Textile Product Design and Development
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Lecture - 28
Tolerance Design

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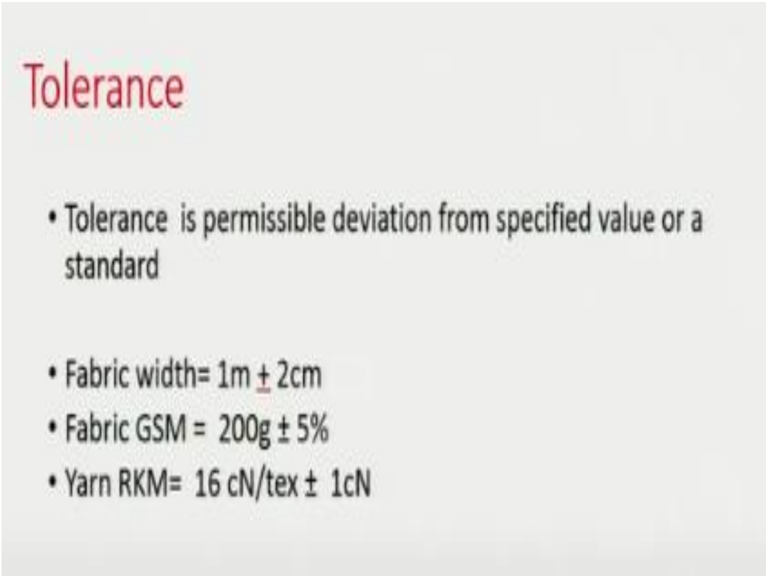

So, today's discussion is on tolerance design. What is tolerance? Tolerance is a permissible deviation from the specified value or standard. For example, the nominal value of fabric width could be 1 m. the tolerance could be given as ± 2 cm. Similarly, for fabric areal density, it could be represented as 200 g ± 5 %. Another example of Yarn RKM could be 16 cN/tex ± 1 cN. From the given examples, it is observed that the tolerance part could be in the same unit or could be given in terms of percentage also.

So, whenever specifications are provided, the tolerance also needs to be given because it is otherwise extremely challenging to produce a fabric which is exactly 1 m in width or 200 g in areal density because the process has its own variations. There are a lot of reasons why the variation comes. We always try to minimize the variation but depending upon the capability of the process and the variability that exists in the raw material. There will always be some deviation from the nominal value that you expect in each parameter.

The tolerances are always suggested by a buyer and or by a customer. The producer must produce goods within the specified tolerance.

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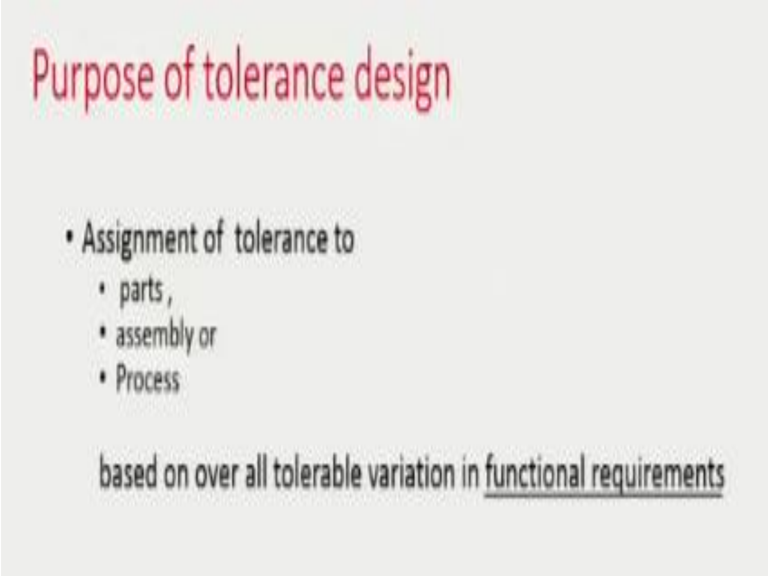

What is the purpose of tolerance design? The purpose of tolerance design is to assign tolerance to parts, to assembly or to processes. The final product may have many parts or assemblies. If we should meet the tolerance of the final product, then the tolerances of individual parts must be considered. For example, a product may consist of a few fabrics which could be an inner layer, middle layer and outer layer. So, these are different parts of the fabrics.

If somebody mentions that the nominal weight of the uniform or the jacket is some 'X' value with a tolerance of, let us say, 4%, 5%, or even 1%. When we are trying to manufacture a product, it consists of so many different components, and we must also consider how much tolerance should be given to those individual parts or components which contribute to the total weight of that garment; it could be uniform or jacket, whatever it is.

So, the tolerance design is based on the overall tolerable variations in functional requirements. If the functional requirement is the garment weight; then the weight becomes very important in certain cases. So, the functional requirement becomes weight, and the tolerance must be given to the total weight of the garment. So, what will be the weight of individual component parts and how can we design that?

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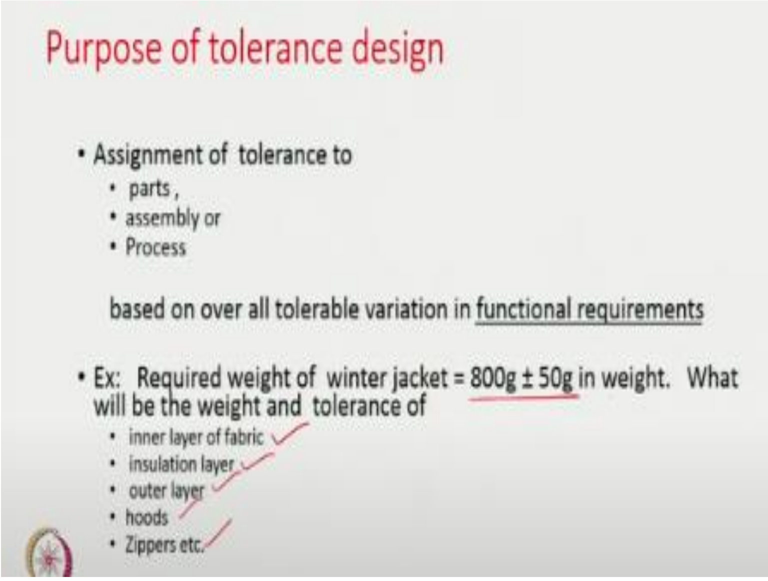

As given by an example in the slide, the required weight of winter jackets is 800 g ± 50 g. What will be the weight and tolerances of the inner layer fabric, insulation layer, outer layer, hoods, and zippers? Because all of them, i.e., the weight of all those individual items, contribute towards the total weight of the winter jacket.

The functional requirement could be any other property as well, which may be insulation property, strength, air permeability, or size, whatever it is. We need to find out the tolerances of individual components which are contributing to the overall tolerances of the product.

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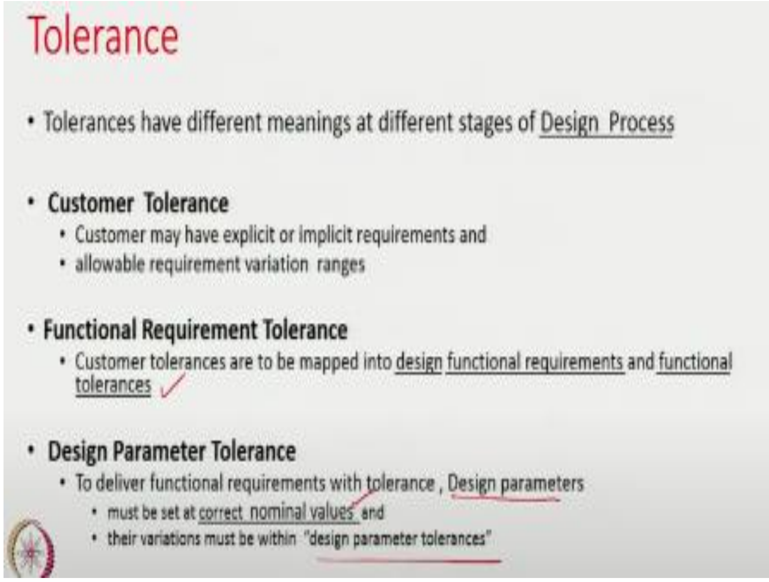

Tolerances have different meanings at different stages of the design process. The first one is customer tolerance: the customers may have explicit or implicit requirements and allowable requirement variation ranges. So, the customer may specify the acceptable tolerance beyond which it is not accepted. For example, let us consider the count of a yarn. We want to buy 10 tons of cotton yarn from a factory, and we specify that the count of cotton yarn must be 20's Ne with a tolerance of ± 1 Ne.

So, while evaluating the yarn count, the allowable range of yarn count lies between 19 and 21 Ne, so it will be acceptable. Usually, the tolerance is given by the customer, and the manufacturer must meet the specified tolerance given by the customers. So, he/she must design the process so that the product can meet the requirements. So, customer tolerance is generally given by the customers.

Functional requirement tolerance: Customer tolerances are to be mapped into design functional requirements and functional tolerances. So, customer tolerances must be mapped to the functional requirements; the customer may say the fabric should last for 2 years. When we say the fabric lasts for 2 years, it is known as the customer statement. We must map it to the property of the fabric that contributes to the life of a fabric.

The important considerations could be the application areas of the fabric and what are the factors responsible for damaging the life of the fabric. So, keeping in mind those things, we must decide the fabric properties that affect the durability of the fabric, and the properties could be the strength, UV resistance, abrasion resistance, etc. We must also determine the tolerances of these properties as we should maintain them during design. So, functional tolerances must be derived from the requirements of the customer.

Design parameter tolerances: To deliver functional requirements with tolerance, the design parameters must be set at the correct nominal value, and their variation must be within the design parameter tolerances. So, for every design parameter, what should be the nominal value and how much tolerance we should give to it are very important; that is the design parameter tolerance. For example, we understand that the strength of a fabric depends upon the ends/inch and picks/inch. So, what should be the exact tolerance in ends and picks/inch? So, the ends and picks/inch become my design parameters.

By manipulating these parameters, we can vary the strength requirement of the fabric in both warp and weft directions. The count of the yarn can also become the design parameter as it affects the strength of the fabric. So, we need to know what the nominal value of every design parameter is, i.e., the target value that we should try to achieve and how many tolerances we should have there.

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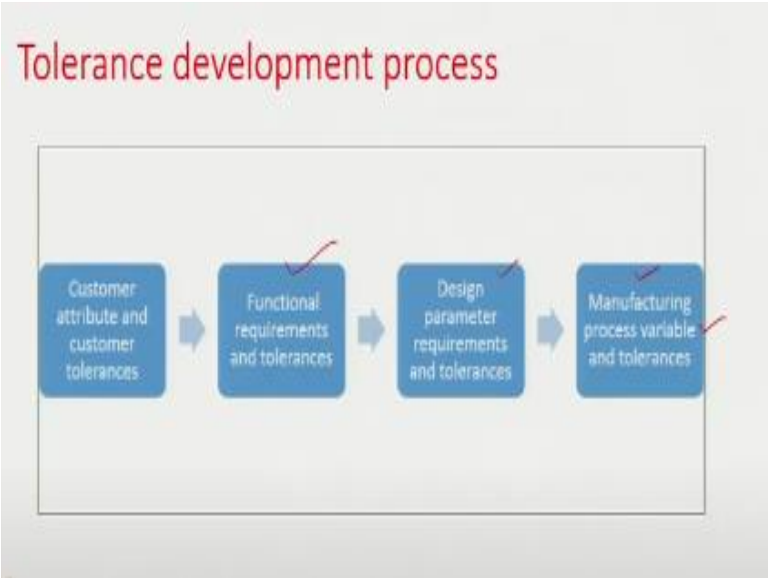

Tolerance development process: A diagram shown in the slide represents the tolerance development process. The first one in the sequence is the customer attribute and customer tolerances. First, we need to know what the customer is looking for. A customer is not necessarily a person who has a technical background. So, customers may be unable to always specify the requirements in scientific language. So, the customer statements may not be very scientific in nature.

The customer statement needs to be translated into a scientific statement which satisfies the requirement of the customer. The next step is the functional requirements and their tolerances. Next comes the design parameter requirements, as we have already stated earlier. So, once we have the functional requirements and their tolerances, we can go to the next step, that is, design parameter requirements and their tolerances.

After the design parameter requirements and tolerances, manufacturing process variable and their tolerances should be decided. Ultimately, we must manufacture the product and hand it over to the customer. So, after doing the necessary calculations to find out what are the design parameters, the machines are set to manufacture the product.

The setting of the machine parameters, which could be speeds or any other machine parameters. So, we also need to know those manufacturing process variables and their tolerance so that the process does not go out of control and keeps producing to meet the requirements of the customer.

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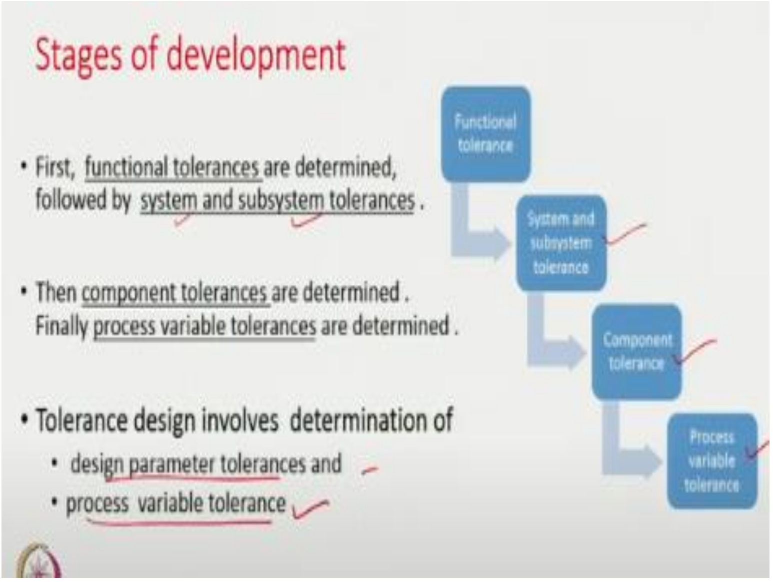

The stages of development are stated in this slide. The first stage is the determination of functional tolerance. From there, the next stage is the system and subsystem tolerance. Then comes component tolerance, followed by process variable tolerance. As discussed, first, we investigate the functional tolerances, and following that, we look for system and subsystem tolerances in a product. Because a product can also be, as we have already discussed earlier, the products sometimes have many subsystems within them.

It all depends upon the product type and, therefore, functional tolerances, and system and subsystem tolerances need to be found out. Then, the component tolerances are determined, and finally, the process variable tolerances are to be determined. Every component has a specific purpose, and therefore, keeping in mind that purpose, what sort of values we expect that satisfy the requirement that we need to find out and how much tolerance to be given to that.

So, tolerance design involves the determination of design parameter tolerances and process variable tolerances. These are the two main activities; that is, design parameters and process variables, and their tolerance; this is the most important part of the development of tolerance.

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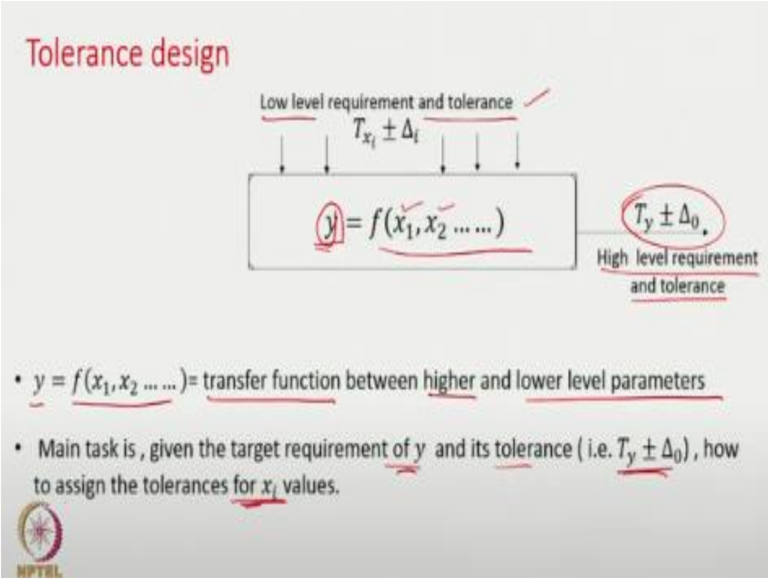

Tolerance design: There is a high-level requirement and tolerance, and there is a low-level requirement and tolerance. The low-level requirement means the property; they are basically an independent variable that affects a dependent variable. The dependent variable depends upon the independent parameters. For example, if the higher-level requirement is the strength of a fabric, then the lower-level requirement could be the strength of the individual yarns. The second low-level requirement could be an end and pick density, and the third thing could be the type of weave, whether it is a plain weave, a satin weave or a twill weave.

So, high-level requirement depends upon the low-level requirements. So, therefore we can write that 'y' is a function of 'x_1', 'x_2' up to 'x_n', and these are the independent parameters that are going to affect the dependent variable, which is 'y'. So, as an example, we have taken 'y' as a fabric strength and x's could be the count of yarn, tenacity of yarn, ends density of yarn, pick density of yarn, type of weave, and more.

So, the relationship between these stages is known as a transfer function between higher and lower-level parameters. Sometimes, some relationships may be available because, from the research literature, we can find out how 'y' and x's are related to each other. Sometimes, we need to do some experiments to find out what is the exact relationship between dependent and independent variables with the help of the design of experiments.

The design of experiments can establish a relationship based on multiple regressions. It gives what sort of relationship exists between dependent and independent variables. So, assume that '$y = f(x_1, x_2, ...)$' is the transfer function. So, the main task is given the target requirement of 'y' and its tolerance. If both are given; the fabric strength should be, let us say, 1.5 kg-f, and the tolerance is ± 0.2 kg-f. So, tolerance is also given, and the target value has been given.

We need to find out what should be the tolerances of x_i's and what should be the target value of x_i's also. Especially here, we are discussing tolerance. Therefore, we are focusing more on tolerance. So, if the tolerance of the output is given to us, how do I find out the tolerance of the individual independent parameter that affects the output parameter?

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Aspects of tolerance designs: one is to manage variability. If the tolerance is tight, if we narrow down the tolerance, then we must have very strict control of the process. So, any reasons for variability or anything that can affect variability, all those factors must be under proper check because when you study variability in the properties of a product, then one can also find out why this variability exists.

What is the role of the process on the variability? What is the role of raw material on the variability? or how the process might get disturbed? So, there are many factors which can affect variability; some of them can be under control, and some variability also remains uncontrollable; that is something where we have no control over them.

Variability control becomes very important in tolerance design. Understanding the source of variability and controlling the same could be very challenging. Achieving functional requirements satisfactorily and keeping design costs low is another objective for the designer. Attempt for tolerances which are difficult to achieve, and if we try it, the cost might go high.

So, considering the cost also, the tolerance limits could be decided. Some textile mills are there that can maintain the count of yarn within a very narrow limit with a 5% Uster value. At the same time, there are textile mills that will keep the count variability, which falls into 10% Uster value, which means a mill that produces a yarn; the nominal count may still be the same, but the variability is different.

So, the variability of yarn in one mill comes under the Uster standard of 10%, and the other one comes in at 5%. So, why the difference is there? So, the difference must be because the processes in one of the two cases may not be proper. In one case, it is a highly controlled process; in the other case, it is not at all controlled, and therefore, the differences are observed. When we want to go for a narrow range of tolerance, obviously the cost will be more because we must have better process controls.

For example, one may need to have more sensors and need to have better machines, so obviously, the cost will be high. So, keeping in mind the cost part also, we must decide whether it is achievable or not.

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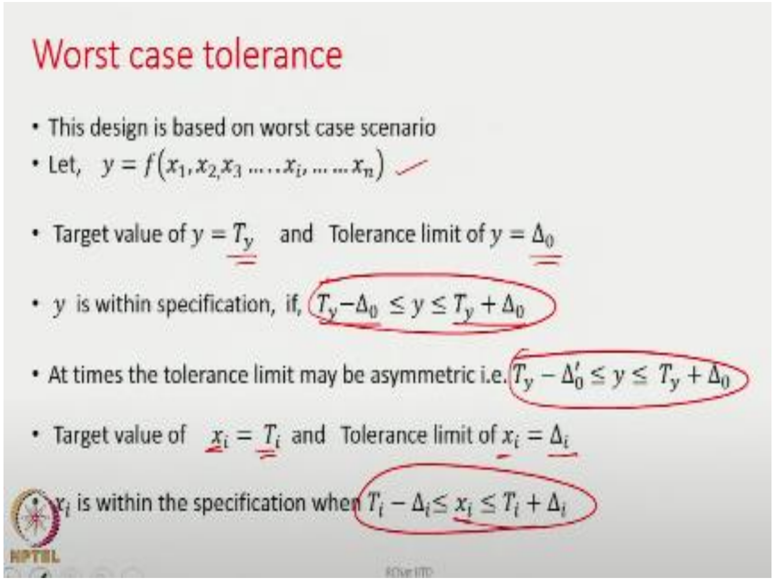

Worst case tolerance: this design is based on the worst case scenario. Let 'y' be a function of 'x_1', 'x_2', 'x_3', up to 'x_n', where 'x_1', 'x_2', 'x_3', 'x_n' are basically lower-level parameters, and 'y' is the higher-level parameter, that means 'y' depends upon 'x_1', 'x_2', 'x_3', 'x_n'. Let the target value of 'y' is 'T_y', and the tolerance limit of 'y' is 'Δ_0'. So, 'y' is within the specification if 'y' remains within the limits: '$T_y + \Delta_0$' and '$T_y - \Delta_0$'.

But the tolerance limit may be asymmetric also; that is, we may have a situation where the right-hand side limit does not equal the left-hand side of the limit. As shown in the slide on the right-hand side, the limit is '$T_y + \Delta_0$', left-hand side '$T_y - \Delta'_0$'. So, the two limits of tolerance, the right-hand side and the left-hand side may not be the same. For example, it may not be a ±2, instead, it could be +2 on the one end and -1 on the other end. So, this is called asymmetric tolerance, and many situations could be like this.

There could also be some situations where the tolerance is only on the lower side or only on the upper side, i.e., we can have only upper tolerance or lower tolerance. For example, customers want a yarn of RKM 18, i.e., 18 cN/tex. The customer accepts if the RKM value falls between 17 and 18, i.e., one cN/tex is the lower acceptable limit, but there is no upper limit. If the supplier supplies yarn with an RKM value higher than 18, it will be beneficial for the customer.

There are situations where there could be only an upper limit. For example, let us say the customer needs yarn with a uniformity level of Uster 13%. If it is up to 14%, i.e., 1% upper limit, the customer accepts it. However, there is no limit for the lower level. If the supplier supplies yarn with a uniformity level of less than 13%, it will benefit the customer because the lower the percentage, the more uniform the yarn will be.

There are situations where the limit is equal to the nominal value, or they may be asymmetric; also, all sorts of things might be there in the tolerance. So, the target value of 'x_i' is 'T_i' and the tolerance limit of 'x_i' is 'Δ_i'. 'x_i' is within specification when 'x_i' remains in this range: '$T_i - \Delta_i$' and '$T_i + \Delta_i$'.

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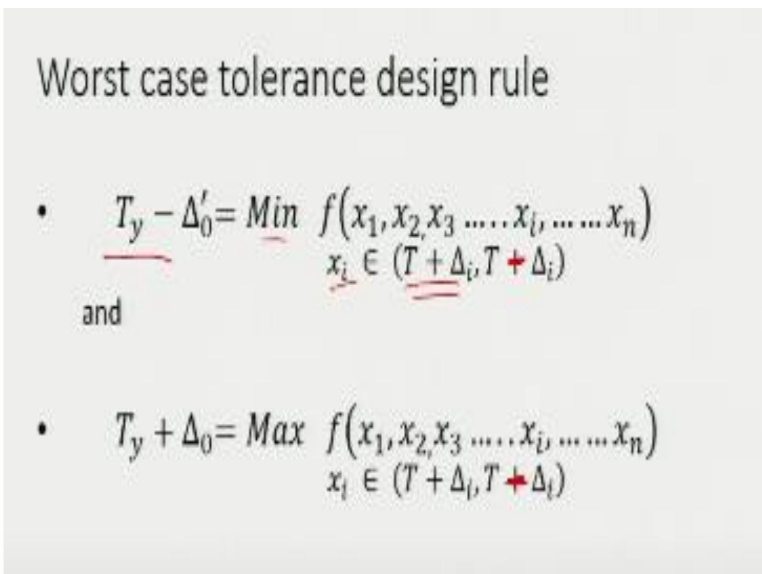

Worst case tolerance design rule: it can be represented by two equations as shown in the slide.

$$T_y - \Delta_0' = Min\, f(x_1, x_2, x_3 \ldots . x_i, \ldots .. x_n), x_i \in (T - \Delta_i, T + \Delta_i)$$

Each value of 'x_i' is within the limit '$T - \Delta_i$', and '$T + \Delta_i$'. Similarly, it can be written as,

$$T_y + \Delta_0' = Max\, f(x_1, x_2, x_3 \ldots . x_i, \ldots .. x_n), x_i \in (T - \Delta_i, T + \Delta_i)$$

The lower tolerance limit is a minimum function, and the upper limit of tolerance is a maximum function.

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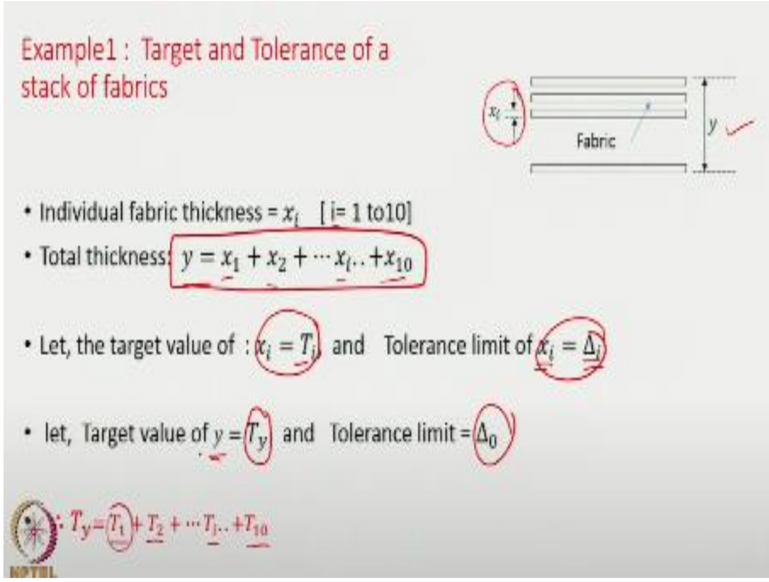

Let us discuss an example of the target and tolerance of a stack of fabrics. A stack of fabrics is placed on top of the other, and each individual fabric has a thickness of 'x_i'. The stack consists of, let us say, 10 fabric layers, and the total thickness is 'y'. In the garment industry, fabrics are laid on top of each other, and then they are cut together. So, the total thickness is,

$$y = x_1 + x_2 + \cdots x_i \ldots + x_{10}$$

because 'i' is varying from 1 to 10.

Let the target value of 'x_i' is equal to 'T_i', and the tolerance of 'x_i' is equal to 'Δ_i'. Similarly, the target value of 'y', i.e., the output total is 'T_y' and tolerance is 'Δ_0'. So, the target and tolerance values of both 'x_i' and 'y' are already taken. 'y' is a function of 'x'.

'y' is the sum of the individual layers of thickness 'x_i'. As 'y' is the sum of 'x_1' to 'x_{10}'; So,

$$T_y = T_1 + T_2 + \cdots T_i \ldots + T_{10}$$

because the target values of x_i's are T_i's.

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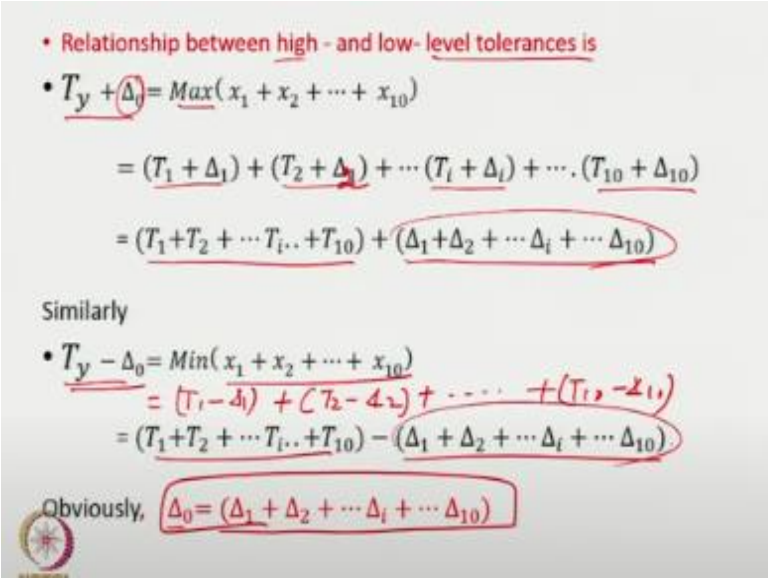

So, the relationship between high and low-level tolerances is,

$$T_y + \Delta_0 = Max(x_1 + x_2 + \cdots + x_{10})$$

So, this will be,

$$(T_1 + \Delta_1) + (T_2 + \Delta_2) + \cdots (T_i + \Delta_i) + \cdots (T_{10} + \Delta_{10})$$

We can separate all 'T' together, and all tolerances together. Therefore,

$$(T_1 + T_2 + \cdots T_i \dots + T_{10}) + (\Delta_1 + \Delta_2 + \cdots \Delta_i \dots + \Delta_{10})$$

The target value of thickness 'T_y' is the addition of individual layer thickness, and the target tolerance 'Δ_0' is the addition of individual tolerances. Similarly, for '$T_y - \Delta_0$' also, the same equations can be written as shown in the slide.

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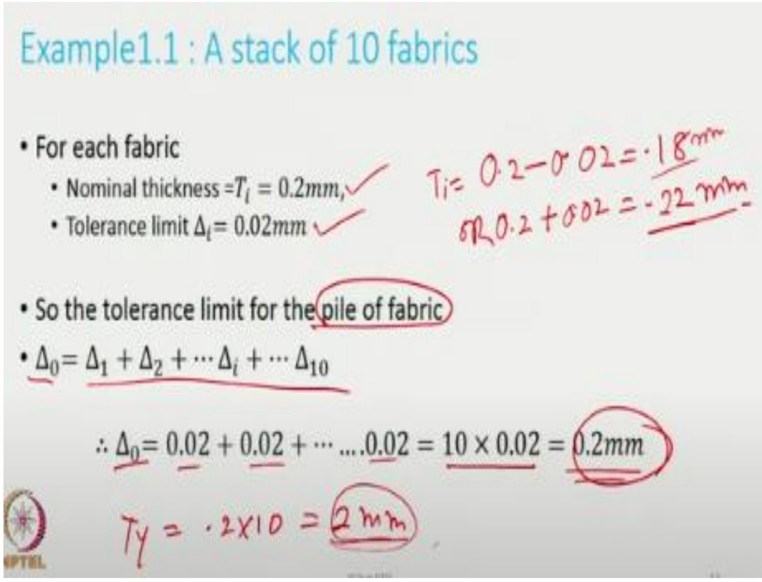

As discussed, the tolerance of the stack is the sum of the individual tolerances of the fabrics. Let the nominal thickness of a fabric be 0.2 mm. All the fabrics are assumed to be the same fabric pieces, so they all have a nominal thickness of 0.2 mm. The tolerance limit is 0.02 mm, i.e., the fabric thickness lies in the range between 0.18 mm and 0.22 mm. In the worst case, some fabrics may have a thickness of 0.18 mm, and in other worst cases, the fabrics are thicker, about 0.22 mm; these two are the two different extreme scenarios.

So, the tolerance limit for the pile of fabrics is the sum of the individual tolerances. The tolerance limit of individual fabric layers is 0.02. There are 10 fabric layers; so, the tolerance limit ($\varDelta_0$) for the pile of fabric is '10×0.02', i.e., 0.2 mm. So, for this stack of fabrics, the tolerance is 0.2 mm, and the nominal thickness (T_y) is '10×0.2', i.e., 2 mm.

In most of the time, we will be interested in the reverse way, i.e., given the tolerance value, what should be the tolerance of the individual fabrics? Every machine has a permissible setting to handle the fabric thickness range. For example, the cutting machine may only handle the fabric of 2 mm thickness; just as an example. It may handle a higher thickness as well.

Obviously, there is a range of thicknesses in which the machine should be able to operate. If we operate beyond that thickness, the machine may fail; it may not be able to cut the fabrics, so such kind of situation could be there. So, many times, we need to know in the reverse way, as stated earlier, that if the tolerance is given, what will be the individual fabric thickness and what should be the tolerance of the individual fabrics?

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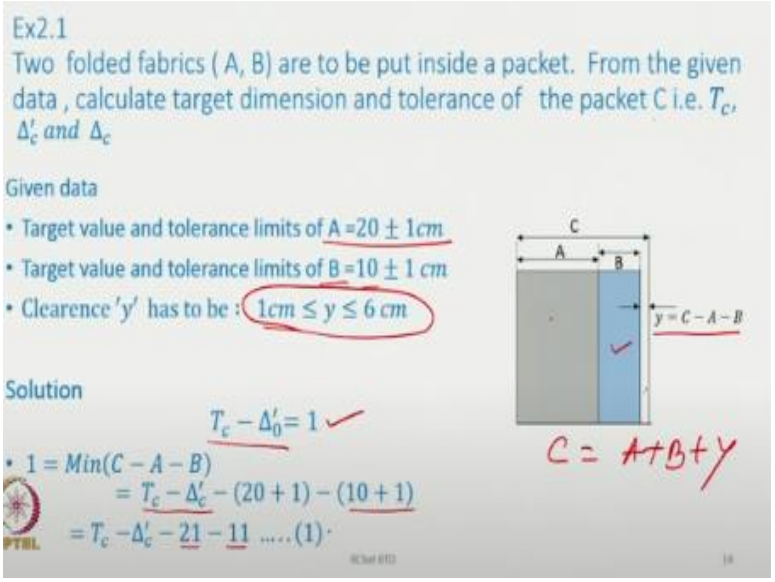

Another example is given in the slide. The question is that two folded fabrics are to be put inside a packet. From the given data, calculate target dimensions and tolerance of the packet. As shown in the diagram, there are two fabric pieces, and are in folded state; one fabric piece is 'A', and the other one is 'B'. Now, we must design a packet within which these two fabrics will be inserted.

If the folded fabric is too short, it is very difficult to put the fabric inside the packet; if it is too large, then also within the packet, the fabric moves. So, the packet that we are going to design should have a certain nominal value and tolerance so that neither we have difficulty in placing these two folded fabrics inside the packet nor does the fabric move too much within the packet.

In this case, the target value and the tolerance of fabric 'A' are given as 20 ± 1 cm, and the target value and tolerance of fabric 'B' are given as 10 ± 1 cm. The clearance of folded fabrics within the packet is in the range of 1 to 6 cm. So, how to solve this problem? What should be the target value of 'C', and what would be the tolerance? The difference between the target and tolerance of 'C' should be 1, i.e., '$T_C - \Delta'_0 = 1$'. Therefore, '$1 = Min(C - A - B)$'.

What is the clearance 'y'? It is '$C - A - B$'. How? Actually, 'C' is '$A + B + y$', where 'y' is the clearance as shown in the diagram. So, 'y' is '$C - A - B$'. So, if the tolerance minimum value is 1, then,

$$1 = T_C - \Delta'_C - (20 + 1) - (10 + 1)$$

After simplification, it is going to be,

$$1 = T_C - \Delta'_C - 21 - 11$$

It is considered as an equation (1).

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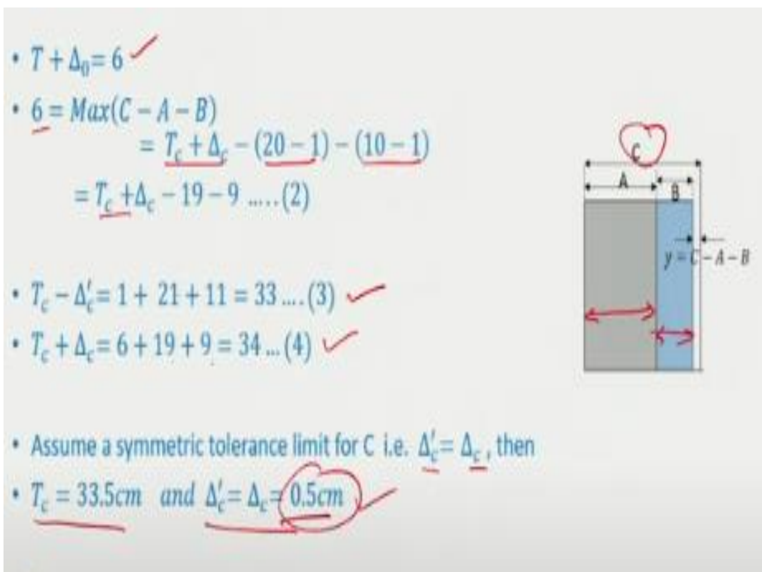

On the higher limit side, '$T_C + \Delta_0 = 6$'. Therefore, '$6 = Max(C - A - B)$'. So, if the tolerance maximum value is 6, then,

$$6 = T_C + \Delta_C - (20 - 1) - (10 - 1)$$

After simplification, it is going to be,

$$6 = T_C + \Delta_C - 19 - 9$$

It becomes equation (2).

Now, we have two equations. i.e., '$T_C - \Delta'_C$' which is the lower side of the limit, and '$T_C + \Delta_C$' is the upper side of the limit. Therefore,

$$T_C - \Delta'_C = 33$$

$$T_C + \Delta_C = 34$$

which are the equations (3) and (4). From there, if we assume a symmetric tolerance limit, i.e., '$\Delta'_C = \Delta_C$'. In that case, we can solve the equations (3) and (4) to determine the value of 'T_C'. If we want to calculate 'T_C', it is the average of 33 and 34, which gives the 'T_C' value around 33.5 cm.

From the 'T_C', we can easily find out the tolerance value using the same equations. The tolerance comes to around 0.5 cm. So, the packet 'C' must be designed in such a way that the target width must be 33.5 cm with a tolerance of 0.5 cm.

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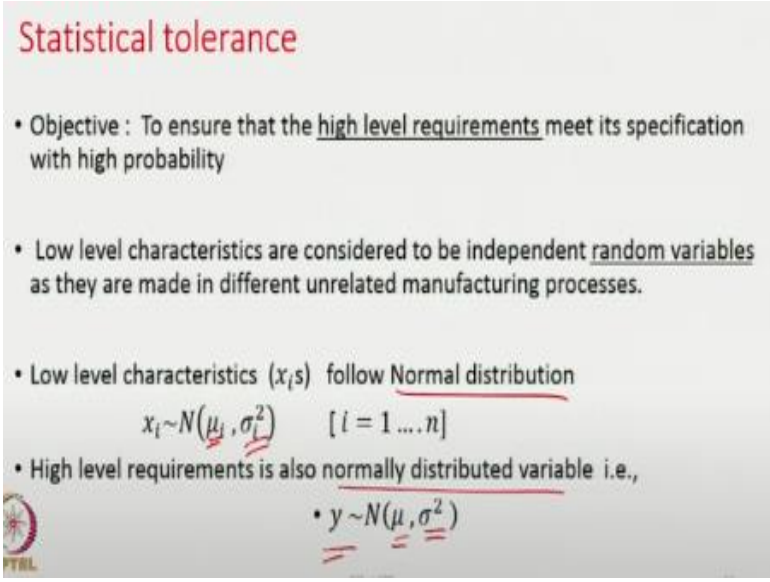

Statistical tolerance: the objective is to ensure that the high-level requirements meet a specification with high probability, and low-level characteristics are independent random variables as they are made in different unrelated manufacturing processes. The different components are made in different manufacturing processes by different machines it could be, and therefore, they can all be independent random variables.

So, low-level characteristics exist, which is x_i's; we assume they follow normal distributions as assumed in most of the time. So, low-level characteristics (x_i's) are following a normal distribution with mean 'μ_i' and standard deviation 'σ_i'. High-level requirements are also normally distributed with 'y', and means are normally distributed with a mean 'μ' and standard deviation 'σ'.

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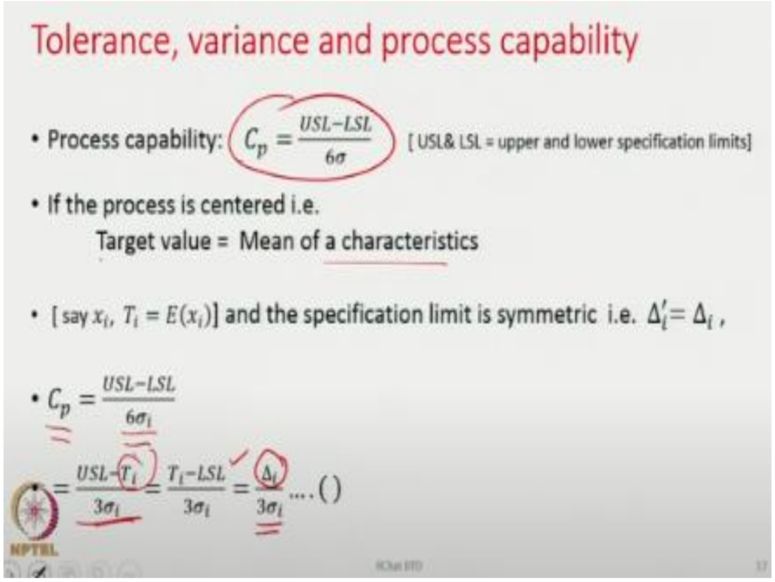

Tolerance, variance and process capability: What is process capability?

$$C_p = \frac{USL - LSL}{6\sigma}$$

where 'USL' and 'LSL' are upper and lower specification limits that the customer has provided. These are the specification limits, and the product specifications must be within these two limits.

If the process is centred, i.e., the target value and mean of characteristics are the same, then 'C_p' can also be written as,

$$\frac{USL - T_i}{3\sigma_i} = \frac{T_i - LSL}{3\sigma_i}$$

where 'T_i' is the target value of 'x_i'. 'x_i' could be any parameter. As the process is centred, one-half of the curve must be equal to the other half with respect to their mean. The difference between either '$USL - T_i$' or '$T_i - LSL$' is known as tolerance (Δ_i) for ith lower-level parameter.

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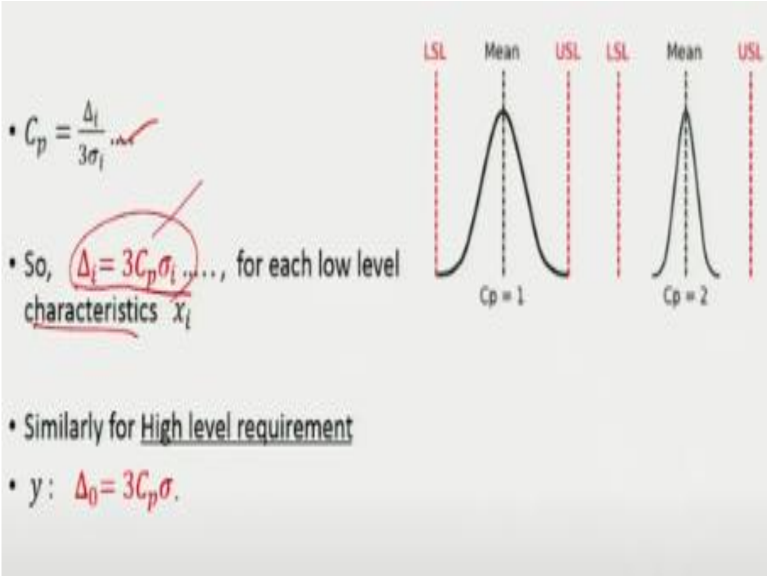

If 'C_p' is '$\frac{\Delta_i}{3\sigma_i}$', therefore, '$\Delta_i$' is '$3C_P\sigma_i$', for each low-level characteristic, 'x_i'. The tolerance of lower-level characteristics (x_i) is connected to the 'C_p' value of the process, i.e., the capability of the process. Is the process capable of giving the specified tolerance or not? If somebody claims the machine produces a nonwoven fabric, where the 'C_p' value is around, let us say, 10%. So, we know that whatever we do, this nonwoven machine always gives a 'C_p' value of 10%.

If we know the 'C_p' value, we can find the standard deviation also. Therefore, if somebody expects the process to be capable of producing a demand of 5% value, then the process may not be able to achieve. So, if customers expect that we need to produce a product where the 'C_p' value is 5%, and if we know that the process can only give at the most 10%, therefore, we should conclude that the process is not capable.

There is no use in taking that order and trying to execute it because we will fail as the process is incapable. So, we must understand what the capability of a process is, whether it is a spinning process, weaving process or garment manufacturing process, whatever process it is. We must have an idea of the capability of the process. For high-level requirements, similarly, we can write,

$$y: \Delta_0 = 3C_p\sigma$$

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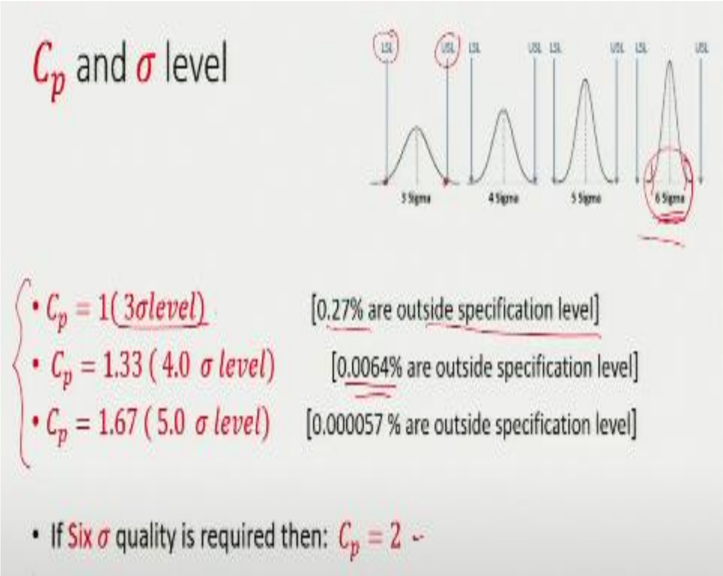

'C_p' and standard deviation: 'C_p' value is the process capability value, which is a part of Six Sigma. So, when 'C_p' is 1, it is a 3σ level, and the corresponding diagram is shown in the slide. In the diagram, the difference between the lower specification limit (LSL) and upper specification limit (USL) are in terms of 'σ'. In 3σ level, the distance between 'LSL' and 'USL' is 6σ; divided by 6σ gives the 'C_p' of 1. In the case of 3σ level, 0.27% of the product will be outside the specification level, close to 0.3%, if the process 'C_p' value is 1.

Similarly, the next one is 4σ level. If 'C_p' value is 1.33, which is equivalent to 4σ, 0.0064% of the product will be outside the specification level. Next, 'C_p' value of 1.67 is for 6σ level and the specification limits are given in the slide. The specification limits in a 3σ level is much more than the 6σ level. The process is much narrower in 6σ level, and hence, in these situations, hardly any defective material will be produced.

So, in this way, the relationship between 'C_p' and σ's has been established. So, 'C_p' value of two produces very low defects, which is almost difficult to achieve. However, high-end industries are trying to achieve this value. In our case, even a 'C_p' value of 1 is very good, but usually, it may go below 1, or it could be 0.8, 0.6, 0.7, and so on. So, we produce more defective material that will exceed the specification limit.

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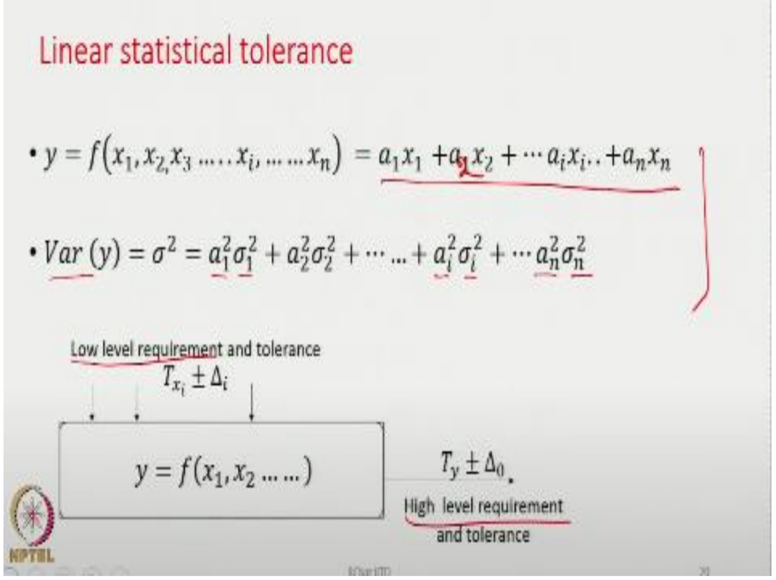

Linear statistical tolerance: if 'y' is a function of 'x_1', 'x_2', 'x_3' and 'x_n', and is a linear function,

$$y = a_1x_1 + a_2x_2 + \cdots a_ix_i \ldots + a_nx_n$$

The variance of 'y'can be written as,

$$Var\ (y) = \sigma^2 = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \cdots + a_i^2\sigma_i^2 + \cdots a_n^2\sigma_n^2$$

These two formulae are very important in the case of linear statistical tolerance, i.e., when there is a linear relationship between 'y' and 'x'.

However, many times, it could be non-linear, and that can be tackled by establishing a non-linear relationship. But to make the case simple, we are assuming that it is a linear relationship. As discussed, x_i's are low-level requirements, and y_i's are high-level requirements. 'y' is a function of 'x'; in other words, we can say 'y' is a dependent parameter, and x_i's are an independent parameter. 'y' depends on 'x' whereas 'x' does not depend upon 'y'.

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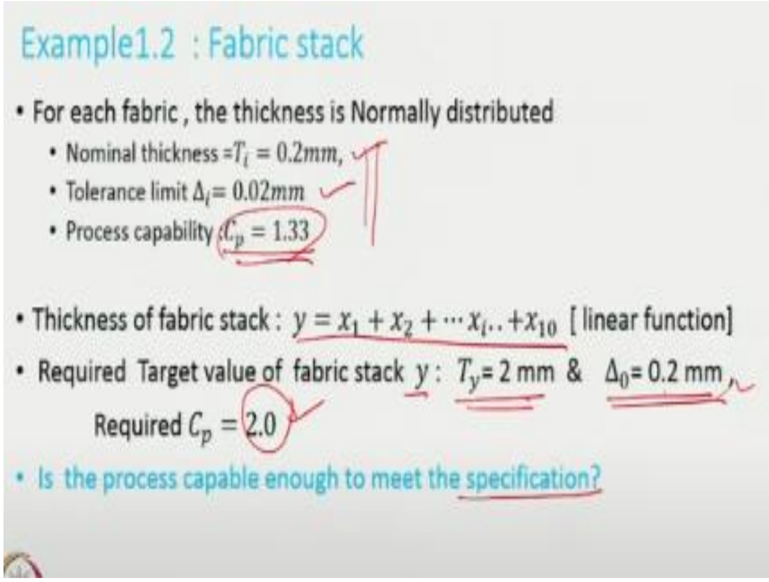

Again, let us take a previous example of fabric stack: 10 fabrics stacked together, the nominal thickness is 0.2, tolerance is 0.02 mm, and the process capability is 1.33. the process capability value, in this case, indicates a very good process, i.e., a 'C_p' of 1.33 means most of the fabrics which are produced meet the requirement.

It is equivalent to almost 4σ level, and therefore, a very small percentage of fabrics will be defective. The thickness of the fabric stack is a linear function. The required target value of the fabric stack 'y' is 2 mm, and 'Δ_0' is 0.2 mm. Suppose the required value of 'C_p' is 2, which is a very high value. Is the process capable enough to meet the specifications or not?

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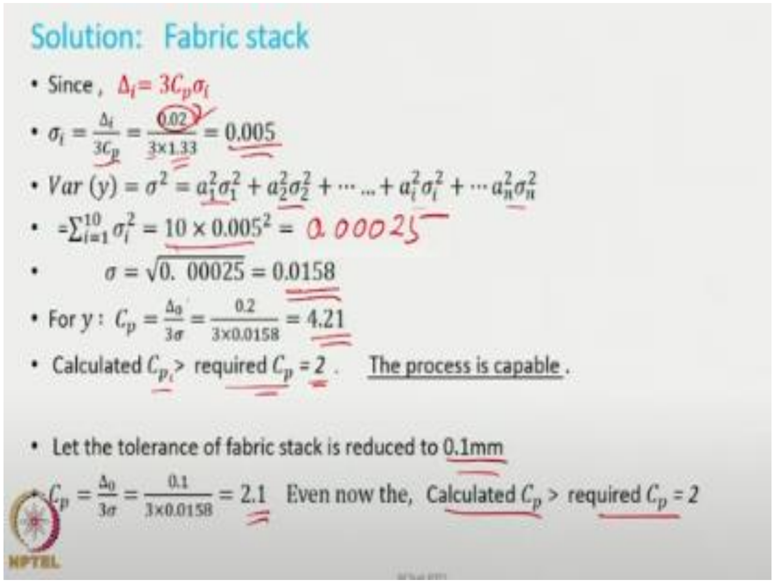

We have already seen that '$\Delta_i = 3C_p\sigma_i$'. Therefore, the standard deviation of the x_i's is,

$$\sigma_i = \frac{\Delta_i}{3C_p}$$

So, 'Δ_i' is 0.02 mm and 'C_p' value of the process is 1.33 as given already. Therefore, the 'σ_i' value is 0.005 mm. The formula for variability in 'y' is stated in the slide. The coefficients of individual layer variances in the equation are one in this case because in the function 'y', the coefficients all x_i's, i.e., 'a_1', 'a_2', 'a_3', etc., are one.

So, it is going to be '$(1^2 \times 0.005^2) + (1^2 \times 0.005^2) + \cdots upto\ n\ terms$', where '$n$' is 10. So, it is simply can be written as,

$$\sum_{i=1}^{10} \sigma_i^2 = 10 \times 0.005^2$$

The variance comes around 0.00025 mm^2. Therefore, the standard deviation of 'y', '$\sigma_y = \sqrt{Var(y)}$', calculated as 0.0158 mm. The process capability of y is,

$$C_p = \frac{\Delta_i}{3\sigma}$$

After substitution and simplification, the value of process capability (C_p) comes at around 2.1. The calculated 'C_p' is more than the required 'C_p' of 2. So, the process can achieve the requirement.

Suppose the tolerance given to the fabric stack is reduced to 0.1 mm; in that case, the calculated 'C_p' is 2.1. Here also, the calculated 'C_p' is more than the required 'C_p' of 2. So, if the tolerance is reduced from 0.02 mm to 0.1 mm, the process is still capable of achieving the requirement. Earlier, it was a very tight tolerance, and now we have increased it to 0.1 mm, which means we have reduced the processing difficulties.

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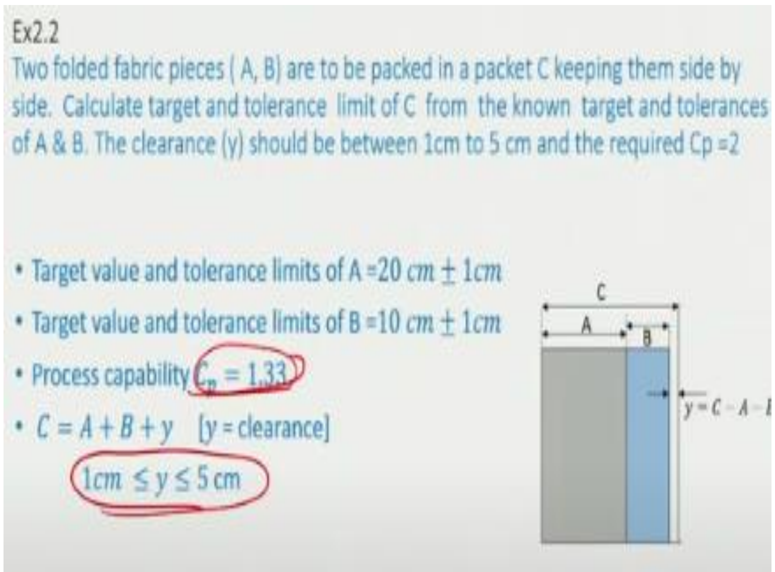

The other example which we discussed earlier was also chosen to understand the process capability. The 'C_p' value is 1.33 for fabric 'A', and 'B'. The packet size 'C' is 'A' plus 'B' plus 'y', where 'y' is the clearance; between 1 to 5 cm.

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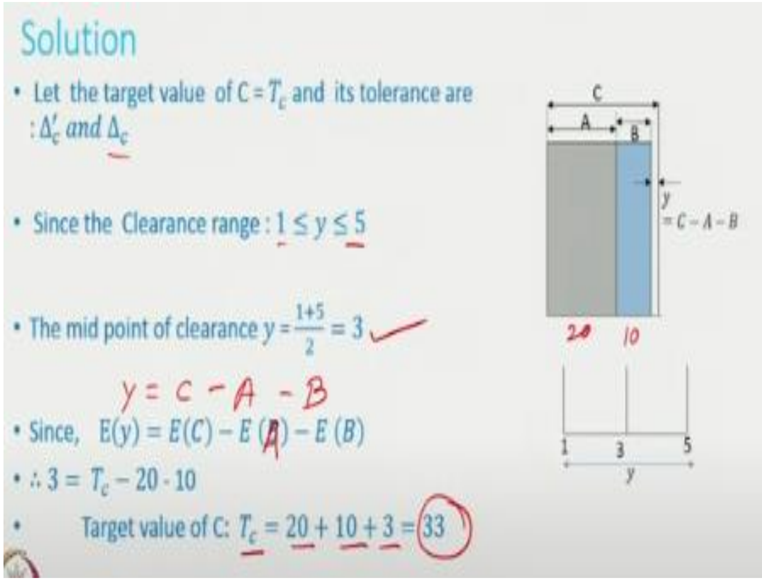

Like the previous case, the target value of 'C' is 'T_C' and its tolerances are 'Δ_C'' and 'Δ_C'. The clearance 'y' is between 1 and 5 cm, as already stated. So, the mid-point of the clearance is 3. The clearance can be written as '$y = C - A - B$' and hence, '$E(y) = E(C) - E(A) - E(B)$'. The expected value of 'y' ($E(y)$) is the middle point of 'y' is 3. So,

$$3 = T_C - 20 - 10$$

Therefore, the target value of 'C' is 'T_C' is '$20 + 10 + 3$', i.e., 33. The target value or nominal value must be 33 cm.

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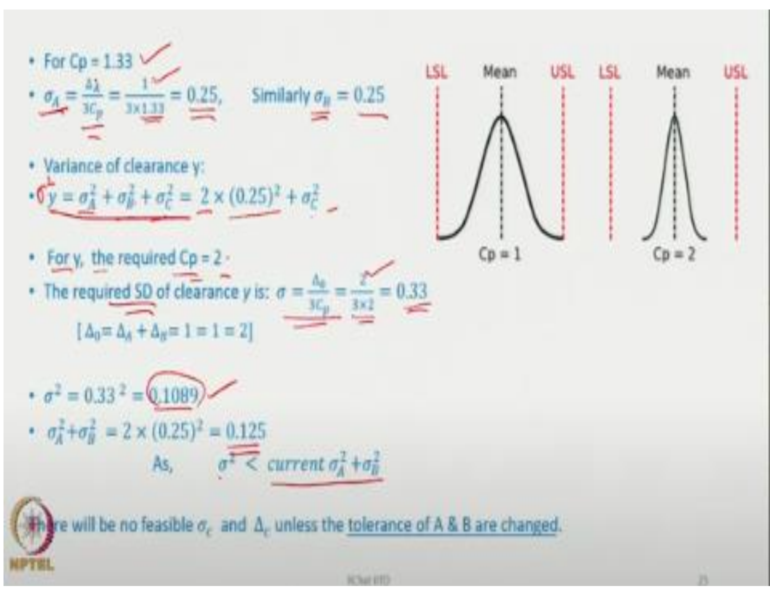

So, we have found out the target value of the packet, i.e., 'T_C'. It is given that the 'C_p' value is 1.33 for the process, which is folding the fabrics and making them 20 cm and 10 cm for the fabrics 'A' and 'B', respectively. What is the standard deviation of that process? Suppose fabric folding is done automatically by some machine, and the folding machine has a process capability of 1.3. So, the standard deviation

$$\sigma_A = \frac{\Delta_A}{3C_p}$$

So, if we go by this, the tolerance has been given to be 1 cm. Therefore, the standard deviation of 'A' is 0.25 cm, and similarly, the standard deviation of 'B' is also 0.25 cm because the tolerance of 'B' is also the same as tolerance of 'A'; both are ± 1 cm. How much is the variance of clearance 'y'? 'y' is a function of 'A', 'B', 'C', and therefore, the variance of 'y' is,

$$\sigma_y^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2$$

In this equation, 'σ_A' and 'σ_B' are already known to us; both are 0.25 cm. So, it will be '$2 \times (0.25)^2 + \sigma_C^2$'. What is the standard deviation for the dimension of the packet which is still unknown to us. But for the 'y', the required 'C_p' must be 2. If it is 2, then the standard deviation required of that process will be how much? Again, we use a similar formula.

$$\sigma = \frac{\Delta_0}{3C_p}$$

'Δ_0' has been given to be 2 because it is a summation of 'Δ_A' and 'Δ_B', which is going to be '$1 + 1$', that is going to be 2. Therefore, the value of 'σ' is 0.33 cm. Now, if 'σ' for the packet to be 0.33 cm, how much is 'σ^2'? 0.33^2, i.e., 0.1089. The sum of 'σ_A^2' and 'σ_B^2' is '$2 \times (0.25)^2$', which is 0.125 cm^2. The σ^2 is less than current '$\sigma_A^2 + \sigma_B^2$' and therefore, there will be no feasible 'σ_C' and 'Δ_C', unless the tolerance of 'A' and 'B' are changed, we are not getting a solution.

Because if the variance of 'y' is 0.1089, which is on the left-hand side, and on the right-hand side, the summation of 'σ_A^2' and 'σ_B^2' is 0.125, then 'σ_C^2' must be negative. The variance cannot be negative, and therefore, there is no feasible solution. So, what we need that we need to change the tolerance of 'A' and 'B', i.e., 'Δ_A' and 'Δ_B'. Otherwise, it does not give any solution.

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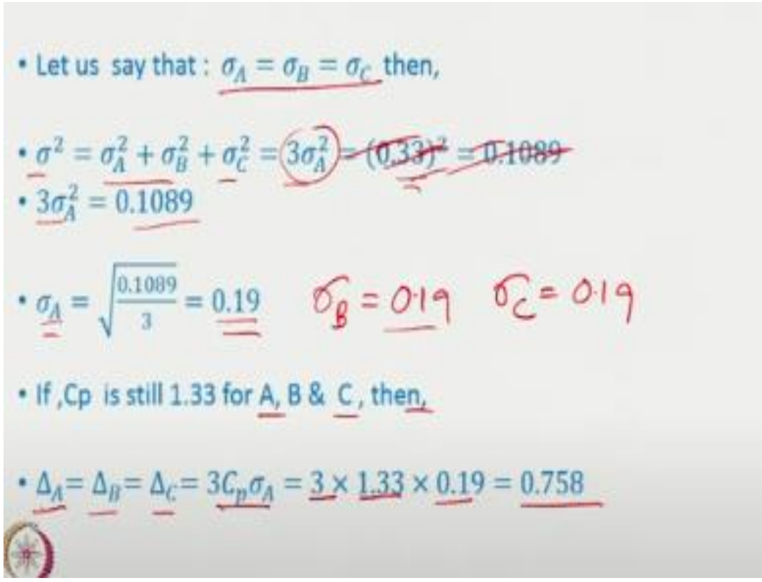

Let us assume that '$\sigma_A = \sigma_B = \sigma_C$' because '$\sigma_C$' was negative in the previous case. Now, the standard deviations are the same. Therefore, '$\sigma^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 = 3\sigma_A^2$'; from which, the value of 'σ_A' can be calculated as 0.19. Similarly, 'σ_B' and 'σ_C' are also 0.19, as they are all the same. If the 'C_p' is still 1.33 for 'A', 'B' and 'C', then,

$$\Delta_A = \Delta_B = \Delta_C = 3C_p\sigma_A$$

i.e., '3 × 1.33 × 0.19', to be around 0.758. So, these are the tolerances for 'A', 'B' and 'C'.

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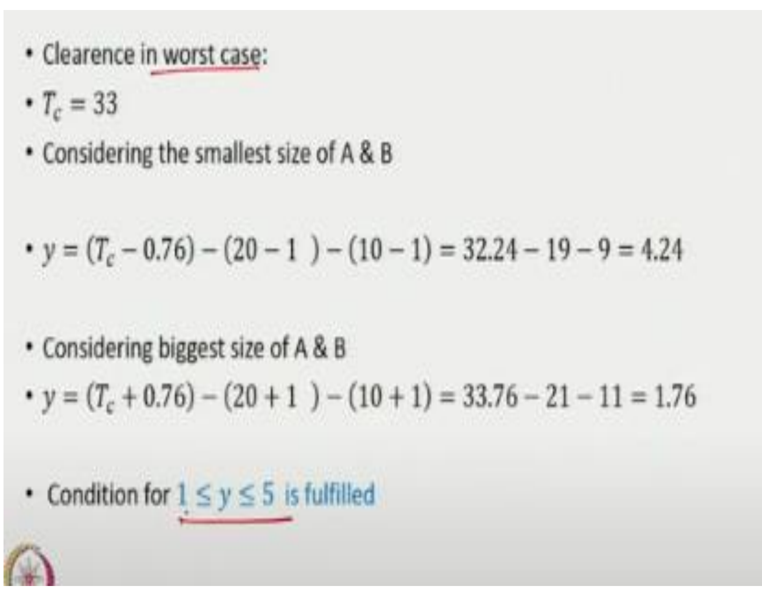

Next is the verification stage. We assumed that the sigma 'A', sigma 'B', sigma 'C' are the same, and the target value of 'C', i.e., 'T_C' was already found to be 33 cm. Considering the smallest size of the 'A' and 'B', how much is going to be the clearance 'y'?

$$y = (T_C - 0.76) - (20 - 1) - (10 - 1) = 32.24 - 19 - 9$$

The clearance 'y' is 4.24 cm.

Next, considering the biggest size of the 'A' and 'B', 'y' is,

$$y = (T_C + 0.76) - (20 + 1) - (10 + 1) = 33.76 - 21 - 11$$

The clearance 'y' is 1.76 cm. What was the requirement? The requirement was that 'y' should be between 1 and 5 cm. So, the requirement is fulfilled.

So, with this, we now close this discussion, which is on tolerance design. So, in some cases of textile product development, tolerance design will be very important. In many cases, it may not be that important because we can have wide tolerances and because it is not a life-threatening product. But there could be some products where it could have an implication on the life of a person. In those situations, meeting tolerances or designing as per tolerances becomes extremely important. Ok. With this, let us close today's session. Thank you.