Advanced Optical Communications

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Lecture No. # 05
Wave Model – I

In the previous lectures, we investigated propagation of light inside an optical fiber using the simplest model, which was the ray model. We saw that the ray model gives some understanding of propagation of light inside the fiber, but if you wanted to have the quantitative numbers about like the velocity with which the energy is going to propagate inside the fiber or quantitatively, if you have a single mode optical fiber, from a dispersion would be then, these questions cannot be answered by the ray model. And that is the reason now we go to the next level of model, which is what is called the wave model.

Now, in this essentially we treat light as an electromagnetic wave and investigate the propagation of electromagnetic wave inside a bound structure like optical fiber. So, we can say that the optical fiber is essentially a dielectric wave guide, in which the light propagates as an electromagnetic wave. Then, we are interested in finding out specifically the relationship between what is called the phase constant and the frequency. So, that, we can get the velocity of the mode propagation inside the optical fiber.

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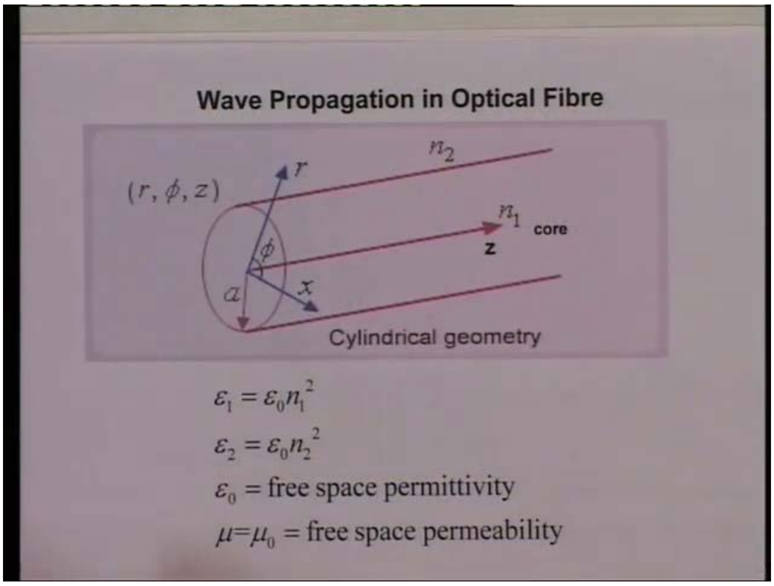

Before we do that, first we have to choose the coordinate system, which is appropriate for investigating propagation of electromagnetic wave inside the fiber. Since fiber is a cylindrical rod essentially we choose a cylindrical coordinal system to investigate this problem. To make the analysis simple first we assume that the cladding is of size which is much, much larger compared to the wavelength or for theoretical analysis we say that the cladding is of infinite extent. So, essentially the problem is only of the one interface which is between the core and the cladding.

So, you have here this glass rod, which we call as core with a refractive index n 1 and surrounding this, the whole medium now is having a refractive index n 2, because we are assuming that the cladding is of infinite extent. Here, a cylindrical core in the system, where the radial distance is given by r. The angle with respect to some reference direction in the cross sectional plain is given by phi and the direction along the axis of the fiber is given by z. So, we have a co-ordinate system which is r, phi, z and we assume that the radius of the rod is equal to a.

So, for our basic electromagnetic understanding, we know this that the dielectric constant of a medium is equal to the square of the refractive index of the medium. So, the permittivity of medium is equal to the free space permittivity multiplied by the square of the refractive index. So, when we analyze the wave propagation, normally we write the wave equation in terms of the permittivity and permeability of the medium, and that is the reason, we write here epsilon 1 which is the permittivity of medium 1, which is core, that is equal to the permittivity of the free space which is epsilon 0 multiplied by the square of the refractive index of the core which is n 1. Similarly, we have a permittivity of medium 2, which is the cladding, and that is equal to the permittivity of free space multiplied by the square of the refractive index of the cladding which is n 2.

Then we assume that the medium which we are talking about is essentially non magnetic, where we are essentially talking about material like glass. So, the permeability of this medium is same as the free space. So, the mu permeability which is equal to mu 0, which is the free space permeability. So, with these now basic definitions, now we can go to analysis of wave propagation inside the cylindrical rod and wherever we start a problem like this we start with the Maxwell's equations. So, let us now goes a problem. There suppose the light has to propagate inside the structure, without worrying about how the light was generated. That means, you do not have any source inside this rod.

Somehow the light was put inside this structure and light propagates in this. So, we ask if we consider now the electromagnetic wave which is source free. In what form this electromagnetic wave would exist inside the cylindrical rod, which is a dielectric rod surrounded by a dielectric material. So, the structure essentially is a dielectric wave guide. So, we are asking, if we consider a source free situation, what kind of electric and magnetic field distributions would survive inside this dielectric wave guide?

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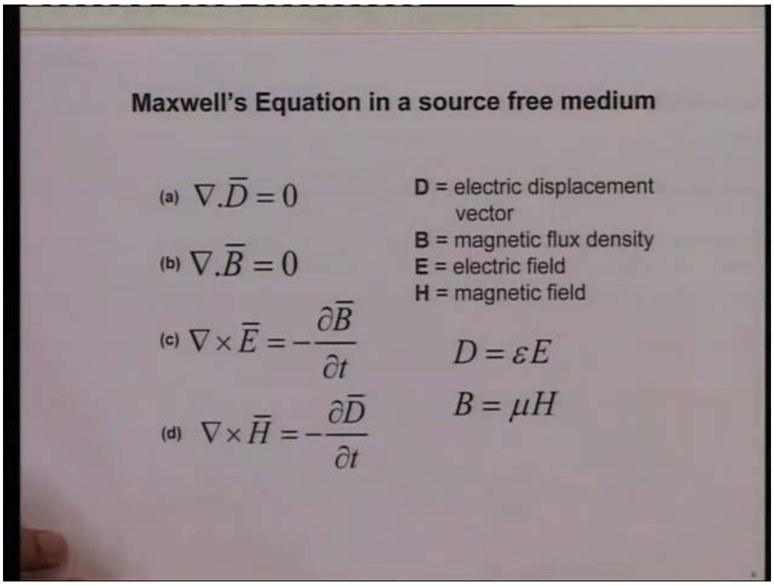

So, if you write down now the Maxwell's equations for the source free condition then we get the four Maxwell's equation as del dot D is equal to 0. Where D is the electric displacement vector, del dot B is equal to 0, B is the magnetic flux density. These equations correspond to the Gauss law. Then we have Faradays law of electromagnetic induction which gives you this Maxwell's equation. Which is del cross E is equal to minus dB by dt and then we have the Amperes law which tells us del cross H is equal to minus dD by dt. There is no minus here it should be plus dD by dt and in electric displacement vector this is related to the electric field through this relation.

So, electric displacement vector is equal to permittivity of the medium, multiplied by the electric field and magnetic flux density is equal to permeability of the medium, multiplied by the magnetic field. Since we are assuming that we are pure dielectrics there is no conduction current flowing in anywhere in the medium and that is the reason in the Amperes law the conduction current density j is 0.

So, essentially we are asking now a simple question, that if we have this set of Maxwell's equations, what field would exist inside the dielectric rod which is optical fiber. So, to solve this equation essentially what we do, we take curl of along these equations and try to first out separate out E and H by here this quantities are coupled. You can see E is related to B and B is related to H. Say, essentially this equation E is related to H and in this equation H is related to E because D is epsilon times E. So, these two equations are the coupled equations for the electric and magnetic fields.

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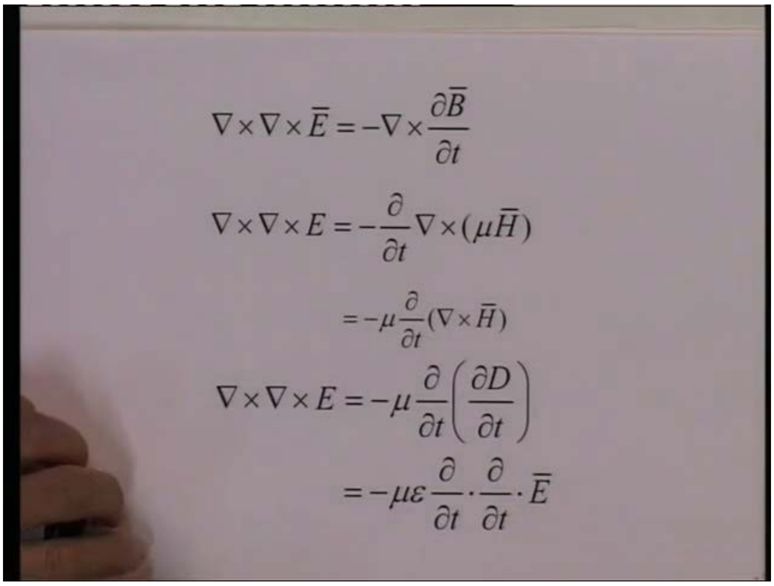

So, firstly we try to decouple these equations and what we do is we take the curl of this equation and we can write now the del cross del cross E is equal to minus del cross d B by d t. If we interchange the two operators d by d t and the del operator then we can write here minus d by d t del cross mu H and since permeability is not a function of space, you are assuming a homogenous medium, we can take mu outside. So, we can get here minus mu d by dt del cross H.

Then from second equation, we can now substitute for del cross H and here we can substitute for D which is epsilon times E. If you do that then we get del cross del cross E that is equal to minus mu d by d t of d d by d t, we got this quantity is nothing but d D by d t. So, I can combine this to get here minus mu epsilon we got D is epsilon times E say you get mu epsilon D by d t D by d t of E. So, this is nothing, but the second derivative of E with respect to time.

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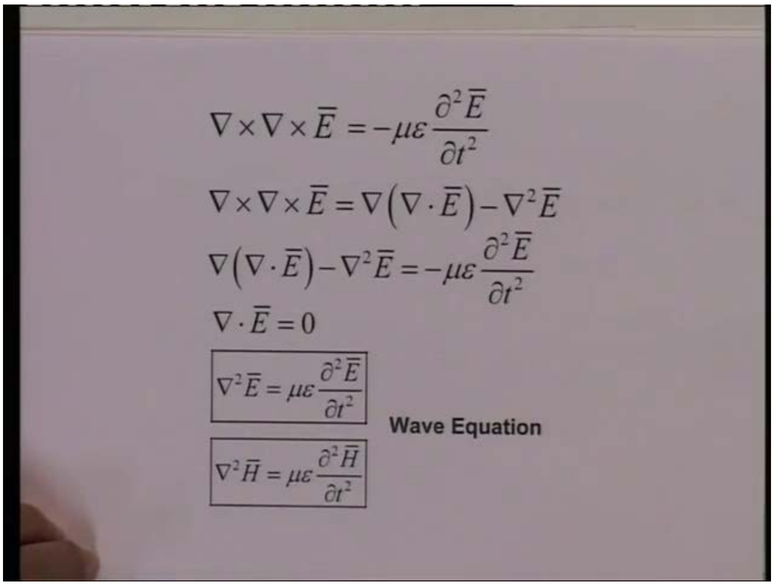

So, you can get the equation which is del cross del cross E, that is equal to minus mu epsilon d 2 E by d t square. Now, we can expand this quantity del cross del cross E by using the vector identity, which says that del cross del cross E is equal to del of del dot E minus del square E. Then using the Gauss law which says del dot d is equal to 0, this quantity and since D is epsilon times E and that epsilon is not a function of space. So, we can take epsilon out you get from this equation del dot E equal to 0.

So, for a homogenous medium, source free medium we get del dot E also equal to 0. So, if I substitute now for del dot E equal to 0, this quantity goes to 0. So, this quantity del cross del cross E is nothing but minus del square E. So, by saying this then we get a equation for electric field which will be del square E equal to mu epsilon d 2 E upon d t square. If I done the similar exercise by taking the curl of this equation and then substituting for del cross E from this equation to this you would get a identical equation for the magnetic field also which will be del square H is equal to mu epsilon d 2 H upon d t square. These equations are nothing but what are called the wave equations.

So, what we find is that, if I consider the time varying electric and magnetic fields then they constitute the wave phenomena. So, these fields are going to propagate in this medium in the form of waves. So, now to find out the behavior of electric and magnetic field inside the optical fiber, essentially we have to solve the equation which is the wave equation for the electric field or for the magnetic field. And once you get any of these quantities then we can go to the Maxwell's equation and substitute in to it and you can find out the other quantity.

So, now what we do is we essentially start to solve the wave equation with the physical understanding which we have developed using the ray model. Apply appropriate boundary condition to this and then get the analytical form of the electric and magnetic fields which would survive inside the core of the optical fiber. In general this quantity E and H are the vector quantities. That means, E has three components and H has three components. So, as such we have six quantities here. All of them satisfy the wave equation. So, we essentially need to find the solution for the six components three for electric field and three for the magnetic field.

However, what we would notice is that we have this four Maxwell's equations which are relating the electric and magnetic fields, what that means is that all six components of electric and magnetic fields are not independent. So, two components of the electric and magnetic fields can be taken as independent components and then by using the Maxwell's equations we can find the analytical expressions for the remaining four components. So, we do not have to solve the wave equation for all six components, essentially we have to solve the wave equation for two components and the remaining components can be derived from the two components.

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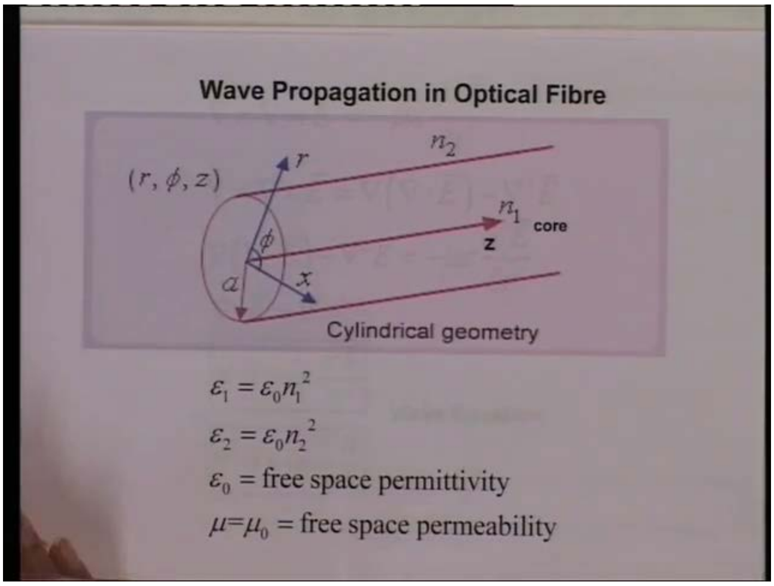

Now, if I look at the structure which we have taken here, which is the core of the optical fiber. The propagation takes place along the direction of z that is where the net energy is going to flow. So, this direction is a special direction. Any direction which is perpendicular to this, if you call as the transverse direction. That is nothing special about it in this cross sectional plain, but this direction is a special direction.

So, what normally we do, we take the two field component which are in the direction of propagation as the independent components and try to expose the remaining four component in terms of this components. So, the component which are oriented in the z direction or along the axis of the optical fiber we call as the longitudinal components of the electric and magnetic fields.

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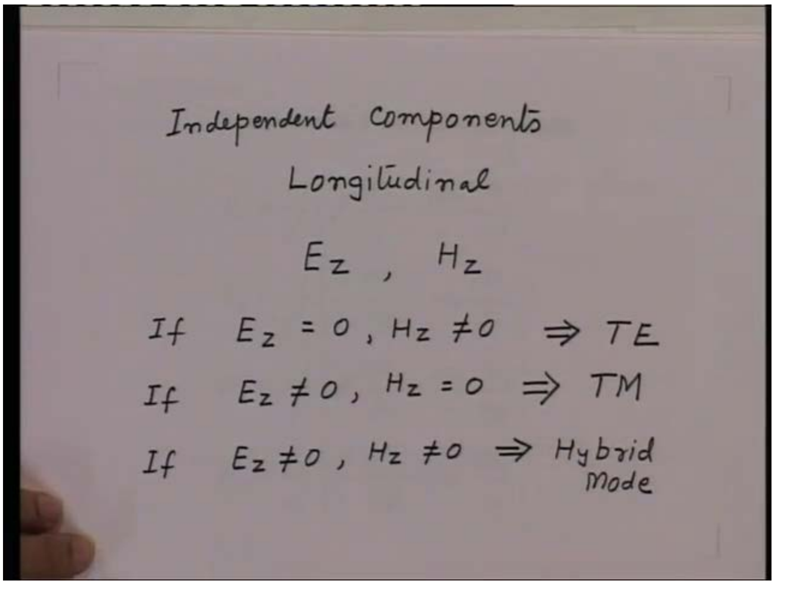

So, what we have we have this independent components, which are longitudinal E z and H z. So, we can solve the wave equation essentially for these two components the E z and H z.

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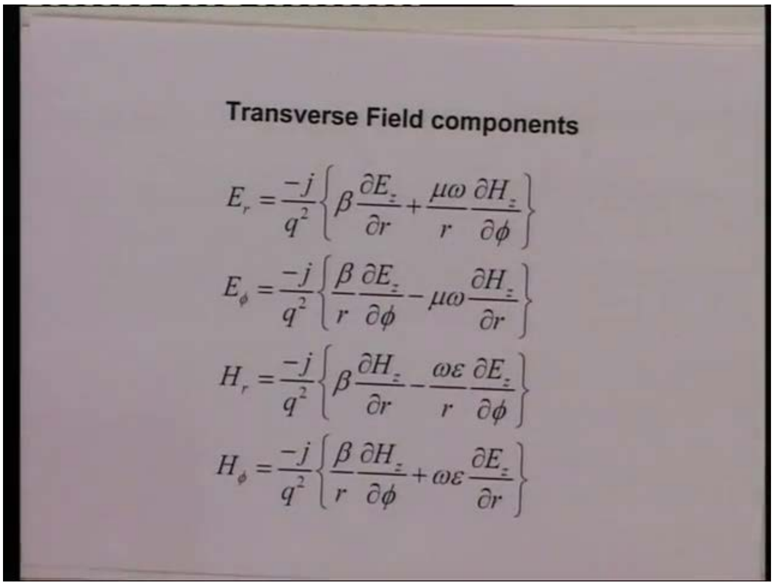

And once you get these component then we can go to the Maxwell’s equations and can get the transverse component which can be written like this. So, since we are talking about here the cylindrical coordinal system the transverse components will be E r, E phi H r and H phi. So, when having the knowledge of the longitudinal components E z and H z and its partial derivatives with respect to r and phi. Then we can find out the remaining four components. So, the whole set of six components essentially are solved.

So, first your problem reduces to finding the solution, analytical solution of the wave equation for the longitudinal component of electric and magnetic fields. Few things can be noted from this expression and that is all the four components are expressed in terms of derivatives of E z and H z. And there is a possibility that even if one of the component is 0 that means suppose H z is 0, this quantity will be 0, this quantity will be 0, this will be 0, this will be 0. I still can have the transverse component when H z is 0.

Similarly, I can have the transverse components when E z is 0, then H z is not 0. So, I have three possibilities here now. The set of fields which we get when H z is 0, the set of fields which we get when E z is 0 and the set of fields which we get when both of them are not 0. This one precisely would give three difference types of fields which we call as the modes. So, if we take E z is equal to 0 then H z is not 0, this gives you the field distribution which will be transverse electric in nature because E z is not 0. That means, we have a component of magnetic field in the direction of the net propagation along the axis of the fiber, but there is no electric field component in the direction of net propagation.

So, we call this mode as the transverse electric mode, same as what we saw from the ray model that there is possibility of finding out the rays which could give the electric field which will remain always transverse to the direction of net propagation. Similarly, if we take E z not equal to 0 , but H z is equal to 0 then this would give us the field distribution which would have a electric field component in the direction of propagation because E z is not 0 , but it will not have any magnetic field component in the direction of net propagation. So, this one then we can designate a transverse magnetic mode.

And if both the components are non zero, then we will have combination of two that mode is neither transverse electric nor transverse magnetic and as we saw earlier that this mode can be called as the hybrid mode. So, these two mode essentially correspond to the meridional rays in terms of ray model and when both of them are non zero, E z is not equal to 0, H z not equal to 0 that gives you what is called the hybrid mode. So, inside the optical fiber when we now carry analysis, we want to carry analysis in general and then you take the specific cases by putting a z equal to 0 or E z equal to 0 or both of them equal to non zero.

So, as we saw even for ray model there are three types of filed distributions would exist inside the optical fiber which we call as modes, which could be transverse electric in nature, transverse magnetic in nature or hybrid.

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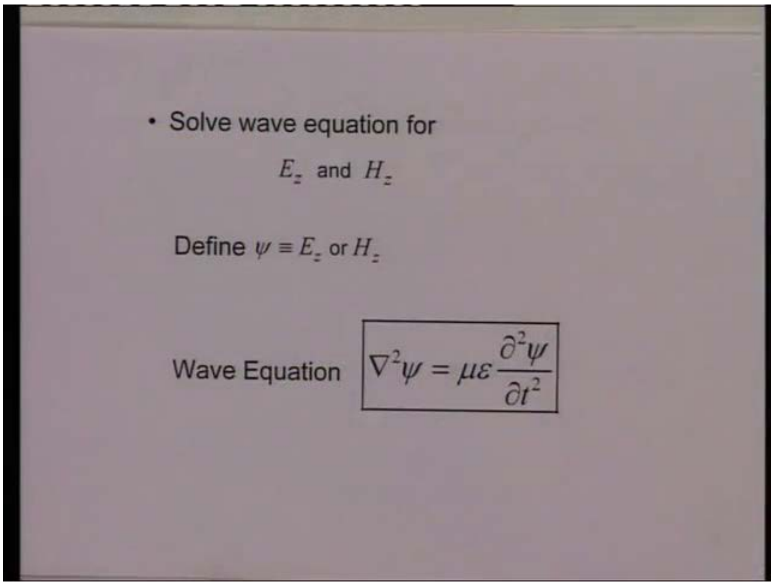

With this understanding now, let us go and try to solve the wave equation for the longitudinal components E z and H z. So, the steps involved are as follows, first you solve the wave equation for E z and H z. Once you get E z and H z then you go to these expressions and there find out the transverse component E r, E phi, H r, H phi. Once you get these components then you apply the boundary conditions at the core cladding boundary and then you get what is called the characteristic equation for a particular mode. So, since for solving E z and H z these quantities are scalar quantities, you have only one component either E z or H z.

Let us define some quantities psi which can denote either E z or H z. So, what we are saying is that we want to solve the wave equation which is scalar in nature of this type, where psi can represent either E z or psi can represent H z. So, the wave equation now is del square psi is equal to mu epsilon D 2 psi upon d t square, and this equation we have to solve in the cylindrical coordinate system because that is the appropriate coordinate system for the circular core optical fiber. So, first let us expand this quantity del square in the cylindrical co-ordinate system and if you do that we get the wave equation which will look like that.

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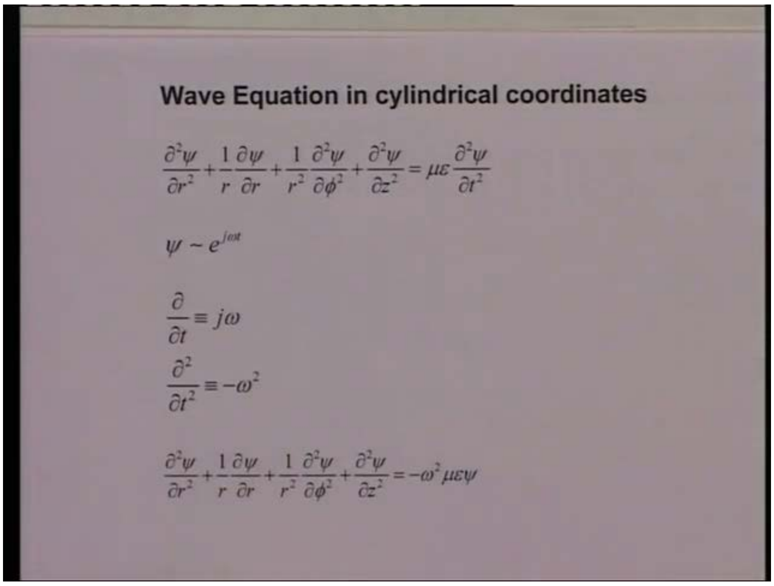

So, we have got here d 2 psi upon d r square plus 1 upon r d psi by d r plus 1 upon r square d 2 psi by d phi square plus d 2 psi upon d z square that is equal to mu epsilon d 2 psi upon d t square. So, the same wave equation which is written here in the compact form can be explicitly written for cylindrical coordinate system like this.

Now, if you assume that all the fields are time harmonic fields with a angular frequency omega, then this quantity psi has a variation which is e to the power j omega t, wave omega is the angular frequency. So, if I take a derivative of psi with respect to time that is equivalent to multiplying a quantity by j omega, if I take a second derivative of this that will be multiplying by one more j omega. So, I have a d 2 upon d t square that is equivalent to multiplying the quantity by minus omega square.

So, any quantity which is varying time harmonically with a angular frequency omega, if second derivative is equivalent to multiplying that quantity by minus omega square. So, I can write this wave equation from here to this where the second derivative now d 2 upon d t square is minus omega square. So, I got the wave equation which will look like that, these things remain same. This is now minus omega square mu epsilon phi. These equation we can solve by using what is called separation of variable technique. So, we assume that phi is a product of the functions, each function is a function of either r or phi or z.

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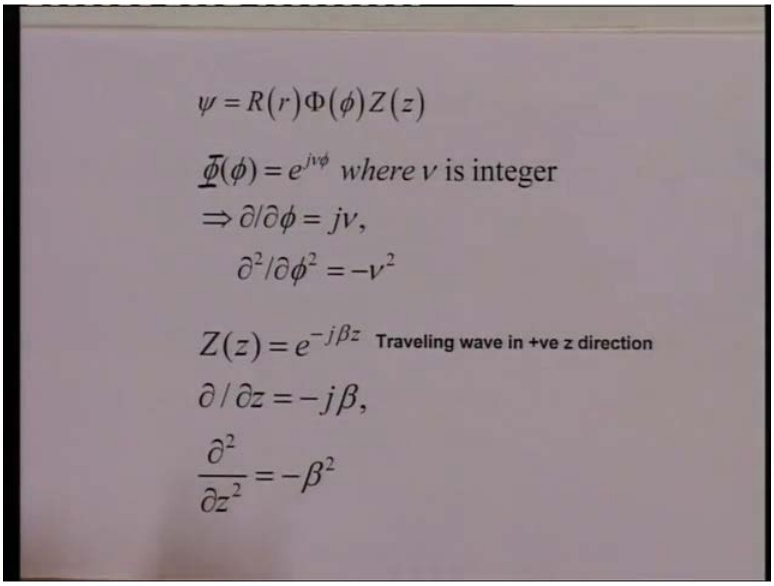

So, we say that if we apply a separation of variables, let us say the phi can be represented by some function capital R of r some function phi of angle phi and some function Z of distance z around the axis of the fiber. Now, you have to put one by one our understanding and try to see what this functional form of this quantities could be. So, firstly what we note here is that this quantity capital phi or phi this quantity, show the variation of this quantity phi in the azimuth or in the cross sectional plain of the fiber at a given value of r at a given value of z.

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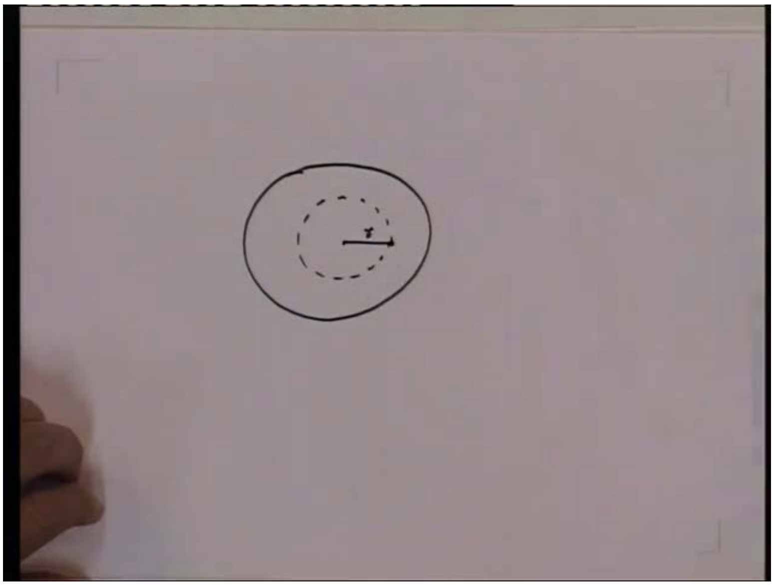

That means if I consider a cross sectional plain like this of the fiber at a given point r as we move phi, it moves along the circle of radius r and the center of the circle is same as the center of the fiber. So, as the phi goes around the circle, when the phi changes by two pi you receive a same point you come here. That means, the functional form is capital phi or phi should be such that it is periodic over two phi and that can happen if we choose a function of this type and if I consider this function capital phi as E to the power j mu phi where mu is an integer where phi becomes two pi you get this quantity multiples of two pi or in other words you will reach to the same point.

So, from this physical understanding that by changing the angle phi by two pi the function must repeat itself, we have a variation of phi which can be given as E to the power j mu phi and mu is the integer. Once you have that then any derivative with respect to phi would be equivalent to multiplying by j mu and second derivative with respect to phi would be equivalent to multiplying the quantity by minus mu square. So, this function now is defined from our physical understanding that if I move along r equal to constant and z equal to constant when pi's phi changes by 2 pi you must reach to the same point, all field must repeat itself when phi changes by 2 pi.

For defining this function z again we can understand, that we are talking essentially about energy propagation inside the fiber which is in z direction or in other words we have a wave which is going to propagate in the structure and if we assume that this fiber is of infinite length there is only one wave which travels inside this fiber. And we know that this travelling wave behavior can be expressed by function E to the power minus j beta z. So, if I have any phase function that we earlier saw in earlier lectures there are quantity E to the power j omega t minus j beta z gives you essentially wave phenomena travelling in z direction.

So, functional form of E to the power minus j beta z represents a travelling wave in the positive z direction. So, again if we define this function then d by d z is equivalent to multiplication by minus j beta d 2 by d z square is equivalent to multiplying the quantity by minus beta square. So, what we have done now, essentially we have defined functions out of this three just by using our physical understanding of the problem, we have defined this two functions, one function is phi which is a azimuthal variation of the fields then you have to find the function in the direction of z which is the travelling wave nature.

So, only this quantity r now remains to be defined. And this essentially we can find out by substituting this quantity phi and z in the wave equation. Whatever equation we get therefore will govern this quantity R.

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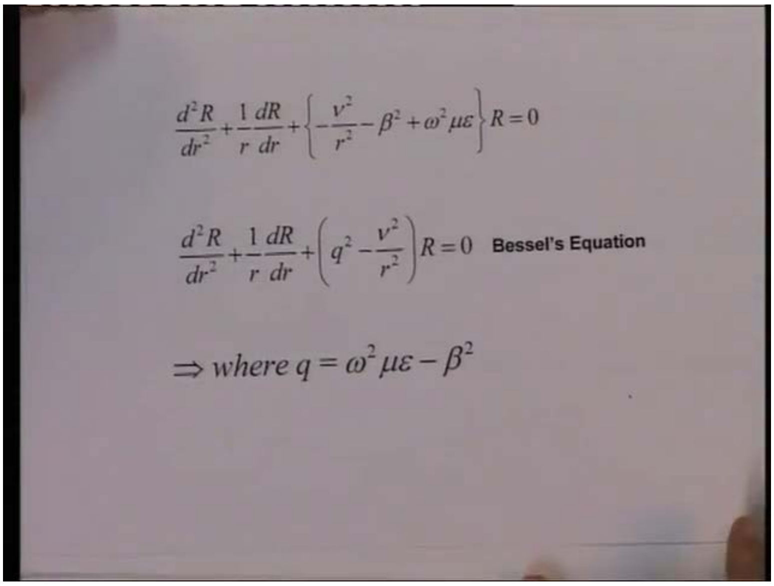

So, what we do is we substitute now this quantity here psi into wave equation and if I divide by psi you get now the equation which governs this function R by this. Note here this minus beta square is because of B 2 psi upon d z square, this minus mu square is because of d 2 upon d phi square this quantity.

So, the wave equation now essentially reduces to this equation for R capital R as a function of r. This equation is what is called the Bessel's equation, where for gravity we define this quantity q square which is equal to this omega square mu epsilon minus beta. Note here that this quantity beta which is nothing, but, the phase constant of the travelling wave, this quantity is to be determined by applying the appropriate boundary conditions. So, this is the quantity essentially which is the unknown quantity in this expression.

But if I knew this quantity beta then this quantity q is known I can substitute for q inside this and then I got this equation which can be solved which is the Bessel's equation and the solution to this equation are what are called the Bessel functions. So, what we find now is that the radial variation of the electric and magnetic field distribution is given by the Bessel functions. Note here the Bessel function is not a property of optical fiber. In fact, this is purely because of the coordinate system which we have chosen.

So, in a cylindrical co-ordinate system, if you solve the wave equation then we will get the radial variation which will given by the Bessel functions. So, with this now the wave equation is solved where once I solve this equation I get Bessel functions. Then I can go back and substitute into this, I get Bessel function multiplied by this quantity which is e to the power j mu phi, this quantity e to the power minus j beta z. So, I got the expression for psi which can represent either the electric or magnetic fields. So, the problem is solved.

But just by saying that this quantity is Bessel function, let us still give us any hold on what I have essentially we are looking for because if I look at the solution of this equation depending upon the value of q you have a variety of functions. For example, if this quantity is q is real I get what are called the Bessel functions. If this quantity q becomes imaginary or if this q square become negative, I get another set of Bessel functions. If this q with a complex quantity I get another set of Bessel functions.

So, though at this point it looks once I know these are Bessel functions the problem is solved, still I do not have any analytical solution in our hand, unless we appropriately choose the solution which this equation gives and that is only possible if we appropriately put the physical understanding in choosing the solutions. So, now, essentially what we have to do is we have to look now the behavior of the Bessel functions for different values of q and then use our physical understanding and then appropriately choose the set of Bessel functions.

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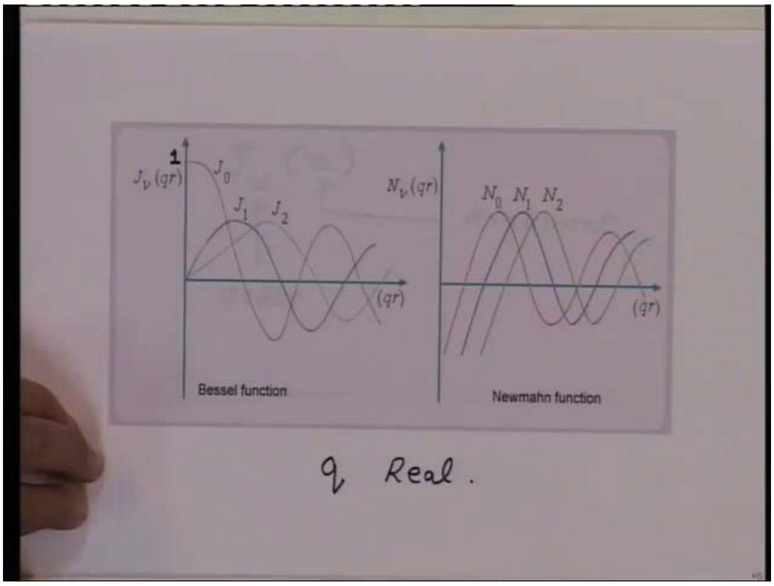

So, let us first look at what will be the behavior of the Bessel functions. So, what is Gauss say is that if this quantity q is real, then we have a Bessel function and since there is second ordered equation differential equation there are two solutions to this, first solution is what is called the Bessel function and a second solution is what is called the Newmahn function. These functions are denoted like this. So, this is j nu q of r, this is n nu q of r, this quantity nu is what is called the order of the Bessel function. So, and the quantity q r is what is called the argument of the Bessel function.

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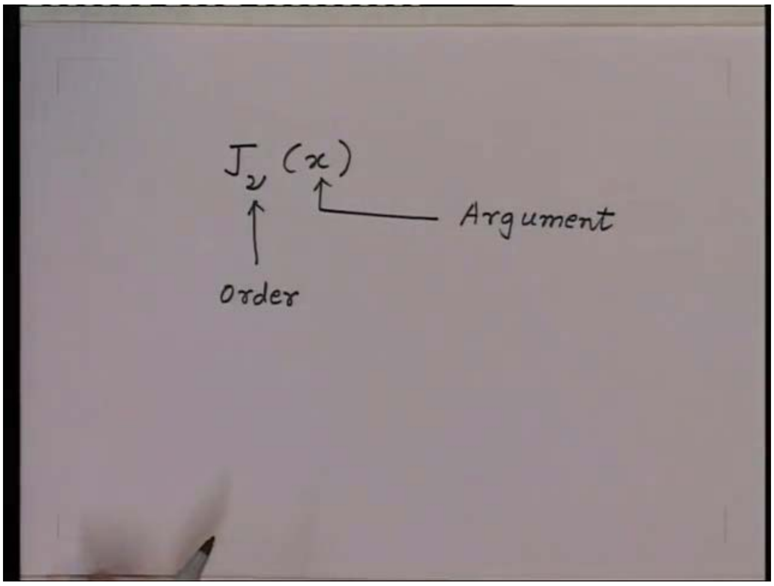

So, essentially we have a Bessel function which is J nu of x. This quantity is what is called the order of the Bessel function and this quantity is what is called the argument of the Bessel function. So, here what we have is a plot of the different Bessel functions. That means, for different values of nu or different arguments. So, for a any given value of nu if I vary the argument I get a plot of the Bessel function of that order. So, if I put nu equal to 0 I get what is called j 0 Bessel function which looks like that.

If I put nu equal to one and vary the argument q r I get this j 1 Bessel function, then I get j 2 Bessel function and so on. Similarly, for the Newmahn function the nu is the order of the function, this quantity is the argument. So, again if I plot the Newmahn function for different orders as a function of argument you get typically thing like this is n 0, this is n 1, n 2 and so, on. Now, note here that this variation whatever we have here is representing the behavior of electric or magnetic field in the radial direction, that is what is because quantity representing this r and this capital R is nothing, but the spatial variation of electric or magnetic field in the radial direction.

So, few things to be noted here. Firstly, except j 0 Bessel function, all other Bessel functions are 0 and argument goes to 0, they all start from 0 here, except j 0 Bessel function which is 1 at the argument going to 0 and all these functions are oscillatory in nature. So, they have maximum, they go to 0, they become negative maximum 0 and so on. So, both the function j 0 and j 0 j 1 j 2 and n 0 n 1 n 2 all these functions are oscillatory in nature, j 0 as the argument goes to 0 has a value 1 and all other values are less than 1.

If I go to the Newmahn function then as the argument approaches 0 all this functions approach to minus infinity. So, if I go to q r equal to 0 for a given q r equal to 0 that means, if I reach towards the core of the optical fiber, center of the optical fiber, axis of the optical fiber then r will go to 0 and you will approach to this point argument going to 0. So, you say this Newmahn function essentially will tend to minus infinity. Whereas, for the Bessel function will either tend to 0 or it will tend to 1.

So, one thing is clear that if I take the q real then I get set of solutions which is Bessel function and Newmahn function. So, this here this quantity q is real.

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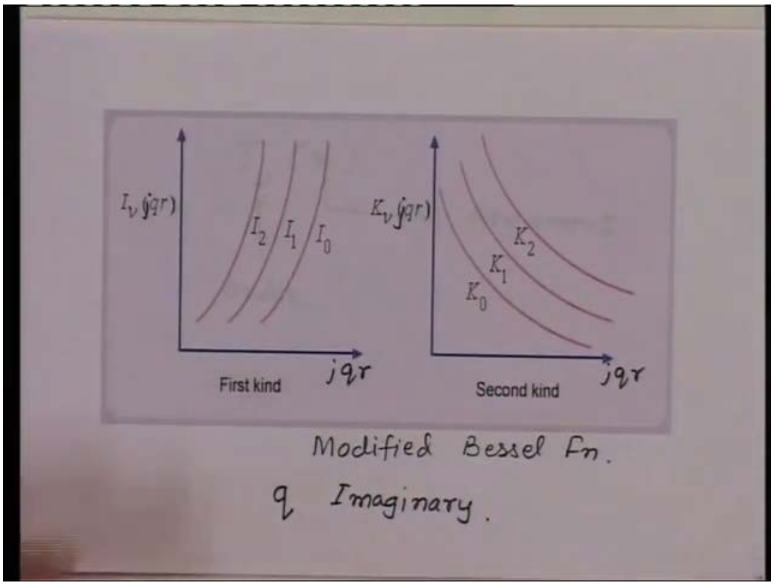

If the q is imaginary then I get the Bessel functions, what are called the modified Bessel functions. So, we have here what are called the modified Bessel functions. And this functions are denoted by I and K again the nu is your argument and q is the q nu gives you the order and q r gives you the argument. In this case again this argument is real. So, essentially we can say this quantity is j, put j here.

So, if q is imaginary then we get a solution to the Bessel's equation which are called the first and second kind of modified Bessel functions and are denoted by I and K. If I look at this behavior now in contrast to the behavior or the Bessel functions when q was real, then the striking difference between the two these functions are oscillatory in nature whereas, these functions are monotonic in nature. So, as I change the argument, this is q r. The I function monotonically increases as r increases and the K function monotonically decreases as r increases.

If I have q which is in general complex then we will get function what are called the Hankel functions, but in this case since we are talking about the medium which is lossless, the mu and epsilon these quantities are real quantities, the beta is the phase constant which is the real quantity. So, in our case either q can be real or q can be imaginary depending upon this is greater than this or this is less than this. So, here we have to consider only two cases the q real and q imaginary.

So, now, what we find is that if I choose a set of solution for which q is imaginary then I get the field variation radially which will be monotonic in nature, if I consider the q real then I get radial variation of field which is oscillatory in nature. That is why now you have to put our physical understanding to choose appropriate solution. We have seen from our total internal reflection that inside the core because of the interference of the face fronts we have a variation which is oscillatory kind of solution. We have a maximum intensity, we have 0 intensity, constructive destructive interferences. So, inside the core we had a field distribution which was oscillatory in nature.

Whereas, we had seen for total internal reflection the field in the cladding was exponentially decaying. And it was monotonic in nature, that gives us now a clue to choose the appropriate solutions. So, what we are saying now is that, if you are inside the core since we are looking for the variation which is oscillatory in nature, this cannot be a solution. This does not give you a variation which is oscillatory in nature. So, only possibility is that inside the core you have to have a solution which is this. So, inside the core q has to be real and the solutions should be the normal Bessel function and the Newmahn function. Whereas, if I go inside the cladding we know that fields have to be monotonically varying. So, it has to have a function combination of which is like this.

Having said that one can further go and ask a question that inside the core of course, both are possible, but since r equal to 0 point is included inside the core, this function will go to minus infinity at the axis of the optical fiber if this function is chosen. That means this function cannot be a solution if the problem because this will tell infinite field strength at the axis of the optical fiber. So, what that means, is that the arbitrary constant corresponding to this solution has to be identically 0.

So, only field which can be there inside the core of the optical fiber with a finite value those field can correspond to the geo Bessel function. So, any field which is having a variation j nu q r that can represent the field distribution inside the core of the optical fiber and q has to be real. Whereas, if I go to the cladding then since the energy is confined to the core as we go away from the core in the cladding or as r increases, the field should die down and it should not increase. So, now, this function r which shows you monotonic increase of the amplitude, that means, as r tends to infinity this field will tend to infinity cannot be a solution to the problem.

So, this field arbitrary constant corresponding to this field has to be identically 0, only this is the field or this is the function which can represent appropriately the field variation in the client. So, with the physical understanding which we have developed from the ray model, we can say that inside a core the fields are given by j Bessel function, inside the cladding the fields are given by k modified Bessel function or in other words q has to be real in the core and q has to be imaginary inside the cladding.

So, what we are saying now is that this quantity q is real in the core and q is imaginary inside the cladding. We will come back to this what this physically mean, but essentially what we have said now is that by using this physical understanding now we can write down the fields appropriately inside the core and the cladding and which can essentially be given by this. So, as we said these are the functional form which you got for the quantity psi and psi can represent as an electric or magnetic fields, z component.

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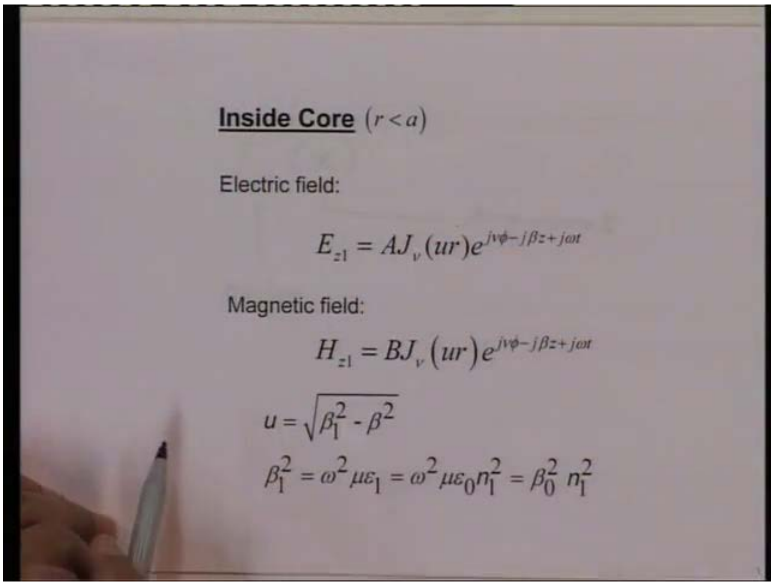

So, we can get the z component of electric field, as some arbitrary constant which define the amplitude of the field j nu function and in this case just to separate out the two regions let us call this quantity q, now is u. So, u r e to the power j nu phi minus j beta z plus j omega t. Similarly, we can write down the longitudinal component of the magnetic field which is H z 1 which is another amplitude constant and j nu u r same variation will face phi and z.

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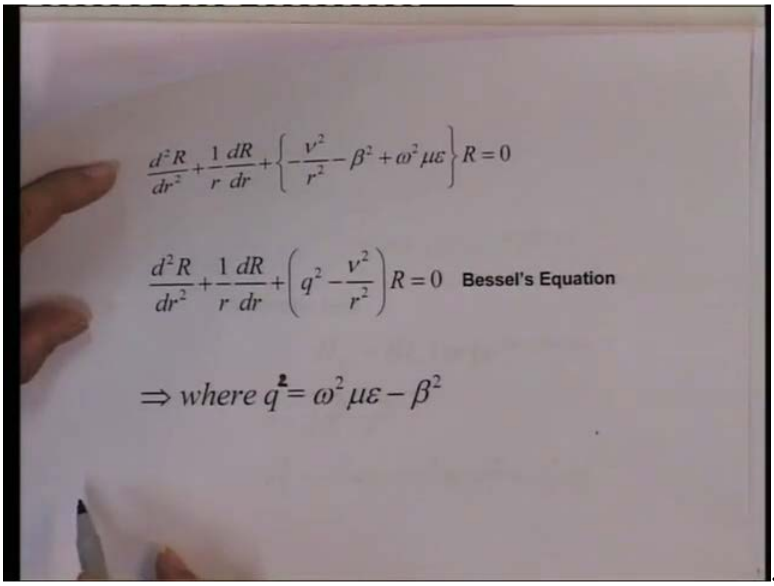

And now since q is which is this has to be real, this quantity has to be greater than this and for the core the epsilon is epsilon 1. So, if I substitute that we can get this quantity u which is a square root of beta 1 square minus beta square where beta 1 square is nothing but omega square mu epsilon 1 which if we write in terms of a refractive index that is equal to omega square mu epsilon 0 into n 1 square is equal to beta naught square into n 1 square. This is nothing but the phase constant intrinsically in the medium of refractive index n 1.

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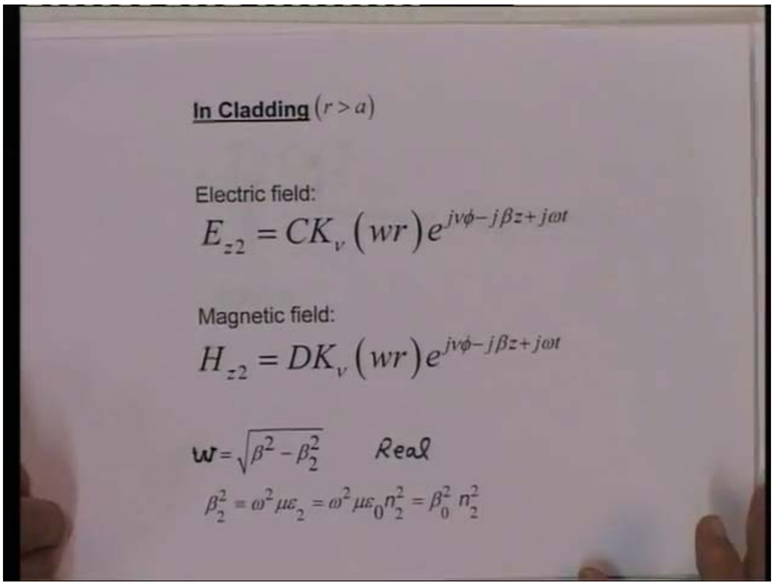

Similarly, we can write for the fields inside the cladding, where C is some arbitrary constant we define the amplitude of the fields and K now is the modified Bessel function with the argument w r where w is defined as this. So, what we have done here we have this quantity q square which is omega square mu epsilon minus beta square. Since, now we are saying that we are looking for modified Bessel function we can interchange the sign here and define this quantity w, which is real quantity which is beta square minus this quantity which is nothing, but beta 2 square.

So, in this case w is real, q is imaginary, but w where I will interchange the negative sign here has become real quantity. So, I can define this quantity now beta 2 square which is omega square mu epsilon 2 which is equal to omega square mu epsilon 0 into n 2 square is equal to beta 0 square into n 2 square. So, by using my physical understanding now I have defined the longitudinal component of electric and magnetic fields inside the core and cladding and we have chose the propagation constant beta now in such a way that w is real and u is real.

So, what essentially now we are saying is that to make w real the beta should be greater than beta 2 and to make u real here beta should be less than beta 1. So, what in this process by choosing the proper functions, unknowingly we have already committed for the range of beta. What we are saying is beta cannot be any arbitrary thing now, since u has to be real the beta has to be less than beta 1, w has to be real beta has to be greater than beta 2.

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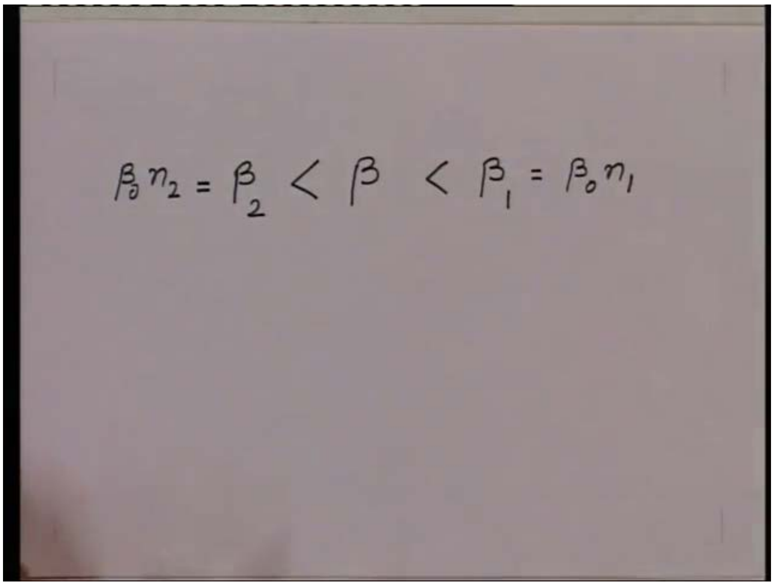

So, essentially we are saying the propagation constant beta is bound by two limits beta 2 and beta 1 and this quantity beta 2 is nothing, but your beta 0 into n 2 and this quantity is nothing, but your beta 0 into n. So, by using appropriate solution and for the choice we have used our physical understanding which we got from the ray model we essentially get bounce on the propagation constant of a mode inside the optical fiber.

So, next time when we meet essentially we will investigate and understand physically what this quantity why this bound is there, what this physically means and then we will try to get the transverse component get the boundary conditions and from there we will derive what is called the characteristic equation of a particular mode inside the optical fiber.