Advanced Optical Communications
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Module No. # 01

Lecture No. # 32

Introduction to non–linear Fiber Optics

Welcome to this course on fiber optic communication and in this course, now you are going to discuss more advance topics the Non-linear Fiber Optics. Up till now, whatever discussed that falls into the category of linear fiber optics, what that means is that the medium properties, the fiber properties do not depend upon the signal. On the other hand when the material properties are modified by the signal itself, then that falls into a category of what is called non-linear fiber optics.

So, now onwards for next couple of lectures, we are going to discuss the propagation of light in the optical fiber taking into consideration the non-linear effects, the simple question one can ask is that if the medium properties change, what will happen to the signal propagation. Take a simple example let us say, we want to transmit a pulse on the optical fiber, when the pulse was propagating in the linear domain, than the properties of the propagation are decided just by the medium properties.

We could write down the wave equation, we can solve the wave equation without varying, what is the amplitude of the pulse and we get the model distribution; we get propagation constant and from that, we can find out the propagation characteristics of a pulse.

Imagine now, that medium properties are modified by the pulse itself and let us say that there refractive index of optical fiber is related to the intensity of flight. And first simple example let us assume, that the refractive indies increases with the intensity of light. What means is that now, the whole pulse is not going to see is the same propagation parameters, because at the center of the pulse that the light is intense, the refractive index seen by the pulse is different, than on the age of the pulse; where the light is less intense and because of that refractive index is lesser, compare to the refractive index seen by the pulse at it is center.

So, you require actually whole new frame work for analyzing this problems and that is what precisely we are going to do in next couple of lectures, that if the medium properties are altered, why the signal itself, what will happened to the propagation of a pulse. Second thing we can see later on, that when the non-linear effects are present in the optical fiber continues signal propagation is unstable, what that means is even if I try to send a continues signal. In the optical fiber any small perturbation will break this signal into pulses; that means without taking into consideration, the pulse nature of the signal the non-linear propagation will not be very effective.

So, in this lecture first up all will basic formulation will also ask a question, why non-linear fiber optics is important, is it only a question of academic interest or is it becoming more and more important to include, the effect of non-linearity into fiber optic communication. And if it all us then we can we make use of this non-linear effects to improve the signal propagation from optical communication point of view.

So, let us take very basic model, what is the dielectric constant of medium, when the electric field is impose on dielectric material, there is induce polarization in the material, which is given by the susceptibility of the medium. And normally, we consider only the first order susceptibility, which give the dielectric constant of the medium; however this is just an approximation. In real situation if the light of intensities large, then the first order term which give the dielectric constant use not adequate and you have to also consider the higher order terms, into the polarization of the material.

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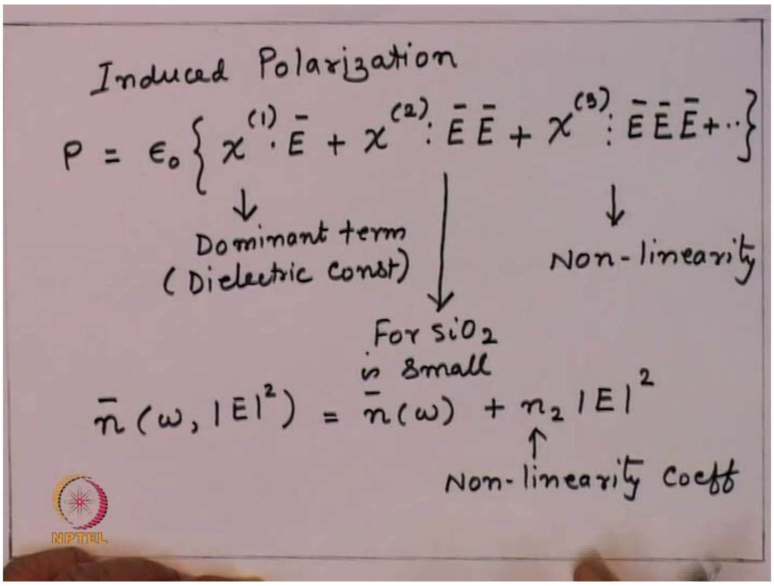

So, in general then one can say that, the induce polarization in the material is P is given as epsilon 0, which is the free space permittivity, the first order susceptibility, which is given by chi 1 E is the impose electric field plus the second order susceptibility given as chi 2 1 a E E plus third order susceptibility E E E plus so on.

So, normally this is the dominant term, which is chi 1 susceptibility and this 1 is contributing to dielectric constant, so this is the dominant term and this contributes to dielectric constant the second term, where you will see now, that the polarization is related to E and E and this quantity now with a tenser. So, basically what we are saying this that the dielectric constant when we include this term has an effect of electric field itself.

However, for material like glass this is a symmetric molecule, so kind to is generally negligibly small; so this quantity for s i o 2 for glass is small, the third term here, which is the third order susceptibility, which now says that the refractive index is proportional to the square of the electric field. And that is the one which is going to contribute to the non-linear effects in the optical fiber, so this term is in the glass will contribute to non-linearity.

So, now if you take simple case, then we can say that the refractive index of the medium in the presence of non-linearity it is a function of omega, because of the dispersion nature of the glass itself. The chi 1 actually the complex quantity it is a function of frequency, which gives dispersion effect at the same it time gives attenuation also, this quantity is a complex quantity. And let us say very simple case where thus this term, the refractive index is proportional to mode of E square.

So, we have this is a term which is non-linear term, where non-linearity's not included plus you are having a non-linearity term is n 2 mod of E square, this term n 2 is what is call the non-linearity coefficient. And this is material dependent, so for different materials you will have different values of of n 2, the non-linearity coefficient and this quantity actually is related to this third order susceptibility a of the medium.

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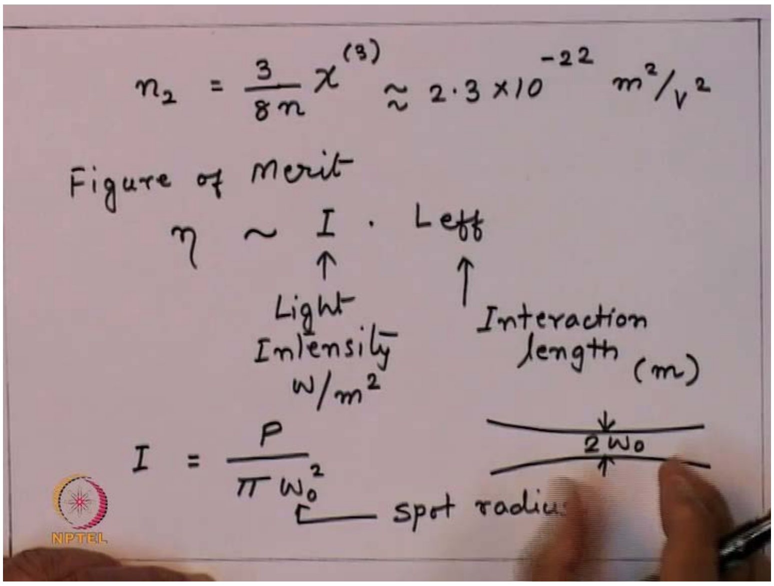

So, if I write in terms of the third order susceptibility this quantity n 2 is equal to 3 by 8 n into chi 3 the third order susceptibility and for the glass this value is approximately 2.3 into 10 to the power minus 22 meter square per volt square. If you really look at this number and compare this number with other materials, you will find that this number is about two orders of magnitude smaller that what you see in many other materials.

One can then ask a simple question, if the glass non-linear coefficient is two orders of magnitudes smaller, then the other well-known material which have good non-linearity, should this non-linearity effect we consider in optical communication or not. And this question can be answered by comparing, how the non-linear effects are manifested in the bulk material. And in the optical fibers, is there any situation difference in non-linear effect, when you see in the bulk material the light beam enters the bulk material there is the size of the bulk material and the non-linearity non-linear interaction, between the light and the matter takes place in that bulk material, whatever size is given. On the other hand light enter the optical fiber it travels very long distance inside the optical fiber.

So, it keeps interacting with with glass, so you have a cumulative effect of the non-linearity in the optical fiber, so one can just do a quick comparison see, how much enhancement of non-liner interaction takes place inside the optical fiber, compare to a bulk material. So, let us say let us define something called figure of merit for non-linear interaction and this figure of merit is call it let us say some efficiency parameters.

So, let us say some eta this is now is decided by the product of two quantities the intensity of light I and the interaction length, so this is the light intensity and this is the interaction length. Now, light intensity is nothing but, the power density per unit area, so this will be watts per meter square and this is the interaction length which is in terms of (())

So, if you want to increase the efficiency of non-linear interaction, either I can increase the intensity of light or I can increase the effective length of interaction or both if if that possible. So, let us see if I have certain light beam available to me and we want to increase the light intensity, what will normally come to your mind, immediately it is likes you that I can take the light beam and focus this light beam I using this some lance or something and then the light intensity is in acts.

So, by focusing of the light thus spot size becomes smaller and smaller and then, the light intensity can be enhanced, so if I have a Gaussian light beam and let us say the spot size which is created by the light beam is a let us say w 0. So, the intensity of light this I will be the power, which is in the optical beam divided by the area of the spot, which is pi w 0 square, where this quantity w 0 is the spot radius.

So, I have a I have Gaussian beam if you like that and that is where the spot size which we have here which is equal to two times w 0 but, now as we are tightly focusing this spot you will that the length over, which this focus region is that is related to this size of the west which is the spot size.

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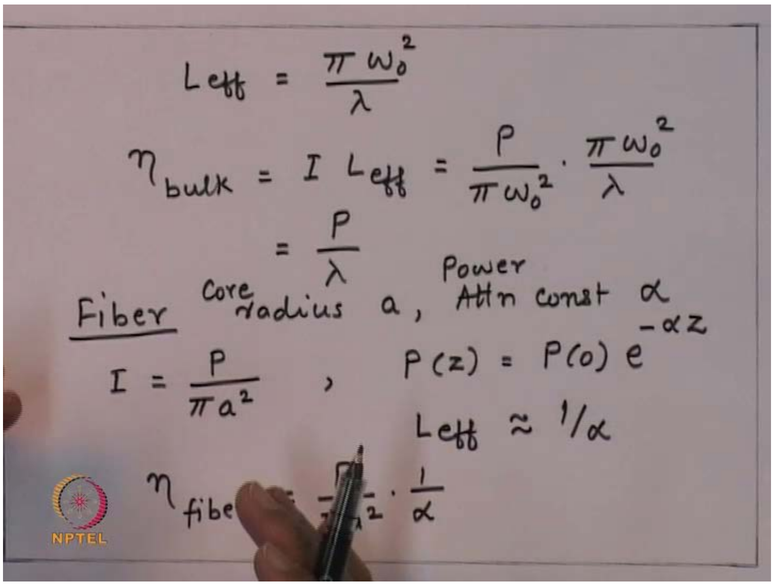

So, in a bulk medium when I tightly tightly try to focus the optical beam the interaction length also get of effected and interaction length in bulk optics L effective is pi w 0 square divide by lambda.

So, the product of the two quantities the figure of merit what you have define in the bulk medium, if we take this is eta bulk that is equal to I times L effective that is equal to the power divide by pi w 0 square times pi w 0 square divide by lambda. So, that is equal to p divide by lambda, what that means is that the figure of merit or the interaction efficiency in the bulk material is independent of the focusing conditions, because if you have a loose focusing the interaction length is more. But, then the spot size is also very large if you try to reduce the spot size you tightly focus it the interaction length also get effected and the product of these two and independent of the the focusing parameter, that is the situation in a bulk material.

So, that means in bulk material if you want to enhance or if you want to find the effect of non-linearity and if you that you want to do a certain wavelength only option you have with you is to increase the optical power. And that is what people do if the want to studies and only n effects in bulk material they require intense optical beams, because by that only the non-linearity's can be induced effectively and you can study those non-linearity's, let us compare the situation with the optical fiber.

So first thing while not is that inside the optical fiber one the light gets in, the light remains focused or confined to a region which is the size of the core. And then since, the loss one optical fiber is very very small that intensity reduces but, it is do this slowly and that remain intensity remains practically constant over some tens of kilometers.

So, in this fiber case if you want to down if the radius of the fiber is let us say a and the attenuation constant is given by alpha the I which is the intensity of light inside the optical fiber is P divided by pi a square, by as a said when the light gets inside the optical fiber it remains confined to the size of the core. So, intensity is P divided by pi a square where a is radius of the the optical fiber core, we can say it is core radius, when the light propagates inside the optical fiber it exponential decays.

So, if I have P as a function of some some z that is equal to some initial value, e to the power minus alpha z, where alpha is attenuation constant we can also quality power attenuation constant.

So, then if you take it distance which is 1 over alpha, that is were the power will reduce to 1 over e of it is initial value, that means effectively one can say that the light is having that intensity over a distance which is approximately equal to 1 over alpha. But, another words for the fiber case, we have L effective which is approximately as 1 over alpha, so if I write down what is the figure of merit for the fiber, that will be equal to this into this say it will be P divided by pi a square into 1 upon alpha.

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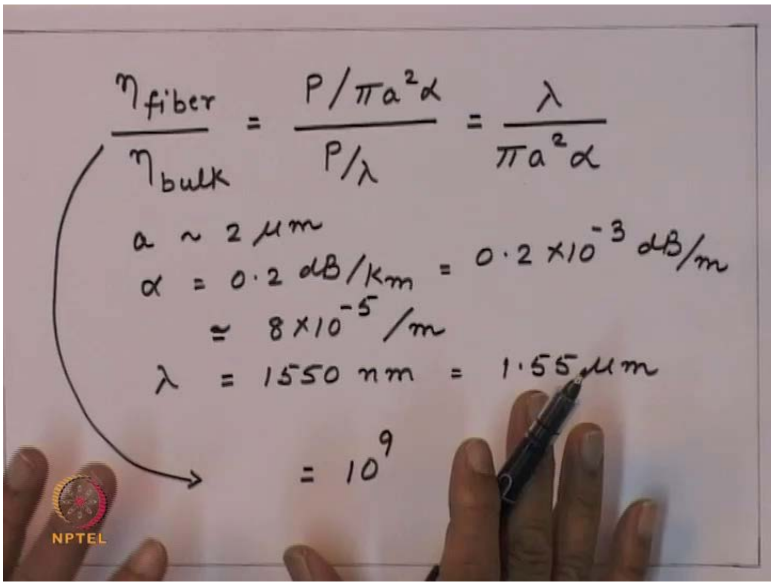

So, if I ask how much interaction efficiency is increase inside the optical fiber compare to the bulk material, I can just take a ratio ratio of this two quantities; so one can say that we have this fiber divided by eta bulk material and that is equal to this quantity p divided by pi square alpha so p divided by pi a square alpha divided by the eta bulk which is p upon lambda.

So, we can put P upon lambda, so this is equal to lambda divided by pi a square into alpha, let us put some parameters, now for atypical optical fiber, so let us say I have single mode optical fiber, where the core size is of the order of about 4 micron. So, 4 define micron something like that, so let us say a I take let us of the order of 2 micrometer, the alpha which is 0.2 d B equal to 0.2 d B per kilometer, that is equal to 0.2 into 10 to the power minus 3 d B per meter this can be converted into the (( )) values, so if I do that that will be let us say approximately about 8 into 10 to the power minus 5 per meter and lambda let us say we take 1550 nanometer.

So, 1550 nanometer which is equal to 1.55 micrometer, so if I take this values for a alpha and lambda into this expression, you will see that this ratio here that will be equal to approximately about 10 to the power 9. Now this interesting, where want that means is for the same parameters, same value of optical power the non-linear effect inside the optical fiber is enhance by one billion times, then what you can see inside bulk material. Just to give numbers, what that means is whatever effect you can seen the bulk material with let us say one water power, the same effect inside the optical fiber can be seen with 1 nano water power.

And normally when people conduct experiments for bulk optics, certainly people have the power in that range of few watts. So, that means the power which we are dealing an optical communication, which could be border of about microwatts, mille watts, the non-linear effects are certainly going to be present, because that power once it is confined to the optical fiber, that will keep interacting with the material over a long distances of 10’s of kilometers.

So, now it makes a very strong case, what that means is that the propagation of light inside the optical fiber ideally is incomplete without taking into consideration the effect of non-linearity, because this non-linear effects are going to be present; and when the lights are propagating inside the fiber, it will keep effecting the signals one way also not that when it was a single channel transmission may be the total power which was confined inside that the optical fiber was small.

The channel use to carry, let us say may be of the order of about few mille watts of power but, when we are talking about the multichannel w d m system, if each channel has to carry a power of mille watt. And let us say 100 channels transmit inside the optical fiber that total power will be watt 100 mille watts, number can go still higher. What that means is that as we are going to go to more complex optical communication system with multichannel transmission, the power density inside the optical fibers certainly is going to be large enough to really vary about the non-linearity inside the optical fiber.

And that is the motivation, that if the non-linear effects are going to be present in the optical fiber, even for the moderate powers, then instead of ignoring this effects the better approach would be to understand, what this non-linear effects are doing to the propagation of signal and try to make use of this non-linear effects for improvement of the quality of the second.

So, in last two decades essentially, there has been significant emphasis here investigating the effect of non-linearity in the signal propagation in the optical fiber, so non-linear effects actually, you various phenomena, they give modification of the pulse, they can also give different generation of frequencies, because that we saw in the polarization you are having a product term of e and e. So, if you are having two difference signals, two different frequency, then the non-linear effect may generates the third frequency.

So, you have new generation of frequencies, you can have a second harmonic generation inside the material, because of non-linearity, so you have verity of phenomena, which are going to be seen inside the optical fiber, once the non-linearity’s in mode. So, let us first look at the simple model with non-linearity, where the refractive index non-linear refractive index is proportional to the mode is square. And mode a square this proportional to the pointing vector or the power density, that means we are considering a simple case, were the refractive index change is proportional to the power density inside the medium, inside the optical fiber and that is what is called the Kerr non-linearity.

So, in our analysis we essentially are going to confine ourselves to the Kerr non-linearity and thus what as we saw here this quantity (Refer Slide Time: 28:06), since we are having the non-linear refractive index it proportional to E square this non-linearity, we call as the Kerr non-linearity. So, in our analysis to start with we will just focus only on this non-linearity and later on then, we will talk about the other effects also as we proceed now a discussion.

So, now to make the analysis of the propagation of of light inside the optical fiber in the presence of non-linearity, we have to again start from the very basic equations the no the maximum field equations. Because now the wave propagation which we are studied so for is now going to be complete altered, because the maximum equation are going to be modified, because now if you look at the displacement which is related to the polarization the polarization has a non-linear term.

So, the displacement will have a non-linear term, that non-linear term will get reflected into the wave equation, so the wave equation will become non-linear. So, you require a fresh approach to investigate the signal propagation in the optical fiber, where non-linear effects are present. So, that means you have to again starts from the basics of writing down the wave equation and then try to make certain approximation, so that the equation becomes solvable and then, we can see how one can investigate the signal propagation in the optical fiber one more (( )) note here that the effect. If we are talking about here, then non-linear effect, when it is stimulate over long distance it becomes large enough inside the optical fiber but, if you consider a short distance, the effect intensically the very weak effect it is not very strong.

So, if for short distance if signal travels one can assume even for the moderate light power inside the optical fiber the non-linearity can be neglect, only thing is the small, small pieces when we accumulate together, then the non-linearity effect will become significant. But, since the intrinsically this effect is weak, the equation can be approximated, there are certain terms higher order terms in the equations can be neglected. So, the equation become little more tractable and then one can solve those equations to study the propagation of light inside the optical fiber.

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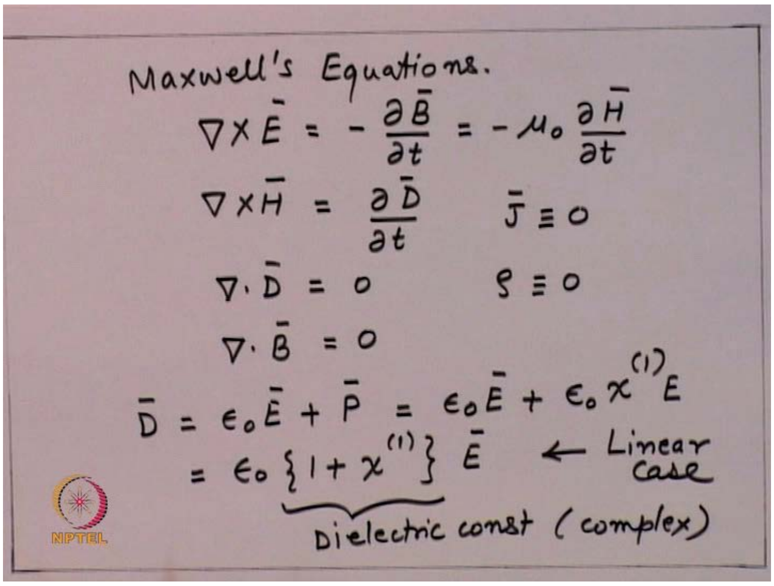

So, let us start the basic equations the Maxwell's equation, so let us start with Maxwell's equations, so we have del cross E that is minus d B by d t and that is if you assume that the permeability is not varying is the function of time. We can write that as minus mu 0 d H by d t also, since we are dealing here with the dielectric media, we can assume that the permeability of the medium is same as the free space permeability, which is mu 0, in second Maxwell equation del cross H is equal to d D by d t because, we are assuming here that the conductivity of the medium is 0.

So, there are no condition can an flowing, so we are saying here J is identically equal to 0, we can have third equation which is del dot D is equal to the charge density but, again we may assume here there are no free charge density. So, this quantities also 0 because the row is also identically taken is 0 and forth equation del dot B is equal to 0; and as we are seen now, in this case that d the displacement, D is now epsilon times E plus this polarization, which is the polarization expression which we talk earlier.

So, now the p (Refer Slide Time: 33:12) is actually given by this, so let us said earlier normally when the intensities are not very large, this terms are negligible. And this is the only term which is their, so epsilon 0 times chi 1 that is the term which will be here and then we can write down 1 one plus chi 1 that is a quantity which will give you dielectric constant of the medium.

So, in normal situation, this will be equal to epsilon times E plus epsilon times chi 1 times E and other terms are neglected, so that will be equal to epsilon 0 arms 1 plus chi 1 times E this is the linear case, because we have neglected the other term which are coming here into the polarization the second order susceptibility and third order susceptibility. So, the displacement now is given by this, so this quantity is nothing but, the dielectric constant of the medium, thus what we been talking all through that the fiber has a dielectric constant. And as we know that this quantity dielectric constant, if you are having loss in the medium then this quantity becomes a complex quantity.

So, in general this quantity actually is a complex quantity, so which is which is a complex quantity and (( )) part of this gives what is called the attenuation constant. Let us now do this same thing, what we normally do for the wave equation you can take curl of this equations of substitute from this.

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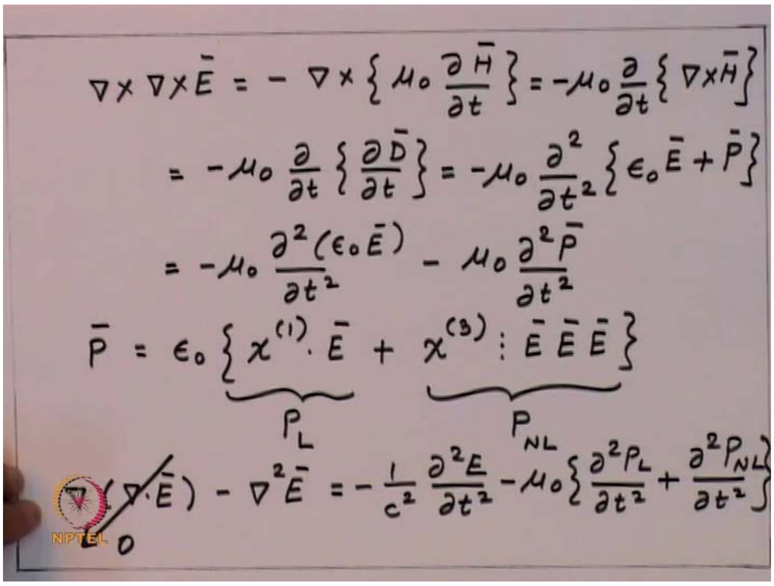

So, we can say del cross del cross E that is equal to minus del cross mu 0 d H by d t by interchanging the del and d by d t, this you can get as minus mu 0 d by d t of del cross H and you can substitute for del cross H from this equation (Refer Slide time: 36:00) from here, which is d D by d t.

So, this will be equal to minus mu 0 d by d t of d D by d t, if I substitute for for d in terms of polarization this will be minus mu 0 d 2 by d t square epsilon times E plus the polarization. We can separate out this two terms, so that is minus mu 0 d 2 epsilon 0 e by d t square minus mu 0 d 2 p by d t square and now as you are seen this p has, now the linear a non-linear term.

So, P now is given as epsilon 0 chi 1 times E plus since we are dealing with the medium which is glass I am saying, the second order susceptibility negligibly small. So let us ignore that, so you have a third order susceptibility which is chi 3 E E E, so we can call this quantity is the linear polarization and this quantity as the non-linear polarization, we can take this quantity. Now P L and P N L and substitute into this and also expand there the third triple product, we can get del of del dot E minus del square E is equal to when I take this quantity here, the epsilon 0 can be taken out.

So we have a mu 0 epsilon 0 but, if you recall 1 upon square you music 0 epsilon 0 is the velocity of light in vacuum, which is c; so this thing can be return as minus 1 upon c square d 2 E by d t square minus mu 0 d 2 P L by d t square minus of plus minus is taken out d 2 P N L by d t square.

Now, from maximum equations this quantity (Refer Slide Time: 39:48) del dot d is equal to 0, because a low is equal to 0 and since epsilon is not varying is a function of space, so this also gives you del dot D is also equal to 0. So this quantity will identically go to 0 and that will give us then the wave equation, which will be.

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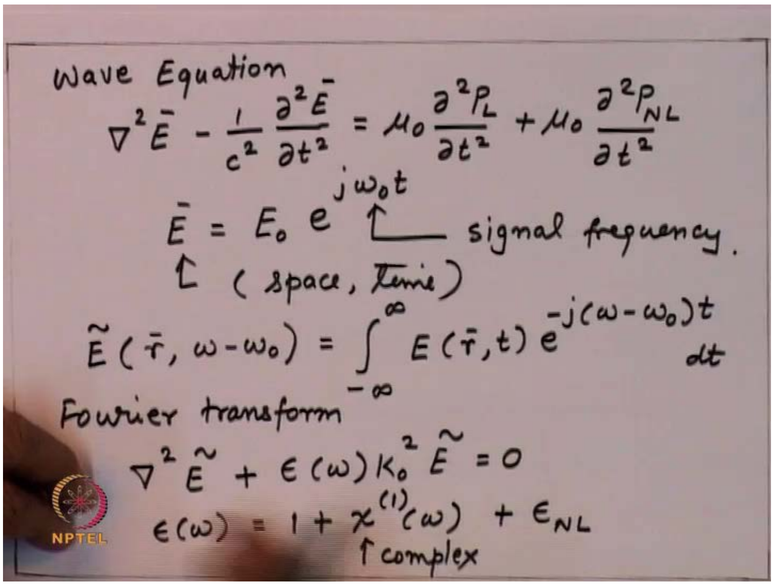

So, wave equation which will be del square E minus 1 upon c square d 2 E by d t square that is equal to mu 0 into d 2 P L by d t square plus mu 0 d 2 P N L by d t square. So, not here, now that when the non-linear terms were not their, this quantity would not be there and then this would reduce to the normal standard equation, which we already solve for the wave propagation in the optical fiber.

However now, we are having a term which is additional term here, which is P N L and P N L as we know depends upon the electric field itself. So, you are having this quantity which is which is given by this (Refer Slide Time: 41:33), so actually this quantity which is the non-linear term thus what now is going to come into picture, because of this third order susceptibility.

To solve this equation, now let us consider the simple time harmonic fields, so let us say the electric field E is given by some some E 0 E to the power j omega 0 t, where omega 0 is the signal frequency. Now one can show that if you are having pulse width, which is small then about 10 nanoseconds, then you have dispersive as well as non-linear both effects present.

If you are having the pulse width which is larger than about 10 nanosecond, then you will see the non-linear effects but, in the range of about 10 nanosecond to about 10 (( )) to seconds; you will see both the effects of dispersion and non-linearity will be present and the pulse propagation will be decided by the simultaneously effects of these two.

So, essentially our interest now is the if you launch pulse of light, inside the optical fiber what way the pulse is evolved as you propagates inside the optical fiber, so the mentioned earlier also that, here we are not now, looking at only the continues propagation of light. We are actually interested in investigating the pulse propagation of light, so the light is having a carrier frequency which is omega 0 but, it also has an envelope in the form of pulse. So, carrier will have certain propagation characteristics but, we are more interested in finding out the evolution of the envelop of this pulse that is what our prime interested.

So, that is what essentially we would like to investigate that when the light pulse is launch inside the optical fiber, what way the pulse evaluation takes place, now to study the pulse evaluation in the in the optical fiber, it is easier to first express the electric field which are in terms of time as a function of frequency or in the spectral domain. So, the electric field actually if you see it is function of a the space and time; however since we are talking about a narrow band signal around this frequency which is omega 0 one can re define this problem in terms of the frequency domain.

So, let us say that I define the Fourier transform of the electric field which is a function of r which is the space and omega minus omega 0, so the frequencies around this carrier frequency omega 0. And that is minus infinity to infinity the electric field with a function of space and time, e to the power minus j omega minus omega 0 t d t. So, this is the Fourier transform, so e tilde is the electric field in the frequency domain, so if I take the electric field as the function of space and time and I am taking the Fourier transform as the function of time.

So, here I see essentially the spectrum of the electric field as the function of distance r, so the wave equation, now can be return if I take this quantity n in terms of the Fourier transform, the wave equation del square E tilde plus epsilon is the function of omega k naught square E tilde equal to 0. Now note here all this quantity is non-linearly effects and dispersion of about, whatever parameters are their essentially we have captured into this quantity which is this quantity epsilon.

So, one can define this effective dielectric constant which takes into consideration the linear as well as the non-linear effects and also the frequency variation. So, here this epsilon omega is equal to 1 plus the chi 1, which is a function of omega and this quantity as we already said is complex, because there we loss in the medium plus your having the non-linear effects, which can be given by epsilon non-linearity.

So, whatever we have return here essentially can be return effectively within this, now we can apply separation of variable to solve this wave equation and there are we we already said that, now you are going to have a signal which is not only a continues signal. But, you have a signal which as an annual up it is form of a pulse, that means the electric field if I as a function of space and omega minus omega 0 has a spectral band.

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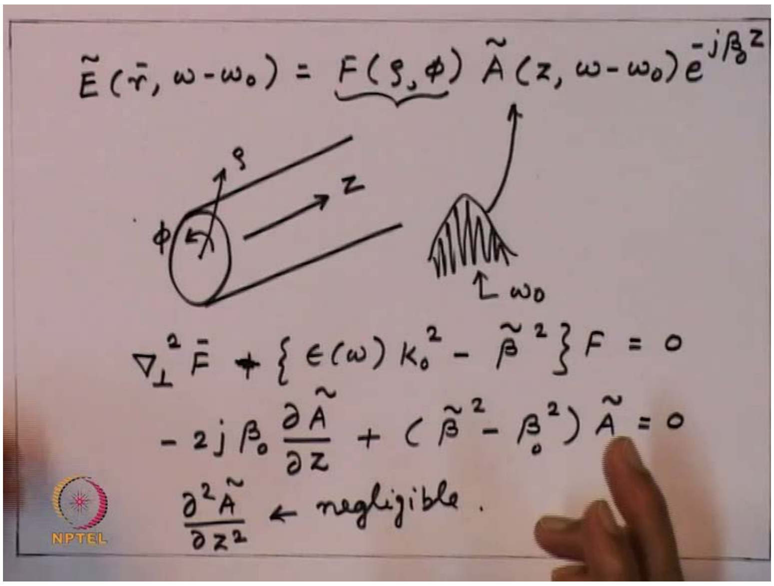

So, that is given as some function have which is of transform coordinates rho and phi as an annual up spectrum, which is a function of z omega minus omega naught and a phase term is with the minus j beta 0 z. So, what you are saying here, let us say this is the optical fiber in the radial direction here, this is rho this direction is phi this direction is z.

So, we are saying the electric field is given by product of a function, which is into transfers plane which is rho and phi that this quantity and this is the annual up which is now evolving as the function of z. So, you are having a band of frequencies, which are going to be altered as the signal propagation takes place along the optical fiber and the whole thing is moving with a phase constant which is equal to beta 0. So, you are having traveling wave kind of phenomena which is taking place here it has an annual up.

So, you have a situation you have a pulse here, which is like this then you are having the carrier frequency inside this, so this is carrier frequencies omega 0 this is the annul up function, which we are seeing here, the frequency domain and this is the phase constant corresponding to the center frequency which is omega 0.

Now, if I substitute this function into the wave equation and separate out you will see two equations, one which will give the transfers fields F, that is equal to plus here, epsilon omega k naught square minus beta F is equal to 0. And second equation which will get for the annual up, that is minus 2 j beta 0 d A by d z plus beta square minus beta 0 square A that is equal to 0; now note here, that this equation is the simplified equation.

So, you have a term here, which is d 2 A by d z square also because, when we are having the wave equation this equation here (Refer Slide Time: 51:09), we also going to get d 2 z by d 2 by d z square of E. However since the evaluation of the pulse is slow as we already said is the weak phenomena, which we are talking about this quantity is negligible small. And therefore, for as approximation we can say that the evaluation of the pulse essentially is given only by this term, the second order effects are negligible small and the transfer fields are essentially given by this equation.

So, essentially we have to know solve this two equations together to get a compressive equation, which will tell us how the pulse evaluation is going to take place, so in the next lecture when we meet essentially, we will take this two equations try to substitute some parameters into this and see what finally, the equation arises, which then can be solved to study the evaluation of the pulse on the optical fiber.

So, let me summarize what we have done today, first of all, we show that, when the light is intense that time the induce polarization has higher order susceptibility terms also, they cannot be neglected for the material like glass the second order susceptibility contribution is negligible, because the s i o 2 molecule is symmetric.

So, the third order susceptibility is the one which contributes to the non-linear effects then, we so that even if the glass is not a very good non-linear material, because its non-linear coefficient is two order of magnitude smaller compare to many well known non-linear materials. Seen the interaction length inside the optical fiber is very large the non-linear effects are very pronounce inside the optical fiber.

So, the compare to the bulk material the interaction efficiency is billion times more inside the optical fiber, approximately to there is a strong case to investigate the non-linearity effects inside the optical fiber. Then we started with a Maxwell equation and wrote down the Maxwell equation and the wave equation, in terms of the polarization which includes the non-linear terms. So, next time when we meet then we solve this non-linear equation to see the effect of pulse propagation.