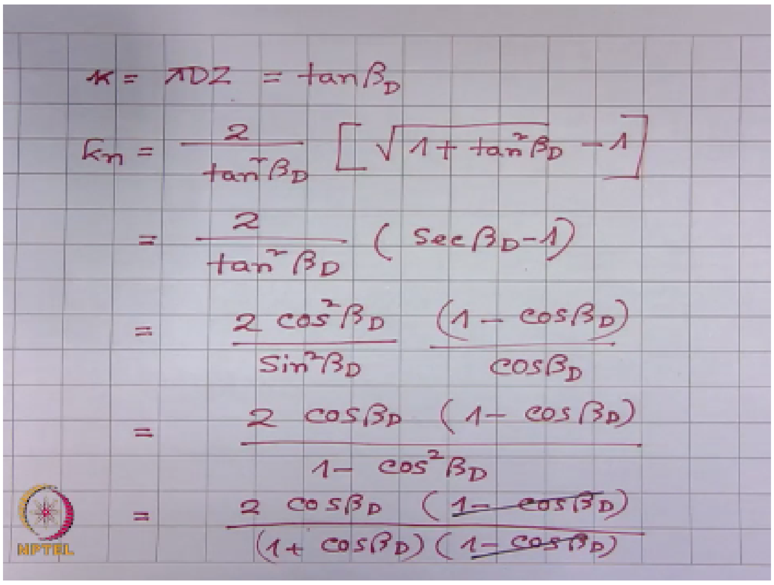

What is your kappa pi DZ and what is your pi DZ tangent of beta D beta D surface fibre twist angle okay then if we substitute here first let us find out for kn kn is 2/pi DZ square tan square beta D within square bracket root over 1+pi DZ square. So pi DZ square is tan square beta D-1 right. Look at the expression within square bracket root over 1+tan square beta sec square beta square root of that sec beta.

So 2 tan square beta D sec beta D-1 what is sec beta D 1/cos beta D so 2 and tan square beta D is sin square beta D/cos square beta D. So 2 cos square beta D/sin square beta D 1-cos beta D/cos beta D right. Further 2 cos beta D 1-cos beta D/sin square beta D sin square beta D is 1-cos square beta D. Further we can write 2 cos beta D 1-cos beta D 1+cos beta D 1-cos beta D. Now what we see is that here these 2 terms are cancelling out.

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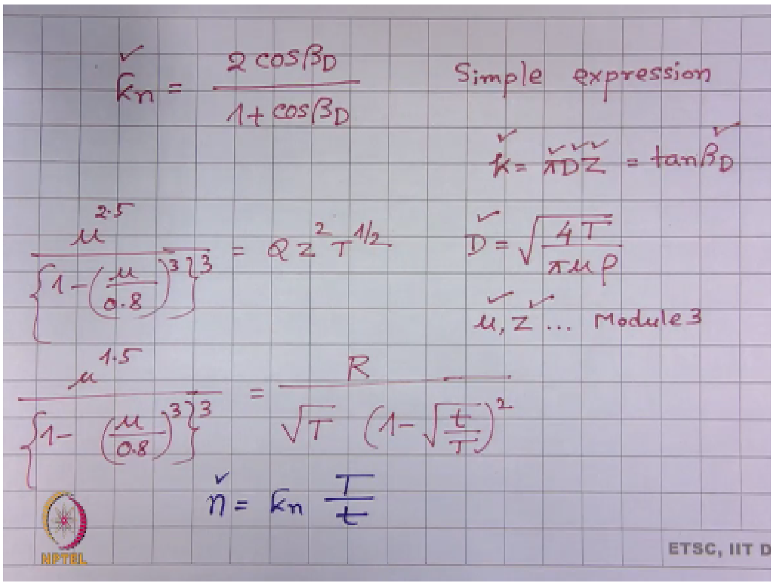

So what do we obtain kn= 2 cos beta D/1+cos beta D is not it simple expression. Hence it is more acceptable. So we found quite simple expression for coefficient kn 2 times cos beta D/1+cos beta D it can be made further simple you can try for that. Now what do we see is that if we know kappa pi DZ. If we know diameter pi mu rho for that you need to know mu. So this mu and this Z this we can know from module 3 right.

So by using those 2 expressions of module 3 we can find out mu and Z remember those 2 expressions. So by using these 2 expressions we can find out mu and Z how first we will find out this value then corresponding mu value we will find out this mu value will substitute here then corresponding Z value we will find out. So theoretically we will be able to determine mu and Z then we will be able to determine D then we will be able to determine pi DZ kappa.

So if we know that is= your tangent of beta D so we will be able to determine beta D if we know beta D we will be able to find out kn. So by using this what we describe just now we will be able to determine the value of coefficient kn and if we know coefficient kn then we will be able to find out n because coefficient kn is known capital T/small t are very well known quantities for yarn.

We will be able to find out this small n number of fibers present in yarn cross-section right. So now you will go and talk about this behavior of this expression.

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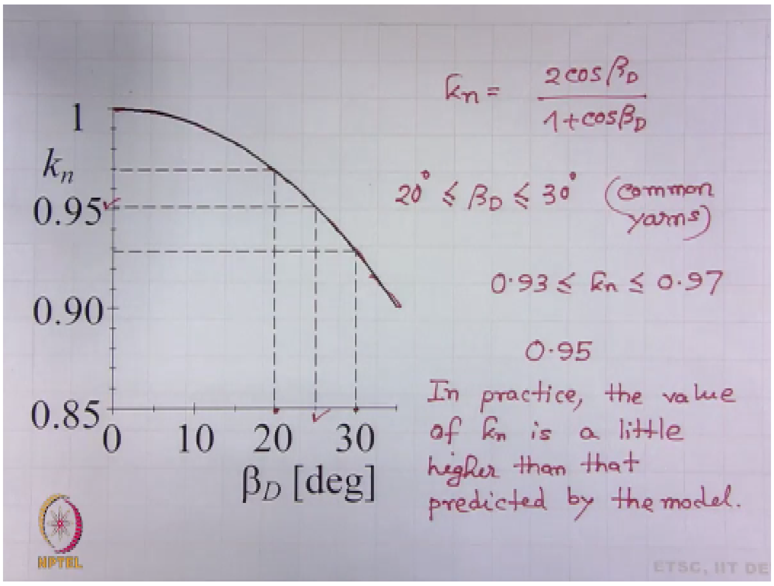

What expression kn= 2 times cos beta D/1+cos beta D. So this expression is graphically plotted here along the x axis beta D in degree is plotted then we calculate kn from here and we plotted kn along this y axis and this is the curve of kn for different value of beta right. Generally, this angle beta D for common yarn will lie from say 20 degree to 30 degree so for common yarns this angle beta D lies from 22 to 30 degree.

When beta D is 20 degrees, beta D is 10 degree here corresponding kn is 0.97 is not it corresponding kn is 0.97 and when beta D is 30 degree corresponding kn will be close to 0.93. So=kn 0.97. So for ring spun yarn this kn lies from 0.93 to 0.97 what is the average, average is 0.95. So average is somewhere here 0.95 that means somewhere here right. So that is why you remember in module 2 when we define coefficient kn that time we told for ring yarn kn is typically 0.95 and for rotor yarn kn is typically is 0.80.

That 0.95 is basically coming from here, but in practice the value of kn is little higher for ring yarn then that predicted by the model little higher that is what we observe why is it so probably our fourth assumption packing density is same at all places in yarn is the reason for this behavior. However, packing density is a function of R and what is that function is still unknown.

So unless and until we characterize this function properly and put it mathematically inside the integration and solve for that. This can be done possibility exist, but before that we have to solve for that function right. In any way so we have demonstrated you how this helical model can be used to find out the number of fibers present in yarn cross-section as well as how to determine coefficient kn in a yarn.

So as I told you this has been a very important contribution from Helical Model of Fibers in Yarn. Another important contribution of Helical Model of Fibers in Yarn lies in explaining the phenomenon of yarn retraction. When you insert twist the length of yarn shortened this behavior is known as yarn retraction. What is the fundamental principles lying behind yarn or underlying yarn retraction can be understood from helical model? This we will talk in the next class. Thank you for your attention.