

**Analog Circuits**  
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**Lecture-22**  
**Active Filters**

We were looking into filters the last time did something about Butterworth, Butterworth will be there polynomial which is given by a function  $B^2(\omega)$  which is equal to  $1 + \zeta^2 \omega^2$ .

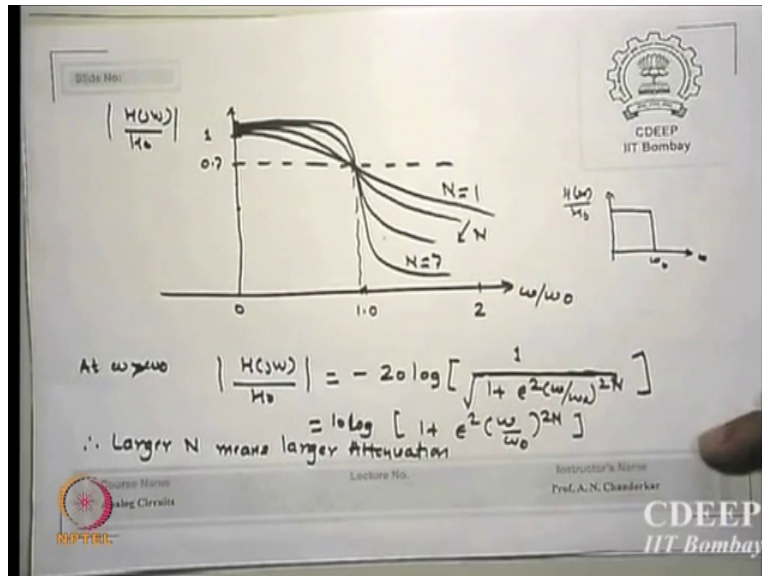
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The slide contains handwritten text and mathematical formulas. At the top right is the IIT Bombay logo and 'CDEEP IIT Bombay'. The text reads: 'Slide No. Butterworth Filter', 'If  $H(s)$  is Transfer F<sup>n</sup>  $V_o(s)/V_i(s)$ ', 'given by  $H(s) = \frac{A(s)}{B(s)}$ ', 'And if this has only Poles but 'no' zeros  $A(s) \text{ const} = H_0$ ', 'Then  $H(s) = \frac{H_0}{B(s)}$ ', 'Then the P<sup>n</sup>  $B^2(\omega) = 1 + \zeta^2 \left(\frac{\omega}{\omega_0}\right)^{2n}$  is called Butterworth Polynomial', 'The Filters using Butterworth P<sup>n</sup> are called 'Maximally Flat' Kind, i.e. Ripple is v. low.' At the bottom left is the NPTEL logo and 'Course Name: Analog Circuits'. At the bottom right is 'Lecture No.', 'Instructor's Name: Prof. A. N. Chandorkar', and 'CDEEP IIT Bombay'.

The real part  $F^n$ ,  $n$  is called the order of Butterworth order of polynomial. Now the idea in filtering is that since you are looking for a typical pass band, stop band, ripples. These functions which are mathematical functions they can actually fit to the required desired response by proper choice up order of the functions. The Butterworth of functions are also called maximally flat filters essentially they mean their ripples are extremely small.

The last time we are doing this something about this I just thought I will repeat where I started and I did derive out expression that a Butterworth function.

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Normalized Butterworth function can be written as  $20 \log 1$  upon  $\epsilon^2$   $\Omega/\Omega_c$  where this  $\epsilon$  is a function of ripple frequencies. So, obviously larger  $n$  means larger attenuation so one can see from this transfer function if I want to attend a sharp larger the  $N$  number I create larger will be fallen d from output to the higher value to the lower value and I already shown you.

As I increase the Butterworth polynomial number order  $N = 1, 2, 3, 4, 5, 6, 7$  the sharper false starts okay. Now this essentially means if you are looking for a very great sharp filters also you need a very low ripple you may have to require seven stages any the number of stages or sections each  $N$  is a pole, each  $N$  will give you a pole. So, each section will have one circuit which will give realizable and if you are larger number of those many sections will have to create.

So, if I see for example for 7, one can see it is a very sharp filtering has been done this is for a low pass similar thing can be done for high pass. So, sharper fall essentially means larger value of the function you are creating on the  $N$  being larger and that is the problem. So, we and as I said last time larger the  $N$  value larger is the issue of money and therefore the cost wise it is better if your  $N$  number is relatively smaller.

So, I will give an example to solve my problem let us say this is the attenuation function please remember at  $H_0$  by  $H_0$  the attenuating function maximum value is 1 and it will start going down

that means is called a attenuating higher and higher and you can see larger and will attenuate at lower frequencies make sharper bands and also you can see the flatness of the curls okay in the pass band. Ideally what is the low pass filter response I am looking I am looking as if something like this.

This is an ideal low pass filter response and I am trying to duplicate it by using a polynomial function which fits closer to this. All that we are trying is trying to fit the function as close to that and doing so we figured out that larger then I put closer I will come to the reality okay what normally I said you are the day there will be a ripple okay and then there is a some kind of a fall which is a transition band so I want to reduce foundation band I want to reduce the ripple I want sharper falls.

So, I require larger polynomial functions larger number means as I said each N will give you as we shall show you the function each end will give one pole larger the number of poles you create one can understand why larger will fall faster each is 20 DB, 20 DB, 20 DB says 7 poles 140 DB it will go down. So, the trick of the trade is how many sections you can tolerate by your money in your pocket okay.

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Slide No. Butterworth Filter

If  $H(s)$  is Transfer FN  $V_o(s)/V_i(s)$

given by  $H(s) = \frac{A(s)}{B(s)}$

And if this has only Poles but 'no' zeros  $A(s) \text{ const} = H_0$

Then  $H(s) = \frac{H_0}{B(s)}$

Then the FN  $B^2(\omega) = 1 + \epsilon \left(\frac{\omega}{\omega_0}\right)^{2n}$  is called  
Butterworth Polynomial  $\epsilon \rightarrow 1$

The Filters using Butterworth FN are called  
'Maximally Flat' Kind, i.e. Ripple is v. low.

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So, this is the Butterworth function and one can see from here if you are noted down I will just show you okay here the HS is the transfer function which is nothing but V0 by VS which has to

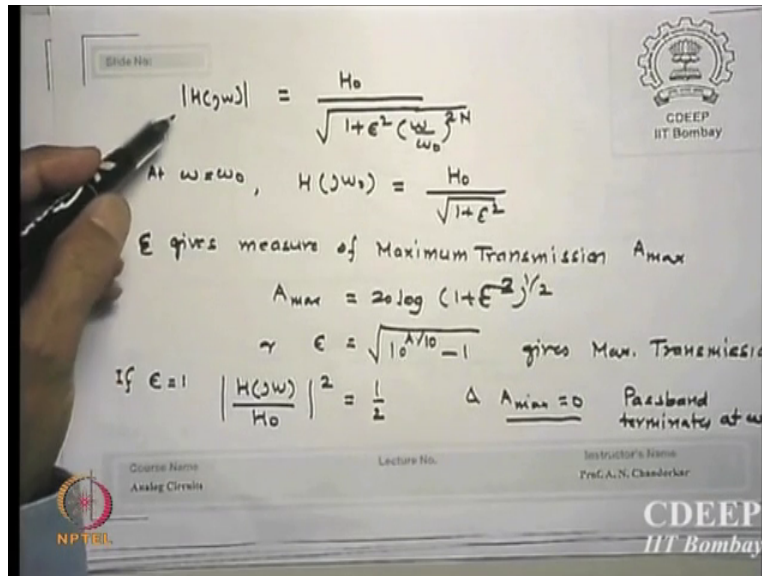
be a AS by BS and if it has only poles or no 0's it is called all pole transfer function. It can be 0 but we normally want to have all pole transfer functions. Then HS let us say there are no 0's so AS is never at except at  $\Omega = 0$ , it is a pole probability 0.

But otherwise it is  $H_0$  upon VS and the function which is called Butterworth polynomial is given by  $B^2$  why  $\Omega$  is  $1 + \zeta^2$  or  $\epsilon^2 \Omega$  by  $\Omega_0$  by  $2n$  where  $\Omega_0$  is the cutoff frequency at which he wants first pole to occur okay. Epsilon is a I would not say constants a variable or rather you may say for the particular problem is a constant which is a function of ripple frequency.

It is some function which has will actually evaluate that value of how much it should be or normally they make you data is typically taken one zeta is typically taken 1 in the case of Butterworth polygon, sharper this zeta has taken one in Butterworth but the zeta will be taken something else in case of the other filters which is Chebyshev okay. So, unless assumed unless not critically specified used always epsilon or theta whatever you feel AS is equal to 1 okay, unless stated otherwise.

So, this is the Butterworth function and what I am trying to do I want to fit this kind of function into the pole 0 functions and I like to see whether it replicate the desired response this is all that I do when I say I am designing a low pass filter for that matter any printer.

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If I calculate and  $H(j\omega)$  function as I gave you there it is a 0 upon  $\omega$  on  $\epsilon^2 \omega^2$  upon  $\omega_0^{2N}$  at  $\omega = \omega_0$  at  $H(j\omega_0)$  is  $H_0$  square upon  $\epsilon^2$  and normally I say absolute stated 1 most cases. At  $H(j\omega_0)$  square by  $H_0$  is half. So, in this  $\epsilon$  if you want the value it is 10 to the power 8 by 10 - 1 is the actual gain at  $H_0$  as it should have been  $H_0$  there.

So, the maximum transmission  $A_{max}$  occur at this volume and therefore absolute value I can calculate but typically as I said I choose the value of 1, so this function goes to a value of half,  $A_{max}$  is a no you say  $H(j\omega_0)$  by  $H_0$  magnitude of that it is a normalized gain function. So, I named separately to that ok did that correct  $H(j\omega_0)$  by  $H_0$  magnitude of that it is 20 log of that is this, so, this absolutely essentially something to relate with maximally flat conditions.

And maximally flat conditions means what the gain which I have only shown the figure as flat instead of ripple as flat as I get any response here that is what I say maximally flat because ideally what do I want sharp constant gain and followed. So, as was I choose  $\epsilon$  I can get more and more flatness but normally the maximum value which all filter people use in  $\epsilon = 1$  okay. I give an example and I think that all things which you will be very clear now okay.

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Slide No. Example: For a LP Butterworth Filter, we need Attenuation of 40 dB and at  $\frac{\omega}{\omega_0} = 2$ . We use  $\epsilon = 1$ .

Then  $\left| \frac{H(j\omega)}{H_0} \right|^2 = \frac{1}{1 + (\omega/\omega_0)^{2N}}$

Given  $\frac{H(j\omega)}{H_0} = \frac{1}{100} = 0.01$

$\therefore 10^{-4} = \frac{1}{1 + 2^{2N}} \quad \text{or} \quad 2^{2N} = 10^4 - 1 \approx 10^4$

or  $2N \log 2 = 4 \quad \text{or} \quad N = \frac{2}{\log 2} = \frac{2}{0.3010} \approx 6.64 \approx 7$

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Here is an example which is very easy to understand let us say I want to design a low pass filter using Butterworth functions and we want an attenuation of 40 DB at frequency of  $\Omega = 2 \times \Omega_0$  is that clear. I am giving an attenuation at  $\Omega$  twice the  $\Omega_0$ ,  $\Omega_0$  there is the first cutoff is that point clear which value I am talking. If this is my  $\Omega_0$  twice this frequency what is the value transfer function value goes down okay that value I say is 40 DB down it should go by.

When the frequency doubles from the cutoff I want how much attenuation 40 DB attenuation. I want okay and we as I say I use epsilon 1 so if I write Butterworth function as  $H(j\omega)$  by  $H_0$  square is called  $1 + (\omega/\omega_0)^{2N}$  and since I am given  $H(j\omega)/H_0 = 1/100$  is that okay by I  $1/100$ , 40 DB is  $1/100$  magnitude and that = .01, if I substitute here in this function I  $H(j\omega)/H_0 = 1/100$ ,  $H(j\omega)/H_0 = 1/100$  square I put 10 to power -2 square 10 to power -4 =  $1/10000$   $\Omega$  by  $\Omega_0$  is 2.

So,  $2^{2N}$  evaluate what I am going evaluation what is that I am doing from this? What is the value I am calculating? Order of polynomial which will give me at twice the frequency from the cutoff the attenuation by 40 dB, so, I calculate N from this value and there this is equal to around 6.64 roughly 7. So, if I have a 7 section polynomial or 7 or 7 third order polynomial then what is the output of a Pinter will be that it will have a cut-off of  $\Omega_0$ .

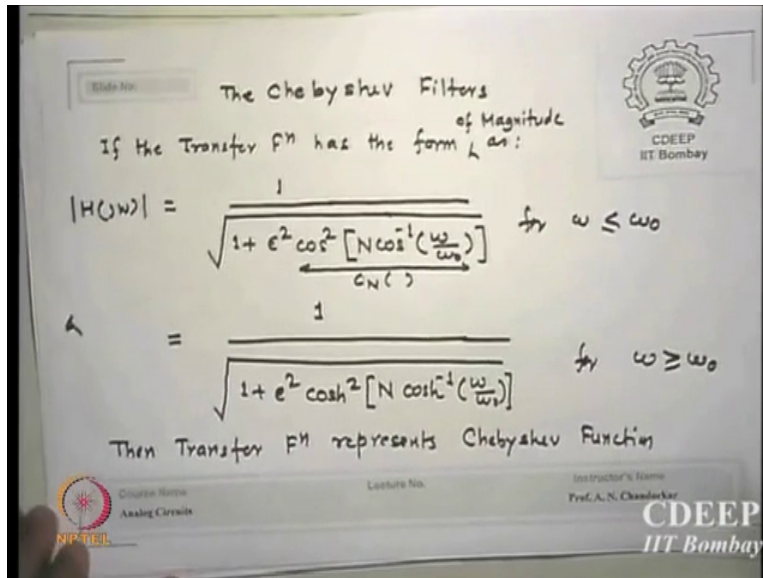
And at twice that frequency the attenuation will be 40 DB that means it will call 40 DB down bias double the frequency is that clear dismissed. So, now you can see from here if I want even more attenuation watch what will happen it says 60 DB, this N will further increase 7 may become 12, 11, 10 what a number it comes. So, larger the attenuation you are expecting at a smaller frequency shifts.

You can say  $\Omega = \Omega_0$  itself that value occur it will be even and maybe even more okay. If that point if I want very sharp at  $\Omega_0$  itself then it will be even larger N number will appear in the real life and that means what is the problem is larger N? So, many poles you know how to realize, so many realizing each pole will be realized at one section as we say. So, so many sections will be required so you have a large low pass filter actually creating a really not great pinter response.

Because most cases you are not really looking for ideal low pass if you are really looking for very ideal low pass yes all that we said 3 DB down if this starts falling, it is fair enough. The problem start how much is that transition magnitude allowed okay and therefore that is where the next when it starts again. Because if you reduce do not reduce transition band then the next filtering frequency will also come immediately.

So, we want to separate the pass band from they are stopped done ok. So, I want to ask sharply I do it that is what I am trying to look at. Now, I shown you one method of designing a filter by using a Butterworth polynomial okay. Now what is the problem I said everything is fine ripples are almost close to 0 but larger number of sections are required here is another function which the similar job and is called Chebyshev okay.

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So, here is another transfer function which a polynomial which can create it has a function which will also do attenuation. The only problem with this transfer function is that it will have a ripple. The typical Chebyshev transfer function can be represented as and it has two possibilities when the frequency is less than  $\omega_0$  and the frequency is larger than  $\omega_0$  the transfer function has this kind of expression 1 upon epsilon square crosses;

This CN is the name given to Chebyshev function this is my Chebyshev function okay. This is  $\cos^2 N$  bracket and  $\cos - \omega$  by  $\omega_0$  is the Chebyshev function okay. And this becomes; can you tell why  $\cos$  become  $\cosh$ , (FL)  $e$  to the power  $J\Phi - e$  to the power  $a + e$  to power  $- J\Phi$  by 2 is  $\cos$  and if by 2  $J$  in case of signs and if you remove that  $J$ 's then it will become  $\cos H$  functions.

So, whenever the function is larger than this according to this theory the function becomes  $\cosh H$  kind this is how the functions have been. This function has nothing to do with filters per C this is a mathematical function which was used by filter theory by Mr. Chebyshev this is not; she only actually got this kind of function. So, how do you get new functions you had a lot of mathematical functions available okay.

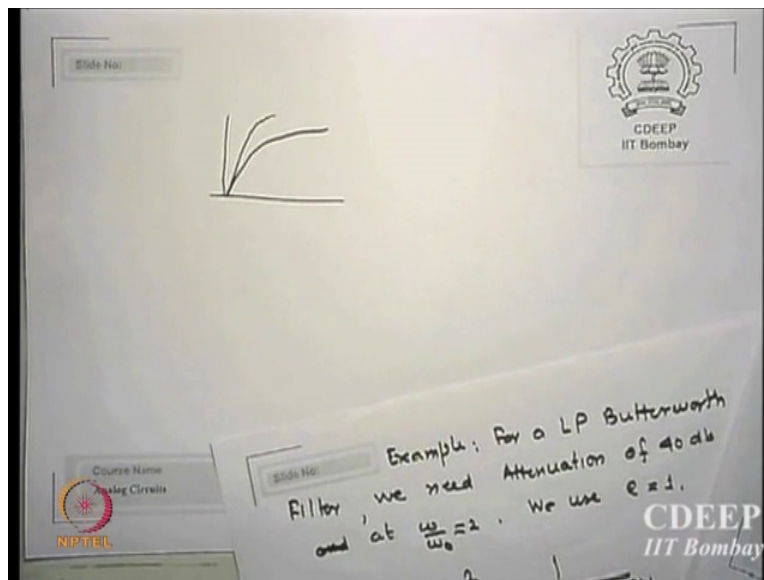
You try to see which one come closer to the kind of actual response you are looking for okay and then many a times in engineering what we do we put a fitting function on that. So, like for



example I told you, you give me any conic section I can always put any order of polynomial  $a_0 + a_1 X + a_2 X^2 + \dots + a_n X^n$  to the millions I can do but everything can be fitted I will get the coefficients. So, I need so many terms to so many coefficients to evaluate. But at the end of end of the day I can fit any kind of shape any random shape coning of course is the relatively periodic shapes but even any kind of shapes.

This idea is therefore valid after you are looking for some kind of this response. So, you know initially it should be constant and then should fall okay or initially it should start rising and then become constant. So, we are looking for some kind of a function suppose easiest (FL)

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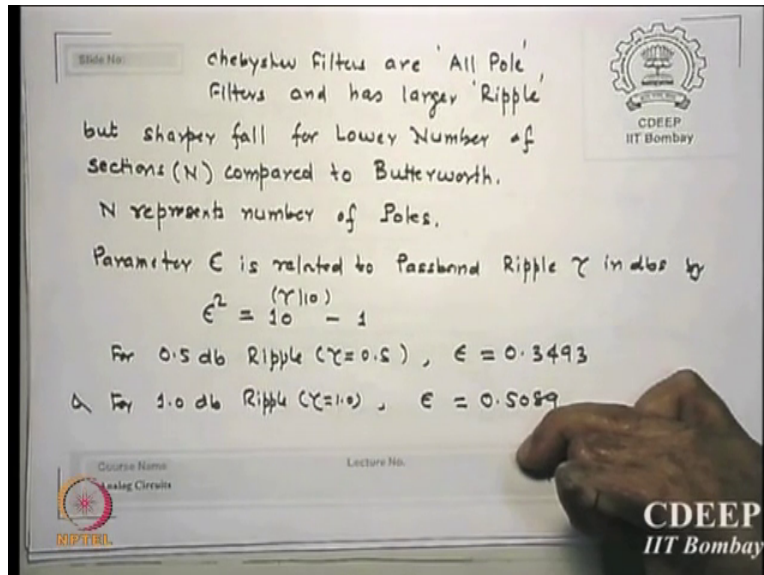


Essentially even an exponential function which is that why cos and cos H function are appearing cuts an exponential function trying to fit something okay. So, any mathematical function which tries to replicate the response you are looking for can be used. But these two became very popular because these can be implemented on a circuit much easier way. Why only these two because they were very easily implementable on circuit performance which we have blocks.

Like OPAMP's, strings with capacities using this many of them can be easily implement. Therefore these two became extremely popular but I am not saying there cannot be any other function better or worse than there are many other functions. These two became only popular

some other I will say okay there are other kinds of function also fits in and maybe fits in better at some other cost. But for this quotes let us take it that these are the only two filters.

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So, if I have this Chebyshev filter and it is always said all Chebyshev which are filters are all pole functions okay there are no 0s in that however. They always cost you on ripple the ripple of all Chebyshev filter is larger compared to Butterworth it is called maximally flat okay. In the Chebyshev, there is going to be a ripple but as in Butterworth sharper fall for lower number of sections probably if you can get compared to Butterworth, at the cost of ripple.

Then you are doing something better one way other something you are losing other is that point clear. I can reduce either the ripple or I can reduce the sharper fall. So, if I say a very sharp fall then I know I am going to increase my ripples. If I want really lower falls a sharper lower sharpness  $\epsilon$  and I may reduce the ripple itself. That ripple is essentially given by what we call epsilon term and it is related to a ripple whatever equivalent to indeed gamma db's.

By this function epsilon square is 10 to the power gamma by 10 - 1 this is the expression given by Chebyshev no proofs needed, epsilon square is 10 to the power gamma by 10 - 1 okay - is outside. So, we say if gamma is half DB if the ripple is half DB is that clear, earlier in curve Butterworth how much epsilon (FL) 1 only. Now I say depending on the gamma I can choose different epsilon values is that correct.

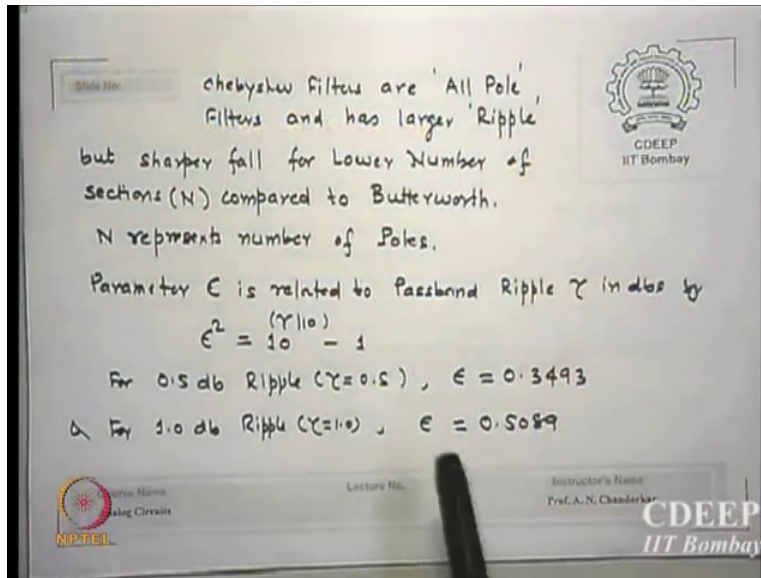
Depending on gamma value I choose I can substitute gamma half DB, 3 4th DB, 1 DB whatever ripple you can tolerate, you substitute it here that dominative is and evaluate that. So, for two cases which is the most popular case in Chebyshev filters one is half DB ripple the other is 1 for which epsilon is 0.3494. The first case and epsilon is 0.5089 this is just math's nothing great and these values are also not evaluated by me I am giving from third Sedra's Smith book.

Hopefully there are max there max is also correct because they also copied from Chebyshev first paper. So, hopefully Chebyshev was right okay. But it is very simple math's, so you can verify on a calculator in a few minutes time, I have not done it this case so plea I do not want to vouch on that but must be right. Yes, for the same 40 DB let us say at the same frequency if I get Chebyshev filter which are then lower than this then I achieve my partner at the cost of ripple.

Is that point what I say I will get earlier example I will use the same 40 DB fall at double the frequency okay and I say the sections which I get how many are there less than the Butterworth, if they, yes I achieved but what cost I am paying these either half DB or 1 DB ripple and which in the case of Butterworth I have known no ripple there. So, if you are really looking only flat bands okay then you will require larger and to use Butterworth's.

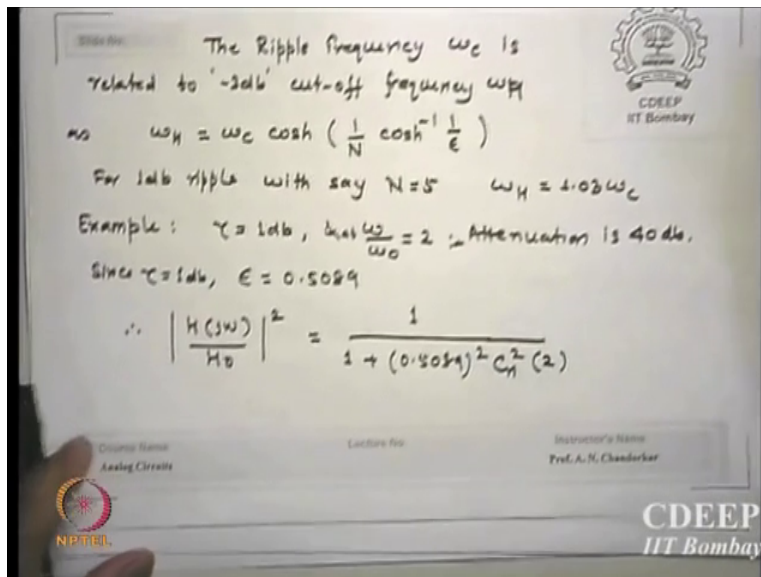
I agree section is a one pole realization what section word I am using N is one pole 7 poles (FL)  
Okay you are you have a point but let us wait for it.

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So, is that value I can calculate epsilon for any DV ripple whichever you are asking or (FL) is that point clear to you is that point clear if epsilon is larger denominator is larger DB's will be shaper falls is that correct. So, 1 DB (FL)

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Okay, the ripple frequency  $\Omega_c$  which is also related to 3 DB point (FL) okay call it  $\Omega_c$  your  $\Omega_c$ . (FL) sorry the cutoff frequency (FL) please remember where is the ripple coming again way let us look at it this is my good band, this is what I am going to get I add this frequency 3 DB point this is related how is that correct this is 3 DB point that is our actual bandwidth up to which pass band exists okay.

And this is the ripple frequency so; this ripple frequency is related by Chebyshev function to this corner frequency by this expression. As I said the function has; when the I change that I have used the transfer function which I given you I start increasing  $\Omega$  from say 0 onwards, so initially because of the function of cost is varying function, it starts rippling okay. I  $\Omega = \Omega_0$  or  $\Omega_H$  whatever 3 DB point.

We say the transfer function net value suddenly starts falling because of the other N to the power N values which argue. As it starts falling I say ripple at that corner whatever is the ripple available I want to make relationship with this corner frequency to the ripple frequency. Essentially ripple is also not universe one frequency term it is there multiple frequency term.

But the fundamental of plot well is chosen as the ripple frequency whatever is the ripple frequency at that point that I am related to the corner frequency by this expression which is from Chebyshev okay, I have now assumed as I am not doing with Chebyshev function because it will take longer time for me. Your next course you should feed them. With the cutoff frequency 3 DB cutoff frequency is related to the ripple frequency.

Repeal it as I say ripple is not constant it is like this, like this (FL) ripple frequency it has a largest component among all others with is returns that frequency to the corner frequency (FL) correlation here okay. so, let us take the same example I have a 1 DB ripple accepted N is 5 this is 1.03 relation (FL) ok what does this mean  $\Omega_C$  and  $\Omega_{ripple}$  frequency is 1.03K meaning event if the cutoff frequency and the frequency of the ripple is very close to each other what is the advantage you will get.

When you substitute in this the H function will start following sharper at that point okay as close you come to thee that means the ripple is (FL) there is a very little other frequency components going through. So, it is almost becoming average value is very close to the maximally flat fan average well that is what we are looking for the frequency of fundamental should be such that its average value is close to flat back value is that correct.

Amplitude (FL) this is what Chebyshev is trying, I calculate for 1 DB that is the same example is that clear what was the last example 40 DB down had twice the cutoff frequency. We want when twice the frequency attenuation is 40 DB and now I am saying the ripple is 1 DB (FL) why this 2 Omega by Omega 0 is 2, I did this math's for you.

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Chebyshev Polynomial  $C_N(x) = \cos(N \cos^{-1} \frac{x}{\omega_0})$   $\omega \leq \omega_0$   
 $= \cosh(N \cosh^{-1} \frac{\omega}{\omega_0})$   $\omega \geq \omega_0$

Given  $\left| \frac{H(j\omega)}{H_0} \right|^2 = (-40 \text{ dB})^2 = 10^4$

$\therefore 10^4 = \frac{1}{4(0.5089)^2 C_N^2(2)}$

or  $C_N^2(2) = \frac{10^4 - 1}{(0.5089)^2} = 3.86 \times 10^4$

$C_N(2) = \sqrt{3.86 \times 10^4}$

or  $196.5 = \cosh(N \cosh^{-1}(2))$

Solving  $N = 4.53 \approx 5$

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I have this Chebyshev function (FL) will say I substituted this value in this CN square is 10 to power 4 - 1 upon this, this (FL) why I am calculating I use this expression substitute attenuation and evaluate the value of CN square 2, is that correct. (FL) transfer function should reduce by a 40 DB so 10 to power - 4 this is 1 + (FL) so, 1 + epsilon square into the Chebyshev function square at 2 at Omega by Omega 0 = 2.

So, I went to evaluate this value from this I know this I know this I calculate CN squared - 2 B 3.86 into 10 to power 4 that means CN the Chebyshev function at Omega = twice Omega 0 is 3. under root on this into 10 to power 2, this function is key value a 196.5 and the Chebyshev function is cos h, why I am using cos h Omega greater than Omega 0 this is CN2 is 3.86, this is 196.5 = cos h N cos h inverse 2 (FL)

So, N becomes 4.53 is that clear (FL) is that correct one D because ripple (FL) so, there is no point in than using Chebyshev separately is that clear, so Chebyshev (FL) because corresponding Butterworth may require 12,15 larger number, Chebyshev (FL) so, then you better

use Chebyshev filters, if you are looking they daledylee 40db or something of that kind you may as well use Butterworth with no class, no ripple. (FL) is that correct this is what the design of pinter is all about is that correct. (FL)

We will just tell you that I want the attenuation at this frequency so much okay. Then you will use both functions is that correct and evaluate how much N that is giving, how much N this is giving for both values of gamma also half DB as well as 1 DB. And then figure out which one you should choose I first take once I am given at this frequency this is the attenuation I am asking for you then you say okay.

I will use Chebyshev formula for both gamma = half DB and gamma = 1 DB, I will also use Butterworth function fine for Dibble 0 how much is in, if N is much lower and Butterworth further you want then you need not go Chebyshev at all okay. But let us say you are getting Chebyshev much smaller than the Butterworth for a even for a 1 DB ripple you better go for it is that clear. So, the choice of field kind of function choose and kind of sections you can use is decided by what specifications filter is asking from is that correct.

So, the set is the were clear, why I say design because what will be specified to us so many DB's down at this, this is specified then we calculated what normally what we should do okay any spy substitute get the value say okay it will use 43 DB, this is not what we have to do some one will tell this is the filter I want this should do like this. Then I should reverse where go and figure out what should I do should I use Chebyshev with half DB ripple 1 DB ripple or should I use Butterworth you have a choice.

So, calculate with this kind of thing all three and choose whichever cost wise you feel is cheaper and acceptable is that, this is what the design is all about. So, I this course I am trying to again and again bring to your notice that at the end of the day we are not analyzing things. But to do this I must know the function then sir I must know how we are using them. So, I analysis is only required to know what options I hold okay.

Because then I will be given by customer as we say all designs are custom designs what does that custom design means? The customer specifies this is what I want so you go to a shop and you make a child I want this I do not want this is that, so that is the choice a customer has manufacturer has no choice he has to provide he said okay. You want this (FL) and they know it is even costlier. (FL) what you want that money is according okay. (FL)

That is the trick all designers follow, please remember I my attempt in this course all through was to bring to your notice the designers how designers look at the theory which we do okay. Always remember one because whatever people believe that epsilon 1 (FL) gamma is almost 0, DB flatness it is related to ripple. So, gamma is higher which means epsilon is higher means you are closer to the kind of ripples you are looking for okay.

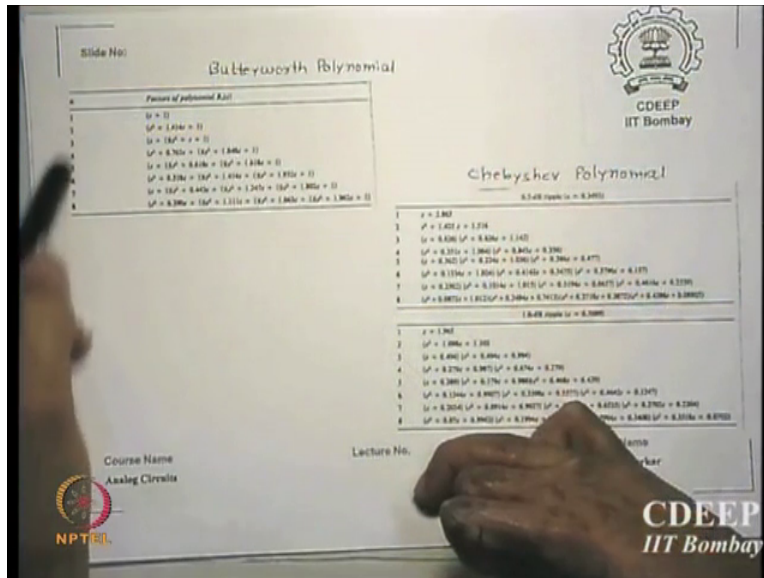
Now in but also repel magnet if it is given then you say ok if maximally flat filters gives much lower ripple anyway is not it and it still gives you lower sections so I do want to have a higher ripple with a lower you already got a lower value of them. The choice is yours you can stay near Chebyshev but if I can you equivalent sections are along the same section that (FL) I may actually work for 6 but then I say I do not have to worry about.

Because if I am giving you 0 DB ripple which is anywhere better than half DB ripple you are asking for whichever value are this. So, you cannot; that is the maximum allowed to you below anyway that is allowed is that correct. So, if you say that your 6 section and 5 sections you have to make a choice, you better make a choice of 6 because you say okay ripple is now no more there. Think the choice is something like this if I say half DB ripple then I say okay it is = that 10 to the power gain by 10<sup>-1</sup>, you evaluate epsilon from there.

So, that value will tell you what is the flatness on the gain function you want okay that flatness may ripple on that the flat A max (FL) that is your value of epsilon you are choosing you should say 0 fine okay ripple 0 fine, so epsilon is 1 for that, so use 1 they are always related by the same function okay is that okay.

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Here is I just copied it the Xerox , this is the order of polynomial Butterworth polynomial 1, 2, 3, 4, 5, 6, 7, 8, this is normalized polynomial BNS (FL) I am realizing in butter well it  $S + 1$  that is a pole,  $S + 1$ , if  $N = 2$ ; I am realizing  $S$  square + root  $2 S + 1$  okay, if  $N = 3$  it is  $S + 1$  times  $S$  square +  $S + 1$ , if  $N$  is 4  $S$   $2S$  squared terms will appear 4 means (FL) so 4 poles is that correct. These are number of poles, 5 means  $S + 1$  into  $2S$  square bracketed term is that correct.

5 poles, so you can see whenever there is a odd number of polls there will be  $S + 1$  single pole (FL) is that correct (FL) the other two always will be a conjugate poles whereas this will be a real pole okay. So, what should I have I had to realize I must be able to realize the conjugate poles as well as a real pole. So, if I have a circuit which realizes a single pole which is real and also can replicate a  $S$  square  $S$  is kind of terms.

Then I am having a transformation so how many; if this is the this third one; so what is the kind of function I am saying  $1$  upon  $S + 1$  into  $S$  square + root  $S + 1$  this is the kind of function I am having from the Butterworth is that correct, each I should be able to realize now, what is the method I said if you are with 3 functions  $1$  upon  $S + 1 + 1$  upon  $A S + B$ ,  $1$  upon  $S + 1$  what they can be 3 transfer function this into this into this.

So, I realize first function and output of that I give it to the input of the next I have product of the 2 now output a value to the third stage with product of the third product. So, all that I do is each

is now clearing what is the section, section me realization of a single pole okay. so, one of 1 upon  $S + 1$  is 1 transformation of function into 1 upon  $S +$  something is second transformation function, so I realize individually in the each pole is that clear.

And since I can realize a pole I just shown you the theory earlier (FL) so, we should be able to then realize any number of such sections in series of that, is that that for  $N$  (FL) 7 when you are understood 7 means there will be 7 sections of realizing individual poles is that correct that means there are so many series combinations are going on to realize that function and as I said the penalty essentially is the power.

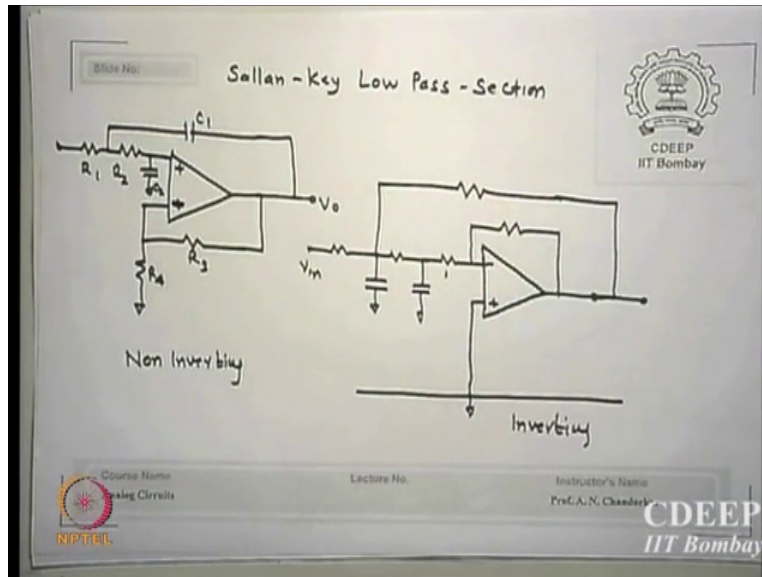
Larger the sections each will drop over okay okay. So, once you get either you are going for Chebyshev or going for Butterworth you are decided this is the function you am implement this is known that I will him if it is a Chebyshev function I will implement this, if it is a Butterworth I will implement that is that clear. So, that is the; I have started with AS upon VS is the transformation function is all pole is 0, constant.

So, I am actually realizing that functions which is the trans function I said is that clear okay. (FL) if I am asking you to design something I will provide you this table is that clear I will you do not have to remember, I will provide you this table only thing is that  $N$  you must get it and choose among them which function to realize is that correct. So, how to design a filter I repeat given the spec find which filter and which order.

And choose expression from here or there this is for half DB this is for 1 DB okay (FL) so do not go before it is a much larger than to the worse okay. This is as I said please do not have to remember anything this is just too show you  $S +$  something, you can see the functions are not very different here only thing it was  $S + 1$  here between  $S + 2.863$  the next term is  $S$  square 1.425  $S$ , this they say they have been derived from the Chebyshev and Butterworth will function these values okay.

So, please look into book or other as I said just for the heck of it I just tell you these are the functions I am going to realize.

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This is called Sallen and Key Low Pass section okay, now this words section is clear. How many Poles this is going to realize each of them inverting and non-inverting two poles (FL) is that clear the constants which you are derived expression I mean you have got from functions had to be now implemented through  $R_1 C_1, R_2 C_2$  values but that is normally taken higher you can see now your gain is now reduced. (FL)

You have a point this is one, two pole theory may  $N = 2$  is the other two poles. So, one is  $S$  square sections is that correct  $N = 5$  means  $1$  upon  $S + 1$  into  $S$  square bracket (FL) please take it this value just to give an idea let us take I am doing Butterworth  $N = 4$  or  $5$  (FL) is that correct each  $S$  square term can be realized by one section. So, you require two sections per  $2 S$  square terms the third (FL) is that clear.

So, given the transfer function once you make a choice you can actually start realizing using Sallen keys (FL) is that clear (FL) is that okay a function wise, is that clear.  $N$  (FL) is that clear so section wise I am sorry I made a mistake section wise  $S$  square term can be realized by one section, another  $S$  square term (FL) is that correct so three section. I will show you an example. (FL)

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Slide No. CDEEP  
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### Creation of a Real Single Pole

$$\frac{V_o}{V_{in}} = H(s) = \frac{1}{1 + RCs} = \frac{1}{1 + s/(1/RC)} \rightarrow \text{Non Inv.}$$

$$\frac{V_o}{V_{in}} = -\left(\frac{R_2}{R_1}\right) \cdot \frac{1}{1 + s/(1/RC)} \quad \text{Inverting}$$

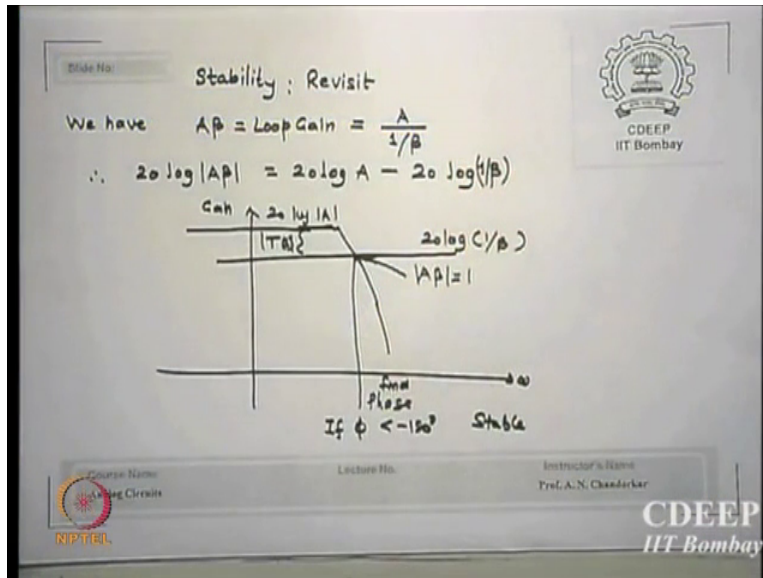
CDEEP  
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Okay, (FL) is that current real pole requirement. So, here is a single real pole (FL) RC this the filter input is not loading input is coming here with high impedance on that so it is not getting loaded by this (FL) if you want a gain fine if you do not want to put  $R_2 = R_1$  so that will become 1 upon s by 1 upon R thing with a - sign is that clear choice have RR yours.

So, you can always get rid of us so this is called creation of a real pole is that a real single pole why it is called real again and again (FL) okay. So, this finishes the filters do not pass (FL) is that correct replace R by CC by off accept the feedback (FL) if you see the function (FL) okay. We will get that high pass is that okay so, I had now I can create a band pass, band reject, low pass, high pass using Sallan Key filters fiction's okay either using Chebyshev defines are using this;

When I have (FL) if you see it is trying to do the same thing is that clear, (FL) is that clear there I did not name that is then Sallan Keys section but it is essentially a Sallan key section that (FL) So, this finishes filters, if; (FL)

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I figured out there is something which I did not tell you so I thought maybe quickly I will tell you something more interesting what they did for stability. So, that is why I named it revisit what is the way I has told you to find the stability of the amplifier (FL) this is very interesting math's has been done by Sedra Smith and I thought are you sure you can read that more detail this is just to give his idea which is fantastic.

Why I should have also this we do in other method but I hear they think this is the loop gain is A beta is that clear how can I write A beta A upon 1 by beta, A beta can be written as A upon 1 by beta, I take log the 20 log A beta is nothing but loop gain in DB's which can be written as 20 log AOL - 20 log 1 upon beta. So, (FL) you do not find 20 log A beta at all (FL) AOL versus Omega bode plot for open loop amplifier is that point clear.

Do not get A beta just plot Bode for magnitude of AOL versus Omega and phase corresponding is that clear. Then I want to subscribe this is that 20 log M is bode plot of that, beta is constant in this case, so 20 log 1 upon beta is a constant value okay (FL) so, 20 log 1 upon beta (FL) this is over without this, this is our AOL transfer function okay. Normal open (FL) which has a value of 20 log 1 by beta okay. (FL)

This is our AOL this is 20 log 1 by beta. This is 20 log A, there difference (FL) is that point clear for different feedbacks beta values I can have different straight lines which means the cutoff

point will change is that correct. If the cutoff point change, (FL) so, the 1 upon 20 log 1 upon beta (FL) the choice of beta can be actually pre find predefined (FL) is that current AOL no, no, no plot the phase of AOL (FL) is that correct. All that I am saying what is this point loop gain one but; (FL) so, a given trans function (FL) is that clear. If this is 1 this is 0 A beta 1 is 20 log that is 0 that means this must be equal and this point both are equal okay.

Given feedback Network (FL) that means this value is equal to this value that means A beta loop gain is 1 okay. So, this is the point where loop gain is 1 and we said instability wherever loop gain becomes 1 find the phase okay. That is phase margin should be +1 (FL) so, that 20 log 1 beta lower value now that is feedback is larger. Let us see, per see (FL) these are equal that is loop gain is 1(FL) you will have to find when they are equal find the value of such value of beta where the phase it is 180 degrees at that point; (FL)

That is the minimum that is the minimum beta you must use so that the at least stability start (FL) this method has been provided as an alternate method in Sedra's Smith book alternate (FL) this is much easier in my opinion but if you are using my earlier technique of finding loop gain and plotting is nothing wrong anyway (FL) but this is easier way of doing it, is that clear, this is what they have suggested as I thought you should know this.

**(Refer Slide Time: 56:15)**

The slide is titled "Sinusoidal Oscillators" and features a handwritten diagram of a feedback loop. The input is  $X_i$  and the output is  $X_o$ . The forward path contains a block labeled 'A', and the feedback path contains a block labeled 'β'. The transfer function is derived as follows:

$$H(s) = \frac{X_o}{X_i} = \frac{A(s)}{1 + A(s)\beta(s)}$$

The loop gain is given by  $T(s) = L(s) = A(s)\beta(s)$ . The slide notes that if  $1 + A\beta = 0$ , then  $H(s) \rightarrow \infty$  or Loop Gain = -1, which leads to oscillation.

Logos for NPTEL, GDEEP IIT Bombay, and CDEEP IIT Bombay are visible on the slide.

Next analog block which is required by each and every one of us in our system is oscillators. (FL) Once I said give us some joke that whenever I design an amplifier it oscillates and whenever I design an oscillator it amplifies or other attend units (FL) so, let us say what is that criteria. This is our standard feedback network the only difference now I am saying compared to the earlier feedback network (FL) any one now  $X_Y$  is the input an  $X_Y$  is the output that's fine (FL)

Beta is constant okay now I say beta can be a function of frequency what does that mean beta (FL) now you realize that you are seeing oscillators (FL) RNC this is exactly what it means the beta network has now frequency dependence. All amplifiers we award that we want to (FL) we already said loop gain TS or in some book LS, AS into beta S (FL) Hence if  $1 + A\beta$  is 0 or  $A\beta$  is -1 (FL) infinite if  $A\beta + 1$  is 0 new denominator is 0 transfer function of the value of infinite is that clear.

What does that mean in amplifier  $V_0$  by  $V_N$  is infinite what does that mean without having input  $V_{in}$  I can create  $V_0$  is that by  $V_0$  by 0 is infinite, is that correct. So, without having an input I have an output is that clear. By using it feedback theory which I said ok is that point clear without having any input what is gain  $V_0$  by  $V_{in}$  okay. If I say gain is infinite which means without input I have output that clear.

This term without having actual input provided I actually can get constant value of  $V_0$  is called oscillator the condition will be true oscillations is the principle clear to you. So, what we are saying I pass some input to a which becomes  $X_0$ , beta of that I returned it to  $X_I$  and if it is such that this add to this then increase  $X_0$  afterwards I had, if I have at a certain value even if I remove  $X_I$ , it will still start reading some of input output (FL)

When that have occurs then we say we are having your oscillations was initially (FL) is that clear what this  $X_I$  will be initially (FL) no actually are in oscillator you never give any input (FL) noise so little bit of noise can start oscillations and no it will grow to a standard frequency of it is that correct. So, that is why no real inputs are actually required to start oscillations as soon as you switch on the power supply. (FL)



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Slide No. The Barkhausen Criterion

For Sinusoidal Oscillations will be possible when  $T(s) = -1$

$\therefore |T(2\pi jf_1)| = 1$  and  $\angle T(2\pi jf_1) = -180^\circ$  (A)

At Oscillator Frequency  $f_1$

$|T(2\pi jf_1)| = \text{Magnitude}$  &  $\angle T(2\pi jf_1) = \text{Phase}$

OR Real Part of  $T(2\pi jf_1) = -1$   
& Imaginary Part of  $T(2\pi jf_1) = 0$  (B)

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Lecture No. Instructor's Name: Prof. A. N. Chanderkar  
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NPTEL

So, there is a person called mister Barkhausen he suggested a criteria okay which will give you a certified oscillations (FO) without any given frequency say upon the magnitude of this transfer function is  $2\pi jf_1 = 1$  magnitude and what is its phase because of its -1 value - 180 degree is that correct, -180 degrees. So, what Mr. Barkhausen says that at any frequency if the magnitude of this loop gain is unity and its phase is 180 degree then you will have oscillations?

This is one possible condition (FO) let us say A is 180 degree phase out input to output (FO) is that correct, so the return signal (FO) in phase to the input is that correct (FO) is that correct now this is what oscillator is trying to see this criteria says the magnitude of loop gain at a given frequency if it is one unity and its corresponding phase is - 180 degree then oscillator oscillation and  $f_1$  can be seen is called alternative. (FO)

If you take the transfer function here and you say it is the real value is - 1 an imaginary value is 0 even then this condition is satisfied is that clear to you. Please take it Matt Smith (FO) that is how much is the phase 0 fits fair enough okay but the imaginary part is 0 and the magnitude is 1 which is also is you have a problem situation in which system may become unstable are oscillatory this is what essentially Barkhausen suggested.



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Slide No. That is to say for Oscillations Conditions be —

i) Phase Shift through Amplifier and Feedback Network must become  $360^\circ$  (In Phase of Input)

ii) And  $|A\beta| = 1$

These conditions are called Barkhausen Criterion.

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And real life how do I achieve that (FL) that is to say oscillation conditions will be there phase shift through amplifier and feedback network must be such that it is either 0 or 360 degree that is the feedback return is in phase with the input and the magnitude of A beta is 1 then you have a condition of oscillation called Barkhausen criteria is that the 0, 180, - 180 - 180 - 180 which is also 0 (FL) so, 180 + one assumption is that amplifier use you 180 degree is that correct, yes. So, this criteria is called Barkhausen criteria, so we amplify (FL)

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Slide No. Phase-Shift Oscillator

The diagram shows an inverting amplifier with a feedback network consisting of three RC stages. The output is labeled  $V_o$ . A hand is pointing to the circuit diagram.

The feedback network is shown as a block with a phase shift of  $180^\circ$ , followed by an amplifier with a phase shift of  $180^\circ$ .

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Essentially saying feedback if again 180 this is giving 180 return path is 0 DB 0 degrees amplifier (FL) total 360 or 0 whatever you call it (FL) phase shift oscillator why is called phase shift 180 is shifted by another 180 to create a oscillatory conditions (FL) so in RC constant code time constant (FL) is that clear we will come back to it again but I just want to show what is Barkhausen criteria.

How is it different from this case this condition is start of instability and amplifier is that pointer this was the point (FL) this was the point A  $\beta = -1$  (FL) so, is that no point clear why sometimes amplifier do not because like amplifiers and oscillators do not because they are fed may not become exactly 180 then it will not it will further they broke it out diode.

So, oscillator oscillating (FL) that is what the stability theory suggestiveness that you must get your phase and magnitude in fact okay is that clear to you. This is what essentially Barkhausen suggested based on it will have some 3, 4, 5 kinds of this then we will start moving from after few oscillator I will not go all of them.

Next time I will start few of them and then I will start for you most important part in this A to D and D to A converters, (FL) because (FL) thank you.