Textile Product Design and Development
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Lecture – 20
Bungee Jumping Cord

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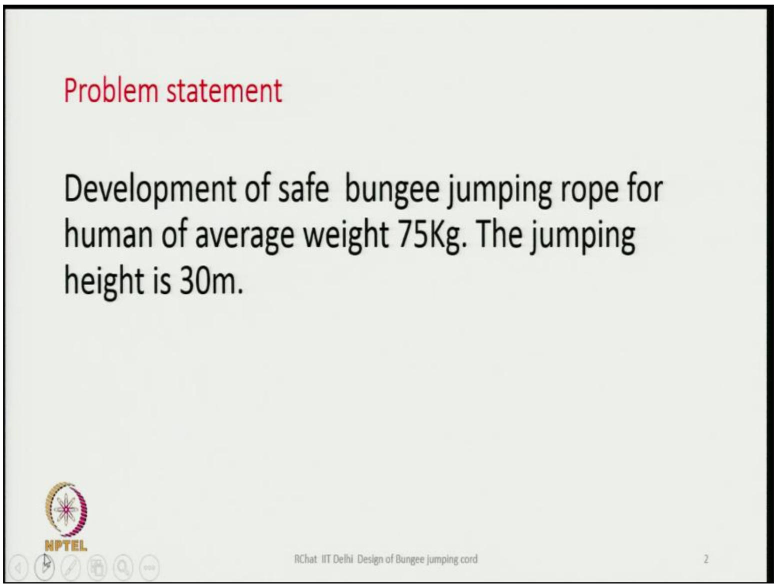

We will discuss the design of a bungee jumping cord, a specialized sports product. Let us consider developing a bungee cord for a person of average weight, around 75 kg, with a jumping height of 30 m.

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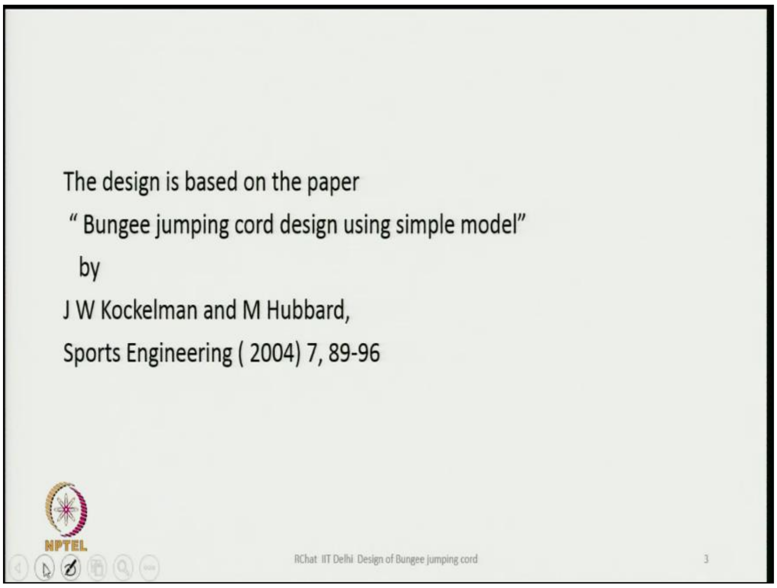

The discussion is based on a research paper by J.W. Kockelman and M. Hubbard, titled 'Bungee Jumping Cord Design Using a Simple Model'.

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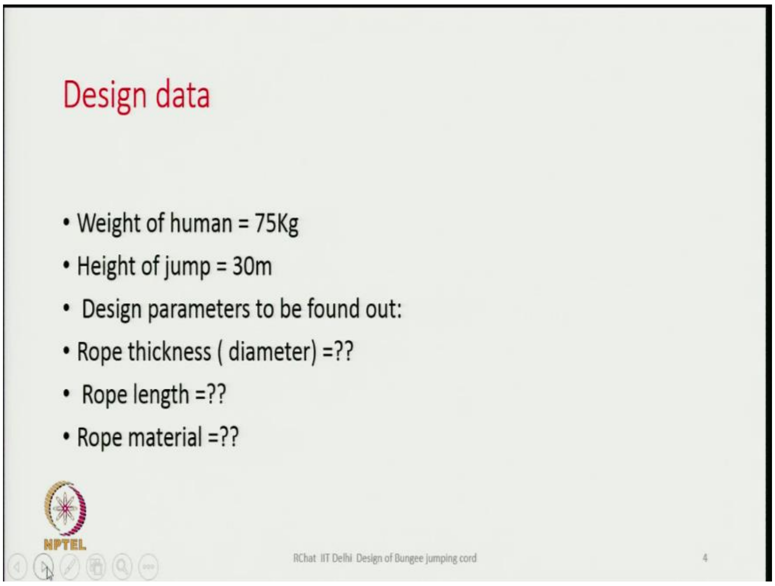

Let us delve into this research paper, which offers valuable insights into designing a product like a bungee jumping cord. This principle can be used in other types of similar products. The design data provided includes the weight (or mass) of the person, which is 75 kg, and the jumping height, which is 30 m. The jump could be from a cliff or a bridge over a river, ensuring enough space downwards. With 30 meters of space available, several design parameters for the bungee cord must be determined. These include the rope thickness or diameter, the required length of the rope, and the appropriate material for the rope.

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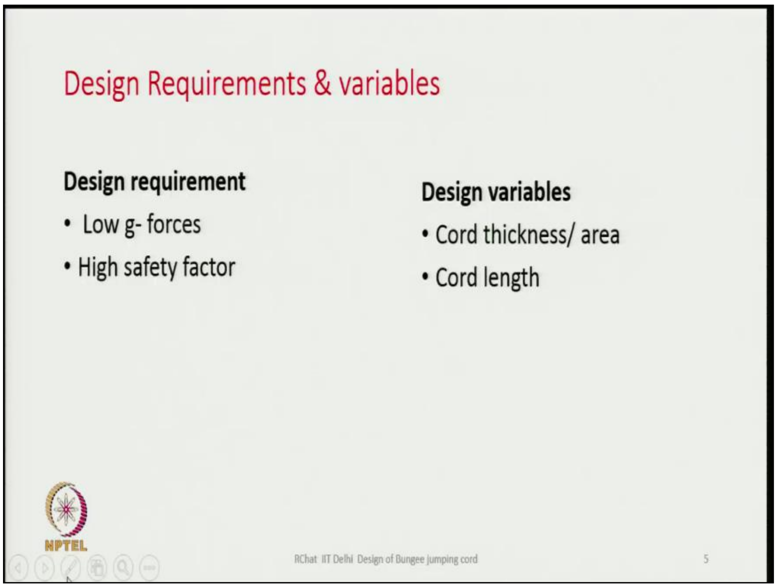

The key requirement for the bungee cord design is to minimize g-forces. High g-forces can be harmful to the individual, so it needs to be ensured that they remain at tolerable levels. The design must incorporate a high safety factor to prevent the cord from failing during use. The primary design variables in this case include the cord thickness, the cross-sectional area of the cord, and the length of the cord that should be utilized.

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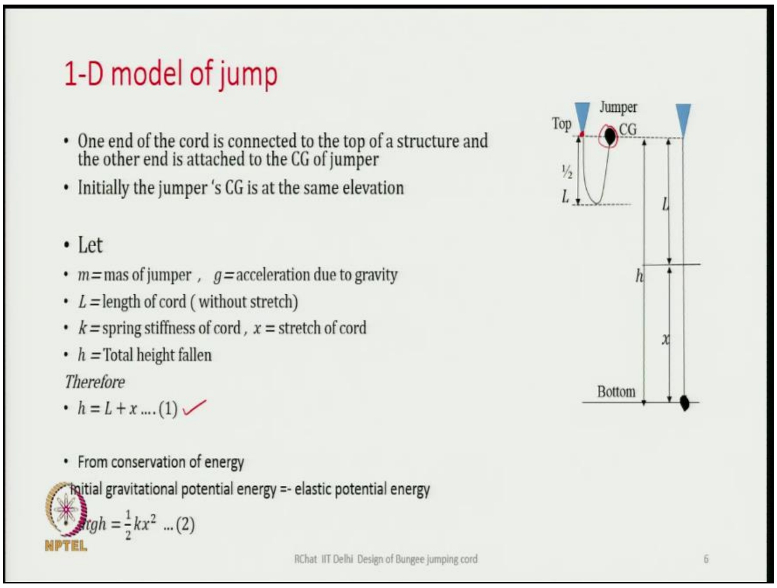

Here, the schematic on the right-hand side illustrates a one-dimensional model of the jump. In this diagram, the jumper is seen, with the cord attached at one end to the jumper's centre of gravity (CG) through a harness and the other end connected to the top of a structure. The diagram also shows the length of the cord, its extensibility, and the total length, including any extensions during the jump.

Initially, the centre of gravity (CG) of the jumper is at the same elevation as the point where the cord is attached. Let 'm' represent the mass of the jumper, 'g' be the acceleration due to gravity, 'L' be the unstretched length of the cord, 'k' the spring stiffness of the cord, 'x' the stretch of the cord, and 'h' the total height has fallen. From the diagram, the equation is written as '$h = L + x$'.

As the person jumps, they first fall by a height equal to 'L'. Beyond that point, the cord begins to elongate until the entire gravitational potential energy is absorbed by the cord's extension. Therefore, the total height fallen is '$h = L + x$', as shown in the diagram. By applying the conservation of energy, the initial gravitational potential energy must equal the elastic potential energy of the cord. Hence,

$$mgh = \frac{1}{2}kx^2$$

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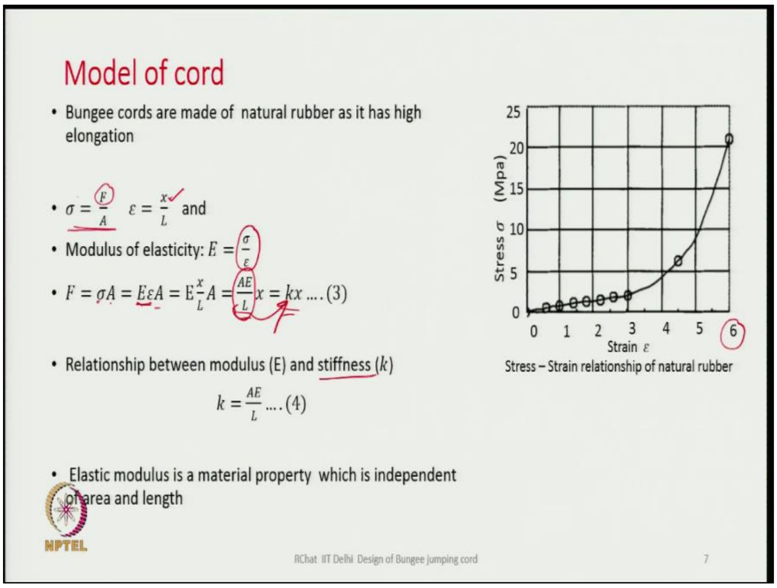

Next is the model of the cord. Bungee cords are typically made of natural rubber due to its high elongation. This extends because the energy of the person who is jumping has to be absorbed. Natural rubber or similar synthetic rubbers are ideal materials for this purpose, as the extension is an important requirement. On the right-hand side, a stress-strain diagram of natural rubber illustrates the relationship between strain and stress in the material.

The natural rubber has a breaking strain of around 6, meaning it can stretch up to 600%, which is stretchable. Synthetic rubber, which is one of the extendable materials that is used in textiles, is spandex or lycra. They also have high extensibility and are used in many textile products like stretch denim, which are very popular. Similarly, in textiles, whenever there is a need for stretch to enhance comfort, Lycra or spandex is used.

However, natural rubber is typically used for bungee cords. When examining the stress-strain diagram of natural rubber, it can be seen that it exhibits a non-linear relationship. It is known that stress is defined as force per unit area, while strain is the ratio of extension to the original length. The modulus of elasticity, or Young's modulus, is the ratio of stress to strain.

From here, the force can be expressed as '$Force = Stress \times Area$', which is '$F = \sigma A$'. Since stress is equal to the modulus of elasticity multiplied by strain, the equation can be written as '$F = E\varepsilon A$'. Strain is written as '$\frac{x}{L}$', where 'x' is the extended length and 'L' is the original length. Substituting this, '$F = kx$', where 'k' is a spring constant. In the case of a spring, the force required to stretch it by 'x' is '$F = kx$', with 'k' representing the spring constant. This is a similar situation here.

The relationship between the modulus (E) and stiffness (k) is given by

$$k = \frac{AE}{L}$$

Hence, in the equation, '$\frac{AE}{L}$' is replaced with 'k'. The elastic modulus (E) is a material property which is independent of area and length. However, stiffness 'k' depends on the elastic modulus, the cross-sectional area, and the length of the material.

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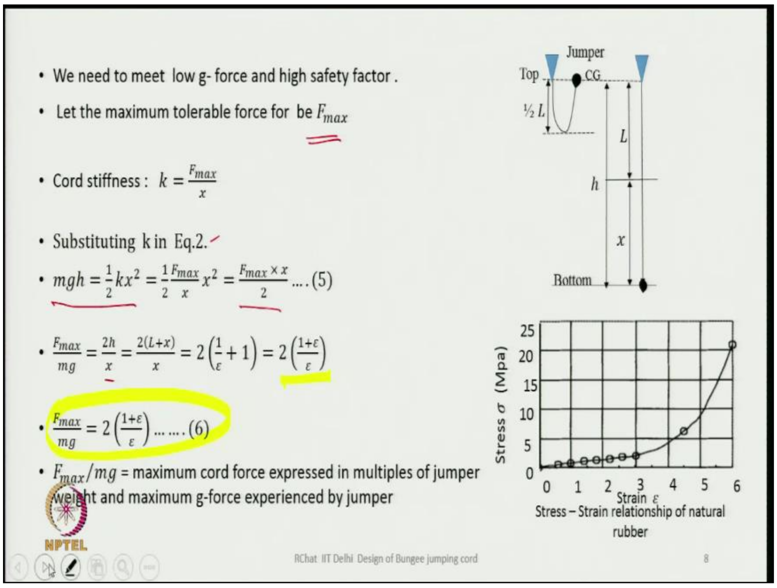

It is necessary to ensure that the g-force experienced during the fall is very low, g-force refers to the acceleration a person experiences while falling and the reverse force they feel because the cord will extend. To meet this requirement, the design must achieve low g-forces and maintain a high safety factor. Let the maximum tolerable force be denoted as 'F_{max}'. The stiffness of the cord 'k' can then be expressed as

$$k = \frac{F_{max}}{x}$$

Substituting the value of 'k' in the equation

$$mgh = \frac{1}{2}kx^2$$

this gives the equation of

$$mgh = \frac{F_{max} \times x}{2}$$

By simplifying further, it becomes

$$\frac{F_{max}}{mg} = \frac{2h}{x}$$

Since, '$h = L + x$', the equation can be written as

$$\frac{L}{x} = \frac{1}{\varepsilon}$$

This yields an equation,

$$\frac{F_{max}}{mg} = 2\left(\frac{1+\varepsilon}{\varepsilon}\right)$$

Hence, '$\frac{F_{max}}{mg}$' is the g-force that the person will experience as their velocity is decelerated by the cord. Hence, the human body has a certain tolerance to g-forces. If the g-force experienced during the jump is too high, it can lead to problems, and in extreme cases, the jumper could even be injured. Therefore, it is crucial that the ratio '$\frac{F_{max}}{mg}$' (which represents the maximum force experienced relative to the person's weight) remains within a tolerable limit. This ratio is the function of the strain of the bungee cord.

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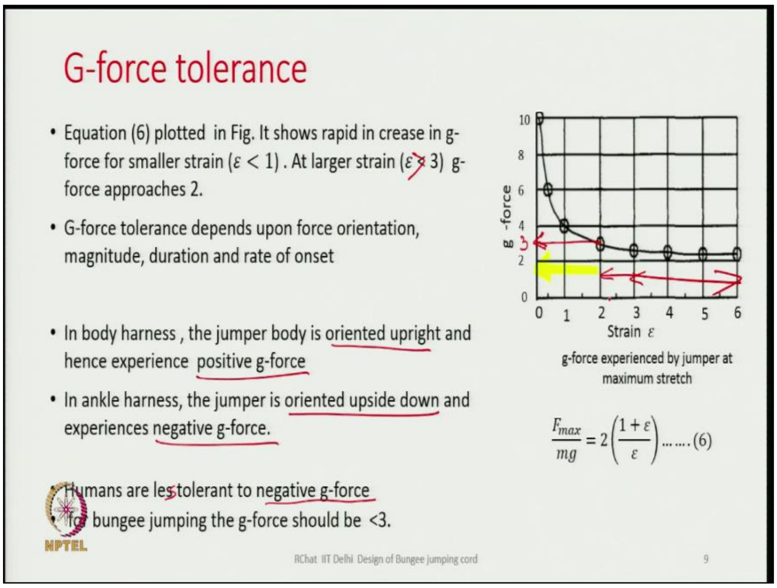

The graph shows a critical aspect of bungee cord design: the relationship between strain and the g-force experienced by the jumper. It shows that when the strain is low, there is a drastic increase in the g-force. When the strain is low, the g-force increases rapidly from 2 to 4, 4 to 6 to 8, and even up to 10 times the person's weight. This means that the cord does not extend, and the deceleration experienced by the jumper will be higher, leading to a high g-force. Therefore, that could cause injury to the person.

At larger strains, specifically when the strain is greater than 3, the g-force approaches a value close to 2. Hence, the tolerance to g-forces, however, depends on the force orientation, magnitude, duration and rate of onset. When the jumper's body is harnessed and oriented upright during the jump, i.e., when jumping, the jumper's head is up, and legs are down, and hence, it experiences positive g-forces. In the ankle harness, when the harness is attached to the ankle, the jumper orientation is upside down and experiences negative g-force. Humans are less tolerant of negative g-force. Hence, for bungee jumping, the g-force should be less than 3. It is evident that the strain level should be two or more to keep the g-force within safe limits.

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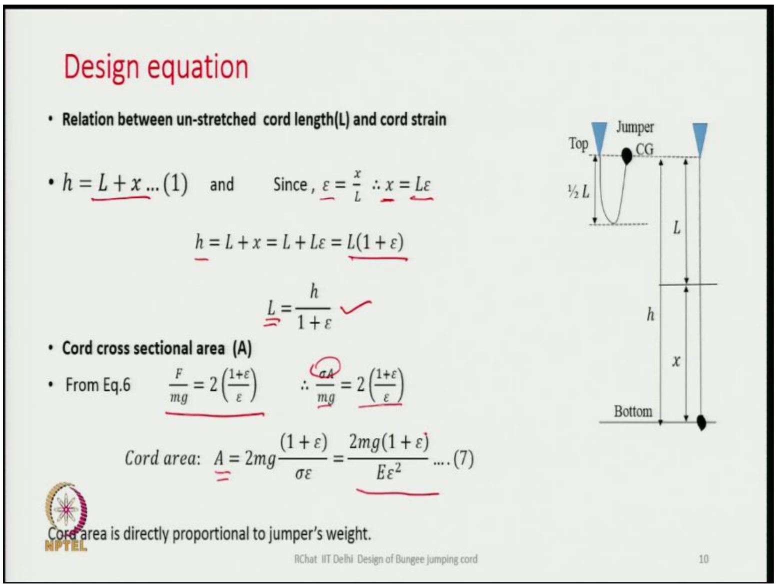

The design equations can be written based on the relation between unstretched cord length and cord strain. It has been seen that '$h = L + x$'. Since '$\varepsilon = \frac{x}{L}$', the equation becomes,

$$h = L + L\,\varepsilon$$

i.e., the original length of the chord multiplied by strain. Substituting the value of 'x', the length of the cord in an unstretched state is given by '$L = \frac{h}{1+\varepsilon}$'. For calculating the cord cross-sectional area, the equation,

$$\frac{F}{mg} = 2\left(\frac{1+\varepsilon}{\varepsilon}\right)$$

in which 'F' is replaced by 'σA'. Hence, the equation becomes

$$\frac{\sigma A}{mg} = 2\left(\frac{1+\varepsilon}{\varepsilon}\right)$$

By substituting the value of force and solving for the cord area 'A', this yields the equation,

$$A = \frac{2mg(1+\varepsilon)}{E\varepsilon^2}$$

This shows that the cord area depends on several factors: the weight or mass of the person, the initial modulus of the cord, and the strain level, i.e., the amount of strain the cord experiences. These parameters determine the required cord area.

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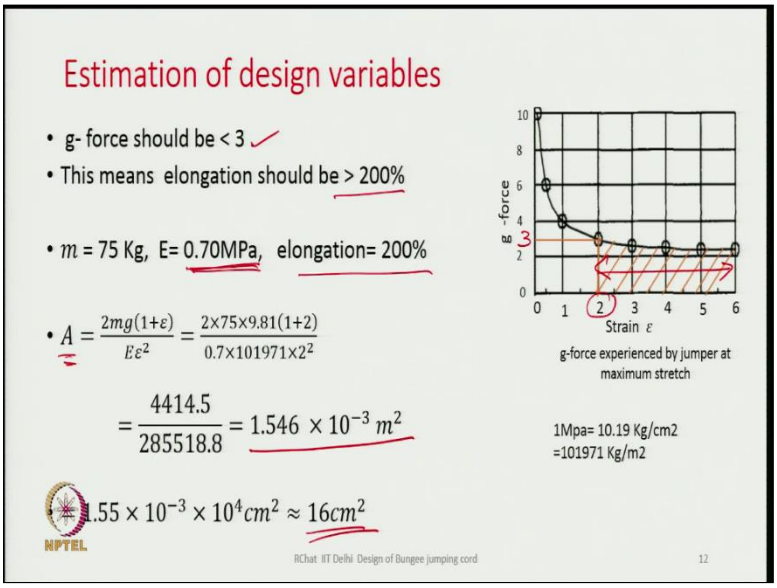

Next is the estimation of design variables, specifically the required area of the rubber cord we need to select. The g-force should be kept below 3 to ensure that it remains within the tolerable limit for humans. This implies that the elongation of the cord must be more than 200%, as shown in the diagram, where '$g = 3$' is indicated. Therefore, the elongation region should extend from this point, with a minimum elongation of 200% or greater.

In the equation of the area of the cord, the value of 'm' is substituted as 75 kg. Modulus has to be determined based on the type of rubber which is selected. In the example provided, the stress-strain diagram we observed earlier indicates that the modulus in the initial region, up to a strain level of about 3, is nearly linear. Therefore, this portion of the stress-strain curve is utilized, where stress and strain are proportional to one another. Beyond this range, the relationship becomes non-linear.

From there, the value of 'ε', the elongational limit is kept at 2, which means 200% strain, as it needs to be at least 200% or higher. Substituting this 200% elongation corresponds to a lesser strain of 2. By substituting these values in the equation for the area of the cord, it is found that

$$A = 1.546 \times 10^{-3} m^2$$

which is equivalent to 16 cm^2. Therefore, a rubber cord having an area of 16 cm^2 has to be selected. From there, the diameter of the cord also can be calculated by the formula

$$A = \frac{\pi\, d^2}{4} = 16$$

From this equation, the required diameter of the rubber cord can be determined.

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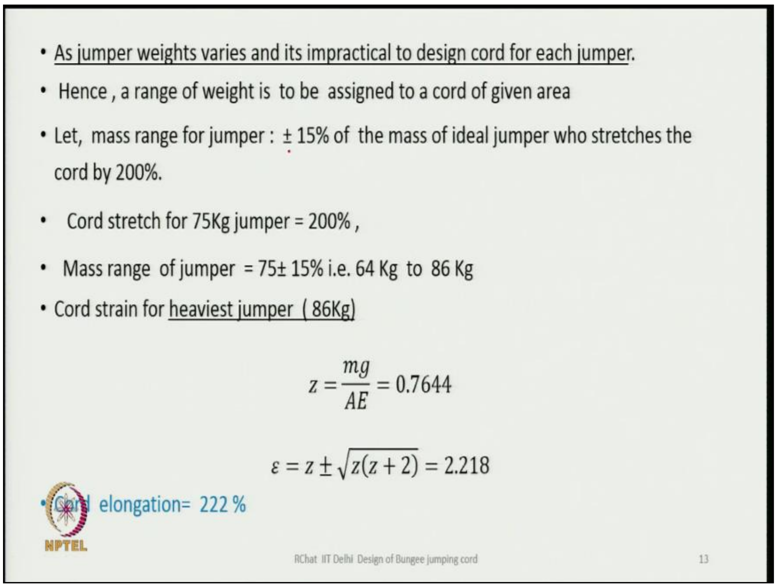

This slide presents a problem that the weight of the jumper can vary. Initially, the weight of the jumper was unknown, so the design started by assuming that the weight of the jumper was 75 kg. to satisfy the wide range of jumpers, the weights may differ. The cord designed for a jumper of 75 kg will also be suitable for any jumper weighing less than 75 kg. For jumpers weighing 70 kg, 65 kg, or 60 kg, the design will work without any issues since it was initially based on a weight of 75 kg. However, heavier individuals also need to be considered. Let's define the mass range of the jumper as ±15% of the mass of the ideal jumper, which stretches the cord by 200%. Hence, the mass range for the jumper is 75 ± 15. This results in a weight range from 64 kg to 86 kg, which includes the potential weights of the jumpers.

For jumpers weighing 75 kg or below, this design is sufficient. However, for heavier people, it has to be assessed whether it accommodates heavier weights. The cord strain for the heaviest jumper is given by the equation,

$$z = \frac{mg}{AE}$$

Based on the weight of the jumper and the area of the cord, it has been previously calculated that

$$\frac{mg}{AE} = 0.7644$$

This value is used to determine the resulting strain value of the cord.

In the equation, which was previously seen, the strain equation is related to the area of the cord, weight of the cord, and strain. By expanding and rewriting the equation for the area of the cord, the equation becomes

$$\varepsilon^2 = \frac{2mg(1+\varepsilon)}{EA}$$

Replacing '$\frac{mg}{AE}$' with 'z', the equation simplifies to

$$\varepsilon^2 = 2z(1+\varepsilon)$$

This is a quadratic equation in terms of strain (ε). From this quadratic equation, the value of strain can be found.

The equation of strain can be expressed as

$$\varepsilon = z \pm \sqrt{z(z+2)}$$

where 'z' is a constant which is equal to '$z = \frac{mg}{AE}$'. To calculate the mass per unit length of the cord, the formula used is

$$mass\ per\ unit\ length = s\rho_w AL$$

Here, 's' is the specific gravity, 'ρ_w' is the density of water, 'AL' represents the volume of the cord. Since it is a circular cord, the weight of the cord is determined by multiplying volume and density. Thus, multiplying the volume by the density gives the weight per unit length of the cord.

The strain is then given by equation as

$$\varepsilon = z \pm \sqrt{z(z+2)}$$

In this equation, to find the value of strain, substitute the value of 'z' and it yields a value of 2.218. This means the cord elongation for an 86 kg jumper will be approximately 222%.

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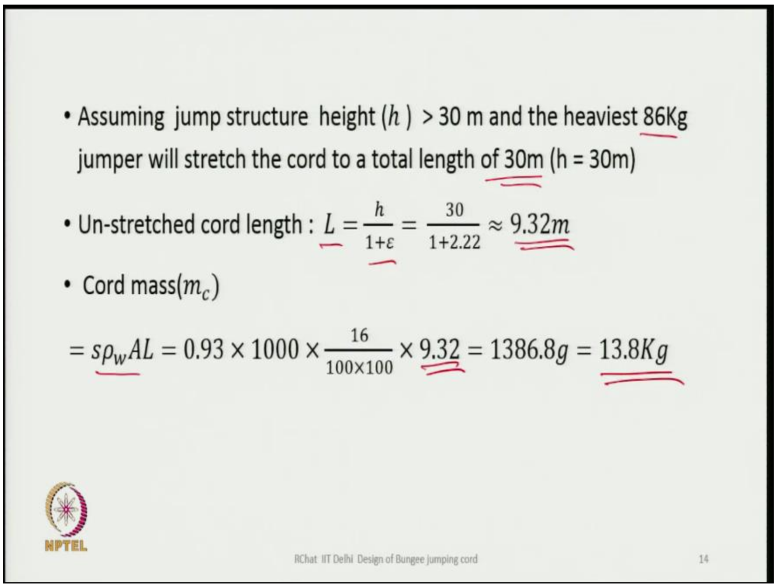

Let us assume that the jump structure has a height of more than 30 m, which is necessary for the person to jump and the cord to fully extend. Given that the total height available is at least 30 m, and the heaviest jumper will stretch the cord by 30 m, the unstretched length of the cord can be calculated. Using the equation for the unstretched length of the cord, it is found to be approximately 9.32 m. This means that the required cord length is 9.32 m.

Additionally, the mass of the cord is calculated, which comes out to be 13.8 kg for the entire cord of 9.32 m in length. With this cord length and weight, even the heaviest person will be able to survive because he will not experience a high g-force greater than 3, and it remains under the critical limit of 3.

This design approach gives an interesting insight into the approach to designing. These principles, which have been applied to bungee jumping cords, can be extended and applicable to mountaineering ropes as well. Similar situations might also exist. The rope which has been discussed here is only the rubber part of the bungee jumping cord, which is the primary load-bearing element. This plays an important role, and the rubber part must be covered with a protective layer to enhance durability and performance under different conditions.

To protect the rubber core, which is susceptible to sunlight and abraded fast when it comes to contact with an abrading surface, it is typically covered with textile material. To protect the core, it is covered by a sheath, which is made up of textile material, generally filaments. Hence, the rubber part is protected inside from abrasion by a sheath. Nylon is commonly used because it offers excellent abrasion resistance and durability.

The sheath also plays a crucial role in protecting the rubber core from not only abrasive damage but also from UV radiation and chemicals which come into contact with it. While the design of the sheath part has not been discussed, the focus is on the design of rubber cords, which are the main load-bearing elements. Thank you.