Advanced Optical Communications

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Lecture No. # 06

Wave Model - II

We are discussing propagation of light inside the optical fiber. In the beginning, we took the light in the form of rays and investigated the propagation of light inside the optical fiber by using phenomena, what is called total internal reflection. Then we found the limitations of that model, then we started investigating the advance model, what is the called the wave model for propagation of light inside the optical fiber.

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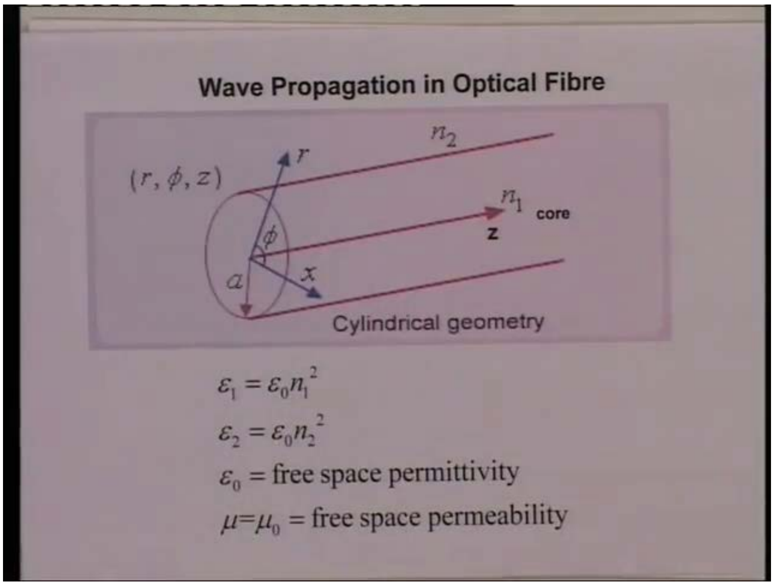

So, for wave model essentially we define first appropriate coordinate system for the optical fiber, which is cylindrical coordinate system. So, we assume that the core of the optical fiber is a solid glass rod having refractive index n 1 and then we assume that the cladding is of infinite size with a refractive index n 2.

So, essentially we are solving the wave equation for this structure which has only one interphase, but in the core and cladding. Then by using the physical understanding which we develop from the ray model, we try to solve the wave equation and then we found that the radial variation of the electric or magnetic field is given by the Bessel functions inside the core and it is given by the modified Bessel function inside the cladding.

So, the phenomena essentially which we utilize for establishing this, was that inside the core we have a interference phenomenon and because of that we expect that the field variation would go through maxima minima. Whereas, if you go inside the cladding then the field should die down monotonically as we go away from the core cladding boundary and that variation was appropriately given by the modified Bessel function k. Whereas, the variation which was of oscillatory in nature, that was given either by the Bessel function or by the Neumann function.

However, we found that for the Neumann function, if r equal to 0 point is included, then the field would go to infinity and because of that the Neumann function would not appropriately represent the field variation inside the core.

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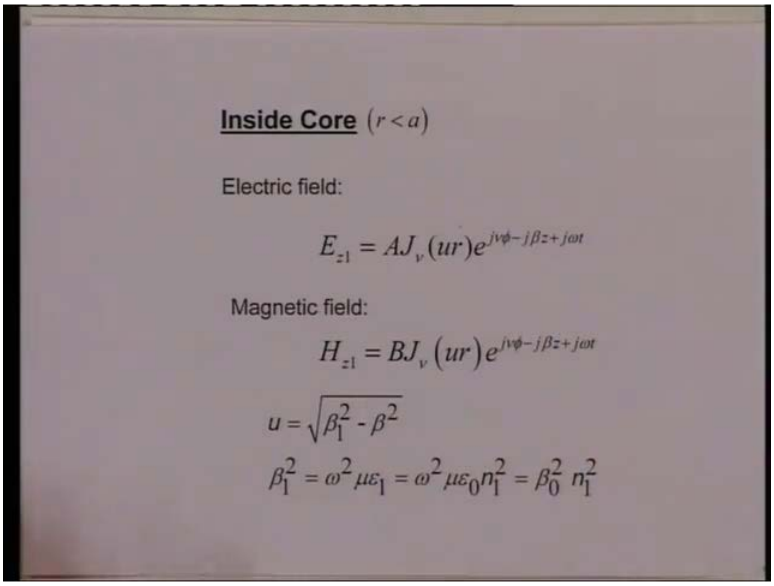

So, with this understanding essentially we got the expressions for the longitudinal component of the electric and magnetic field inside the core which is given as A J nu u r e to the power j nu 5 minus j beta z plus j omega t. Similarly, if the magnetic field also is given by some on arbitrary constant B and the same variation J nu ur e to the power j nu 5 minus j beta z plus j omega t. This quantity u, what we can call as the radial propagation constant inside the core is given by the square root of beta once square minus beta square, where this beta is the face constant of the mode or the field distribution which is going to propagate inside fiber and beta 1 is the phase constant of the medium which is feeling the core, that is refractive index n 1. So, if you assume that you have an infinite medium of refractive index n 1 and if you ask what to do the phase constant of a wave in that medium? That essentially would be equal to beta 1 and that is essentially given by this

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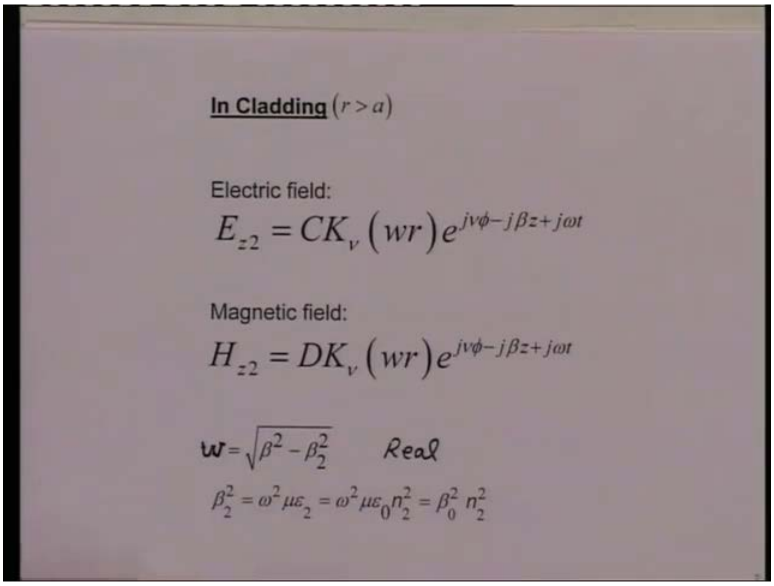

So, beta 1 square is beta naught square multiplied by n 1 square n beta naught square is the phase constant of a wave in vacuum. Inside the cladding we had the field variation which was given by modified Bessel function. So, we had this arbitrary constant C and then we have this K which is modified by Bessel function of order mu and then we have this argument which is w r, where w is given by beta square minus beta 2 square, where beta 2 is the phase constant intrinsic to the medium of refractive index n 2. And we saw that this quantity u and w both have to be real for appropriate representation of the field.

So, if u become imaginary then the J function will not remain oscillatory function, in fact it will become modified by Bessel function. Similarly, if the w becomes imaginary then the modified Bessel function will become normal Bessel function and they will not represent appropriately the fields.

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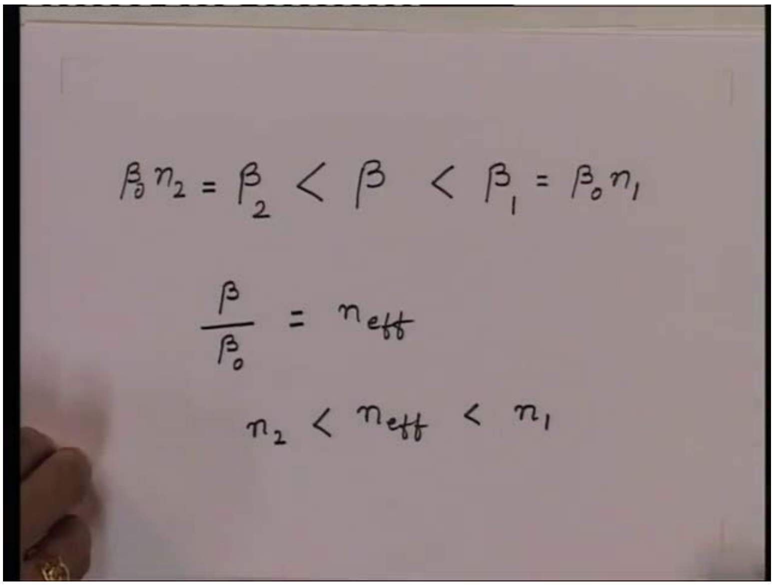

So, what you find is that once we have chosen this functional form for the fields inside the core and cladding, we are essentially put the condition on beta in an indirect fashion. That is beta has to lie between the two bounds which are given by beta 2 and beta 1. So, the lowest value of beta is equal to beta 2, which is nothing but beta 0 into n 2 and the highest value of beta is beta 0 into n 1. So, now if I get the face constant beta and if I divide this quantity beta by the free space constant which is beta naught, we get a number which would lie between n 1 and n 2.

So, essentially we can say that if I define this quantity beta divided by beta naught as something what is call the effective refractive index of the model propagation. So, let us say this is n effective then the n effective would lie between n 1 and n 2. So, you have this quantity n effective its lowest value will be n 2 and its highest value will be n 1. So, either we can talk the propagation characteristic in terms of the phase constant beta or we can talk in terms of what is call the effective model index, what is called n effective, which gives you the velocity of wave on their structure. So, we will later understand the physical meaning of, why you have this two bounds here? n 2 and n 1, but let us complete our analysis of the wave equation.

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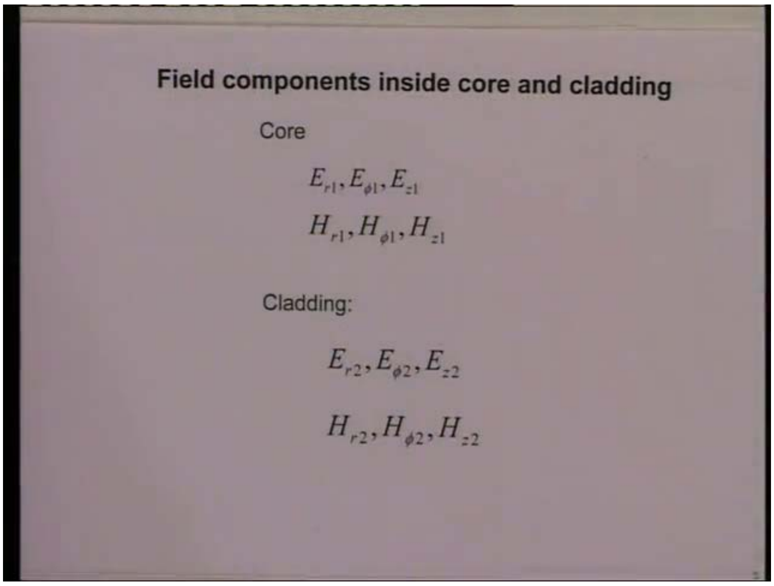

So, once you have got a longitudinal component of electric and magnetic fields then as we seen earlier, we can use the expressions for transverse fields in terms of longitudinal components and we can get the total electric and magnetic field inside core and cladding. So, essentially now we have got six components E r, E phi, E z, H r, H phi, H z and suffix 1 denote this quantity inside the core. Similarly, you have E r, E phi, E z, H r H phi, H z this suffix 2. Theses quantities represent inside the cladding. Once you get the six components, then we can apply appropriately the boundary conditions at the core cladding interferes that is at r equal to a

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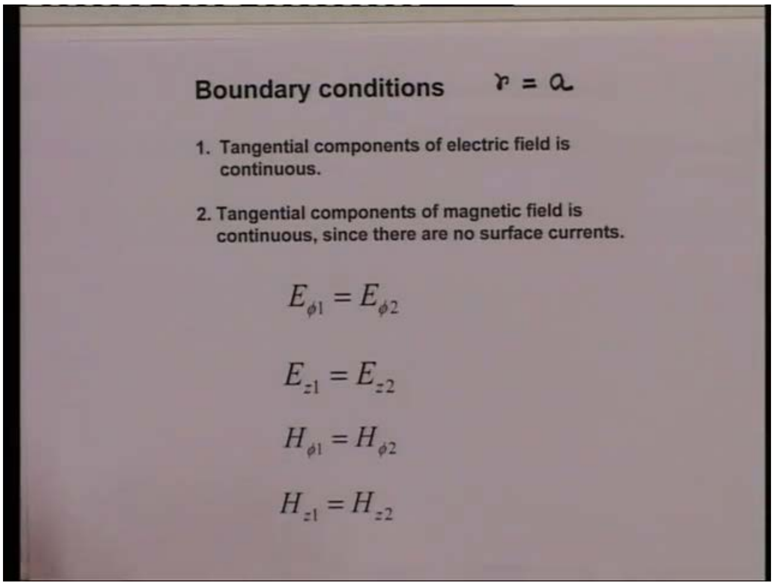

So, we have a boundary conditions now at r equal to a, which is the core cladding interference. And from our basics of electro magnetics we know that there are four boundary conditions which you can impose, we can apply the continuity of the tangential component of electric field, we can apply the continuity of normal component of magnetic field and so on. However, in this case since we are talking about the media which are purely dielectric media, there are no surface currents. For the conductivity, for the cladding and the code is 0. In this situation not only the electric field, tangential component is continues across the boundary, but even the tangential component of magnetic field is continue across the boundary because the surface currents are 0.

So, we essentially apply two boundary conditions here. So, we said tangential component of electric field is continues at the core cladding interphase and then we said tangential component of magnetic field is continuous since there are no surface currents. So, now if I go to my geometry here, the cylindrical geometry, you will see that the two components are tangential to this interphase. Now here when I say the interphase, they interphase essentially defined by this cylindrical surface. So, any field which is along the z direction is tangential to this cylindrical surface; that means, the z component of either electric field or magnetic field is tangential to the interface, between the core and the cladding; similarly, if I consider if a field, which is phi oriented that will be tangential again to this interphase.

So, when I apply the boundary conditions, essentially we have this four quantities here that the phi component of the electric field is continues, z component of electric field is continues, phi component of magnetic field is continues and z component of magnetic field is continues. So, we get here if E phi 1 equal to E phi 2, E phi 1 equal to E phi z 2, H phi 1 equal to H phi 2 and H z 1 equal to H z 2. So, once I get the expression for the longitudinal components and the transpose component, by equating this essentially I have got four equations now. And how many unknowns we have? We have five unknowns overall, we have four arbitrary constants a b c and d which are representing the magnitude of the electrical magnetic fields and the component beta which is the phase constant of that field is distribution in the fiber.

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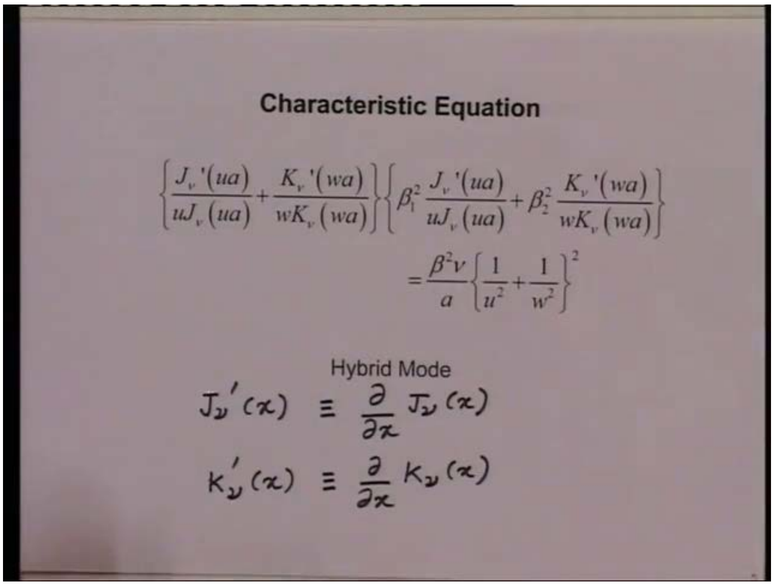

So, overall we have five unknowns and from the boundary condition we get four equations. So, essentially what we can do is we can eliminate these quantities a b c d from this four equations and then what we get is the characteristic equation of the model propagation in the optical fiber. So, if I equate the tangential components and eliminate this arbitrary constant a b c d then we get the characteristic equation of a mode which essentially given by this. Here you have this quantity here J nu prime u a and K nu prime u a, where the prime denotes the derivative of the Bessel function or modified Bessel function with respect to argument.

So, essentially the J nu prime x is d by d x of J nu x. And similarly, K nu prime x that is d by d x of K nu x; so, just for the simplicity, we write down in the characteristic equation in this from. So, you have here the derivative of Bessel function, you have derivative here for the modified Bessel function and this equation essentially now represents the characteristic equation for a general mode which is having all six components, because we have taken all the six components, started with a two longitudinal components E z there H z and all these four components, which are transverse. We got a characteristic equation now which is in general true for any arbitrary field distribution with six components.

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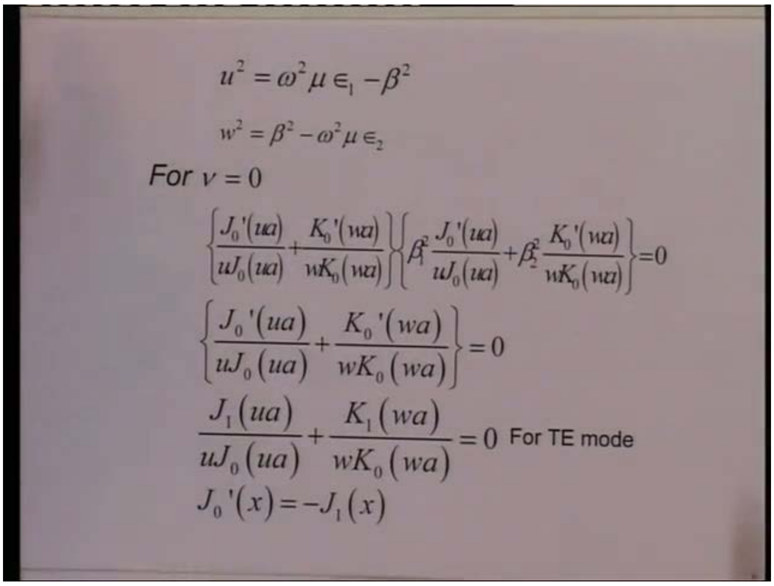

So, this mode as we saw earlier that if all the six components are present, then we designate this mode as the hybrid mode; so, this is nothing but the characteristic equation for the hybrid mode. The quantities u and w as we defined earlier, the u square is omega square mu epsilon 1 minus beta square and w square is equal to beta square minus omega square mu epsilon 2. Now, if I consider the special case of this equation, that is for nu equal to 0 and what is nu representing here? The nu represents the variation of the field in the azimuthal direction or in the phi direction, because we have the functional form for this field which is it the power J nu 5. So, we get a variation of electric or magnetic field in the phi direction which is given by e to the power J nu phi.

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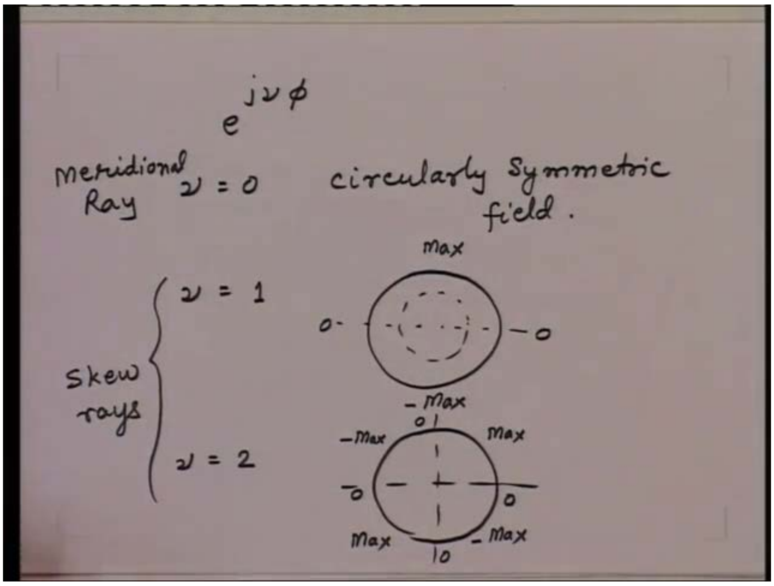

So, if I take nu equal to 0 then that represents a circularly symmetric field or field we does not have a variation in the azimuth direction. So, if I look at the function which is into the power J nu phi, the nu equal to 0 gives me circularly symmetric field and if nu is not equal to 0 then we do not guess circularly symmetric field, it will have a variation in the phi direction. For example, if I take nu equal to 1 then I will have 1 cycle variation as I move in the direction of phi if I take nu equal to 2, I will have 2 cycle variation in the azimuth and so on. So, for nu equal to 1, if I consider the fiber like this, it will have a one cycle variation in the azimuth. That means, this will be, let us start with 0 here, it will be maximum in this direction, it will go to 0 here again and go to minus maximum in this direction. So, as I move along this in the phi direction the field will see a variation in going from 0 to maximum, to 0 to negative maximum.

Similarly, if I take nu equal to 2 then I will get the variation which will be going into two cycles now. So, it will be the like this, it will be 0 going through positive maximum, will be 0, will be negative maximum, will be 0, will be positive maximum 0 and negative maximum. So, this quantity the index nu essentially is telling us, how the field amplitude is going to very in the azimuthal direction for a given value of r in given value of z. One thing you immediately notice here is that, if I consider no variation which is this circularly symmetric case then, the field will have a maximum that the center of the fiber. Whereas, if I consider any value of nu which is not equal to 0 then on this side of this line, you have the value which is positive and on another side you have value which is negative. It should have a continuity of the fields from this part to this part.

So, essentially the field must be identically 0 at the center of the fiber. So, if I take a nu equal to 0, then we get circularly symmetric fields and these fields would have a maximum intensity at the axis of the fiber. Whereas, if you take new equal to 1 or 2 or any other value of nu then essentially the field must go to 0 at the center of the fiber.

Now, if you recall the discussion in the ray model, we had two types of rays. One was the meridional ray and other was the skew ray. And the property of meridional ray was that all the ray start the again coming join the axis. So, you have a constructive interference of the rays at the axis of the fiber. So, you have intensity maximum at the axis of the fiber. Whereas, if you go to the skew rays, then we saw that the skew ray always spirals around the axis of the fiber and because of that you have low intensity at the axis of the fiber.

Precisely that is what these two quantity are telling us that if I consider, a circularly symmetric case which corresponds nu equal to 0, then these modes or these field essentially correspond to the meridional rays. Whereas, if you take any other field distribution or modes, which correspond to nu equal to 1 or any other higher values then they correspond to the skew rays. So, these are the one skew ray. Where as this one is the meridional ray. So, you can see one to one correspondence between the ray model and the wave model. And that is always good, because whatever understanding you have to developed from the ray model, that should support the mover deeper understanding what we are going to develop for the wave model.

So, now this set of rays which correspond nu equal to 0 these are circularly symmetric and these are special rays. So, that is the reason we want to find out what would be the characteristic equation for nu equal to 0. Because they are the one which are going to represent the meridional rays. So, if I put nu equal to 0, essentially the equation, the right hand side of the characteristic equation goes to 0, because here nu is appearing here. So, this quantity is 0 and then we get the characteristic equation essentially reduce to this. Seen the product of these two term equal to 0, we may have two solutions either this bracket going to be 0 or this bracket going to 0.

So, what we find here is, that if I taken a transverse electric case or transverse magnetic case and go on through the whole exercise of matching boundary conditions and so on. These brackets equal to 0 would come as the characteristic equation for the transverse electric mode. Whereas, this bracket equal to 0 will come as the characteristic equation for the transverse magnetic mode.

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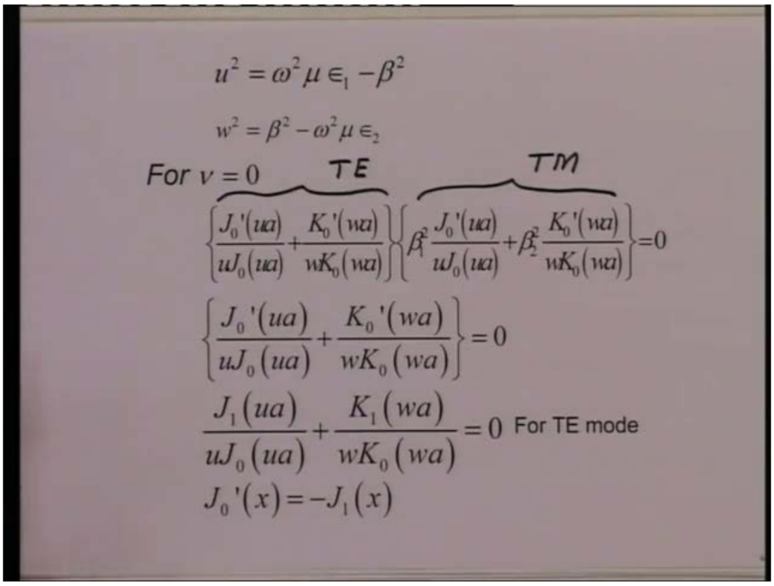

So, this bracket here essentially is representing the T E mode and this bracket here, equal to 0 is representing the characteristic equation for the transpose magnetic mode. So, if I take this bracket and equated it to 0, this will be the characteristic equation for the transverse electric mode. And what we can do is we can use the recurrence relation for finding out the derivative of J function. So, the J 0 prime x is nothing, but minus J 1 x and same is also true for the K also. So, if I substitute now into this, we get the characteristic equation for the transverse electric mode which is given by this. And important thing is that the transverse electric mode is always going to be circularly symmetric.

Similarly, if I take this bracket and equated to 0, we get the characteristic equation for the transverse magnetic mode. So, we are a very important conclusion and same thing we are seen for the ray model also, that transverse electric mode and a transverse magnetic mode have field distributions which are circularly symmetric. Whereas, if you take a hybrid mode, the hybrid mode has a field distribution which is essentially circularly non symmetric, because that corresponds to the nu equal to 1 nu equal to 2 and so on. So by this now, one can write down the field distributions for various modes.

So, if I know take this characteristic equation here and I saw this characteristic equation, immediately it will be clear that this equation has multiple solutions. And why multiple solutions? Because J 1 and J 0 both this functions or oscillatory functions. So, they cross 0 they become negative and so on. So, essentially depending upon the value of a, you will get multiple solutions to this problem.

So, for a given size of the optical fiber, one has multiple solution for the problem. So now I have, for designating a mode, have two quantities now. One is, what is this azimuthal index nu, which tells how many cycle variations the field has in the azimuthal direction and second which solution of the equation is representing the mode. So, if I get the first solution then I have a combination of some nu and the first solution. If I take second solution, then I have combination of nu and second solution and so on.

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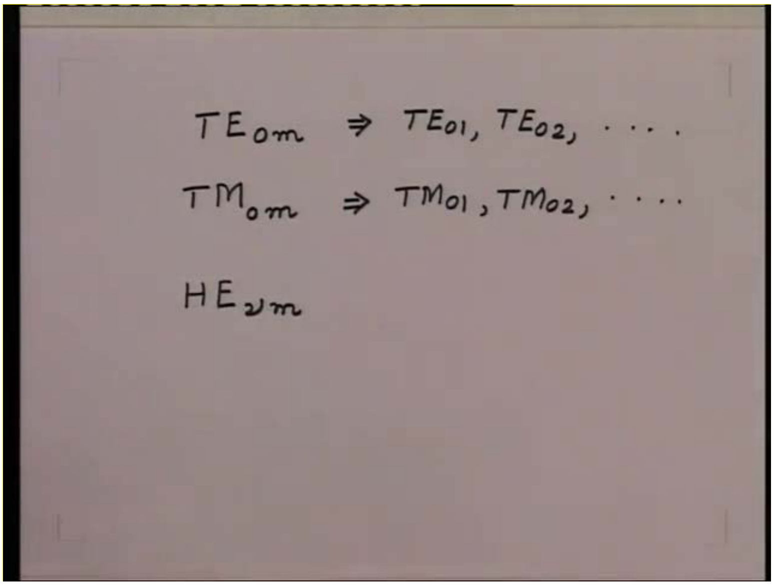

So in general, then we can designate a mode which is T E and for T E this is the nu always equal to 0, first index will also put as 0. And then depending upon the solution, whether it is the first solution or second solution or third solution, we put some index M. So, we get the modes for the transverse electric, which could be T E 0 1 mode or T E 0 2 mode and so on. The same argument is true for the transverse magnetic case also. So, we have a transverse magnetic case, again the nu is 0 for transverse magnetic case. So, this index is 0 and then depending upon with solution we have, you get index m. So, you can get a mode which could be transverse magnetic 0 1 are transverse magnetic 0 2 and so on.

Similarly, if we take a general mode which is given by this characteristic equation, then we know for this one, now the nu essentially has to be now 0 and then we can designate this hybrid mode by a g mode . So, we get a mode which is H E and here the index is nu, which is not equal to 0 and the solution which could be the first solution or second solution or third solution. So, now inside the optical fiber, we have three sets of mode which are propagating, the transverse electric mode with the first index 0 and first index here gives number of cycles in the azimuthal directions and second index tells the solution number of the characteristic equation. Also this quantity essentially tells us, how many maxima we have in the function in the radial direction.

So, if I consider the first solution, then we have only one maximum which is at a center if I take m equal to 2 then one 0 would have cross in the Bessel function and so on. So, now this two indices which we have for each of the mode either T E or T M or hybrid which is H E, the first index tells the variation of the field of the amplitude in the azimuthal direction or in the phi direction. And this one, second one index, tells us how many is 0 crossing you have in the radial direction. So, 0 crossings are theses quantity minus 1. So, if m equal to 1 that is no crossing and radial direction, if m equal to 2 will be one 0 crossing and radial direction and so on.

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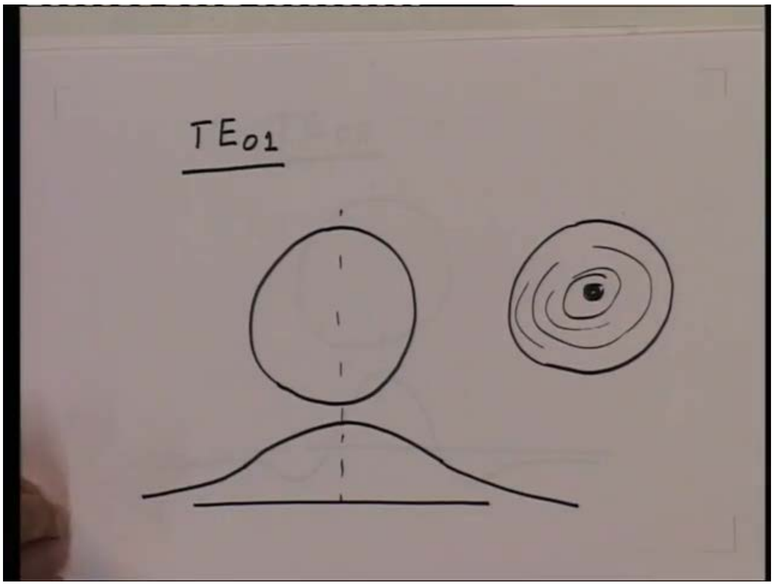

So, if I say that I have a T E 0 1 mode and then this mode is circularly symmetric and does not say 0 crossing an radial direction. So, if I take this and try to draw the fields for this, it is a circularly symmetric. So, if I draw the field distribution this the function of radial direction, the field distribution would go something like this .No 0 crossing is reach till the end and them the field is decays exponentially in the cladding.

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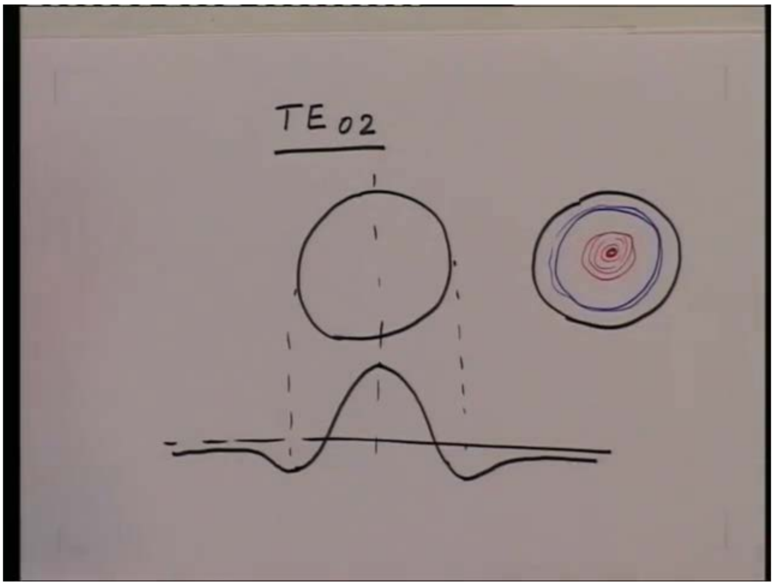

Whereas, if I consider a more which is let us T E 0 2 mode, then the field variation is circularly symmetric, it does not have an variation in the phi direction, but one 0 crossing is there in the radial direction So, if I will look at the field for this, the field would be it will be maximum at the center somewhere it will cross through 0 till be reduce to this point, 1 0 crossing as taken place and the speed exponentially decays as I go inside the cladding. So, the field distribution would look something like this. If I look at the intensity pattern of this mode, then this will show me intensity maximum at the center and then the intensity will slowly reduce toward the core cladding boundary and it will keep reducing as I go inside the cladding.

So, if I draw here, the intensity variation I get a very bright spot here at the center and then slowly the intensity will reduce as I go away from this and then it will simply merge with the intensity in the cladding. Whereas, if I take the intensity distribution, in this case there it has a very bright spot at the center which corresponds to this then, the field in intensity reduces it become 0 at this and here the field become a negative that mean the field direction reverses.

So, will get very bright spot here, it will reduce slowly here, go to 0 and then it will become a negative field here, I think like this. So, we will get a bright region at the center then there will be dark ring somewhere between and again we will see bright ring of light, with a electric field orientation opposite to what you are having at the center because the functional has become negative. So like that, we can visualize now the field distribution inside the optical fiber by the nomenclature which we have define that the first index tells us the variation of the field amplitude in the azimuth direction and second index minus 1 tells us the number of 0 crossings in the radial direction.

With this understanding, then now one can go and look at some more features of this and if I look at this quantity u square which you define here.

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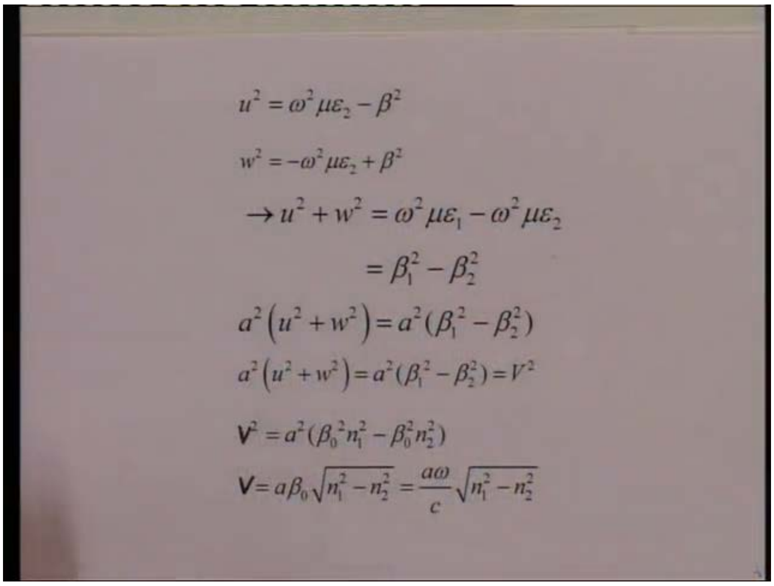

Which is u square is equal to omega square mu epsilon 1, beta 2 and w square is equal to beta square minus omega square mu epsilon 2. You find that some of these quantities is independent of beta. So, u square plus w square if I take, that is nothing but omega square mu epsilon 1 minus omega square mu epsilon 2 and this quantity as we know is beta 1 square; so, this beta 1 square minus beta 2 square.

So, for a fiber of radius a, now we can define a quantity which is a square multiplied by the this one, which is a square multiplied by beta 1 square minus beta 2 square and if I write down explicitly beta 1 and beta 2 as beta 0 square to n 1 square minus beta 0 square and n 2 square, we get this parameter which is a characteristic parameter for the optical fiber. Why it is characteristic parameter? Because there are only three quantities involved in defining this optical fiber, the refractive index of core, the refractive index of cladding and the radius or the diameter of the optical fiber core.

So, you recall that the parameter which we define the numerical aperture that was a characteristic parameter for optical fiber, but the size of the core was not coming into picture while defining numerically aperture. The numerically aperture is purely defined in terms of the refractive indices of core and cladding.

But we are seen, that just the difference in the refractive index does not completely characterize the fiber propagation because depending upon the size of the core, that different modes or different numbers of modes can propagate in the fiber. That means, just parameter which is defined on the basic of refractive indices, does it completely describe the propagation characteristics. So, now we find here this parameter is a more complete parameter for describing the fiber. If I simplify this quantity v, will be equal to a, you can take beta common from here; so, a into beta 0 square root of n 1 square minus n 2 square. And a 0, beta 0 is the free spaces propagation constant which is given by omega upon c, where c is a velocity of light in vacuum. You get this quantity v, which is a omega upon c, square root of n 1 square minus n 2 square.

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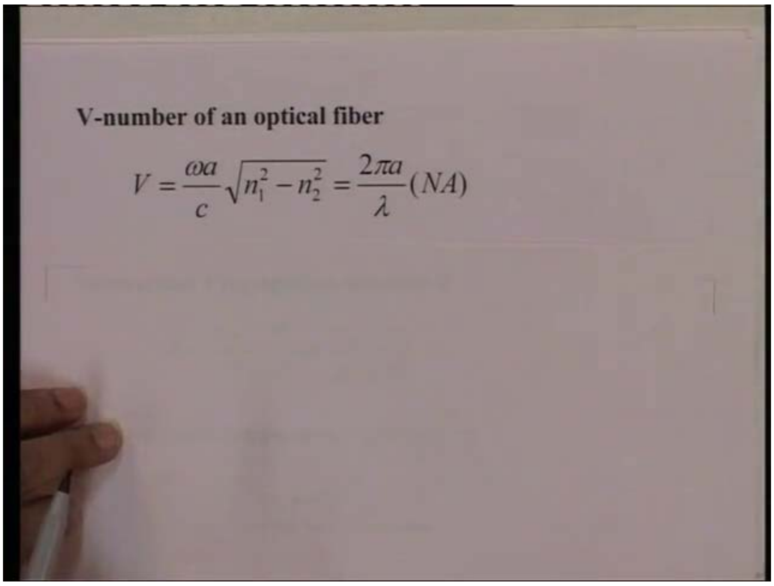

So, if I write this quantity, realizing that this quantity we already know, which is nothing but the numerical aperture of the optical fiber. We can define this characteristic parameter for the optical fiber which is what is called the V number of the optical fiber. So, V number is a more comprehensive parameter for the optical fiber because it has is radius included in this, it also has a numerical aperture included in this and radius is normalize with respect to the wave length. Where if I write down this quantity here c, omega is 2 point to frequency c upon frequency will lambda. So, you get V number which essentially is given by this.

Now, this V for a given parameter since n 1 n 2 is constant, the radius is constant, the V number of an optical fiber is directly proportional to the frequency omega. That is the reason, many times this parameter is also refer to has the normalize frequency. And later on, we will see this is the parameter which essentially used to compare different types of fibers. So, this parameter V number of an optical fiber is one of the extremely important parameters, that is one which is going to describe the propagation characteristics inside the optical fiber.

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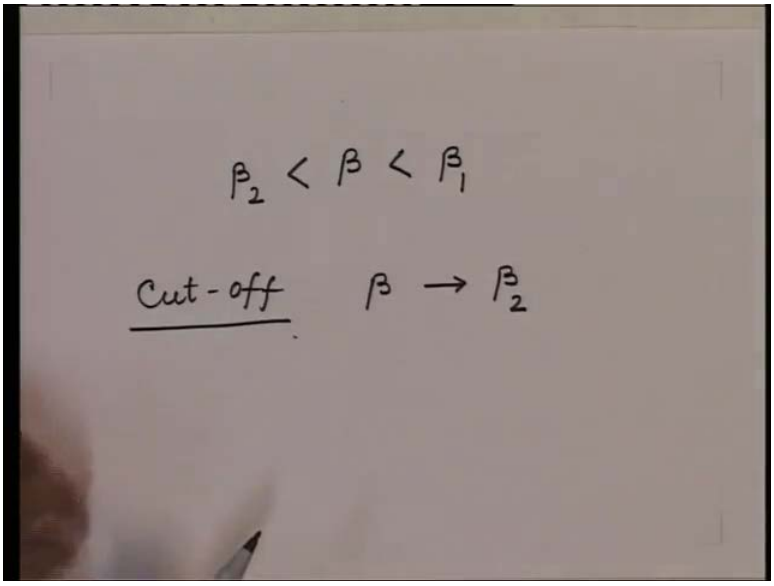

Now, before you make use of this parameter, the V number, let us come back to our understanding that your beta had two bounds, which were given by your beta 2 and beta 1. And you have is, relationship between u and w which is essentially given by this. So now, if I look at this quantity here w and w times r, is the argument of the modified Bessel function. If w becomes 0 or if this w becomes imaginary, then the modified Bessel function does not remain modified Bessel function, that what we saw earlier. That means, if this quantity w becomes imaginary, then the field variation will not be appropriately described like a exponentially decaying fields in the cladding and we will say that, these mode is no more remain a guided mode, then energy will start leaking inside the cladding.

Since we want that there has to be sustain propagation energy only along the axis of the optical fiber, the field must die down monotonically in the cladding or in other words as we saw that this quantity beta has to be greater than this quantity. So, when beta approaches this quantity or when beta becomes equal to this quantity or beta becomes equal to beta 2. That time, your modified Bessel function gets converted into the normal

Bessel function and then there is no confinement of the energy inside the core because the field does not die down monotonically in the cladding.

So, we define this condition as what is called the cut of condition for more. So we define a condition, what is called a cut of condition, when beta tends to beta 2. What does this physically tell me now? That you let us say, beta is equal to beta 2. Suppose what that means is, that whatever energies propagating inside this medium its sees, is basically refractive index n 2. That is why the propagation constant which is beta 2 or in other words it is telling us that most of the energy associated with these fields is lying in a medium, which is having refractive index n 2. That means, most of the energy is lying inside the cladding.

So, as we, beta approaches beta 2 more and more energies spreaded inside the cladding. Similarly, if I situation where beta approaches beta 1, then more and more energies confined inside the core because most of the energies see the refractive index which is equal to n 1. Now we can understand this physical picture very easily, that why beta lies between these two limits beta 1 and beta 2 because, for an optical fiber part of the energies lying inside the core, part of the energy is lying inside the cladding.

If you are left to themselves in the fields which are lying inside the cladding would travel with a phase constant which is beta 2, the field which are lying inside the core would travel which a phase constant which is equal to beta 1, but now these fields are tied together by this mode, by the boundary conditions.

So, the field inside the cladding and core, they cannot move with arbitrary phase constant way they like, but they have to come to an agreement that the phase constant for the fields in the cladding has to increase little bit, for the fields which are inside the core have reduces little bit, so that they can travel together with the same phase constant beta

So, essentially the limit of beta, beta 2 and beta 1 is telling us that the energy is there in this two regions and because the energy which is outside the core has a tendency it will travel with phase constant beta 2 and energy in the core has tendency to travel with a phase constant beta 1, there has to be a mutual agreement so that, they can travel with a phase constant that lie between these two limits.

But it also tells you something more, what it tells is, if I look at the behavior of beta. If beta is very close to beta 2, then most of the energy lies inside the cladding. So, without even worrying or without even finding out, how to the total fields are distributed and so on. If I can get this parameter beta and if this beta lies close to beta 2, we can immediately say that most of the energy is the lying inside the cladding compared to inside the core. If we can get a situation, where beta lies very close to beta 1 then immediately we can conclude that most of the energy lying inside the core and very little energy is lying inside the cladding.

So, as beta goes towards beta 1 the energy gets more and more confined inside the core and very little energy remains inside the cladding. And that is a very desirable feature, that is what we want in the propagation of light because if the light has to be free from interference from the external world, the light should be very well confined inside the core. So, we have to create a situation, we have to operate in a domain, where beta should be as closed to be beta 1 as possible and as away from beta 2 as possible.

So, this condition what is called the cut off condition, at which the mode propagation essentially seizes or the mode starts now leaking out inside the cladding beyond that the mode propagation does not remain any meaning full, because we very quickly loose the energy inside the cladding.

So, for a good welcome find propagation of light inside the core, we should make beta as large as compare to beta 2 and as close to beta 1 as possible. Now with this stand, one can say that, if I now look at the variation of beta as function of frequency, if forever the interest falls. Let me recall, we started this analysis to understand how the beta is related to omega. Because once we understand the relationship between beta and omega, then we can get find out the phase velocity, which is omega by beta, we can find out the group velocity which is d omega by d beta.

So, the whole exercises was essentially to find out the relationship between the phase constant beta and frequency omega. And what we found here is that the V number now is related to the quantity which is the frequency, which is proportional to frequency. So, x of the personality constant for given fiber, we can use for omega the number V. Similarly, for beta what we can do is that we can define a parameter, what is called normalize propagation constant, which can be defined like this.

Now, we are seen earlier that this quantity the effective refractive index which we got has a limit which goes between n 1 and n 2. So, when the mode approaches cut off, the n effective essentially approaches n 2 and when the mode is far from cut of then n effective will approach n 1, most of the energy will get confine inside the core. So, instead of plotting now beta as a function of frequency, we can get a plot of beta which is respect to v because v is proportional to omega or we can have a normalize plot of this quantity, which we call as b which is defined as n effective square minus n 2 square upon n 1 square minus n 2 square.

And this will now always between 0 and 1 because at the cut off n effective will tell to n 2. So, this quantity will go to 0, when we go very far from cut off n effective will tend to n 1 and that times this quantity will become equal to 1. And that is the reason we can call this quantity, the normalize propagation constant.

So, beta will vary from beta 2 to beta 1, but is normalize propagation constant will always vary between 0 and 1 irrespective of what more we are talking about.

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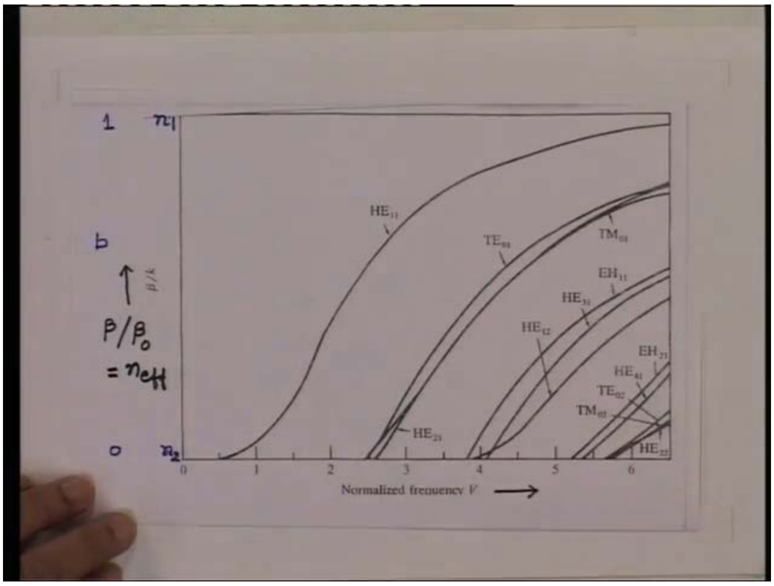

So, whether we have a T mode or we have a T M mode or we have a hybrid mode or whatever the index is, the limits of this is always between 0 to1 and limit for beta is always between beta 2 and beta 1. So now, instead of having a plot or variation of propagation constant beta, as a function of frequency, we can get a variation of this normalize propagation constant as the function of V number because V number is proportional to frequency and this quantity essentially is quantity beta.

So, here we have plotted here, the quantity beta divided by beta naught, which is nothing but your n effective as we seen n effective will range between n 2 and n 1. For a particular mode, we can vary the V number and ask how the beta varies. So, essentially the characteristic equation which we have got, you have to solve that characteristic equation numerically, get the value of beta for various values for V numbers. And note here the size or the refractive index of core and cladding, they individual do not matter, what matters is the total together what is called the V number.

So, we can have this quantity V number which is the representation of frequency on this axis and on in this axis we have the normalize propagation constant b, which will vary between 0 and 1 or a effective refractive index, which is going to vary between n 2 and n 1. So, what we find is that, as the V number increases, all these graphs are monotonically increasing graphs. This is the graph which corresponds to H E 11 then we have the graph here, which are T 0 1, T M 0 1 and H E 2 1 and so on. So, first thing to note here is, that this quantity V, which is proportional to frequency. There is only one mode which can propagate down to V equal to 0 there is the value here is very small, essentially this curve can extend up to 0.

So, no matter how small value of frequency is, this mode H E 1 1 mode is the one which is always going to propagate on the optical fiber. Where as if I consider a mode T E 0 1 mode which is the first mode in transpose electric or T M 0 1 mode, then the V number has to be greater than certain value. And this quantity here is 2.4. What is special about 2.4? 2.4 is the first root of J 0 Bessel function. So, the cut off for the T or T M is given by the roots of J 0 Bessel functions.

So, a first 0 since it is 2.4, you see that for T E or T M mode to propagate, the V number has to greater than certain value or for a given fiber the frequency has to be above certain value. If it is not above certain value than the T and T mode will not propagate. Whereas, this mode which is the H E 1 1 mode, this will always propagate because its cut off frequency is 0. So, we find something very interesting that between the ray in between V number 0 and 2.4 only one mode propagates.

And that mode is not transverse electric or transverse magnetic, that mode is a hybrid mode. So, it is very interesting that the mode which is predominantly propagating on the optical fiber, has in general all six components associated with it. It neither transverse electric not transverse magnetic, if I compare this with the metallic wave guides that a metallic wave guides, the dominant modes are transverse electric modes.

Whereas, if I go to the dielectric wave guide like optical fiber, then the dominant mode is not transverse electric, but, the dominant mode is hybrid. So now, if I check this back with my ray model essentially what it tells me is that, this mode which is going to propagate here, between this and this numbers that must correspond to the ray which was going along the axis of the optical fiber. Because all other rays had to satisfy certain face condition for total internal reflection. Only the ray which was going along the axis of the optical fiber was independent of all these quantities. So, even if I take the size of the optical fiber very small, one ray which goes along the axis of the optical fiber will always go.

So, this mode H E 1 1 mode, is the mode which corresponds to the ray which travels along the axis of the optical fiber. So, as very important conclusion, that the ray which goes along the axis optical fiber, if I see only in terms of ray, it would look as if its transverse electromagnetic light which is going to propagate. So, from the wave of analysis we find that, that ray is not transverse electromagnetic in nature, this wave is hybrid in nature.

So, now we got a very important conclusion and that is we got essentially a analytical answer to the range of parameters or the frequency over which only one mode will propagate inside the optical fiber or the condition for which the fiber will be single mode optical fiber. Recall we have said, when the large number of rays propagate, there is going to be more broadening of the signal for a dispersion; whereas on single mode there was no dispersion because of the multiple ray propagation.

So, we wanted to know what is the quantitative measure, which tells us whether the fiber is going to single mode or the fiber is going to be multimode; here is the quantitative answer, that if the V number of the optical fiber is less than 2.4 then the fiber will be single mode. If the V number of the optical fiber is greater than 2.4 then the fiber will be multimode. And as the V number increases higher and higher, more and more modes starts propagating inside the optical fiber. So, in conclusion essentially our analysis which we have carried out, has given us a very important information about the quantitative parameter of the fiber what is call the V number, that if the V number is less than 2.4, then the fiber remains single mode optical fiber.