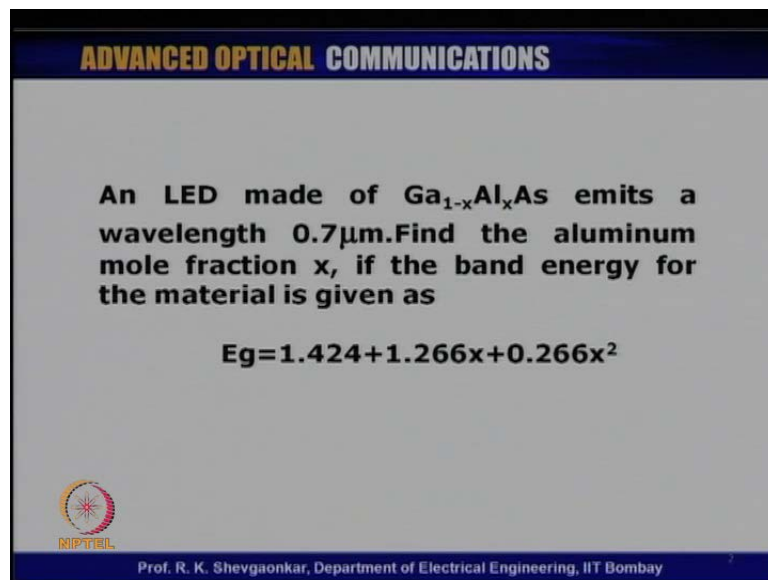


**Advanced Optical Communications**  
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**Module No. # 01**  
**Lecture No. # 31**  
**Tutorials**

In this tutorial we are going to discuss the problems related to the optical sources. We have seen while discussing theory that there are two types of sources which are used for optical communication. One is LED the light emitting diode and other one is ILD which is the injection laser diode we have seen the operation of LED the basic principles behind the emission of light from LEDs. We also saw the principle related to the lasers; so in this session we are going to solve some problems related to LEDs and the laser diodes.


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**ADVANCED OPTICAL COMMUNICATIONS**

**An LED made of  $Ga_{1-x}Al_xAs$  emits a wavelength  $0.7\mu m$ . Find the aluminum mole fraction  $x$ , if the band energy for the material is given as**

$$E_g = 1.424 + 1.266x + 0.266x^2$$

 **NIPTRIL**

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So, let us look at the first problem here ((no audio 01:08 to 01:38)). This problem is essentially related to designing a material for a emission of light at certain wavelength. We have seen by discussing LEDs that, there are these ternary materials which when combined in right proportion they form an alloy and the band gap of the material can be

manipulated by changing the mole fraction of the ternary materials. So, here is an example where we are having a ternary material which is gallium aluminum arsenide.

The problem says that this material gallium  $1 - x$  aluminum  $x$  arsenic emits a wavelength of 0.7 micrometer; find the aluminum mole fraction  $x$  if the band gap energy for the material is given as and that is band gap energy is given as  $1.424 + 1.266x + 0.266x^2$ . So, this problem is very simple problem so essentially what we are going to do first we convert the wavelength of emission to the corresponding band gap energy.

So, it is given that  $\lambda$  for the LED is 0.7 micrometer we know from the very basic relation that the band energy is given as  $1.24$  divided by the wavelength of emission in micrometers this band gap energy is given in terms of electron volts say if you substitute 0.7 in this we get the band gap energy which is  $1.24$  divided by  $0.7$  that is equal to  $1.77$  electron volts. Once we get the band gap energy then the problem is very simple we substitute this in the expression which is given there and then we find out the corresponding value of  $x$  so that becomes a simple algebra.

So, then we have here  $1.77$  that is  $E_g$  that is equal to  $1.424 + 1.266x + 0.266x^2$  square by rearranging we get a quadratic in terms of  $x$  square. So, that gives us  $0.266x^2 + 1.266x - 0.346$  that is equal to  $0$ . By solving this quadratic you will get the value of  $x$  and that is equal to  $0.259$  so this is one of the most simplest problem which says that if you are having a material gallium  $1 - x$  aluminum  $x$  and arsenic then, for this value of  $x$  which is  $0.259$  this ternary material will emit a wavelength which will be 0.7 micrometer.

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$$\lambda = 0.7 \mu\text{m}$$
$$E_g = \frac{1.24}{\lambda (\mu\text{m})} \text{ eV} = \frac{1.24}{0.7}$$
$$= 1.77 \text{ eV}$$
$$1.77 = 1.424 + 1.266x + 0.266x^2$$
$$0.266x^2 + 1.266x - 0.346 = 0$$
$$x = 0.259$$


This essentially say that whenever we try to design the LEDs corresponding to different wavelengths you have to find out a proper combination of this materials and by manipulating this mole fraction of the constituent elements of this composite material one can design a material to find out the wavelength of emission and which can be tailor made depending upon the requirement. So, this is 1 of the very simple problems related to the LED.

Let us consider the second problem ((no audio 06:11 to 06:41)) now this problem is related to finding out the radiative recombination lifetime of a semiconducting material see if you get an LED and if you inject current inside the LED then the recombination takes place and the depending upon the recombination rate the speed at which the LEDs can be modulated that is decided so here the problem is that LED has a 50 micrometer diameter emitting area and operates at a peak modulation current of 100 mille amperes. It is also given that this current is falls in the domain of what is called the high injection. Find the bandwidth of LED having 2 micrometer active area thickness the recombination constant  $B_r$  is given as  $10 \text{ to the power minus } 10 \text{ centimeter minus } 3 \text{ per second}$ .

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**ADVANCED OPTICAL COMMUNICATIONS**

**An LED has a 50  $\mu\text{m}$  diameter emitting area and operates at a peak modulation current of 100mA (high injection). Find the bandwidth of the LED having 2  $\mu\text{m}$  active area thickness. Take recombination constant  $B_r = 10^{-10} \text{ cm}^{-3}/\text{s}$ .**

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Let us start with the basics of finding out the recombination equation so let us say we are having inside the LED the  $\Delta n$  carriers are injected and let us say that equal number of holes are injected so  $\Delta n$  is equal to  $\Delta p$  now the rate of recombination or decrease in the injected carrier per unit time that is given as the recombination constant  $B_r$  times the product of the electron density and the hole density so this is  $n_0 + \Delta n$  times  $p_0 + \Delta p$ .

These are the carrier which are injected inside the junction however there are going to be thermal generation which you have to take care of so you subtract the thermal generation which is  $B_r \times n_0 \times p_0$  now we can do little algebra here which gives that  $-\frac{d}{dt} \Delta n$  that will be equal to the recombination constant  $B_r$  times this  $n_0 + \Delta n$  times  $p_0 + \Delta p$ . This term will cancel with this so you get here  $n_0 + p_0$   $\Delta n + \Delta n \times \Delta p$  and if assuming  $\Delta n$  is equal to  $\Delta p$  this is  $\Delta n^2$ .

So, from here now if you take the high injection the  $\Delta n$  is much larger compared to this quantity so we get for high injection  $\Delta n$  is  $n_0$  or  $p_0$  and then you get the rate of decrease of the injected carriers that is equal to  $B_r \times \Delta n^2$  we can write that as  $B_r \Delta n \times \Delta n$ . So, then the radiative recombination lifetime essentially driven upon this quantity this simple first order differential equation which gives that the carriers injected carriers are exponentially decay with a lifetime which is  $1$  over this quantity.

So, you get here then the  $\tau_{rr}$  that is equal to  $1$  upon  $B_r$  into  $\Delta n$  but, we also know that this  $\Delta n$  if we consider a junction that that is the region where the carriers are injected which is the active region and that is the area and that is the depth so in this volume essentially you are going to have the excess carriers so the injected carrier density  $\Delta n$  that is equal to the injected current  $I$  divided by the charge of the carrier  $q$  divided by the volume of this region that is the area of cross section and the  $d$ .

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The whiteboard contains the following handwritten equations and notes:

$$-\frac{\partial \Delta n}{\partial t} = B_r \left\{ (n_0 + \Delta n)(p_0 + \Delta p) \right\} - B_r n_0 p_0$$

$$-\frac{\partial \Delta n}{\partial t} = B_r \left\{ (n_0 + p_0) \Delta n + \Delta n^2 \right\}$$

For high injection  $\Delta n \gg n_0, p_0$

$$-\frac{\partial \Delta n}{\partial t} = B_r \Delta n^2 = (B_r \Delta n) \Delta n$$

$$\tau_{rr} = \frac{1}{B_r \Delta n}$$

$$\Delta n = \frac{I}{q A d} \tau_{rr}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$\tau_{rr} = \sqrt{\frac{q A d}{B_r I}}$$

A small diagram shows a rectangular junction with thickness  $d$  and area  $A$ , with a current  $I$  flowing through it.

You have area cross section here that is a the thickness of the active region is  $d$  and the lifetimes are going to remain there the carriers are going to remain there for the lifetime which is  $\tau_{rr}$ . So, this  $\Delta n$  will be given by this so if I take this  $\Delta n$  and substitute into this I will get a term which is  $\tau_{rr}^2$ . By substituting this then we can get  $\tau_{rr}$  that is equal to square root of  $q a d$  divided by the recombination constant  $B_r$  into  $I$  now all this quantities are given here the  $q$  is electronic charge which is given so  $q$  is  $1.6 \times 10^{-19}$  coulomb.

The area of LED junction is given because, it says that the LED has an emitting area of  $50 \mu\text{m}$  diameter so  $A$  is given as  $\pi (50 \times 10^{-6})^2$  divided by  $4$   $d$  which is the thickness of the active region is given as  $2 \times 10^{-6}$  meters. This is meter square and  $B_r$  is the quantity which is given recombination constant that is  $10^{-3} \times 10^{-10}$  centimeter minus  $3$  per second. So,

this will be equal to  $10$  to the power minus  $10$  into  $10$  to the power  $6$  meter minus  $3$  per second.

Now, we know all the quantities in this expression we know we know  $q$  we know the area of cross section we know the diameter we know the thickness of the active region we know the recombination constant. So, from here then we can calculate this value of  $\tau_{rr}$  so  $\tau_{rr}$  can be calculated once you know  $\tau_{rr}$  then the bandwidth of this device  $\Delta\omega$  is approximately  $1$  upon  $\tau_{rr}$  so substituting this values of  $A$   $d$   $B_r$   $q$  we can calculate this value of  $\tau_{rr}$  and taking  $1$  upon that then we can find out approximate bandwidth which the LED will have.

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$$A = \frac{\pi (50 \times 10^{-6})^2}{4} \text{ m}^2$$

$$d = 2 \times 10^{-6} \text{ m}$$

$$B_r = 10^{-10} \text{ cm}^{-3}/\text{s} = 10^{-10} \times 10^6 \text{ m}^{-3}/\text{s}$$

$\tau_{rr}$  can be calculated.

$$\Delta\omega \approx \frac{1}{\tau_{rr}}$$

So, one can do simple calculations to get the bandwidth corresponding to the LED the third problem is related to the injection laser diode ((no audio 15:31 to 16:03)). It says that when you are having if you write the red equation for the injection of the carriers inside the junction then, there are 2 processes which are going to take place 1 is the injected carrier which has the excess carrier which are going to be in the region these carriers are going to exponentially decay as the time goes with the recombination lifetime and then the net change in the carrier density in the region is the balance between these two.

This problem essentially is the equation giving the balance between the injected carrier and the recombination of the carriers so the problem is when a current pulse  $I_p$  is

applied to an unbiased laser diode unbiased means that when the current is not applied when the pulse is not applied the current injected inside the laser diode is 0. So, pulse  $I_p$  is applied to an unbiased laser diode the injected carrier density  $\Delta n$  in the recombination region changes as then the equation is given here which is  $d$  by  $d t$  of  $\Delta n$  is equal to  $I_p$  divided by  $q a d$ , where  $q$  is the charge electronic charge  $a$  is a area cross section  $d$  is the depth of the active region.

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**ADVANCED OPTICAL COMMUNICATIONS**

When a current pulse  $I_p$  is applied to an unbiased laser diode, the injected carrier density  $\Delta n$  in the recombination region changes as

$$\frac{\partial(\Delta n)}{\partial t} = \frac{I_p}{qAd} - \frac{\Delta n}{\tau}$$

Where  $\tau$  is the average carrier life time, and  $A$  and  $d$  are area and thickness of the recombination region respectively. If the laser threshold current is  $I_{th}$  show that the time needed for the onset of the stimulated emission is

$$\tau_d = \tau \ln \frac{I_p}{I_p - I_{th}}$$

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This is the excess carrier which are injected inside the junction minus the recombination of the carriers which are having a lifetime of  $\tau$  so minus  $\Delta n$  divided by  $\tau$  so that is the equation which governs the carrier density inside the active region so it is asked that if the laser threshold current is  $I_{th}$  show that the time needed for the onset of the stimulated emission is by the expression which is  $\tau_d$  equal to  $\tau \ln \frac{I_p}{I_p - I_{th}}$ .

That is problem also is essentially related to solving the differential equation which governs the excess carriers in the recombination region so here we get  $d \Delta n$  by  $d t$  that is equal to the injected current which is  $I_p$  divided by the electronic charge area of cross section  $d$  is the thickness of the active region minus  $\Delta n$  divided by the carrier recombination lifetime  $\tau$ . Now, this equation can be solved very simply so the solution of this equation as we know as a function of time;  $n$  is a function of time can be given as  $I_p \tau$  divided by  $q a d$  into  $1 - \exp(-t/\tau)$ .

One can verify that when  $t$  is equal to 0 the time they were the injected carriers are 0 and when  $t$  goes very high then essentially you reach to a steady state and that is the carrier density you are going to have in the steady state a domain. So, it is the carriers number of carriers per unit time multiplied by the recombination lifetime so that is the steady state density you will see inside the a junction of the laser diode. Now, it is said that so then if you look at the laser characteristic we know that if the laser was unbiased the initial current is 0 and then for lasing to take place the electron density has to increase to a certain value where population inversion takes place and then the laser will start lasing.

So, it is going to take certain time to build the electron density in the recombination region to a minimum required value. So, let us say the time which takes to build that electron density is given by  $t_d$  so then, we have at  $t$  is equal to  $t_d$  that much delay is there in building that electron density. The current reaches to the threshold current so if I take this and substitute into that we will see that the  $\Delta n$  threshold you calculate that will correspond to the  $I_{th}$  threshold tau divided by  $q a d$ .

So, after  $t$  equal to  $t_d$  then the threshold the current should reach to threshold current that means the carrier density should reach to this value. Now, the problem is very simple; we can take this and substitute into this equation. So, you get  $I_{th} \tau$  divided by  $q a d$  that is equal to  $I_p \tau q a d$  into  $1 - e^{-t_d / \tau}$  divided by  $\tau$ . This term will now cancel the tau will cancel the  $q a d$  will cancel and we will get a simple expression in terms of  $I_p$   $I_{th}$  and the lifetime of the carriers tau.



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$$\frac{\partial (\Delta n)}{\partial t} = \frac{I_p}{q A d} - \frac{\Delta n}{\tau}$$
$$\Delta n(t) = \frac{I_p \tau}{q A d} (1 - e^{-t/\tau})$$

At  $t = t_d$ ,  $I = I_{th}$   $(\Delta n)_{th} = \frac{I_{th} \tau}{q A d}$

$$\frac{I_{th} \tau}{q A d} = \frac{I_p \tau}{q A d} (1 - e^{-t_d/\tau})$$
$$e^{-t_d/\tau} = \frac{I_p - I_{th}}{I_p}$$
$$t_d = \tau \ln \left( \frac{I_p}{I_p - I_{th}} \right)$$

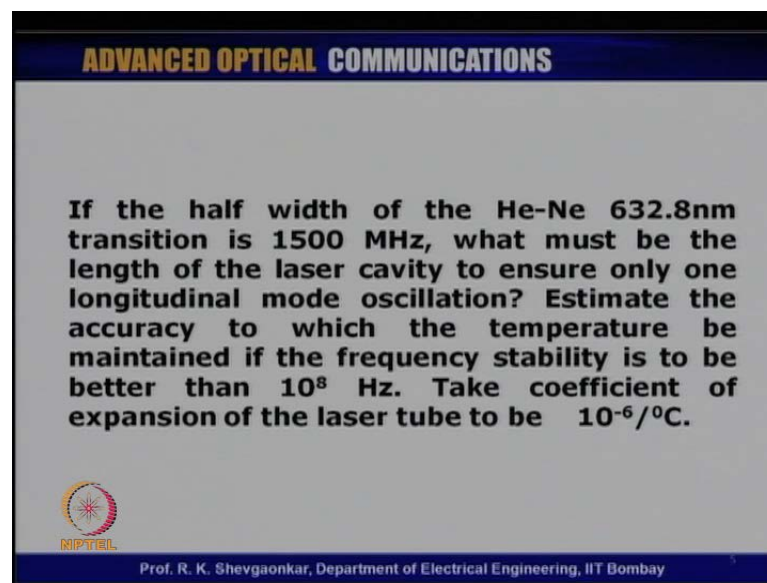
So,  $e^{-t_d/\tau}$  is equal to  $(I_p - I_{th})/I_p$ . Inverting this, we get the delay which is  $t_d$  that is equal to  $\tau \ln(I_p / (I_p - I_{th}))$ . Taking log on both sides,  $I_p / (I_p - I_{th})$ . That is what we actually wanted to show that the delay which is going to take place in this - in building the population inversion up to a threshold value that actually is given by this relation is important relation because, what it tells you is that if the laser was biased at 0 current then it will require certain time to build the population inversion.

So, if you wanted to modulate at a faster rate you will not be able to lase it at a very faster rate because this much time is required to get the laser into action so this parameter essentially tells you about the speed of the device or modulation speed which the laser would have so of course, if the threshold current was equal to was large. Then, this quantity will become smaller and smaller and then, you will see that as you will see that the other speed increases what this quantity essentially goes down.

So, whenever you want to operate laser we must make sure that the laser is biased at the threshold current but, this problem essentially tells you that it was not biased properly then it requires certain time to get into action because the laser has to build a population inversion and that will require certain time. So, this a simple problem which is related to the basic operation of a laser diode; let us look at now a problem related to the helium neon laser.


((no audio 24:33 to 25:04)) we all know the helium neon laser is used in pointers in laboratory experiments because, that is the laser which gives the red light the optical communication uses optical spectrum which is in infrared but, in laboratory when we conduct the experiments we want to have a light which is visible and helium neon gives a wavelength which is 632.8 nanometer which is a red color so this is the laser which is very commonly used.

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**ADVANCED OPTICAL COMMUNICATIONS**

**If the half width of the He-Ne 632.8nm transition is 1500 MHz, what must be the length of the laser cavity to ensure only one longitudinal mode oscillation? Estimate the accuracy to which the temperature be maintained if the frequency stability is to be better than  $10^8$  Hz. Take coefficient of expansion of the laser tube to be  $10^{-6}/^{\circ}\text{C}$ .**

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The problem here is that if the half width of helium neon laser which is which corresponds to a transition of 632.8 nanometer and the bandwidth of this transition is 1500 megahertz. What must be the length of the laser cavity to ensure only 1 longitudinal mode oscillation? Estimate the accuracy to which the temperature we maintain if the frequency stability is to be better than  $10^8$  hertz take coefficient of expansion of the laser tube to be  $10^{-6}$  per degree centigrade,

As we know that in the lasers we are having this transition energy levels and the electron makes a transition from here to here and that gives you a laser light is the stimulated emission we also know that because of various factors these energy levels are not really sharp so these energy levels are little diffused and because of that you do not get a monochromatic light from the laser. But, you get a certain bandwidth associated with the emission of from the system.

So, what is given here? That you are having a spectrum of a laser which is like this and the center wavelength of this is 600 and 32.8 nanometer which is the red color it is also given that the half width of the spectrum emission spectrum is 1500 megahertz. Now, if you want to have only 1 mode, essentially what we are saying is that the line separation for the mode which are generated inside the cavity of this laser should be more than this or should be at least equal to this if you make sure then only 1 longitudinal mode will propagate inside the cavity and then you will get only single mode output from this laser.

Now, write the simple relation that the mode number  $m$  inside the laser that is equal to length of the cavity divided by  $\lambda$  by 2 and if the medium is not air then, you are having a refractive index for that medium which is given as  $n$ . Now, for helium neon laser since the medium is a gaseous medium the  $n$  can be taken approximately equal to 1. So, take  $n$  approximately equal to 1 so that simply says this is equal to  $2L$  divided by  $\lambda$ .

Now, we find out what is the if and if we consider the adjacent modes then we want to find out what is the change in the wavelength or what is the spacing between 2 adjacent modes in terms of the wavelength. So, we can differentiate this to get  $d m$  is equal to minus  $2L$  upon  $\lambda^2$  into  $d\lambda$ . But, in this case we are not given the spectral width in terms of the wavelength, we are given now the spectral width in terms of the frequency 1500 megahertz.

So, rather than using this relation in terms of  $\lambda$  let us use this relation in terms of frequency so this will be  $2L$  divided by  $\lambda$  we can put as  $c$  into the frequency or if I differentiate now with respect to frequency that will be equal to  $2L$  divided by  $c$  into  $d f$  or  $d f$  now for the adjacent mode the  $d m$  is equal to 1 so for adjacent modes  $d m$  is equal to 1 so from here we can find out what should be the value of  $L$  so that I get the lines which are separated more than 1500 megahertz. Say if I substitute  $d f$  equal to 1500 megahertz I can find out value of  $L$  and that is for  $d m$  equal to 1 so you will get from here  $L$  which is  $c$  divided by  $2 d f$  which is equal to  $3 \times 10^8$  to the power 8 velocity of light 2 into the transition bandwidth which is 1500 multiplied by  $10^6$  or megahertz and meters. So, if I solve this you will get a length of this is 100 millimeter, this tell us that if you take a helium neon laser tube which is about 100 millimeters then, it will generate a single mode propagation if the transition bandwidth is about 1500 megahertz.

So, that is the first part of the problem what should be the length of the tube to get a single mode longitudinal oscillation inside the cavity. The second problem is that if you wanted to have certain stability for the frequency emitted by the laser then the length of the tube should not change. But, as a function of temperature because the expansion coefficient the size of the tube is going to change and when the size of tube changes the resonant frequency changes and because of that the frequency of the laser changes.

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The image shows handwritten notes on a whiteboard. At the top left, there is a diagram of a laser cavity with two mirrors and a central gain medium. Wavy arrows labeled "Stimulated emission" point from the gain medium towards the mirrors. To the right, a graph shows a frequency distribution curve with a peak at 1500 MHz and a width of 632.8 nm (Red). Below the graph, it says "Take  $n \approx 1$ ".

The mathematical derivations are as follows:

$$m = \frac{L}{\lambda/2n} = \frac{2L}{\lambda} = \frac{2L}{c} f$$

$$dm = -\frac{2L}{\lambda^2} \Delta\lambda = \frac{2L}{c} df$$

For adjacent mode  $dm = 1$

$$L = \frac{c}{2 df} = \frac{3 \times 10^8}{2 \times 1500 \times 10^6} \text{ m} = 100 \text{ mm}$$

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

As a question we are asking is how accurate the temperature should be maintained so that we do not have a deviation in the frequency more than 10 to the power 8 hertz so again we can go back to the same relation to find the stability of the frequency so within the same relation here we can write that the frequency is given as  $c$   $m$  divided by  $2L$  so we can ask the deviation in frequency now as a function of  $L$ . So, if I ask  $\Delta f$  how much is going to be because of change in the length this will be minus  $c$   $m$  divided by  $2L$  square multiplied by  $\Delta L$ .

But, this quantity again can be substituted for the center wavelength of the laser. So, this gives  $\Delta f$  divided by the center wavelength of the laser  $f$  naught which is equal to  $\Delta L$  divided by  $L$  and center wavelength or center frequency of the laser  $f$  naught can be calculated because, we know the wavelength of helium neon laser. So, it is  $3 \times 10^8$  to the power 8 velocity of light divided by the wavelength of helium neon  $632.8$  nanometer  $10$  to the power minus  $9$  and we are given that now the stability has to be

better than 10 to the power 8 hertz. That means, this quantity delta f for this problem is 10 to the power 8 hertz, this also hertz. So, if we substitute into this we get delta L upon L which is equal to 10 to the power 8 divided by this quantity f naught which will turn out to be 0.2 1 into 10 to the power minus 6. Now, the length expansion coefficient is given as 10 to the power minus 6 if you substitute that then, we can get temperature accuracy equal to 0.2 1 degrees centigrade because, expansion coefficient is given as 10 to the power minus 6 per degree centigrade.

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$$f = \frac{c}{2L}$$

$$\Delta f = -\frac{c}{2L^2} \Delta L$$

$$\frac{\Delta f}{f_0} = \frac{\Delta L}{L}$$

$$f_0 = \frac{3 \times 10^8}{632.8 \times 10^{-9}} \text{ Hz}$$

$$\Delta f = 10^8 \text{ Hz}$$

$$\frac{\Delta L}{L} = \frac{10^8}{f_0} = 0.21 \times 10^{-6}$$

$$\text{Temp accuracy} \approx 0.21^\circ \text{C}$$

So, this calculation show that if you make a helium neon laser then, if you wanted to do a single mode operation for the kind of parameter which are given then the tube length has to be 100 millimeters which is about 10 centimeters and then this length has to be maintained for this accuracy of 10 to the power 8 hertz. So, the temperature of this laser has to be maintained with an accuracy better than about point 2 degrees centigrade then the frequency stability of 10 to the power 8 can be achieved.

Let us now look at the problem which is a related to the semiconductor lasers ((no audio 36:28 to 37:01)). So, as we know the semiconductor laser we are having the gain function and then we are having the losses which are there inside the laser, we also know that the gain function normally is a frequency sensitive function. But, the losses which take place inside the lasers (( )). I will redo it we have seen from the red equation of the

laser that when the current is injected inside the laser. It requires certain time for building the population inversion and then only the laser gets into the lasing action.

We know that when the laser if the speed of the laser has to be increased then, the laser has to be biased at the threshold current. Now, if it is not biased at the threshold current then from the biasing point the time is required to get in to the, up to threshold current and beyond that then the lasing action essentially starts. there are many reasons why the laser cannot be exactly bias at the threshold current so the bias current normally is different than the threshold current it may be little smaller in the threshold current.

In that situation we have seen that the delay for getting the laser into lasing action is given as  $t_d$  which is equal to the lifetime of the carrier  $L_n$  the peak current which is injected inside the laser divide by  $I_p$  plus  $I_b$  minus  $I_{\text{threshold}}$  where this consists the peak current which is injected this is the biasing current and that is the laser threshold current. Now,  $\tau$  is the lifetime of the carrier and this is to be calculated for the data which is given inside this in the problem.

Now, the problem is the semiconductor laser has a threshold gain of 50 per centimeter and the semiconductor has a refractive index of 4 the threshold current for the laser is 100 mille amperes and the laser is biased at 50 mille amperes the laser is to be modulated by the current pulses of 1 50 mille amperes. What is the highest pulse rate to which the laser can be modulated satisfactorily? So, as we have seen in the earlier problem the speed can be calculated. If I know this time which is  $t_d$  all this parameters are given  $I_p$  is given  $I_b$  is given  $I_{\text{threshold}}$  is given only this is the quantity which is not given directly in this problem.

What is given however is, what is the threshold gain which is  $g_{th}$  that is threshold gain? Essentially, a photon remains inside the cavity for  $1$  over this quantity that is approximately the distance which the photon travels inside the cavity. So, if I want to calculate what is the lifetime inside the cavity this  $\tau$  suppose this  $\tau$  will be equal to  $1$  upon  $g_{\text{threshold}}$ . This is the on average distance travelled by the photon before it decays divided by the velocity of light in the medium which is  $c$  divide by  $n$  where  $n$  is the refractive index of the material.

Now,  $g_{th}$  is given  $n$  is given for this problem. So, this is  $n$  divided by  $c$  to  $g_{th}$  when I substitute the parameters given in this problem refractive index  $n$  is 4 the  $g_{th}$  is given as

50 per centimeter  $c$  is the velocity of light so we substitute this quantities in the expression so we get 4 divided by 3 into 10 to the power 8 meters per second multiplied by 50 is given per centimeter so 10 to the power 2; so if we just solve this we get the tau which is 2.66 Pico second.

So tau d which is the delay is given as  $2.66 L \ln I_p$  is given as 150 mille amperes which is 150 lus the bias current is given as 50 mille amperes and the threshold current of the laser is 100 mille amperes. So, that is equal to  $2.66 L \ln 150$  divided by 100; that is equal to 1.07 Pico second so the pulse rate maximum pulse rate which the laser can be modulated to maximum that is approximately  $1$  upon  $t_d$ . So, that is equal to  $1$  upon  $1.07$  into 10 to the power minus twelve per Pico second; so that will be equal to 943 gigahertz.

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$$t_d = \tau \ln \left\{ \frac{I_p}{I_p + (I_B - I_{th})} \right\}$$

$g_{th} \rightarrow$  Threshold gain

$$\tau = \frac{1/g_{th}}{c/n} = \frac{n}{c g_{th}}$$

$$= \frac{4}{3 \times 10^8 \times 50 \times 10^2} = 2.66 \text{ psec}$$

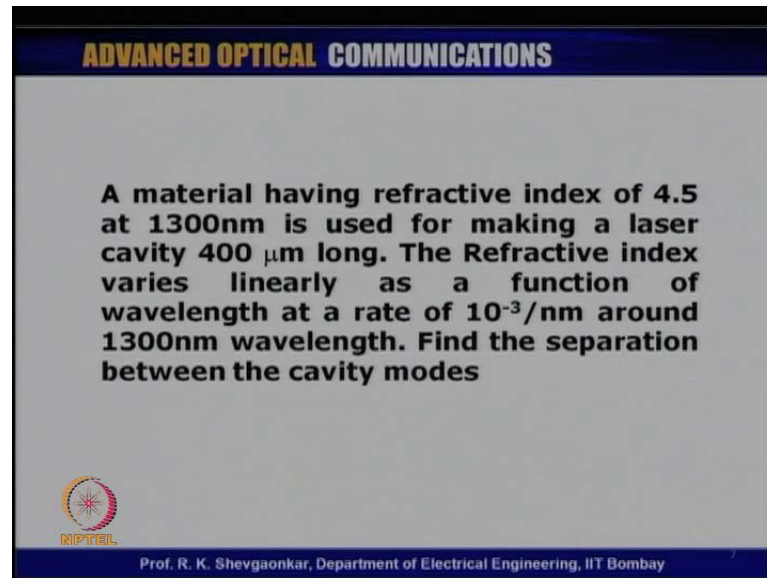
$$t_d = 2.66 \ln \left\{ \frac{150}{150 + (50 - 100)} \right\}$$

$$= 2.66 \ln \left\{ \frac{150}{100} \right\} = 1.07 \text{ psec}$$

Pulse rate  $\approx \frac{1}{t_d} = \frac{1}{1.07 \times 10^{-12}} = 943 \text{ GHz}$


This problem essentially is a problem related to when the lasers are biased how the laser should be bias and depending upon the biasing point what kind of modulation rates can be achieved from the laser let us look at another problem related to the laser ((no audio 44:19 to 44:55)) a material having refractive index of 4.5 at 1300 nanometer is used for making a laser cavity for 100 micrometer long. The refractive index varies linearly as a function of wavelength at a rate of 10 to the power minus 3 per nanometer around 1300 nanometer wavelength find the separation between the cavity modes.

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**ADVANCED OPTICAL COMMUNICATIONS**

**A material having refractive index of 4.5 at 1300nm is used for making a laser cavity 400  $\mu\text{m}$  long. The Refractive index varies linearly as a function of wavelength at a rate of  $10^{-3}/\text{nm}$  around 1300nm wavelength. Find the separation between the cavity modes**

 MPTEL

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We have seen the relation for finding out the separation between the lines inside the cavity but, in those problem we have assume that the refractive index of the medium remains constant as a function of wavelength. Here, we are saying now that the material use for creating the laser that material is a dispersive material because, the refractive index is varying as a function of wavelength. So, in this situation we are asked to find out what should be the line separation for the cavity modes.

Again, we can start with the basic relation that the length of the cavity required is the mode number  $m$   $\lambda$  divided by  $2n$  same relation which you have used earlier. In earlier problems so again from here we can get  $m$  equal to  $2nL$  divided by  $\lambda$  note here however in this case now the  $n$  is a function of wavelength it is not a constant quantity so as you have done earlier to find out the separation between the modes if I differentiate this with respect to  $\lambda$  you will get a change in  $m$  which is the modal index that is equal to  $2L$  I can differentiate this quantity with respect to  $\lambda$ .

So, that is  $\lambda \frac{dn}{d\lambda} - n$  divided by  $\lambda^2$  multiplied by  $d\lambda$  again for the adjacent mode this quantity  $dm$  is equal to 1. So, this quantity you put equal to 1 for the adjacent modes and then inverting the relation then 1 can get  $d\lambda$  which is equal to  $\lambda^2$  divided by  $2L$  which is negative sign here. So,



there is a  $n$  minus  $\lambda \frac{dn}{d\lambda}$  by  $d\lambda$  now we are given all the parameters in this problem where  $n$  is the refractive index of the material.

So, that is given  $n$  is given as 4.5 the rate of change of refractive index as a function of wavelength is also given so this quantity  $d n$  by  $d \lambda$  is equal to  $10^{-3}$  per nanometer so  $L$  which is the length of the cavity is given as 400 micrometer. All the parameters are known now so you can find out what is the corresponding quantity which is less than  $\lambda$  which is the separation between the 2 adjacent modes if you substitute in to this you get  $d \lambda$  that is  $\lambda$  is 1300 nanometer.

This 1300 square divided by 2 into 400 micrometers if you want to convert in to nanometer  $10^{-3}$  into 4.5 minus  $\lambda$  which is 1300 multiplied by  $d n$  by  $d \lambda$  which is  $10^{-3}$ . So, if you solve this you get a separation between the lines for this laser that will be equal to 0.66 nanometer and therefore, happens when in typically for the semiconductor lasers that the material refractive index is not independent of frequency of wavelength.

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$$L = \frac{m \lambda}{2 n}$$

$$m = \frac{2 n L}{\lambda}$$

$$- dm = 2 L \left\{ \frac{\lambda \frac{dn}{d\lambda} - n}{\lambda^2} \right\} \cdot d\lambda$$

$$\downarrow$$

$$1 \quad d\lambda = \frac{\lambda^2}{2 L \left\{ n - \lambda \frac{dn}{d\lambda} \right\}}$$

$$n = 4.5, \quad \frac{dn}{d\lambda} = 10^{-3} / \text{nm}, \quad L = 400 \mu\text{m}$$

$$d\lambda = \frac{(1300)^2}{2 \times 400 \times 10^3 (4.5 - 1300 \times 10^{-3})}$$

$$= 0.66 \text{ nm}$$

This problem essentially is the problem related to the material was dispersive then I can find out what is the precise separation between the modes inside the laser cavity. Let us look at 1 more problem related to semiconductor laser ((no audio 50:50 to 51:22)). As we know inside the laser the gain function is a frequency dependent function. But, the losses which take place inside the cavity normally they are frequency independent and as

we know inside the cavity the 2 mechanisms operates simultaneously the photons are generated because of the gain function and the photons are lost because of the other loss mechanism inside the cavity.

Is the net gain between these 2 which decides whether the mode will go into the oscillation or the frequency will go into the oscillation or not? So, if the gain is dominant then that frequency has a tendency to go in to oscillation. If the gain was less than losses then that frequency dies down exponentially. So, essentially those frequencies over which the gain is more than the losses inside the cavity there is a tendency for amplification or going in to stimulated emission. Thus, the thing which is captured in to this problem.

So the problem is a gallium arsenide laser has a 400 micrometer long cavity with a refractive index of 3.6 the material gain function is Gaussian with its peak at 800 nanometer and its sigma as the Gaussian function is 2 nanometer. The maximum gain at 800 nanometer is 50 per centimeter if the loss in the cavity is 3 per centimeter and is independent of the wavelength find the number of modes which will exist in the laser.

So let us first see what is given here it is given that the gain function of a laser is a Gaussian function the center wavelength for this Gaussian function is 800 nanometer the peak gain which the function has is 50 centimeter minus 1 so the Gaussian function since the sigma is given as 2 nanometer you can write this gain function  $g$  as  $50 e^{-\frac{(\lambda - 800)^2}{2 \sigma^2}}$  since sigma is given as 2 which is square.

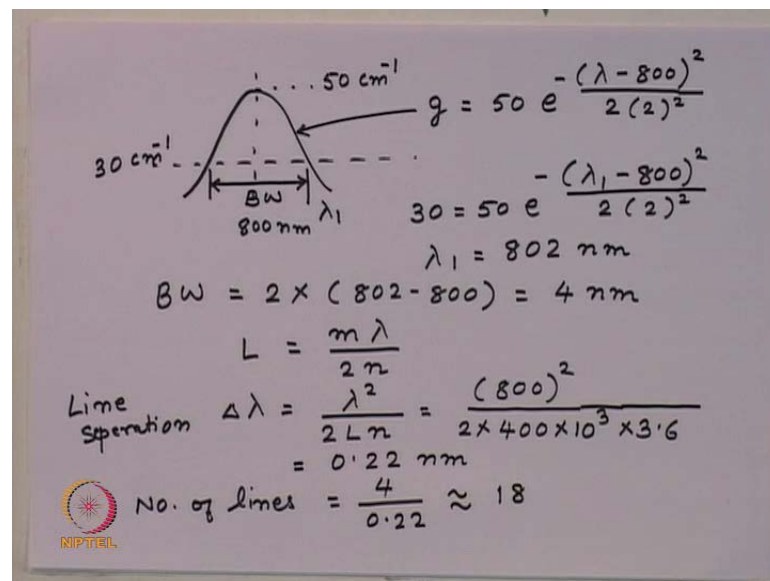
Then we are having a loss function which is independent of the wavelength and that is this which is 30 centimeter minus 1. So, that means over this wavelength range we have a gain more than the losses that means these are the frequencies which lie between this from here to here. They have a gain net gain in to the system so these are the frequency which will go into oscillation. So, let us say this is the  $b$  this is the thing which is the bandwidth over which we are going to get the gain into this is net gain into the system.

So we can ask, what is this wavelength here? let us say this wavelength is  $\lambda_1$  at which the gain equals the loss. So, substituting the  $g$  equal to 30 we can get  $30 e^{-\frac{(\lambda_1 - 800)^2}{2 \sigma^2}}$  because, that that is the wavelength at which the gain becomes 30 minus 800 whole square divided by 2 square. Now, solving for  $\lambda_1$  from here, we

get  $\lambda_1$  is equal to 802 nanometer. So, this quantity bandwidth which is 802 minus 800 which is 2 nanometer here and 2 nanometer here, the bandwidth which we have where the gain takes place is equal to 2 times 802 minus 800 which is equal to 4 nanometers. We want to find out, what are the total number of modes. Now, which are going to get excited in this bandwidth? Again, the exercise is very similar we have to find out. Now, what is the mode separation in terms of wavelength? If we get that then we can find out what is the total number of modes in this bandwidth.

Again, we use the same relation that  $L$  is equal to  $m \lambda$  divided by  $2n$  and from here we can calculate  $\Delta \lambda$  which is  $\lambda^2$  divided by  $2Ln$ . This is the line separation so we can substitute the parameters using 800 nanometer lasers. The length of the cavity is given as 400 ( ). So, this is 10 to the power 3 into 3.6 so we get  $\Delta \lambda$  which is equal to 0.22 nanometer. So, the total number of lines which will exist inside this laser corresponding to this bandwidth which is 4 nanometer.

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We get number of lines equal to bandwidth which is 4 nanometer divided by the line separation which is 0.22 which is approximately equal to 18. So, in this tutorial essentially we saw the problem was related to the sources which are used for optical communication. There are some problems which you have seen regarding LEDs finding out the bandwidth of the LED. Finding the bandwidth of the laser finding out the

switching time of the laser depending upon the biasing of the laser, also the number of lines which exists inside the typical laser.

Normally, whenever we use this component into practice this information normally is required because that tells you approximately what is the effective spectral width of your source this problem also show that whenever we talk about laser even the lasers normal lasers do not have 1 mode propagating inside them there are large number of mode which propagate and the effective width spectral width of a laser is therefore, typically if the order of the order of few nanometers.

As you have seen in the lectures to improve this then there are other laser which are used which are the d f b lasers and so on but, this problem which you have solved in this essentially are for the typical lasers which are used for optical communication. So, this tutorial has given some hands on experience on solving the problems related to the lasers and the light emitting diodes.